Homework 7 Solution

May 8, 2019

Problem 9.1.

(a)
From Equation (9.3.6), page 193:

$$\hat{Y}_t(1) = \mu + \phi\,(Y_t - \mu) = 10.8 + (-0.5)(12.2 - 10.8) = 10.1.$$

(b)
From Equation (9.3.7), page 193:

$$\hat{Y}_t(2) = \mu + \phi\left[\hat{Y}_t(1) - \mu\right] = 10.8 + (-0.5)[10.1 - 10.8] = 11.15.$$

(c)
From Equation (9.3.8), page 194:

$$\hat{Y}_t(10) = \mu + \phi^{10}\,(Y_t - \mu) = 10.8 + (-0.5)^{10}(12.2 - 10.8) = 10.801367 \approx \mu.$$

Problem 9.2.

(a)
From Equation (9.3.28), page 199,

$$\begin{aligned}\hat{Y}_{2007}(1) &= 5 + 1.1Y_{2007} - 0.5Y_{2006} = 5 + 1.1(10) - 0.5(11) = 10.5,\\ \hat{Y}_{2007}(2) &= 5 + 1.1\hat{Y}_{2008} - 0.5Y_{2007} = 5 + 1.1(10.5) - 0.5(10) = 11.55.\end{aligned}$$

(b)
From Equations (4.3.21) on page 75,

$$\psi_1 - \phi_1\Psi_0 = 0 \text{ with } \psi_0 = 1 \text{ so that } \psi_1 = \phi_1 = 1.1$$

(c):
using $\mathrm{Var}\,(e_t(\ell)) = \sigma_e^2\sum_{j=0}^{\ell-1}\Psi_j^2$ from page 203, the prediction limits are $\hat{Y}_{2007}(1)\pm 2\sqrt{\sigma_e^2}$ or $10.5\pm 2\sqrt{2}$. So we are 95% confident that the 2008 value will be between 7.67 and 13.33

Problem 3.

(a)

```
set.seed(1432756);
series <- arima.sim(n=36,list(ma=c(-1,0.6)))+100
actual <- window(series,start=33)
series <- window(series,end=32)
model <-arima(series,order=c(0,0,2))
```

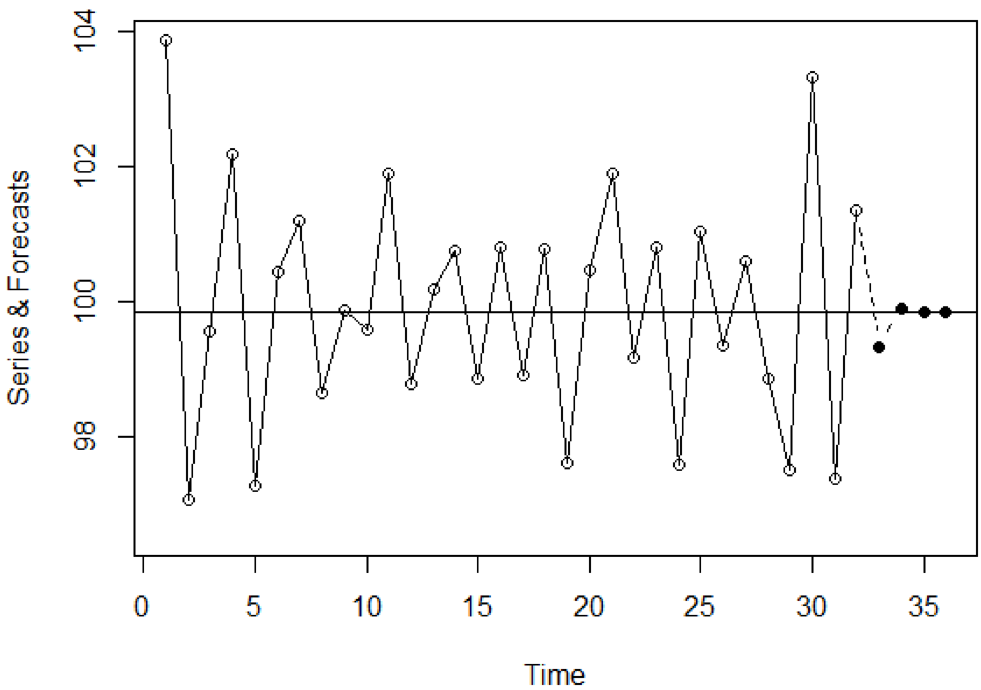
```
Call:
arima(x = series, order = c(0, 0, 2))

Coefficients:
          ma1     ma2  intercept
      -1.0972  0.3257    99.8535
s.e.   0.1539  0.1684     0.0498

sigma^2 estimated as 1.234:  log likelihood = -49.46,  aic = 104.92
```

(b):

```
result <- plot(model,n.ahead=4,ylab='Series & Forecasts',col=NULL,pch=19)
abline(h=coef(model)[names(coef(model))=='intercept'])
```



(c) The forecasts at lead times 3 and 4 are very close to the process mean.

