Coursework 4 — CO496
(Component Analysis and Optimisation)

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General instructions: Regarding the coding part, you should *only* complete the matlab functions required for both parts as described below. No other matlab file will be taken into account during marking. Your code should contain comments about the mathematical methodology you used in your implementation. Regarding the written part, you should produce a pdf named `report.pdf`, where you will include your answers. No aspect of your submission may be hand-drawn. You are strongly encouraged to use LaTeX to create the written component.

Part I:

You are given a facial dataset with identity information per sample (the dataset is part of PIE facial database). You are also given six matlab code files. `demo_PCA.m`, `demo_wPCA.m` and `demo_LDA.m` run a face recognition protocol and report the results in the end (in a form of recognition error). Demo files may *not* be modified. `PCA.m`, `wPCA.m` and `LDA.m` are utilised by `demo_PCA.m`, `demo_wPCA.m` and `demo_LDA.m`, respectively, and are *to be completed* matlab functions that should perform dimensionality reduction techniques (PCA, whitened PCA and LDA, respectively) on Small Sample Sized (SSS) problems (number of samples significantly less that the number of features), according to the notes. You should attach in your report plots of the recognition error versus the number of components kept for each of the methods. Briefly explain which method is the best and why.

(12 marks)

Part II:

i) You are given the centralised data samples $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n]$ and a label per sample $\mathbf{l} = [y_1, \ldots, y_n]^T, y_i \in \{-1, 1\}, \forall i \in \{1, \ldots, n\}$. Assume the covariance matrix $\mathbf{S}_t = \frac{1}{N}\mathbf{X}\mathbf{X}^T$ is invertible.

   a) Formulate the Lagrangian of the optimisation problem (1). Moreover, formulate its dual and find the optimal $\mathbf{w}, b, \xi_i$. Add your solutions in `report.pdf`.

$$\begin{aligned} \min_{\mathbf{w},b,\xi_i} \quad & \frac{1}{2}\mathbf{w}^T\mathbf{S}_t\mathbf{w} + C\sum_{i=1}^{n}\xi_i \\ \text{subject to} \quad & y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1 - \xi_i, \;\; \xi_i \geq 0, \forall i \in \{1, \ldots, n\}. \end{aligned} \tag{1}$$

b) For the coding part of this exercise, you are given a set of centralised samples `X.mat`, together with their labels `l.mat`. Solve (1) numerically on matlab utilising the provided data. You are also given a set of centralised test samples `X_test`. Estimate the labels for each of the test samples and report the accuracy of your trained SVM utilising the ground truth labels for the test data `l_test`. Your code which solves (1) and calculates the accuracy on the test set should be added in the function `SVM.m`.

ii) Propose a way to deal with the case when $\mathbf{S}_t$ is singular.

(13 marks)