Homework 3 Solution

March 3, 2019

Problem 4.2. For this problem, we can use the Equation(4.2.3) , page 63 in the text book or use the R code **ARMAacf(ma=list($-\theta_1$,$-\theta_2$))**. The results are given below:

(a):$\theta_1 = 0.5$ and $\theta_2 = 0.4$.

$$\rho_1 = -0.2127660 \ ; \ \rho_2 = -0.2836879$$

(b):$\theta_1 = 1.2$ and $\theta_2 = -0.7$.

$$\rho_1 = -0.6962457 \ ; \ \rho_2 = 0.2389078$$

(c):$\theta_1 = -1$ and $\theta_2 = -0.6$.

$$\rho_1 = 0.6779661 \ ; \ \rho_2 = 0.2542373$$

Problem 4.3. For a MA(1) process, we have that $\rho_1 = \dfrac{-\theta}{1+\theta^2}$, so that $\dfrac{d\rho_1}{d\theta} = \dfrac{\theta^2 - 1}{(1+\theta^2)^2}$ which is seen to be negative on the range $-1 < \theta < +1$ and zero at both -1 and +1. For $|\theta| > 1$, the derivative is positive. Taken together, these facts imply the desired results:

$$\max_{-\infty<\theta<\infty} \rho_1 = 0.5 \quad \text{and} \quad \min_{-\infty<\theta<\infty} \rho_1 = -0.5$$

Problem 4.19. Notice that these coefficients decrease exponentially in magnitude at rate 0.5 while alternating in sign. Furthermore, the coefficients have nearly died out by θ_6. Thus, an AR(1) process with $\phi = -0.5$ would be nearly the same process since such an AR(1) process could be written in the following way:

$$Y_t = e_t - 0.5e_{t-1} + 0.25e_{t-2} - 0.125e_{t-3} + 0.0625e_{t-4} + \cdots$$

Problem 6.

(i): For an AR(2) process, the roots of the quadratic characteristic equation are $\dfrac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{-2\phi_2}$, and the process is stationary only when the three conditions given by (4.3.11) in textbook are satisfied:

$$\phi_1 + \phi_2 < 1, \quad \phi_2 - \phi_1 < 1, \text{ and } \quad |\phi_2| < 1.$$

We could easily check that the all the conditions are satisfied for the four processes , and thus they are all stationary.

(ii)Using the recursive equation (4.3.21) in the textbook, we could easily calculate the coefficients recursively by defineing and using the following r function:

```
psi=function(k,phi1,phi2)
{
  if(k==0)
  {1}
  else if (k==1)
  phi1
  else
  phi1*psi(k-1,phi1,phi2)+phi2*psi(k-2,phi1,phi2)
}
```

and the results follows: (a)

```
sapply(0:4,psi,0.6,0.3)
```

```
## [1] 1.0000 0.6000 0.6600 0.5760 0.5436
```

(b)

```
sapply(0:4,psi,-0.4,0.5)
```

```
## [1]  1.0000 -0.4000  0.6600 -0.4640  0.5156
```

(c)

```
sapply(0:4,psi,1.2,-0.7)
```

```
## [1]  1.0000  1.2000  0.7400  0.0480 -0.4604
```

(d)

```
sapply(0:4,psi,-1,-0.6)
```

```
## [1]  1.00 -1.00  0.40  0.20 -0.44
```

