CPU Cooling Using Parallel Plate Fin Heat Sink and Forced Convection

535.652 Thermal Systems Design and Analysis Term Project

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Heat sink fin array designs were optimized based on a range of fin spacings with the objective of maximizing the rate of heat transfer. The Lagrange Multiplier method was conducted to determine the optimal fin quantity and thickness, which were contingent on a specific fin spacing. Following the determination of the fin quantity, fin thickness, and heat transfer rate, the volumetric flow rate of a fan was calculated by assuming fixed performance values: operating static pressure, maximum pressure drop, and maximum volumetric flow rate. A fan was determined by filtering through an electronic components vendor's catalog.

I. Nomenclature

N	=	fin quantity
t	=	fin thickness
z	=	fin spacing
$\dot{q}_{out}$	=	heat transfer rate from CPU to heat sink
L	=	plate length
w	=	plate width
ΔP_{fan}	=	fan pressure head
ΔP_{max}	=	maximum fan pressure head
ΔP_{hs}	=	pressure drop across heat sink
Q_{fan}	=	fan volumetric flow rate
Q_{max}	=	maximum fan volumetric flow rate
v_{fin}	=	average air speed between fins
w^*	=	characteristic length of fin entrance
D_h	=	hydraulic diameter
f_{app}	=	apparent friction factor
σ	=	free-flow area ratio
K_c	=	constriction pressure loss coefficient
K_e	=	expansion pressure loss coefficient
Re	=	Reynolds number
$\mathcal{L}$	=	Lagrangian
y	=	objective function
λ_i	=	ith Lagrange multiplier
ϕ_i	=	ith equality constraint
s_i	=	ith slack variable

II. Introduction - Proposed Designs

The *Electronics cooling* section of the *Term Project* document [1] was modified. A Central Processing Unit (CPU) is running a on motherboard sitting on an open-air laboratory workstation. Given the CPU's heat generation rate, determine a heat sink's design parameters to ensure that a necessary amount of heat generated by the chip gets dissipated. Typical design parameters for a heat sink include: fin geometry, fin quantity, distance between the fins, and material composition. The two proposed ways to cool a CPU are described in the subsections below. The second design was chosen to be the focus of this project.

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A. Design 1

The first design concept involves the use of a heat sink with a pin fin array to passively cool a CPU through natural convection. Optimizing the pin fin array will maximize heat dissipation from the CPU to the surrounding environment. The project will require the selection of an appropriate pin fin geometry, such as pin heights, pin diameters, and orientation angles to enhance heat transfer.

B. Design 2

The second design concept involves the development of a parallel plate heat sink system that utilizes forced convection to enhance heat dissipation. The design will involve the use of parallel fins to increase the surface area for heat transfer and a fan to force air over the fins. In addition to designing the fin array, the appropriate fan performance parameters need to be determined to complement the heat sink's design.

III. Background

Heat sinks are widely used to dissipate heat from electronic devices and other heat-generating components into the surrounding air to prevent overheating, ensuring optimal performance, and prolonging lifetime. Parallel plate heat sinks have been used for their low cost and manufacturability. Fans attached to heat sinks provide forced convection and are often designed with fin structures to enhance heat dissipation.

IV. Modeling and Setup

The heat sink fin array design process will be an extension of the work done by Elnaggar [2]. All fins are made of aluminum as it's a common material choice. Figure 1 and 2 depict schematics of the heat sink. The space between the fins are equal because the fin distribution is uniform and are treated as slender channels. All fin plate dimensions are $120mm$ x $25mm$. Only a $120mm$ fan will be mounted onto the fin array. There are 2 copper heat pipes on 2 opposite sides of the heat sink's base (4 total). The base plate is an aluminum-copper stack (lower half is copper and upper half is aluminum).

The work flow overview for designing the cooling system is:

1) obtain t, N, and $\dot{q}_{out}$ for a specific fin spacing from optimization
2) select an arbitrary fan on a vendor catalog that will fit onto the fin array
3) calculate Q_{fan} using t, its corresponding fin spacing, and assumed performance characteristic values from the chosen fan's data sheet
4) repeat steps 2 - 3 if the chosen fan is inadequate based on the calculated Q_{fan}

A. Assumptions

The list of assumptions below were made to greatly simplify the design process.

- isothermal fins
- negligible radiation heat transfer
- the fins' and base plate temperature are equal
- air flow through fins is laminar, incompressible, and steady
- fan exhaust is uniform so there is no "cross flow" at the fin array entrance
- negligible fan vibration
- negligible heat generated by fan
- ignore the presence of the heat pipes within the fin array
- perfect thermal contact between heat sink's base plate and CPU

V. Heat Sink Fin Array Design Optimization

The fluid mechanics and heat transfer of CPU cooling involves solving a combination of nonlinear and transcendental functions. In order to ease the simulation of CPU cooling, the heat sink fin array optimization results were used to determine the fan's performance parameters. This approach focuses on each component individually rather than attempting to determine the specifications for the entire setup simultaneously. The Lagrange Multiplier method was the chosen for optimization as opposed to *search methods* because the design space is small, thus will be more efficient. Lagrange Multipliers provide a systematic way to solve constrained optimization problems.