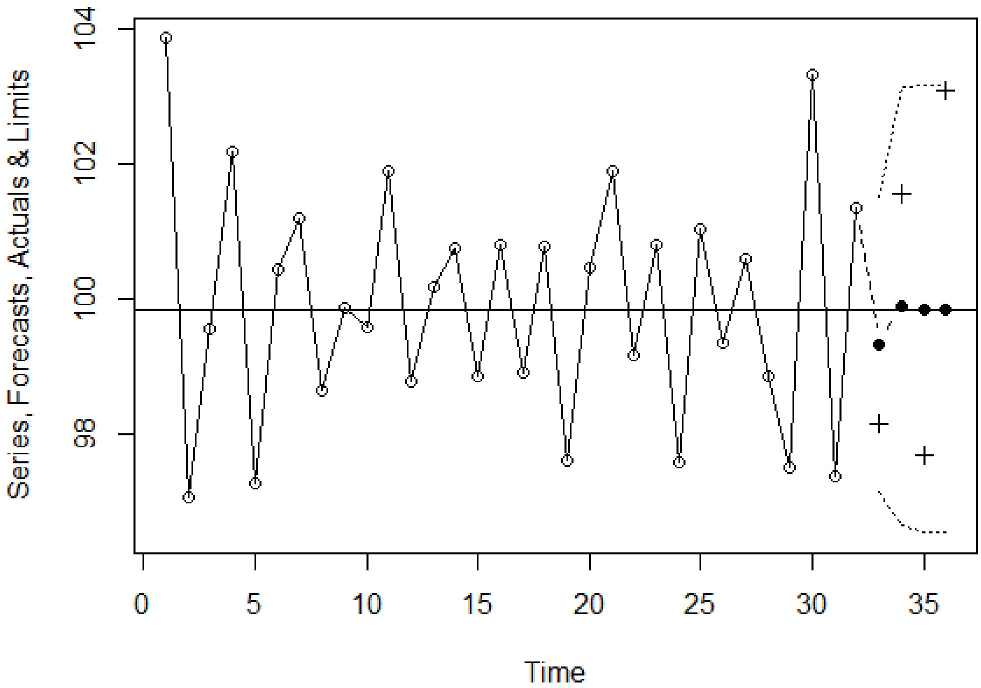
(d):

```
forecast=result$pred; cbind(actual,forecast)

Time Series:
Start = 33
End = 36
Frequency = 1
actual forecast
33   98.16822 99.33236
34  101.56150 99.89905
35   97.68750 99.85353
36  103.08874 99.85353
```

(e):

```
plot(model,n.ahead=4,ylab='Series, Forecasts, Actuals & Limits',pch=19)
points(x=(33:36),y=actual,pch=3)
abline(h=coef(model)[names(coef(model))=='intercept'])
```



The actual series values are contained within the forecast limits. The forecasts decay to the estimated process mean rather quickly and the prediction limits are quite wide.

(f):

```
count=0
for(i in 1:500)
{
```

```
series <- arima.sim(n=36,list(ma=c(-1,0.6)))+100
actual <- window(series,start=33)
series <- window(series,end=32)
model <-arima(series,order=c(1,0,1))
result <- plot(model,n.ahead=4,ylab='Series & Forecasts',col=NULL,pch=19)
actual<-as.vector(actual)
upper<-as.vector(result$upi)
lower<-as.vector(result$lpi)
if(all(upper>=actual & actual>=lower))
{count=count+1}
}
count
[1] 402
```

Note the result 402 (or 371/500) is random.

Problem 9.23.

