Statistics 565 Applied Time Series Analysis   Spring 2019

Homework 5

Due: Mon 03/04/19 @ 6:00pm   rutgers.instructure.com/courses/21204

1. Problem 5.2 (a) and (b). Hint: start by identifying the ARMA process that ∇Y_t follows. Then derive/use what you know about the mean and variance of the ARMA process. Make sure the ARMA process is stationary/invertible, when applicable.

2. Problem 5.13

3. Problem 5.14

4. Using Bartlett's Theorem discussed in class (also page 110 of the textbook), show that for a white noise process,

   (i) The sample autocorrelation r_k, $k \geq 1$, is approximately distributed as $N(0, 1/n)$ for large n,

   (ii) For all $k \neq j$, r_k and r_j are uncorrelated.

5. Simulate a time series of length $n = 100$ from the MA(3) process $Y_t = e_t - 0.7e_{t-1} - 0.5e_{t-2} - 0.6e_{t-3}$. Display the data on your output. Next,

   (i) Compute the sample autocorrelation r_4 from the observed data.

   (ii) What is $Var(r_4)$, the theoretical variance of r_4 under the true model?

   (iii) Simulate the time series 500 times, computing r_4 on each occasion. Plot a histogram for r_4 and compute the variance of r_4 from these simulations. Does the variance match the theoretical variance?

   (iv) Obtain $\hat{Var}(r_4)$, an estimate of the variance from the data you originally generated (the observed data).

   (v) Compute the 95% confidence interval for the true autocorrelation ρ_4 based on your original data.

   (vi) Are the results consistent with a model in which $\rho_4 = 0$?