A. Formulation

The Lagrange Multiplier method is set up as

$$\nabla \mathcal{L} = \nabla y - \sum \lambda_i \nabla \phi_i = 0 \tag{1}$$

The heat transfer rate model analytically determined by Elnaggar [2]

$$\dot{q}_{out} = -9.551 + 52.723t + 1.568N + 1.304tN - 34.207t^2 \tag{2}$$

will be the objective function that gets maximized. Since Elnaggar [2] does not specify the fin spacing, a range of fin spaces between $1.5mm$ to $3mm$ were tested.

To limit the amount of fins, the following constraint equation is used

$$\phi_1 = Nt + (N-1)z - 120 \tag{3}$$

Equation 3 will constrain the fin array to be within the fan's dimensions depicted in Figure 1b.

Manufacturing constraints were implemented to define the minimum and maximum fin quantity and thickness. Slack variables were used to turn inequality constraints into equality constraints.

$$\phi_2 = N + s_1^2 - 56 \tag{4}$$

$$\phi_3 = N - s_2^2 \tag{5}$$

$$\phi_4 = t + s_3^2 - 1 \tag{6}$$

$$\phi_5 = t - s_4^2 \tag{7}$$

Equations 4 and 5 will ensure that the number of fins will fall within the range of $(0, 56]$. 56 is the maximum amount of fins from [2]. Equations 6 and 7 are for setting the fin thickness to be within $(0mm, 1mm]$.

B. Optimization Results

The objective function and the constraint equations, denoted by Equations [2 to 7], were substituted into Equation 1, transforming it into a system of 11 equations and unknowns. By solving this set of equations, the optimal parameters for the heat sink, N, t, and $\dot{q}_{out}$, were determined. For every fin spacing, there is a corresponding optimal amount of fins, fin thickness, and maximum heat dissipated as organized into Table 1. The optimization results are visually represented through a series of plots, shown in Figure 3, for each parameter. A polynomial curve was fitted to each plot.

As the fin space increased, the allowable amount of fins for the fin array decreased (Figure 3b); consequently, the fin thickness had to increase to compensate for the lower available cooling surface area (Figure 3a). Additionally, the lower surface area led to the maximum heat transfer rate to decline (Figure 3c). One verification for the optimization setup working as intended can be seen in Figure 3a; beyond a fin spacing of $2.25mm$, the optimizer is constrained to the specified maximum fin thickness of $1mm$. Upon reaching the limit for fin thickness, the maximum heat transfer rate exhibits a continued decline as the fin spacing widens and the available cooling surface area diminishes.

VI. Fan Performance

A. Formulation

All fans and heat sinks have a pressure vs volumetric flow rate graph to describe their performances. For fans, the pressure decreases as the air flow increases; conversely, for heat sinks, the pressure increases with the flow rate. An example graph from Wang's [3] Figure 4 illustrates the intersections of 1 fan curve and 6 heat sink curves. The intersection, or operating point, between the fan and heat sink PQ curve is mathematically represented by:

$$\Delta P_{hs}(Q_{fan}) = \Delta P_{fan}(Q_{fan}) \tag{8}$$

A fan curve can be linearly approximated by:

$$\Delta P_{fan} = \frac{-\Delta P_{max}}{Q_{max}} Q_{fan} + \Delta P_{max} \tag{9}$$

The graphical representation of Equation 9 can be seen in Figure 5. Substituting Equation 9 into 8 results in Equation 10 for Q_{fan} after rearranging the terms.

$$Q_{fan} = \frac{-Q_{max}}{\Delta P_{max}} (\Delta P_{hs} - \Delta P_{max}) \tag{10}$$

The values for ΔP_{max} and Q_{max} will be assumed to be $88.26Pa$ ($9mmH_2O$) and $0.0483m^3/s$ ($2.9m^3/min$) respectively from the PQ curve in Figure 6 provided by Delta Electronics [4] for the fan model: AFB1212VH-F00. These values come from the fan's data sheet after browsing the product website of Digi-Key Electronics and choosing an arbitrary $120mm$ fan.