(iii)Using the Yule_Walker equations(4.3.30), we could first calculate that

$$\rho_0 = 1 \ and \ \rho_1 = \frac{\phi_1}{1-\phi_2}.$$

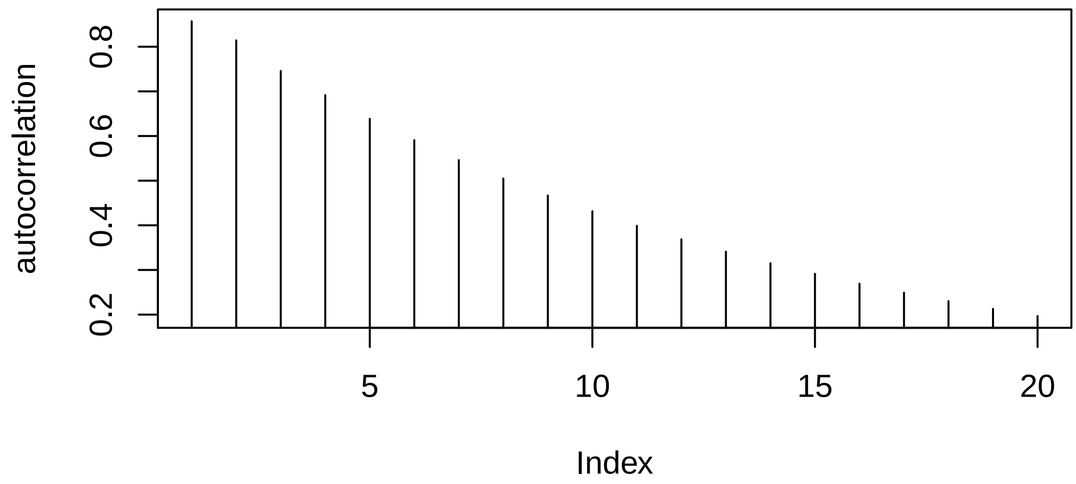
Then we use the follwoing R code to recursively calculate the autocorrealtion function with order larger than 1.

```
rho=function(k,phi1,phi2)
{
  if (k==0)
    1
  else if(k==1)
    phi1/(1-phi2)
  else
    phi1*rho(k-1,phi1,phi2)+phi2*rho(k-2,phi1,phi2)
}
```

Then we plot the autocorrelation coefficients for each of the following process up to lag 20:

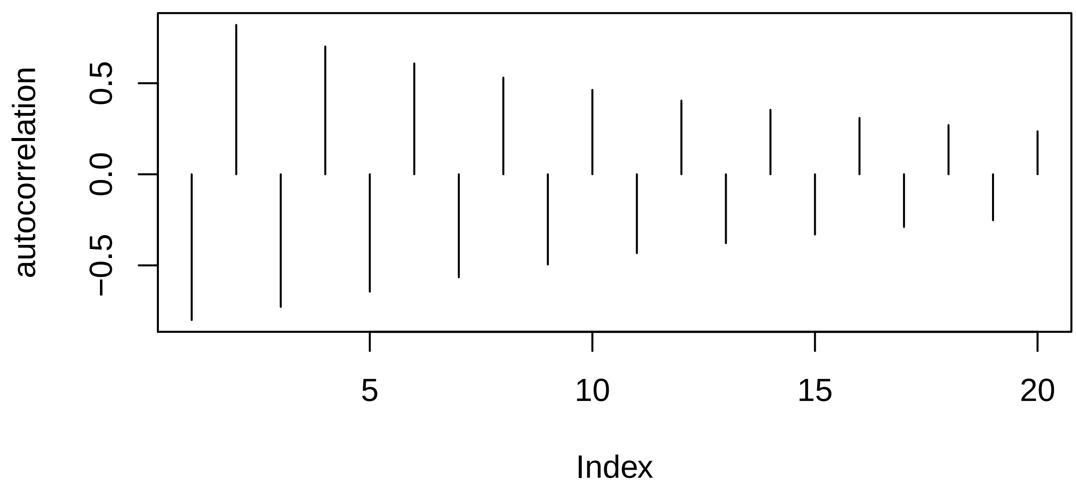
(a)

```
plot(sapply(1:20, rho,0.6,0.3),type="h",ylab="autocorrelation")
```



