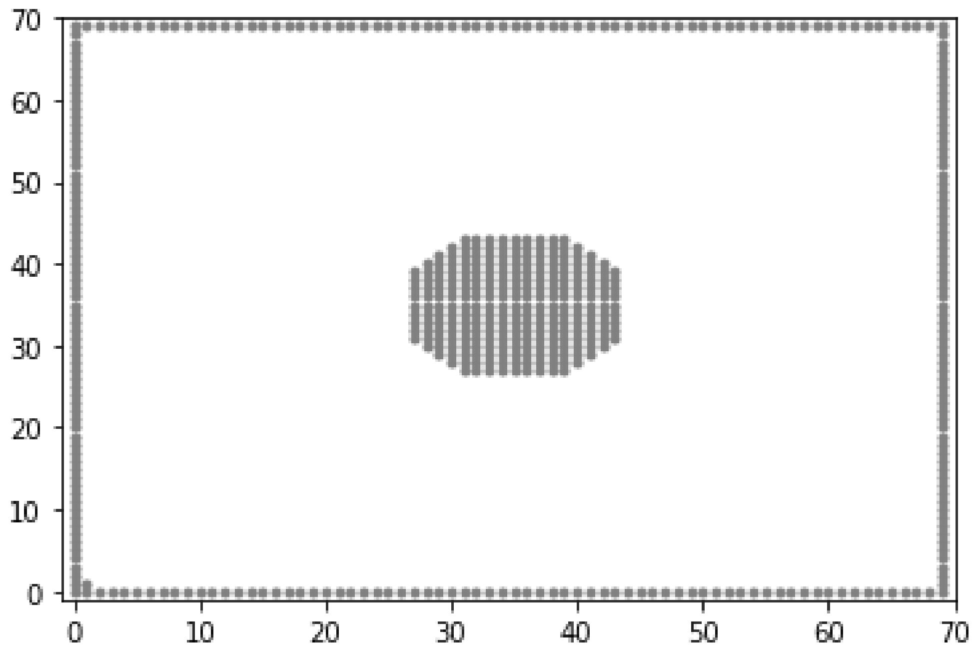
exercise-i

March 20, 2018

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

# Maze dimensions (n by n).
n = 70

# Coordinates of the maze center.
xc = n/2
yc = n/2
```

In [2]:
```
def is_exit(position): # test if position is an absorption site (set A union B)
    i, j = position
    # The set B comprises the maze boundary and the middle circle.
    # The set A comprises the point (1,1).
    return (i == 0 or j == 0 or i == n-1 or j == n-1  # On the maze boundary?
            or is_good_exit(position)                 # In the set A (the point (1,1))
            or np.linalg.norm([i-xc,j-yc]) < 9)       # In the middle circle?

def is_good_exit(position): # test if position is a good absorption site (the set A)
    i, j = position
    # The set A comprises the point (1,1).
    return i==1 and j==1


# Plot the points comprising the elements of A union B (i.e., the absorption sites).

plt.xlim(-1,n)
plt.ylim(-1,n)
plt.scatter([i for i in range(n) for j in range(n) if is_exit( (i,j) )],
            [j for i in range(n) for j in range(n) if is_exit( (i,j) )],
            color='grey', marker='.'
           )
plt.show()
```




```
In [3]: # Construct the transition matrix P.

        P = np.zeros( (n,n,n,n) )  # transition probabilities of the unconditional chain
        for i in range(n):
            for j in range(n):
                if not(is_exit( (i,j) )):
                    # If the position (i,j) is not an absorption site, make all adjacent posit
                    # equally likely to be the next position of the walk.
                    for possible_move in [(i+1,j), (i+1, j+1), (i, j+1),
                                          (i-1,j+1), (i-1,j), (i-1,j-1),
                                          (i,j-1), (i+1,j-1)]:
                        xnew, ynew = possible_move
                        P[i,j,xnew,ynew] = 1

                    # Normalize to get probabilities.
                    s = np.sum(P[i,j,:,:])
                    P[i,j,:,:] = (1.0 / s ) * P[i,j,:,:]
                else:
                    # If the position (i,j) is an absorption site, indicate this in the transi
                    P[i,j,i,j] = 1.0