Equation 11 represents the contributors (inlet flow contraction, friction, and outlet flow expansion) to the pressure drop across the heat sink; $D_h = 2z$ by convention for flow between parallel plates (rectangular ducts) [5] [6] [7].

$$\Delta P_{hs} = \left(K_c + \frac{4 f_{app} L}{D_h} + K_e \right) \frac{1}{2} \rho v_{fin}^2 \tag{11}$$

Equation 12 and 13 are correlations of contraction and expansion pressure loss coefficients due to area change at the inlet and outlet respectively [5] [8].

$$K_c = 0.4(1 - \sigma^2) + 0.4 \tag{12}$$

$$K_e = (1 - \sigma)^2 - 0.4\sigma \tag{13}$$

The free-flow area ratio for frontal area is used to calculate the pressure losses between the inlet and outlet [8]. The fin spacing and thickness define the free-flow area (the space available for air to flow through) and the frontal area (total area of the face of the fin array that the air first comes into contact with) [8] [5] [9].

$$\sigma = \frac{z}{z + t} \tag{14}$$

Apparent friction factor is used to account for the pressure drop due to friction and boundary layer development effects [10] [5] [8]. The apparent friction factor used in [7] [9] for laminar flow through parallel plate heat sinks is:

$$f_{app} = \frac{\sqrt{\left(\frac{3.44}{w^*}\right)^2 + (fRe)^2}}{Re} = \frac{\sqrt{\left(\frac{3.44}{w^*}\right)^2 + (fRe)^2}}{\frac{\rho v_{fin} D_h}{\mu}} \tag{15}$$

w^* is the characteristic channel length [8] [9] [7].

$$w^* = \frac{w}{Re D_h} = \frac{w}{\frac{\rho v_{fin} D_h}{\mu} D_h} = \frac{w\mu}{4\rho v_{fin} z^2} \tag{16}$$

To consider friction of fully developed flow within a rectangular duct, the friction factor Reynolds number group formula from Duan and Muzychka [9] is used for laminar 1D flow.

$$fRe = \frac{24}{\left(1 + \frac{z}{w}\right)^2 \left(1 - \frac{192}{\pi^5 w}\right) tanh\left(\frac{\pi w}{D_h}\right)} \tag{17}$$

The beginning of Section IV introduces the treatment of the space between the fins as slender channels; Equation 18 from Bejan [10] for slender channel flow is used to describe the average air speed between the fins. ΔP_{fan} will be assumed to be $10Pa$ ($1.02mmH_2O$), which is within the performance range of the arbitrarily chosen fan, AFB1212VH-F00, seen in Figure 6.

$$v_{fin} = \frac{z^2 \Delta P_{fan}}{12\mu w} \tag{18}$$

Equations 12 to 18 are substituted into Equation 11. Equation 11 is then substituted into Equation 10.

B. Selecting Fan

The operating fan flow rate from Equation 10 using assumed fixed values for ΔP_{max}, ΔP_{fan}, and Q_{max} are tabulated in Table 1. At a fin spacing of $1.5mm$, $Q_{fan} = 0.045m^3/s = 2.705m^3/min$. At a fin spacing of $3.0mm$, $Q_{fan} = 0.011m^3/s = 0.669m^3/min$. Based on Figure 6, the chosen fan, AFB1212VH-F00, operating at $10Pa$ $(1.02mmH_2O)$ is best suited for a fin array with $1.5mm$ fin spacing, $0.891mm$ fin thickness, and 50 fins which is capable of dissipating $148.988W$. The fan can flow air at $0.045m^3/s$ $(2.7m^3/min)$ on $10Pa$ $(1.02mmH_2O)$, indicated in red in Figure 6, which means the fan can still be used on the other fin array configurations in Table 1 because lower air flow rates are needed. However this would not be efficient as problematic factors would arise, such as higher power consumption and noise generation.

VII. Limitations

1) Equation 2 fitted by Elnaggar [2] is only valid for 120mm fans mounted on aluminum parallel plate heat sinks that have 2 copper heat pipes on 2 opposite sides of a base plate comprised of an aluminum-copper stack.
2) The fin array's and fan's design parameters were not directly interrelated in the optimization.
3) Fan selection is a guess-and-check process.
4) Fan design parameters (blade twist, hub size, airfoil, ect.) were not considered for the analyses.
5) Laminar and uniform air flow were assumed throughout the fin array.
6) The relationship between fan static pressure and volumetric flow rate is linearly approximated, so details regarding undesirable performances (stall, instability, ect.) are lost.

VIII. Conclusion

Design optimization of a parallel plate fin heat sink system for CPU cooling using forced convection was explored in this report. The project involved an analysis of the heat sink fin array design with a focus on optimizing the fin thickness, quantity, and spacing to maximize heat dissipation. The optimization process was based on the Lagrange Multiplier method to solve a constrained problem.

The results indicated as the fin spacing increased, the number of fins decreased, necessitating an increase in fin thickness to compensate for the reduced cooling surface area. This led to a decline in the maximum heat transfer rate as the fin spacing widened and the available cooling surface area also declined. A fan was selected from Digi-Key Electronics's catalog, and its performance were tested using the optimization results to see if it would complement the fin array designs.