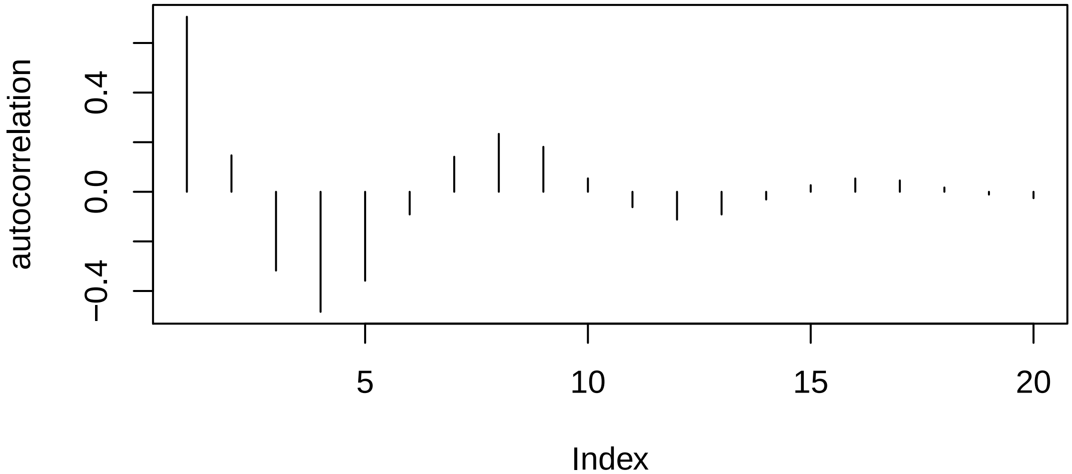
(b)

```r
plot(sapply(1:20, rho,-0.4,0.5),type="h",ylab="autocorrelation")
```



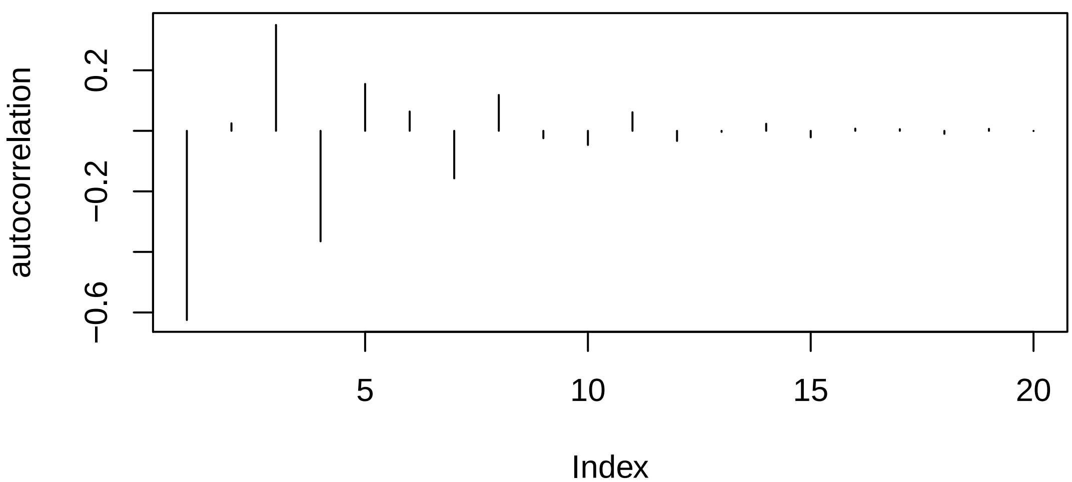
(c)

```r
plot(sapply(1:20, rho,1.2,-0.7),type="h",ylab="autocorrelation")
```



