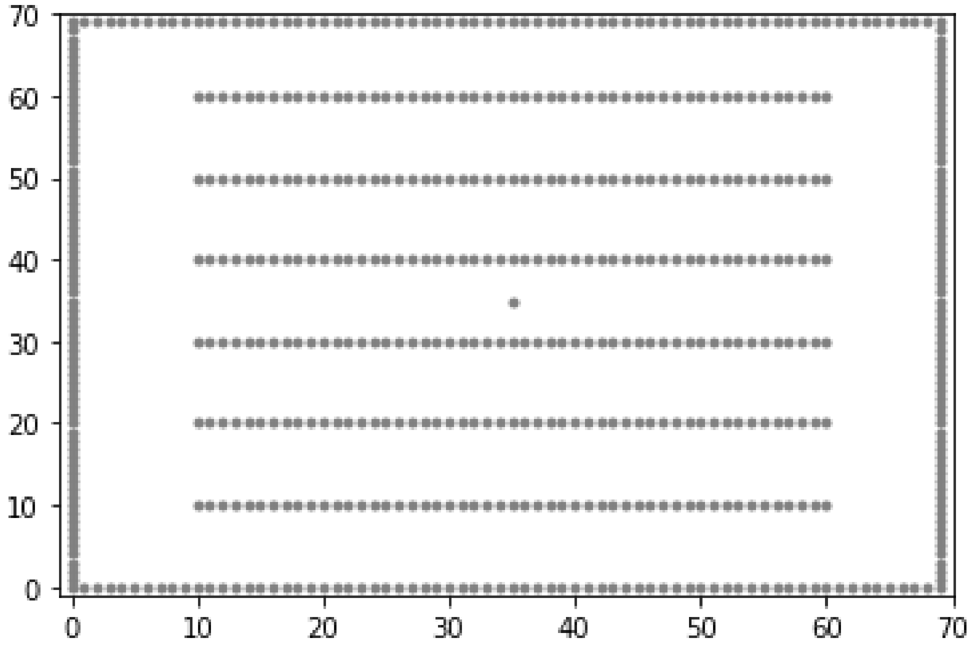
exercise-iv

March 20, 2018

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt

        n = 70

        xc = n/2
        yc = n/2
```

```
In [2]: def is_exit(position): # test if position is an absorption site (set A union B)
            i, j = position
            return (i == 0 or j == 0 or i == n-1 or j == n-1
                    or j % 10 == 0 and 10 <= i <= 60
                    or is_good_exit(position))

        def is_good_exit(position): # test if position is a good absorption site (the set A)
            i, j = position
            return i == xc and j == yc



        plt.xlim(-1,n)
        plt.ylim(-1,n)
        plt.scatter([i for i in range(n) for j in range(n) if is_exit( (i,j) )],
                    [j for i in range(n) for j in range(n) if is_exit( (i,j) )],
                    color='grey', marker='.'
                   )
        plt.show()
```



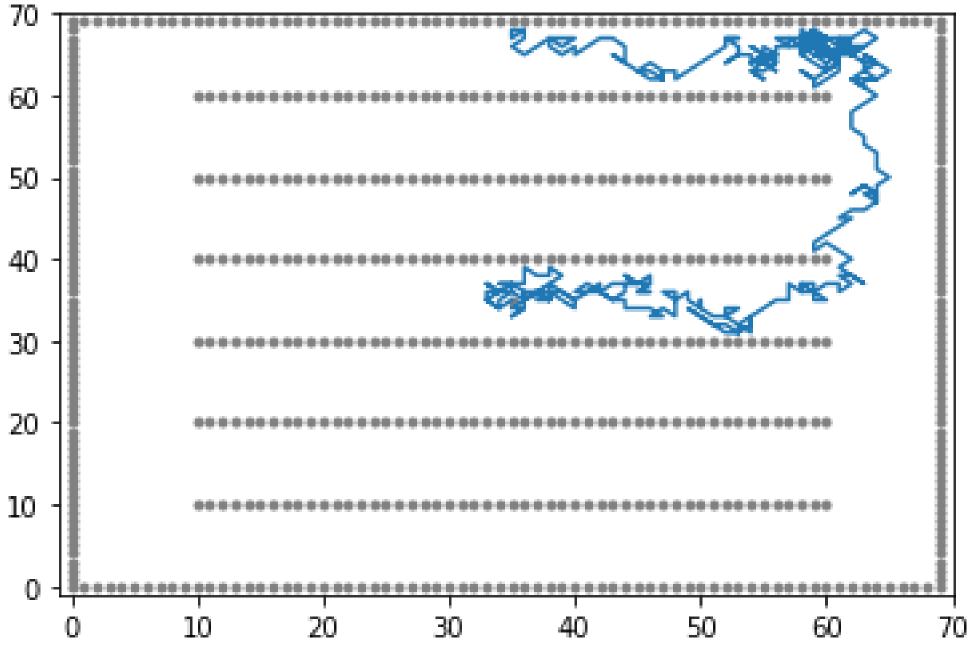
```
In [3]: P = np.zeros( (n,n,n,n) )  # transition probabilities of the unconditional chain
        for i in range(n):
            for j in range(n):
                if not(is_exit( (i,j) )):
                    for possible_move in [(i+1,j), (i+1, j+1), (i, j+1),
                                          (i-1,j+1), (i-1,j), (i-1,j-1),
                                          (i,j-1), (i+1,j-1)]:
                        xnew, ynew = possible_move
                        P[i,j,xnew,ynew] = 1
                    s = np.sum(P[i,j,:,:])
                    P[i,j,:,:] = (1.0 / s ) * P[i,j,:,:]
                else:
                    P[i,j,i,j] = 1.0
```