Future work can address limitations from the overly simplified heat sink and fan model presented by considering additional factors, such as fan design parameters and the impact of turbulence.

Appendix

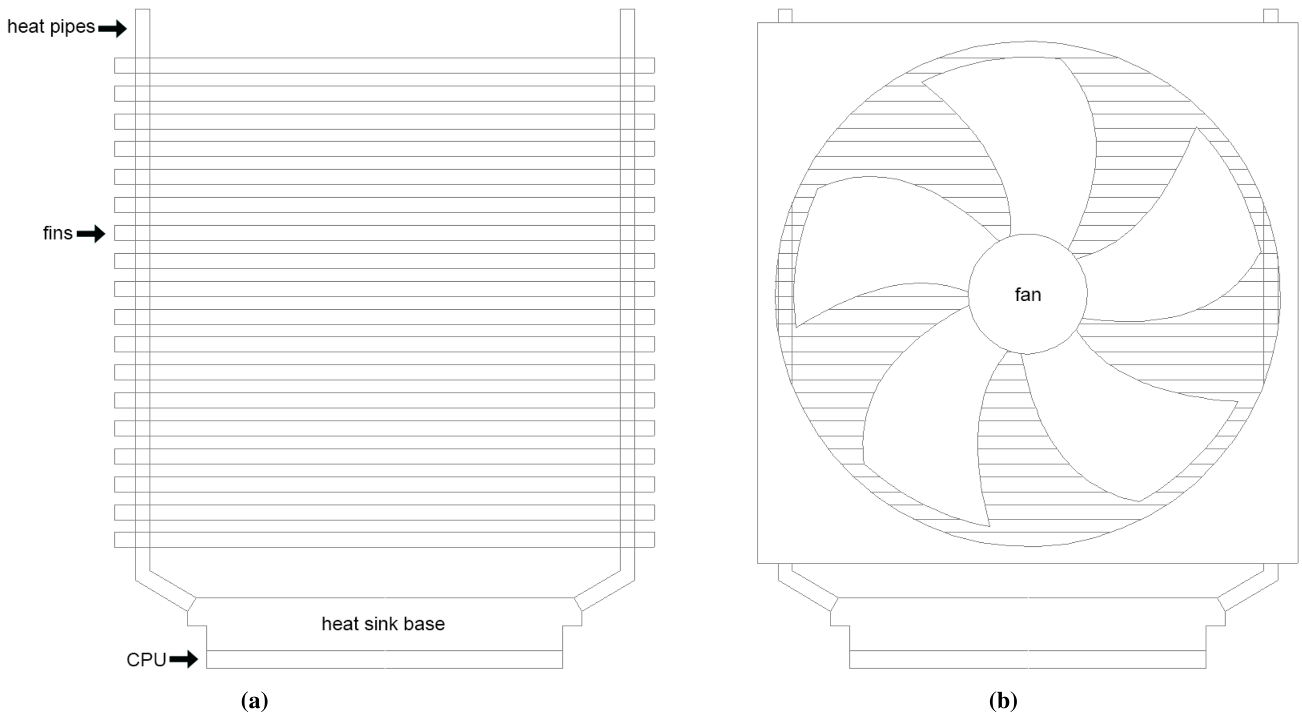


Fig. 1   Schematic of parallel plate heat sink with and without a fan mounted on a CPU

