Statistics 565 Applied Time Series Analysis   Spring 2019

Homework 7

Due: Mon 04/22/19 @ 6:00pm

1. Problem 9.1, ignoring the phrase "in two ways" in part (b). Note: your answers to both this problem and the next one should rely on analytical derivations.

2. Problem 9.2, part (a) (b) and (c).

3. Simulate a MA(2) process with $\theta_1 = 1$, $\theta_2 = -0.6$, and $\mu = 100$ with the following command:

```
set.seed(1432756);
series=arima.sim(n=36,list(ma=c(-1,0.6)))+100
```

   `series` contains 36 simulated values. Set aside the last 4 values in order compare forecasts to these actual values.

   (a) Using the first 32 values of `series`, find the values for the maximum likelihood estimates of the θ's and μ.

   (b) Using the estimated model, forecast the next four values of the series. Plot the series together with the four forecasts. Place a horizontal line at the estimate of the process mean.

   (c) What is special about the forecasts at lead times 3 and 4?

   (d) Compare the four forecasts with the actual values that you set aside.

   (e) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

   (f) Simulate 500 new series using the same MA parameter values and same sample size. For each series, produce 95% forecast limits for the last four observation using the fitted model on the first 32 observations. What fraction of times does the forecast limit cover all four actual values? (Note: make sure you're not using a fixed `seed` for each simulation.)

4. Problem 9.23.

5. An AR model has AR characteristic polynomial
$$(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12})$$

   (a) Is the model stationary?

   (b) Identify the model as a certain multiplicative seasonal $ARMA(p, q) \times (P, Q)_s$ model. That is, specify the values p, q, P, Q, s, and the parameters of its associated characteristic polynomials (i.e. $\phi(x), \Phi(x), \theta(x)$ and $\Theta(x)$, whichever applicable).

A useful R command.   If `model` is a fitted ARIMA object, then

```
plot(model, n.ahead = 5)
```

is the object that returns the vector of forecasts up to lead time $l = 5$, as well as its accompanying upper and lower prediction limits.