

DISSERTATION PROPOSAL

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1. INTRODUCTION

A long time ago my cello teacher pointed out that the way to play a difficult passage of music is not simply to grit one's teeth and keep practicing but to figure out how to make playing those notes easy. The major goal of the proposed dissertation is to reapply the same idea in the context of interactive theorem proving with the Coq proof assistant: I intend to show ways to make the difficult task of using Coq easier by improving the user interface. As well as solving serious usability problems for an important and powerful tool for creating machine-checked proofs, many of the techniques I am developing and testing are widely applicable to other forms of coding.

The research involved in this proposed dissertation, described more fully below, breaks down into two related main parts. Part one is the development of “CoqEdit”, a new theorem proving environment for Coq, based on the jEdit text editor. CoqEdit will mimic the main features of the existing environments for Coq, but will have the important property of being easily extended using Java. Part two is the development and testing of several such extensions.

There are several points that I hope will become clear as I describe the research for the proposed dissertation below. First is that the research will make a significant positive contribution to society. While Coq is a powerful tool, and is already being used for important work, its power has come at the cost of complexity, which makes the tool difficult to learn and use. Finding and implementing better ways to deal with this complexity in the user interface of the tool can allow more users to perform a greater number, and a greater variety, of tasks. At a more general level, the research contributes to a small but growing literature on user interfaces for proof assistants. The work this literature represents can be viewed as an extension of work on proof assistants, which in turn can be viewed as an extension of work on symbolic logic: symbolic logic aims to make working with statements easier, proof assistants aim to make working with symbolic logic easier, and user interfaces for proof assistants aim to make working with proof assistants easier. These all are part of the (positive, I hope we can assume) academic effort to improve argumentative clarity and factual certainty.

In addition, generalized somewhat differently, the research will contribute to our notions of how user interfaces can help people write code with a computer. The complexities of the tool in fact help make it suitable for such research, since a) they are partly the result of the variety of features of the tool and tasks for which the tool may be used (each of which provides an opportunity for design) and b) the difficulties caused by the complexity may

make the effects of good user interface design more apparent. Furthermore, although Coq has properties that make it very appealing for developing programs (in particular, programs that are free of bugs), it also pushes at the boundaries of languages that programmers may consider practical for the time-constrained software development of the “real world”. However, if, as in the proposed research, we design user interfaces that address the specific problems associated with using a language, perhaps making the user interface as integral to using the language as its syntax, these boundaries may shift outward. This means that not only are we improving the usability of languages in which people already are coding, we are also expanding the range of languages in which coding is actually possible.

The second point that I hope will become clear in this proposal is that this research will be an intellectual contribution, i.e. that the project requires some hard original thinking. User interface development is sometimes “just” a matter of selecting some buttons and other widgets, laying them out in a window, and connecting them to code from the back end. While this sort of work can actually be somewhat challenging to do right (just one of the hurdles is that testing is difficult to automate), the project goes well beyond this by identifying specific problems, inventing novel solutions, and testing these solutions in studies with human subjects.

The third and final point is that this work is actually doable. Some of it has already been accomplished and the results will be described below. The remaining work I also describe below, in enough detail, I hope, to make it seem reasonably straightforward.

In the remainder of this proposal I will first give a description of Coq, including its significance, a description of current user interfaces, some examples of theorem proving using the tool, and some usability problems that I find particularly striking. I will continue with a description of a survey, and its results, on user interfaces for Coq that was sent to subscribers to the Coq-Club mailing list. Then, in the heart of this proposal, I will describe jEdit, CoqEdit, three experimental extensions to CoqEdit, and several associated user studies. I will conclude with an overview of related work and a timeline for completing the remaining work.

2. COQ AND THE NEED FOR IMPROVED USER INTERFACES

2.1. Theorem Proving in Coq. Basic theorem proving in Coq can be thought of as the process of creating a “proof tree” of inferences. The user first enters the lemma (or theorem) he or she wishes to prove. The system responds by printing out the lemma again, generally in essentially the same form; this response is the root “goal” of the tree. The user then enters a “tactic”—a short command like “apply more_general.lemma”—into the system, and the system will respond by producing either an error message (to indicate that the tactic may not be applied to the goal) or by replacing the goal with zero or more new child goals, one of which will be “in focus” as the “current” goal. Proving all of these new child goals will prove the parent goal (if zero new child goals were produced, the goal is proved immediately). Proving the current goal may be done using the same technique used with its parent, i.e. entering a tactic to replace the goal with a (possibly empty) set of child goals to prove, and which goal is the current goal changes automatically as goals are

introduced and eliminated. The original lemma is proved if tactics have successfully been used to create a finite tree of descendants—i.e. when there are no more goals to prove.