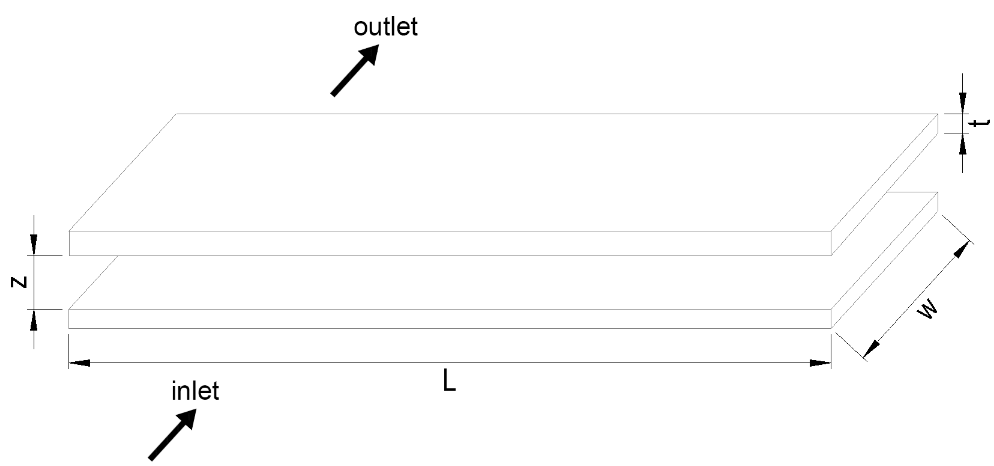
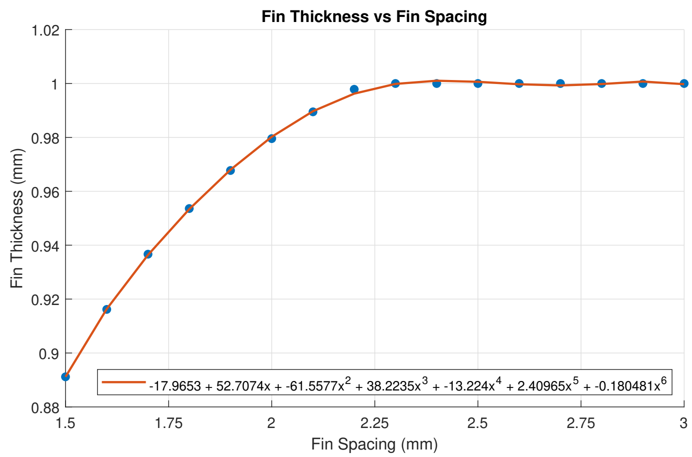
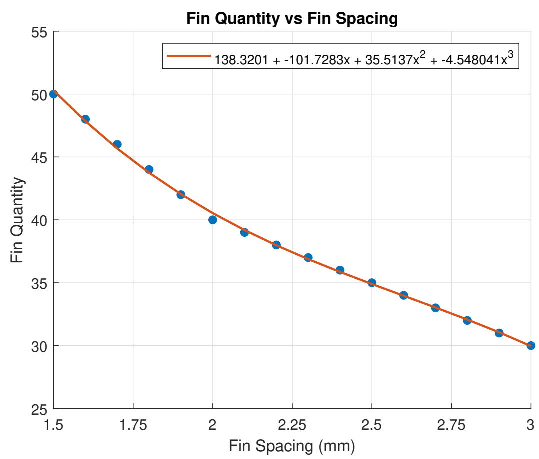


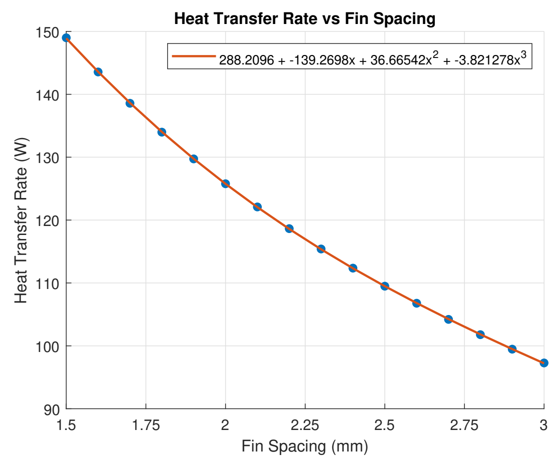
Fig. 2   Geometry of rectangular channel and simplified parallel plate fins



(a) Fin Thickness vs Fin Spacing



(b) Fin Quantity vs Fin Spacing



(c) Heat Transfer Rate vs Fin Spacing

Fig. 3 Polynomial curve fits of optimization results

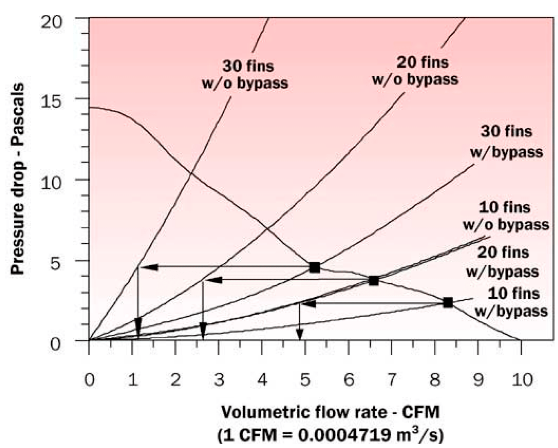


Fig. 4 Example of operating points for fan and heat sink combinations by Wang's [3] Figure 2c

