Gravitational Waves from Self-Ordering Scalar Fields
PHYS 461 Independent Research Report

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I. Context and Motivation

Gravitational waves (GWs) are emitted when a non-homogeneous self-ordering scalar field (SOSF), such as the Higgs field, smooths out. After electroweak symmetry breaking, thermal fluctuations cause the Higgs potential to fall in a variable manner, resulting in different vacuum-expectation values (VEVs) in different regions of space. As these causal regions of space expand, regions with different Higgs field configurations come into causal contact, creating tension in the field. This tension contributes to the energy density of the SOSF, and with the field wanting to reach the lowest possible energy density, the tension gradient naturally becomes less pronounced. The result is emitted energy in the form of GW radiation, ultimately to obey energy conservation.

The energy density of these GWs is not constant throughout time. While GWs are more powerful when emitted at early times of the universe, they spend more time propagating, thus diminishing their energy density. GWs emitted during the radiation-dominated era of the universe are scale invariant. That is to say, the time at which they are emitted and the time taken to propagate to an observer balance each other such that the observed GW energy densities today remain the same.

Previous research has derived the GW energy density from the relaxation of a generic SOSF, though it had yet to be determined what this energy density was for the Higgs field specifically. Thus, the original goal of the project was to determine this value, putting it into the context of sensitivities of contemporary and future GW detectors.

II. Research Methods

In the beginning of the semester, I obtained prerequisite knowledge for the planned project. This included properties and history of the Higgs boson, the Higgs field, the Standard Model of Particle Physics, gravitational waves, and cosmology. With enough foundational knowledge, learning shifted to SOSFs in the middle of the semester, and I familiarized myself with current research being done on the topic. This process involved reading a variety of academic papers from arXiv.org and discussions with my research mentor. A few topics covered included the stress-energy tensor, types of gravitational wave detectors, and mathematical representations of the GW energy density from a SOSF.

Calculations at this stage involved deriving the vacuum-expectation value of the Higgs field and deriving the energy density of a scalar field from the stress-energy tensor. This knowledge prepared me to interpret key formulas found in the research papers I read, culminating in a calculation for the scale-invariant GW energy density for the Higgs field.

III. Quantitative Results

Calculation of the GW energy density from the relaxation of the Higgs field revealed that the value in question was on the order of 10^{-69}. When compared to the scale invariant GW energy density with the Grand Unified Theory scale VEV in Figure 1 - about 10^{-14} - it became apparent that no experiments would be able to detect such an emission from the Higgs field. The following plot was made in MATLAB with preexisting sensitivity curve data [Breitbach et al, hep-ph/1811.11175].

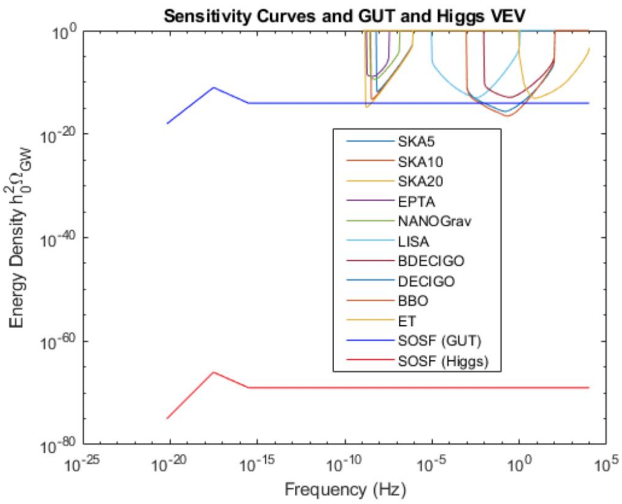

Figure 1. GW Energy Density for GUT/Higgs VEV with Sensitivity Curves for Various Experiments

The scale invariant portion of the SOSFs in Figure 1 can be calculated using

$$h_0^2\Omega_{\rm GW}^{(0)}(f) \simeq \frac{650}{N} h_0^2\Omega_{\rm rad}^{(0)} \left(\frac{v}{M_{\rm Pl}}\right)^4$$

where v is the VEV of the SOSF being used in the calculation [Figueroa et al, hep-ph/2007.03337].

IV. Further Studies and Conclusions

The next step was to incorporate another factor into our calculation, namely the time at which the GWs were emitted in the universe. This would result in a frequency-dependent GW energy density determined across three eras: a kination, radiation, and matter dominated universe. Another ongoing calculation is being performed to derive this piecewise function which will result in a calculable GW energy density that considers a modified cosmic history.

It was not known what the GW energy density would be resulting from a relaxation of the Higgs field. However, after this semester's research, such a value will be obtained not only during the radiation-dominated era of the universe - resulting in the scale invariant GW energy density - but also for any time up to today.