One example useful in making this more clear can be found in a Coq tutorial [1]. Consider a lemma, call it `SimpleLemma` that states

$$(A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C$$

where A , B , and C are propositions (abbreviated in Coq as “`Prop`”), and \rightarrow is interpreted as “implies” and is right associative (so the lemma may equivalently be written

$$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$$

I discuss later how improved user interfaces may assist users, particularly novice users, in dealing with operator associativities and precedences). Coq users generally think of this lemma as the sequent

$$A : Prop, B : Prop, C : Prop \vdash (A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C$$

In general, the statements to the left of the turnstile (\vdash), separated by commas, give the “context” in which the provability statement to the right of the turnstile is to be considered.¹ Another way to think about the sequent is that the statements to the left of the turnstile entail the statement to the right. In this example sequent’s context, the colon indicates type, so for instance “ $A : Prop$ ” just means “ A is a variable of type *Prop*” or, equivalently, “ A is a proposition.”

This is only a very simple example—theorems and lemmas in Coq typically involve many types and operators besides propositions and implications. Other standard introductory examples involve natural numbers and lists, along with their associated operators and the other usual operators for propositions (e.g. negation). In fact, Coq allows users to define their own types and add axioms regarding those types, allowing users to model and reason about, for instance, the possible effects of statements in a programming language, or more general mathematics like points and lines in geometry.

The sequence of tactics, “`intro H`”, “`intros H', HA`”, “`apply H`”, “`exact HA`”, “`apply H'`”, and “`exact HA`” can be used to prove the sequent above (H , H' , and HA are arguments given to `intro`, `intros`, `apply`, and `exact`). The first tactic, “`intro H`”, moves the left side of the outermost implication into the context. The new subgoal is

$$A : Prop, B : Prop, C : Prop, H : A \rightarrow B \rightarrow C \vdash (A \rightarrow B) \rightarrow A \rightarrow C$$

The colon in the statement “ $H : A \rightarrow B \rightarrow C$ ” is generally interpreted differently, by the user, than the colons in “ $A : Prop, B : Prop, C : Prop$ ”. Instead of stating that “ H is of type $A \rightarrow B \rightarrow C$ ”, the user should likely interpret the statement as “ H is proof of $A \rightarrow B \rightarrow C$ ”. However, for theoretical reasons, namely the Curry-Howard correspondence

¹Another list of statements, Coq’s “environment,” is implicitly to the left of the turnstile. The distinction between environment and context is that statements in the context are considered true only locally, i.e. only for either the current “section” of lemmas being proved or for the particular goal being proved. Generally, the environment is large, and contains many irrelevant statements, so displaying it is considered impractical.

between proofs and formulas on the one hand and terms and types on the other,² Coq is allowed to ignore this distinction and interpret the colon uniformly. In fact, in proving a theorem, a Coq user actually constructs a program with a type corresponding to the theorem. This apparent overloading of the colon operator may be a source of confusion for novice users and while there are no plans in this proposal for directly mitigating the confusion, extensions to the proposed user interface might do so by marking which colons should be interpreted in which ways. Furthermore, by clarifying other aspects of the system, we hope to free up more of novice users' time and energy for understanding this and other important aspects of theorem proving with Coq that we may not be able to address.

Tactics allow users to reason “backwards”—if the user proves the new sequent(s), then the user has proved the old sequent. The user is trying to figure out what could explain the current goal, instead of trying to figure out what the current goal entails.³ Above, the (successful) use of the “`intro`” tactic allows the user to state that **if** in a context where A , B , and C are propositions *and* $A \rightarrow B \rightarrow C$ it is the case that $(A \rightarrow B) \rightarrow A \rightarrow C$, **then** in a context containing only that A , B , and C are propositions it is the case that $(A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C$. The fact that the tactic produced no error allows the user to be much more certain of the truth of this statement than he would if checking it by hand⁴.

3. CONCLUSION

I hope to have made several points in this proposal. First, that this is important work, both because the Coq interactive theorem prover is an important tool that could benefit significantly from improved user interfaces and because many of the ideas generalize to other forms of coding. Second, that as an intellectual challenge this work is non-trivial, not only because of the normal programming problems that must be overcome but because designing good user interfaces for complicated systems, which includes the identification of tractable problems and the testing of potential solutions, is non-trivial. Finally, that, despite this non-trivial nature, the work can be accomplished.

²“Terms,” such as the “ A ” in “ $A : \text{Prop}$ ”, are roughly the same as terminating programs or subcomponents thereof; they may be evaluated to some final value. Note also that the types of terms are terms themselves and have their own types (not all terms are types, however). A type of the form $\Phi \rightarrow \Theta$, where both Φ and Θ are types that may or may not also contain \rightarrow symbols, is the type of a function from terms of type Φ to terms of type Θ . For instance, a term of type $\text{nat} \rightarrow \text{nat}$ would be a function from natural numbers to natural numbers.

³When the differences between the old and new sequents are in the contexts, rather than the succedents (i.e. on the left of the turnstiles rather than on the right) we generally say we are doing forward reasoning, even though we are still adding new goals further away from our root goal. This makes most sense, to me at least, when one views a sequent as a partially completed Fitch-style proof.

⁴In general some uncertainty remains, mostly due to the possibility of mistranslating between the user's natural language and Coq's language

REFERENCES

- [1] Gérard Huet, Gilles Kahn, and Christine Paulin-Mohring. The coq proof assistant a tutorial. *Rapport Technique*, 178, 1997.