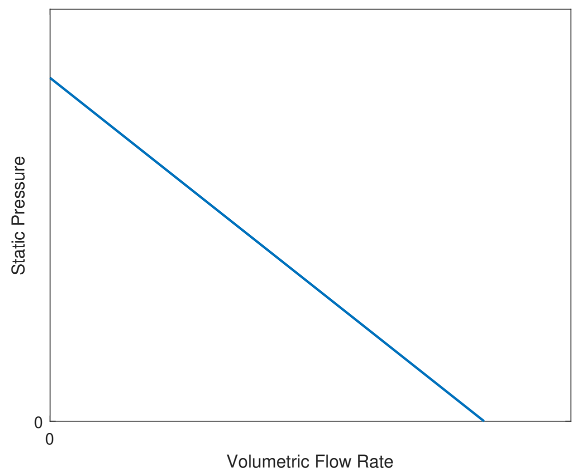
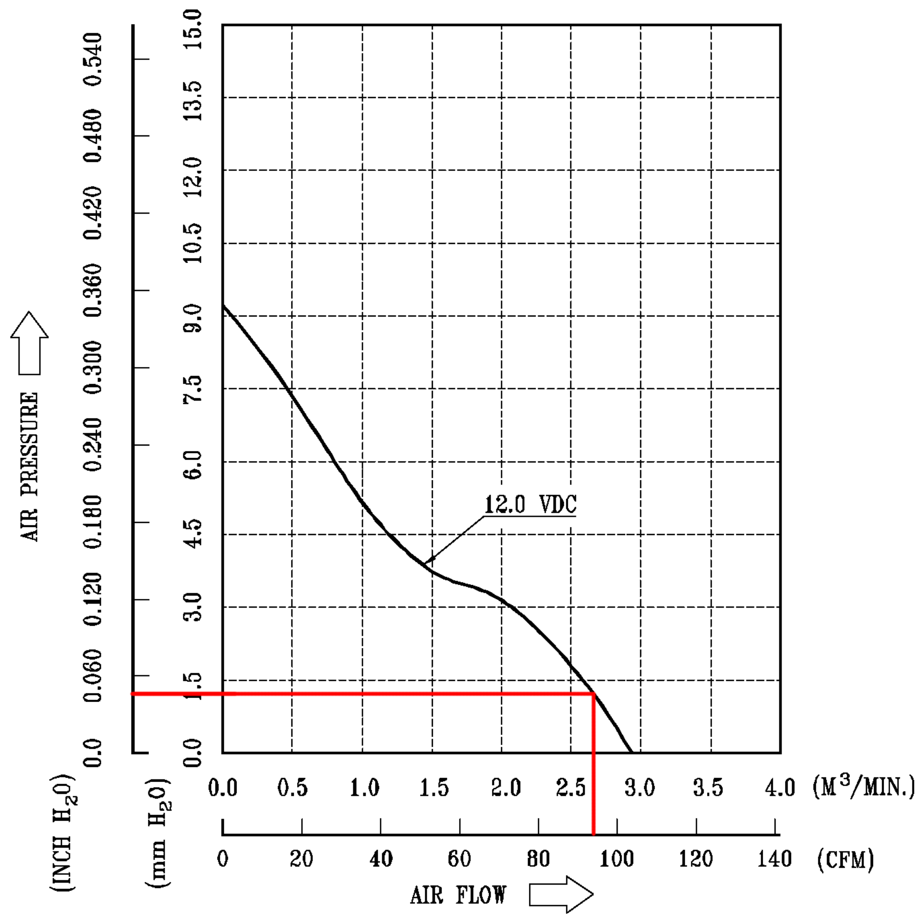


Fig. 5 Linear approximation of fan curve

PART NO:

DELTA MODEL: AFB1212VH-F00

9. P & Q CURVE:



* TEST CONDITION: INPUT VOLTAGE ------- OPERATION VOLTAGE
TEMPERATURE --------- ROOM TEMPERATURE
HUMIDITY -------------- 65%RH

page: 5 A00

Fig. 6 AFB1212VH-F00 PQ curve by Delta Electronics; chosen operating pressure, ΔP_{fan}, and its corresponding flow rate are indicated in red