```
In [4]: a = np.zeros( (n,n,n,n) ) # linear system, see the np.linalg.tensorsolve documentation
        for i in range(n):
            for j in range(n):
                if not(is_exit( (i,j) )):
                    a[i,j,:,:] = - P[i,j,:,:]

                a[i,j,i,j] = 1

        b = np.zeros( (n,n) ) # boundary conditions: 1 for good exists and 0 for others.
        for i in range(n):
            for j in range(n):
                if is_good_exit( (i,j) ):
                    b[i,j] = 1.0


        good_exit_probabilities = np.linalg.tensorsolve(a, b)
```

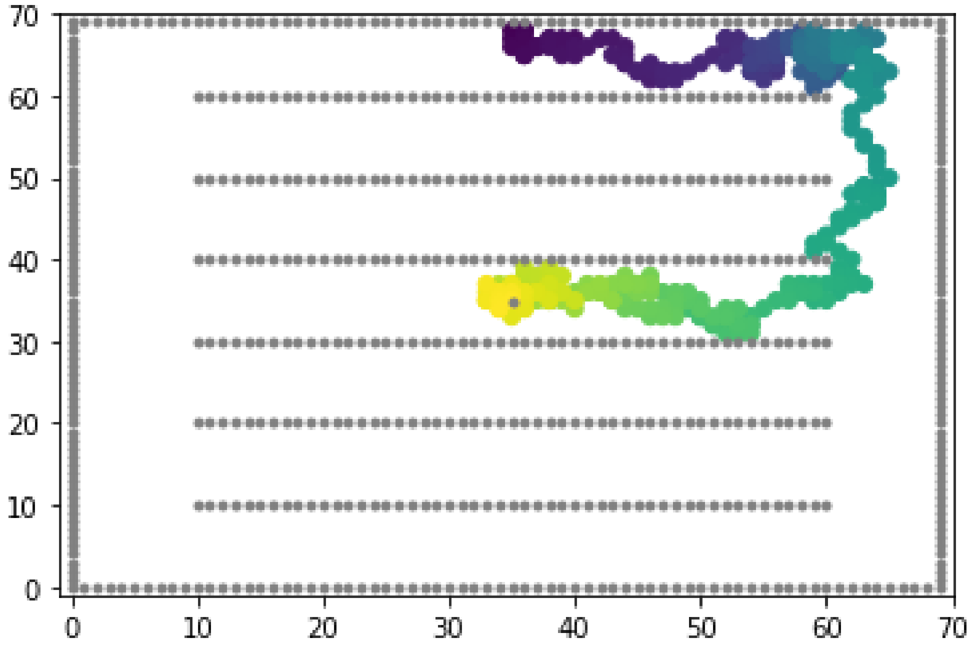
```
In [5]: P_transformed = np.zeros_like(P)
        for i in range(n):
            for j in range(n):
                if not(is_exit( (i,j) )):
                    for x in range(n):
                        for y in range(n):
                            P_transformed[i,j,x,y] = P[i,j,x,y] * good_exit_probabilities[x,y]
                else: # absorption once it reaches an exit
                    P_transformed[i,j,i,j] = 1.0
```

```
In [6]: class Point:
            def __init__(self, x, y):
                self.x = x
                self.y = y

        all_points = [Point(i,j) for i in range(n) for j in range(n)]

        # Run the walk and return the positions visited.
        def run_walk():
            x0 = xc
            y0 = n-2

            current = Point(x0, y0)
            x_visited = []
            y_visited = []
            while(True):
                current = np.random.choice(
                    all_points,
                    1, # return one random element
                    p=np.array([P_transformed[current.x, current.y, p.x, p.y] for p in all_poi
                          )[0]
                x_visited.append(current.x)
                y_visited.append(current.y)
                if is_exit( (current.x, current.y) ):
                    break

            return x_visited, y_visited
```

```
In [7]: x_visited, y_visited = run_walk()
        plt.plot(x_visited, y_visited)

        plt.xlim(-1,n)
        plt.ylim(-1,n)
        plt.scatter([i for i in range(n) for j in range(n) if is_exit( (i,j) )],
                    [j for i in range(n) for j in range(n) if is_exit( (i,j) )],
                    color='grey', marker='.'
                   )
        plt.show()
```



```
In [8]: plt.scatter(x_visited, y_visited,
                    c=[i for i in range(len(x_visited))])

        plt.scatter([i for i in range(n) for j in range(n) if is_exit( (i,j) )],
                    [j for i in range(n) for j in range(n) if is_exit( (i,j) )],
                    color='grey', marker='.'
                   )
        plt.xlim(-1,n)
        plt.ylim(-1,n)

        plt.show()
```


70
60
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10
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60
70