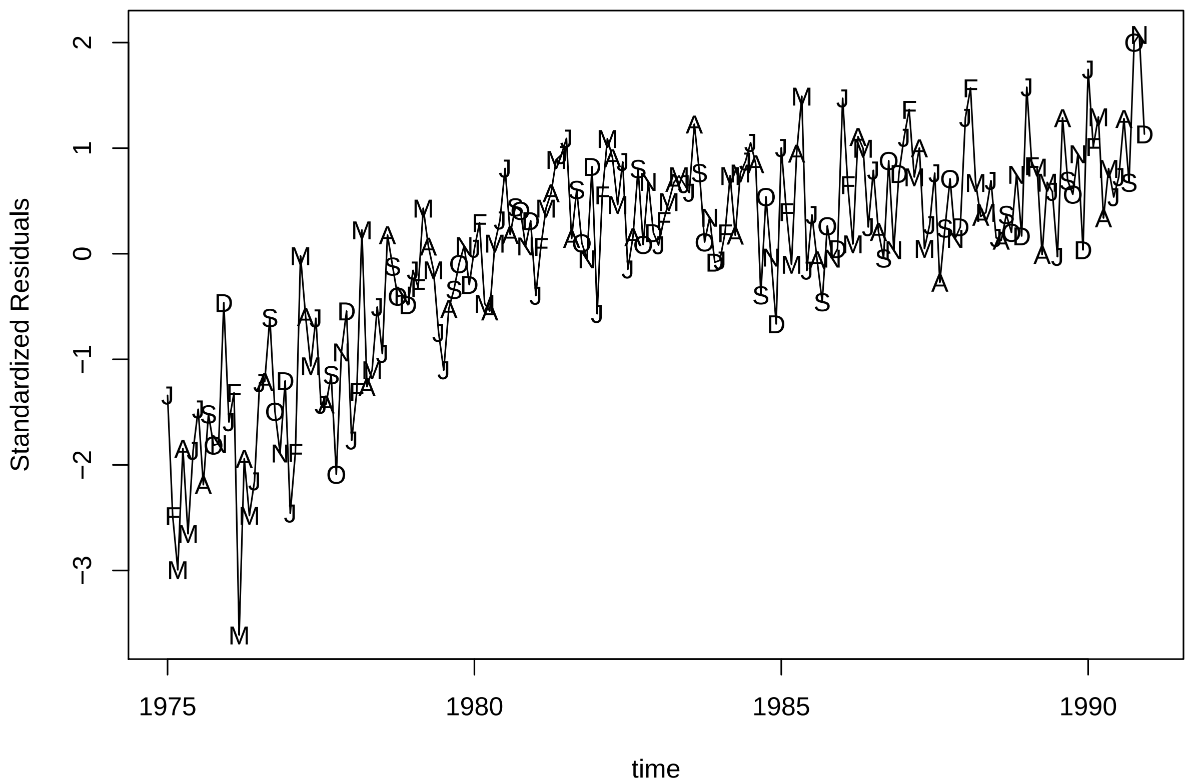
(d)

```
plot(sapply(1:20, rho,-1,-0.6),type="h",ylab="autocorrelation")
```



prblem 3.6 (d)