In [4]: # Solve the linear system defining h in the definition of the transformed transition m

        a = np.zeros( (n,n,n,n) ) # linear system, see the np.linalg.tensorsolve documentation
        for i in range(n):
            for j in range(n):
```

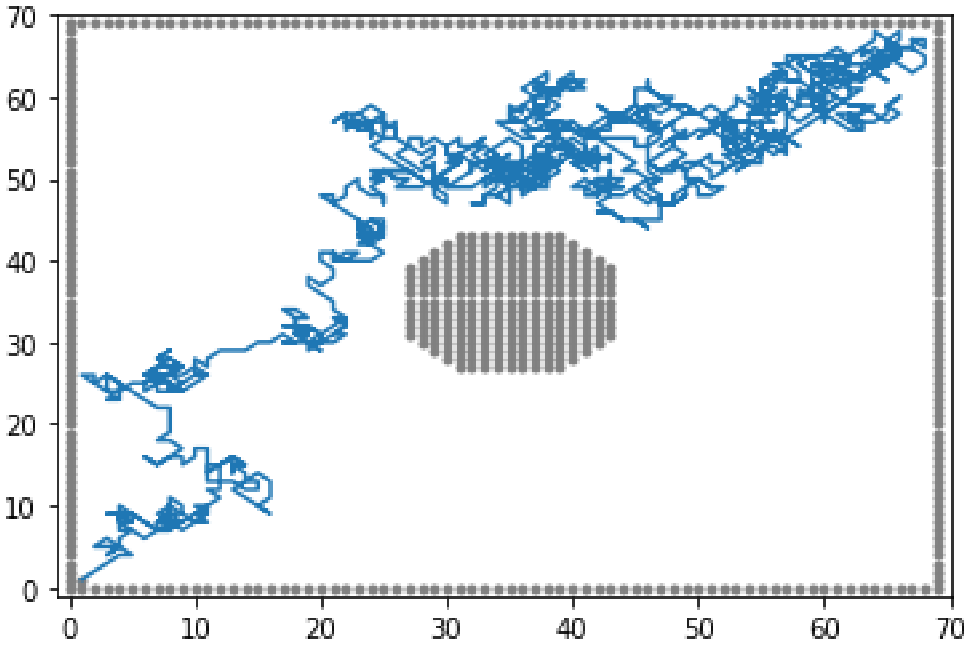
```
            # Indicate that h is harmonic at the state (i,j).
            if not(is_exit( (i,j) )):
                a[i,j,:,:] = - P[i,j,:,:]

            a[i,j,i,j] = 1

    b = np.zeros( (n,n) ) # boundary conditions: 1 for good exists and 0 for others.
    for i in range(n):
        for j in range(n):
            # Indicate the appropriate boundary conditions: if (i,j) is in A, set b[i,j]=1
            # set b[i,j]=0.
            if is_good_exit( (i,j) ):
                b[i,j] = 1.0


    # Solve the linear system for h.
    good_exit_probabilities = np.linalg.tensorsolve(a, b)
```

In [5]:
```
# Compute the transformed transition matrix.

P_transformed = np.zeros_like(P)
for i in range(n):
    for j in range(n):
        if not(is_exit( (i,j) )):
            # If the position (i,j) is not an absorption site, compute using the formu
            for x in range(n):
                for y in range(n):
                    P_transformed[i,j,x,y] = P[i,j,x,y] * good_exit_probabilities[x,y]
        else: # absorption once it reaches an exit
            P_transformed[i,j,i,j] = 1.0
```

In [6]:
```
class Point:
    def __init__(self, x, y):
        self.x = x
        self.y = y

all_points = [Point(i,j) for i in range(n) for j in range(n)]

# Starting coordinates.
x0 = n-2
y0 = n-2

current = Point(x0, y0)
x_visited = []
y_visited = []

# Run the walk until it hits an absorption site.
while(True):
```

```
    # Sample the next position according to the transformed transition matrix and the
    current = np.random.choice(
        all_points,
        1, # return one random element
        p=np.array([P_transformed[current.x, current.y, p.x, p.y] for p in all_points ]
                  )[0]

    # Keep track of where the walk has visited.
    x_visited.append(current.x)
    y_visited.append(current.y)

    # Stop walking if we hit an absorption site.
    if is_exit( (current.x, current.y) ):
        break
```

In [7]:
```
# Plot the maze and the trajectory of the walk.

plt.plot(x_visited, y_visited)
plt.xlim(-1,n)
plt.ylim(-1,n)
plt.scatter([i for i in range(n) for j in range(n) if is_exit( (i,j) )],
            [j for i in range(n) for j in range(n) if is_exit( (i,j) )],
            color='grey', marker='.'
           )
plt.show()
```



In [8]:
```
# Plot the maze and the trajectory of the walk. Each position in the walk is colored b
# last visited that location.

plt.scatter(x_visited, y_visited,
            c=[i for i in range(len(x_visited))])

plt.scatter([i for i in range(n) for j in range(n) if is_exit( (i,j) )],
            [j for i in range(n) for j in range(n) if is_exit( (i,j) )],
            color='grey', marker='.'
           )
plt.xlim(-1,n)
plt.ylim(-1,n)

plt.show()
```