$fin\ spacing\ (mm)$	$t\ (mm)$	N	s_1	s_2	s_3	s_4	λ_1	λ_2	λ_3	λ_4	λ_5	$\dot{q}_{out}\ (W)$	$\Delta P_{fan}\ (Pa)$	$Q\ (m^3/s)$
1.5	0.891	50	2.278	7.128	0.330	0.944	1.142	0.000	0.000	0.000	0.000	148.988	10	0.045
1.6	0.916	48	2.770	6.952	0.290	0.957	1.098	0.000	0.000	0.000	0.000	143.553	10	0.044
1.7	0.937	46	3.137	6.794	0.252	0.968	1.058	0.000	0.000	0.000	0.000	138.572	10	0.043
1.8	0.954	44	3.430	6.651	0.215	0.977	1.021	0.000	0.000	0.000	0.000	133.980	10	0.042
1.9	0.968	42	3.673	6.520	0.180	0.984	0.987	0.000	0.000	0.000	0.000	129.729	10	0.041
2.0	0.980	40	3.880	6.399	0.143	0.990	0.955	0.000	0.000	0.000	0.000	125.776	10	0.039
2.1	0.989	39	4.059	6.287	0.103	0.995	0.925	0.000	0.000	0.000	0.000	122.089	10	0.037
2.2	0.998	38	4.217	6.182	0.046	0.999	0.897	0.000	0.000	0.000	0.000	118.640	10	0.036
2.3	1.000	37	4.352	6.088	0.000	1.000	0.870	0.000	0.000	0.382	0.000	115.403	10	0.033
2.4	1.000	36	4.472	6.000	0.000	1.000	0.845	0.000	0.000	0.844	0.000	112.357	10	0.031
2.5	1.000	35	4.583	5.916	0.000	1.000	0.821	0.000	0.000	1.229	0.000	109.485	10	0.028
2.6	1.000	34	4.684	5.836	0.000	1.000	0.798	0.000	0.000	1.549	0.000	106.773	10	0.026
2.7	1.000	33	4.779	5.759	0.000	1.000	0.776	0.000	0.000	1.811	0.000	104.207	10	0.022
2.8	1.000	32	4.867	5.685	0.000	1.000	0.756	0.000	0.000	2.025	0.000	101.776	10	0.019
2.9	1.000	31	4.948	5.614	0.000	1.000	0.736	0.000	0.000	2.195	0.000	99.470	10	0.015
3.0	1.000	30	5.025	5.545	0.000	1.000	0.718	0.000	0.000	2.329	0.000	97.279	10	0.011

Table 1 Results from Lagrange Multiplier optimization and fan flow rate calculation

References

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```
%%
clear all
close all
clc

%% find t, N, q_out_dot
syms t N s1 s2 s3 s4 lambda1 lambda2 lambda3 lambda4 lambda5

% units: mm
SpacingValues = [1.5:.1:3];

AllResults = zeros(length(SpacingValues), 13);

for i = 1:length(SpacingValues)
    spacing = SpacingValues(i);

    q_out_dot = -9.551 + 52.723 * t + 1.568 * N + 1.304 * t * N - 34.207 * t^2; % units: W
    phi1 = N * t + (N - 1) * spacing - 120;
    phi2 = N + s1^2 - 56;
    phi3 = N - s2^2;
    phi4 = t + s3^2 - 1;
    phi5 = t - s4^2;

    L = q_out_dot - lambda1 * phi1 - lambda2 * phi2 - lambda3 * phi3 - lambda4 * phi4 - lambda5 * phi5;
    e1 = diff(L, t) == 0;
    e2 = diff(L, N) == 0;
    e3 = diff(L, s1) == 0;
    e4 = diff(L, s2) == 0;
    e5 = diff(L, s3) == 0;
    e6 = diff(L, s4) == 0;
    e7 = diff(L, lambda1) == 0;
    e8 = diff(L, lambda2) == 0;
    e9 = diff(L, lambda3) == 0;
    e10 = diff(L, lambda4) == 0;
    e11 = diff(L, lambda5) == 0;


    sol = solve([e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11], [t, N, s1, s2, s3, s4, lambda1, lambda2, lambda3, lambda4, lambda5]);
    results = [double(vpa(sol.t)) double(vpa(sol.N)) double(vpa(sol.s1)) double(vpa(sol.s2)) double(vpa(sol.s3)) double(vpa(sol.s4)) double(vpa(sol.lambda1)) double(vpa(sol.lambda2))
double(vpa(sol.lambda3)) double(vpa(sol.lambda4)) double(vpa(sol.lambda5))];

    filter = any(imag(results) > 0, 2);
    results(filter, :) = [];

    filter = any(real(results) < 0, 2);
    results(filter, :) = [];

    filter = any(imag(results) < 0, 2);
    results(filter, :) = [];

    filter = results(:, 1) <= 0 | results(:, 2) <= 0;
    results(filter, :) = [];

    % check number of rows in results
    if size(results, 1) > 1
        disp('more than 1 row in results');
        disp(['current spacing value: ', num2str(spacing)]);
        break;
    end

    q_dot(results(1,1), results(1,2));

    % Store the results and q_dot in the array
    AllResults(i, :) = [spacing, results(1,:), q_dot(results(1,1), results(1,2))];
end

AllResults = array2table(AllResults);
AllResults.Properties.VariableNames = {'spacing', 't', 'N', 's1', 's2', 's3', 's4', 'lambda1', 'lambda2', 'lambda3', 'lambda4', 'lambda5', 'q_out_dot'};
AllResults.N = floor(AllResults.N);