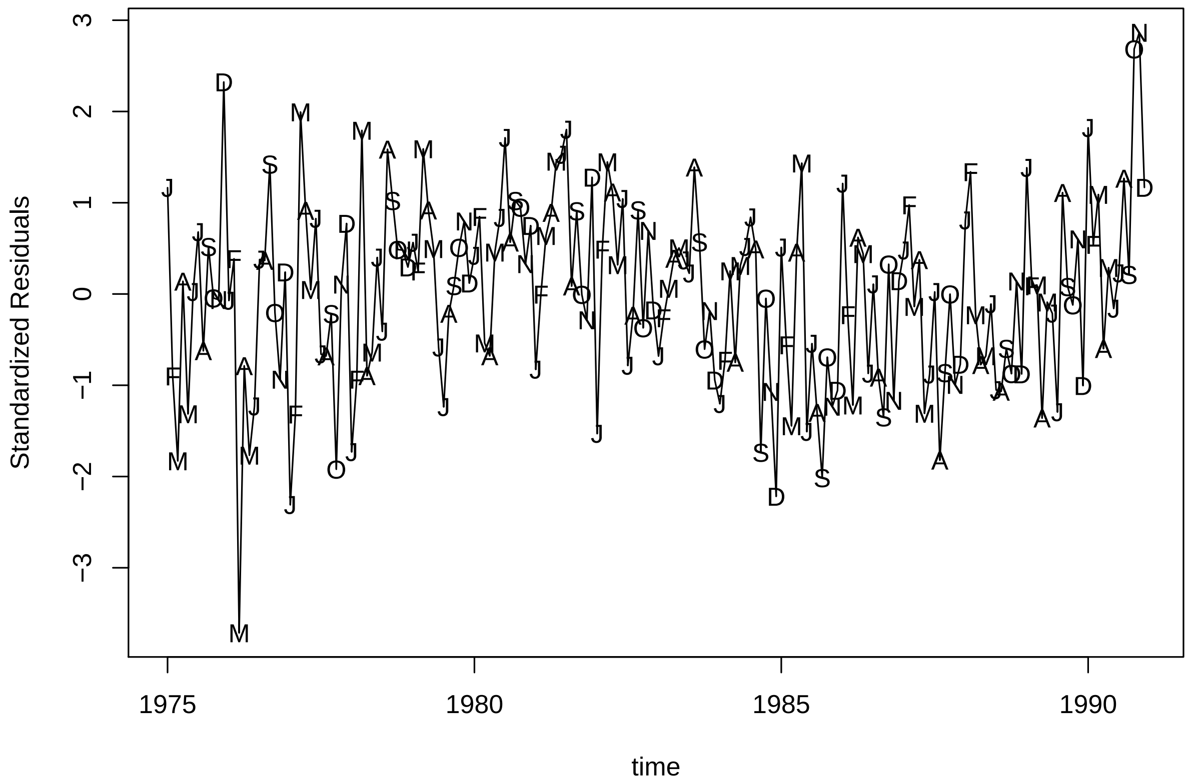
```
library(TSA)
data(beersales)
month.=season(beersales)
beersales.lm=lm(beersales~month.)
plot(y=rstudent(beersales.lm),x=as.vector(time(beersales)),type="l",ylab="Standardized Residuals",xlab=
points(y=rstudent(beersales.lm),x=as.vector(time(beersales)),pch=as.vector(season(beersales)))
```



It is clear that this model does not capture the structure of this time series and we proceed to look for a more adequate model.

(f)

```
beersales.lm2=lm(beersales~month.+time(beersales)+I(time(beersales)^2))
plot(y=rstudent(beersales.lm2),x=as.vector(time(beersales)),type="l",ylab="Standardized Residuals",
     xlab="time")
points(y=rstudent(beersales.lm2),x=as.vector(time(beersales)),pch=as.vector(season(beersales)))
```



This model does much better than the previous one but we would be hard pressed to convince anyone that the underlying quadratic “trend” makes sense. Notice that the coefficient on the square term is negative so that in the future sales will decrease substantially and even go negative!

3.12 (a)

```
library(TSA)
data(beersales)
month.=season(beersales)
beersales.lm2=lm(beersales~month.+time(beersales)+I(time(beersales)^2))
```

(b)

