Statistics 565 Applied Time Series Analysis Spring 2019

Homework 2

Due: Mon 02/11/19 @ 6:00pm rutgers.instructure.com/courses/21204

1. (Recapitulating ordinary least squares regression.) Consider n time series observations $Y_1, \ldots, Y_n$ generated from the additive model

$$Y_i = \beta_0 + \beta_1 t_i + X_i, \tag{1}$$

where $t_1, \ldots t_n$ denote the (possibly unevenly spaced) time points and $X_1, \ldots X_n$ are unobserved realizations of a stationary process with mean 0 and variance σ^2. Show that the ordinary least squares (OLS) estimators of β_0 and β_1 satisfy

$$\begin{aligned} \hat{\beta}_1 &= \frac{\sum_{i=1}^n (Y_i - \bar{Y})(t_i - \bar{t})}{\sum_{i=1}^n (t_i - \bar{t})^2}, \\ \hat{\beta}_0 &= \bar{Y} - \hat{\beta}_1 \bar{t}, \end{aligned}$$

where $\bar{Y} = n^{-1} \sum_{i=1}^n Y_i$ and $\bar{t} = n^{-1} \sum_{i=1}^n t_i$.

The following problems are from the textbook.

2. Problem 3.2.

Hint regarding "unusual results": usually the variance of a sample mean is on the same order with $\frac{1}{n}$. What about here?

3. Problem 3.3.

Hint regarding "autocorrelation in $\{Y_t\}$": Notice that the stochastic component, $\{e_t + e_{t-1}\}$, is a two-point moving average process similar to the one discussed last week. What is its autocorrelation structure?

4. Problem 3.6 (a), (b), (c), and (e).

We are saving (d) and (f) for the next homework. Please save your code!