%% plot
fig = figure;
fig.Position = [1000 918 725 420];
hold on
scatter(AllResults.spacing, AllResults.t, "filled", HandleVisibility = "off")

d = [ones(size(AllResults.spacing)) AllResults.spacing AllResults.spacing.^2 AllResults.spacing.^3 AllResults.spacing.^4 AllResults.spacing.^5 AllResults.spacing.^6];
coefficients = d \ AllResults.t;
sc = string(coefficients) + ["", "x", "x^2", "x^3", "x^4", "x^5", "x^6"]';
y = coefficients(1) + coefficients(2) * AllResults.spacing + coefficients(3) * AllResults.spacing.^2 + coefficients(4) * AllResults.spacing.^3 + coefficients(5) * AllResults.spacing.^4 +
coefficients(6) * AllResults.spacing.^5 + coefficients(7) * AllResults.spacing.^6;
plot(AllResults.spacing, y, DisplayName = strjoin(sc, ' + '), LineWidth = 1.5);

title("Fin Thickness vs Fin Spacing")
xlabel("Fin Spacing (mm)")
ylabel("Fin Thickness (mm)")
xticks(linspace(SpacingValues(1), SpacingValues(end), 7))
legend(Location = "southeast")
grid on
hold off

%
figure
hold on
scatter(AllResults.spacing, AllResults.N, "filled", HandleVisibility = "off")

d = [ones(size(AllResults.spacing)) AllResults.spacing AllResults.spacing.^2 AllResults.spacing.^3];
coefficients = d \ AllResults.N;
sc = string(coefficients) + ["", "x", "x^2", "x^3"]';
y = coefficients(1) + coefficients(2) * AllResults.spacing + coefficients(3) * AllResults.spacing.^2 + coefficients(4) * AllResults.spacing.^3;
plot(AllResults.spacing, y, DisplayName = strjoin(sc, ' + '), LineWidth = 1.5);

title("Fin Quantity vs Fin Spacing")
xlabel("Fin Spacing (mm)")
ylabel("Fin Quantity")
xticks(linspace(SpacingValues(1), SpacingValues(end), 7))
legend(Location = "northeast")
grid on
hold off

%
figure
hold on
scatter(AllResults.spacing, AllResults.q_out_dot, "filled", HandleVisibility = "off")

d = [ones(size(AllResults.spacing)) AllResults.spacing AllResults.spacing.^2 AllResults.spacing.^3];
coefficients = d \ AllResults.q_out_dot;
sc = string(coefficients) + ["", "x", "x^2", "x^3"]';
y = coefficients(1) + coefficients(2) * AllResults.spacing + coefficients(3) * AllResults.spacing.^2 + coefficients(4) * AllResults.spacing.^3;
plot(AllResults.spacing, y, DisplayName = strjoin(sc, ' + '), LineWidth = 1.5);

title("Heat Transfer Rate vs Fin Spacing")
xlabel("Fin Spacing (mm)")
ylabel("Heat Transfer Rate (W)")
xticks(linspace(SpacingValues(1), SpacingValues(end), 7))
```

```
legend(Location = "northeast")
grid on
hold off

%% find Q
FinSpacing = AllResults.spacing * 10^-3; % units: m; (convert from mm to m)
FinThickness = AllResults.t * 10^-3; % units: m; (convert from mm to m)

rho = 1.225; %units: kg / m^3
mu = 1.81e-5; % units: Pa * s
L = 120 * 10^-3; % units: m
w = 25 * 10^-3; % units: m
dP_fan_max = 9 * 9.80665; % units: Pa (convert from mmH2O to Pa)
StaticPressure = 10; % units: Pa
Q_max = 2.9 / 60; % units: m^3 / s; (convert from m^3 / min to m^3 / s)
v_fin = FinSpacing.^2 ./ (12 .* mu) .* (StaticPressure ./ w); % units: m / s

w_star = w .* mu ./ (rho .* v_fin .* 4 .* FinSpacing.^2);
fRe = 24 ./ ((1 + FinSpacing ./ w).^2 .* (1 - 192 ./ (pi.^5 .* w) * tanh(pi .* w ./ (2 .* FinSpacing))));
f_app = sqrt((3.44 ./ w_star).^2 + fRe.^2) ./ (rho .* v_fin .* 2 .* spacing ./ mu);
sigma = FinSpacing ./ (FinSpacing + FinThickness);
Kc = .4 * (1 - sigma.^2) + .4;
Ke = (1 - sigma).^2 - .4 .* sigma;
dP_hs = (Kc + 4 .* f_app .* L ./ (FinSpacing .* 2) + Ke) .* 1 ./ 2 .* rho .* v_fin.^2;
Q = (dP_hs - dP_fan_max) * (-Q_max / dP_fan_max);

AllResults.StaticPressure = ones(size(AllResults.spacing)) * StaticPressure;
AllResults.Q = Q; % units: m^3 / s

%%
function W = q_dot(t, N)
    W = -9.551 + 52.723 * t + 1.568 * N + 1.304 * t * N - 34.207 * t^2;
end
```