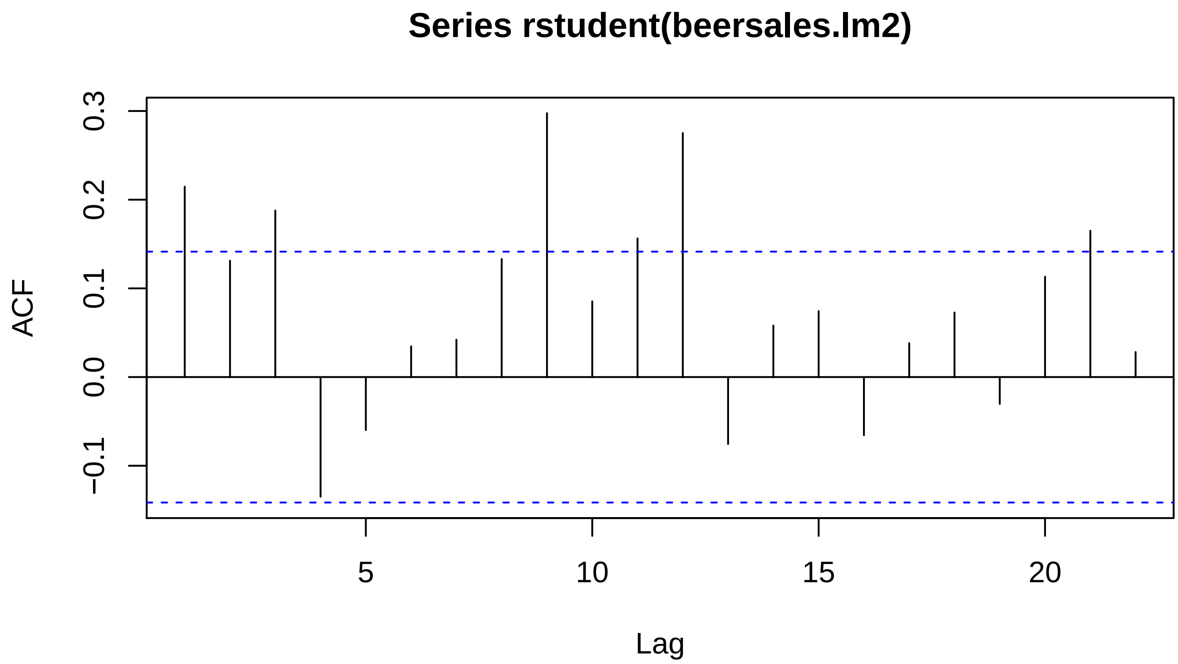
```
runs(rstudent(beersales.lm2))
```

```
## $pvalue
## [1] 0.0127
##
## $observed.runs
## [1] 79
##
## $expected.runs
## [1] 96.625
##
## $n1
## [1] 90
##
## $n2
## [1] 102
##
## $k
## [1] 0
```

We would reject independence of the error terms on the basis of these results.

(c)