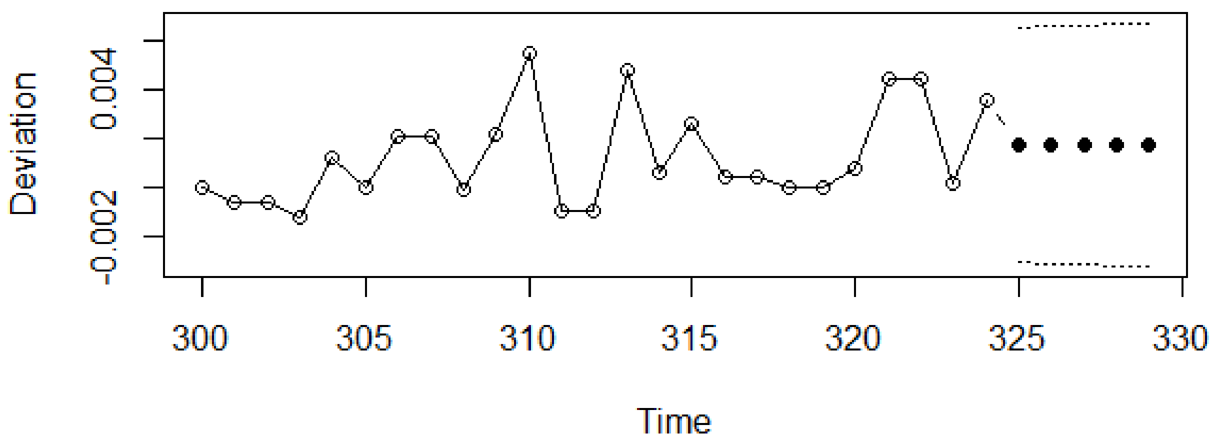
(a):

```
data(robot)
model<-arima(robot,order=c(0,1,1))
plot(model,n.ahead=5)$pred
plot(model,n.ahead=5)$upi
plot(model,n.ahead=5)$lpi

[1] 0.001742672 0.001742672 0.001742672 0.001742672 0.001742672
[1] 0.006571344 0.006611183 0.006650699 0.006689898 0.006728790
[1] -0.003086000 -0.003125839 -0.003165355 -0.003204555 -0.003243446
```

(b):

```
plot(model,n1=300,n.ahead=5,ylab='Deviation',pch=19)
```

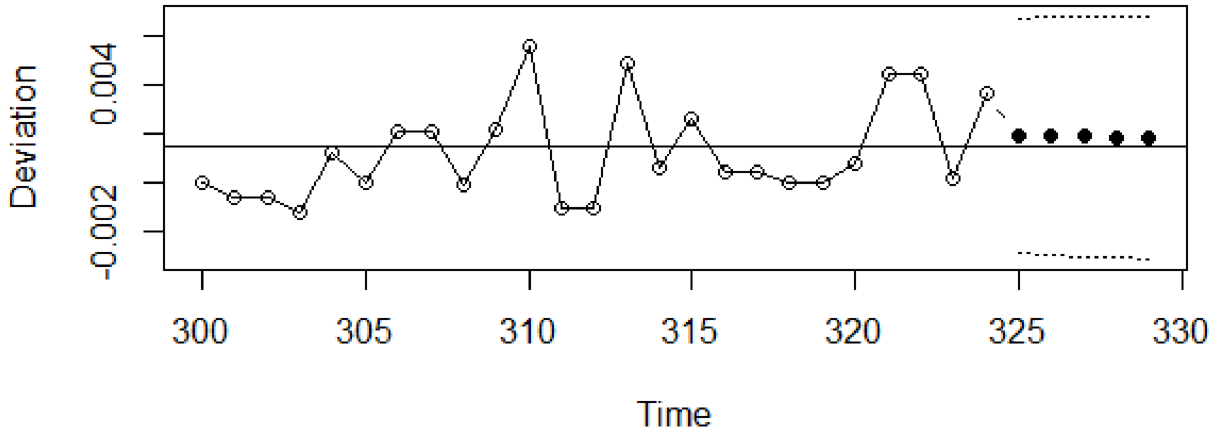


(c):

```
model=arima(robot,order=c(1,0,1))
plot(model,n.ahead=5)$pred
plot(model,n.ahead=5)$upi
plot(model,n.ahead=5)$lpi

[1]  0.001901348  0.001879444  0.001858695  0.001839041  0.001820424
[1]  0.006681473  0.006706881  0.006728193  0.006745972  0.006760700
[1]  -0.002878776  -0.002947994  -0.003010803  -0.003067889  -0.003119851
```

```
plot(model,n1=300,n.ahead=5,ylab='Deviation',pch=19)
```



Both of these models give quite similar forecasts and forecast limits.

Problem 5.

(a):
Since $\Phi = 0.8$, the seasonal part of the model is staionary. In the nonseasonal part, $\phi_1 = 1.6$ and $\phi_2 = 0.7$. These parameter values satisfy Equations (4.3.11) on page 72. Therefore the complete model is stationary.

(b):

$$\phi(x) = 1 - 1.6x + 0.7x^2,$$
$$\Phi(x) = 1 - 0.8x^{12}$$

with $p = 2, q = 0, P = 1, Q = 0, s = 12$.