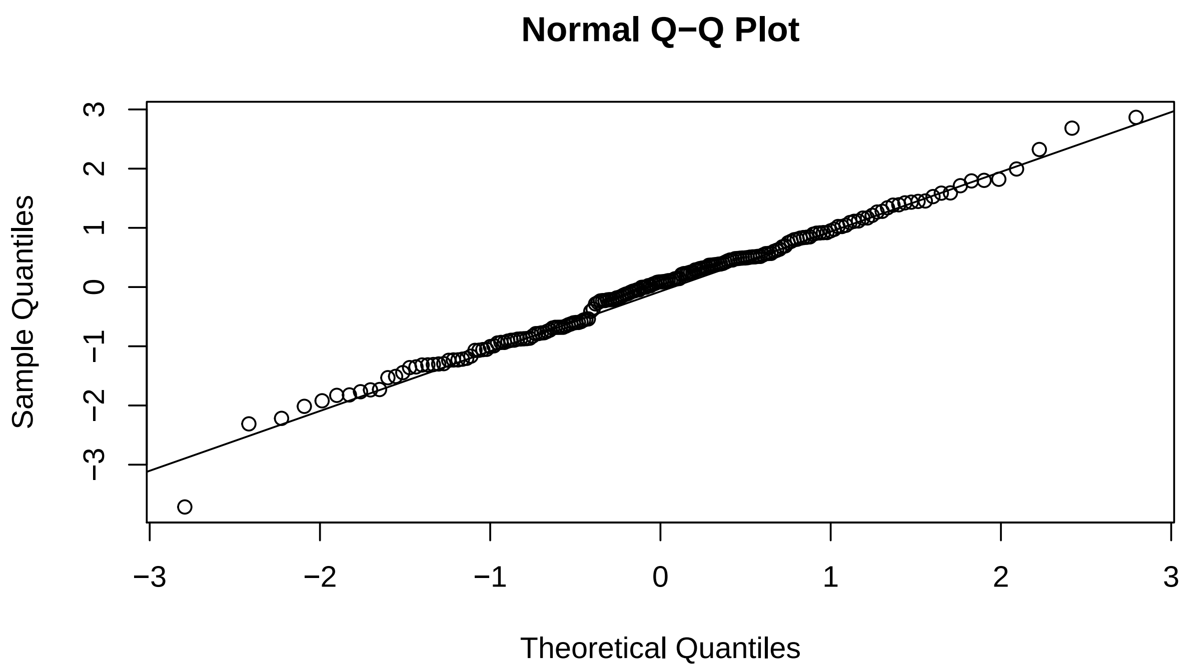
```
acf(rstudent(beersales.lm2))
```



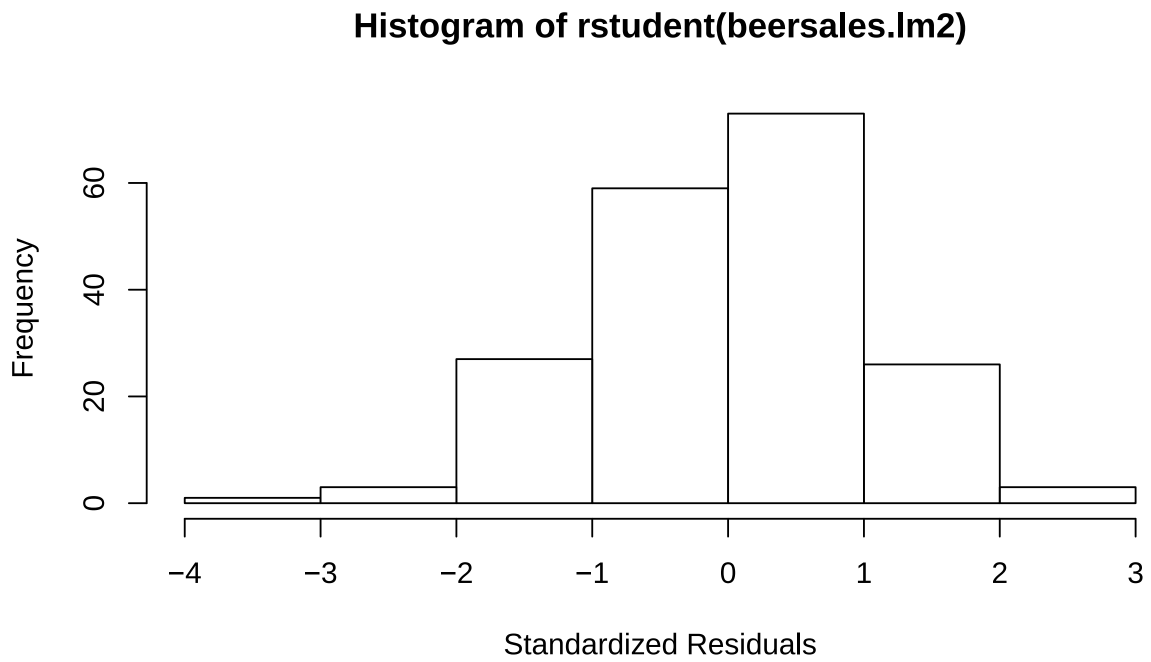
These results also show the lack of independence in the error term of this model.

(d)

```r
qqnorm(rstudent(beersales.lm2))
qqline(rstudent(beersales.lm2))
```



```r
hist(rstudent(beersales.lm2),xlab="Standardized Residuals")
```




```r
shapiro.test(rstudent(beersales.lm2))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  rstudent(beersales.lm2)
## W = 0.99237, p-value = 0.414
```

All of these results provide good support for the assumption of normal error terms.