

# Multi-Query Optimization in Federated RDF Systems

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**Abstract**—This paper revisits the classical problem of multiple query optimization in the context of federated RDF systems. We propose a heuristic query rewriting-based approach to share the common computation during evaluation of multiple queries while considering the cost of both query evaluation and data shipment. Although we prove that finding the optimal rewriting for multiple queries is NP-complete, we propose a heuristic rewriting algorithm with a bounded approximation ratio. Furthermore, we propose an efficient method to join intermediate results during multiple query evaluation. The extensive experimental studies over both real and synthetic RDF datasets show that the proposed techniques are effective, efficient and scalable.

## I. INTRODUCTION

Many data providers publish, share and interlink their datasets using open standards such as RDF and SPARQL [4]. RDF is a self-describing data model that represents data as triples of the form (subject, property, object) for modelling information in the Web, while SPARQL is a query language to retrieve and manipulate data stored in RDF format. Although many data providers publish their RDF data, they often store the actual triple files at their own *autonomous* sites some of which are *SPARQL endpoints* that can execute SPARQL queries. An autonomous site with a SPARQL endpoint is called an *RDF source* in this paper.

To integrate and provide transparent access over many RDF sources, federated RDF systems have been proposed [19], [22], in which, a control site is introduced to provide a common interface for users to issue SPARQL queries. Based on the metadata, the control site takes care of rewriting and optimizing the query. In particular, a SPARQL query  $Q$  is decomposed into several subqueries that are evaluated on relevant sources; then, these subquery results are joined together to form complete results that are returned to users.

Federated RDF systems have been widely used in many applications [9]. For example, European Bioinformatics Institute has provided a uniform platform<sup>1</sup> for users to query multiple bioinformatics RDF sources, including BioModels, Biosamples, ChEMBL, Ensembl, Atlas, Reactome and UniProt. It also provides a unified way to query across sources using the SPARQL query language.

So far, many federated RDF systems have been proposed [19], [22], [1], [7], [20]. A popular federated RDF benchmark—FedBench [21]—is often used to evaluate the

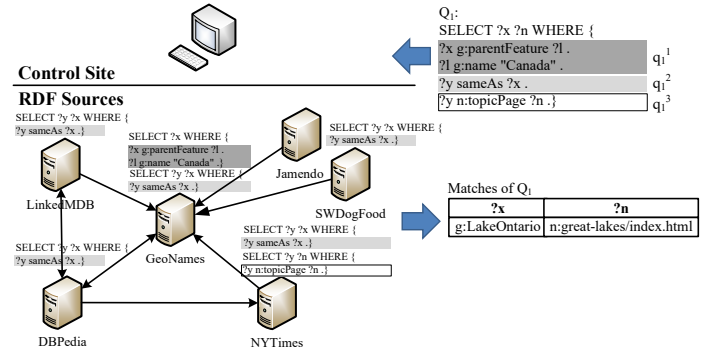


Fig. 1. A Federated SPARQL Query

performance of the federated RDF systems. Example 1 shows a sample federated query (in FedBench) involving multiple RDF sources.

**Example 1: (A Federated SPARQL Involving Multiple Sources)** There are six interconnected RDF sources of different domains, such as NYTimes, GeoNames, SWDogFood and DBPedia, in Fig. 1. Assume a person wants to find out all news about Canada. From NYTimes, s/he finds all news pages associated with news locations (such as cities or towns), but, NYTimes does not explicitly identify all Canadian places. Fortunately, the places in NYTimes are linked to the counterparts (using “sameAs” property) in GeoName, a worldwide geographical RDF database that states explicitly which places are located in Canada. Therefore, the federated SPARQL query  $Q_1$  that ask for can be formulated over the federated RDF system.

To evaluate  $Q_1$ , it is decomposed into three subqueries  $q_1^1$ ,  $q_1^2$  and  $q_1^3$ , which are sent to relevant sources as discussed in Section III. These subquery results are sent back to the query originating site and joined together to form complete results. □

However, existing federated RDF systems only consider query evaluation for a single query and miss the opportunity for multiple query optimization. Real SPARQL query workloads reveal that many SPARQL queries are often posed simultaneously, so this is a real issue. Let us consider a real SPARQL query workload over DBPedia<sup>2</sup> that includes 8 million SPARQL queries issued over 14 days. On average, there are more than six SPARQL queries per second. Take

<sup>1</sup><http://www.ebi.ac.uk/rdf/>

<sup>2</sup><http://aksw.org/Projects/DBPSB.html>

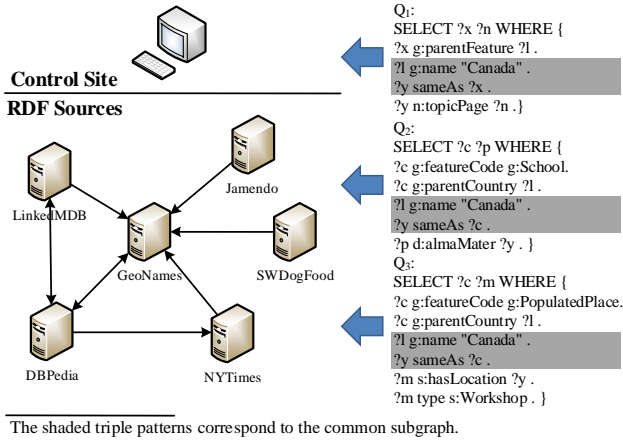


Fig. 2. Multiple Federated SPARQL Queries

another example. According to the workload of SPARQL queries over Linked Geo Data<sup>3</sup>, more than 600 queries are issued per second at peak time. Therefore, it is desirable to design a multiple SPARQL query optimization strategy.

Consider a batch of queries (e.g.,  $Q_1$ ,  $Q_2$  and  $Q_3$  in Fig. 2) that are posed simultaneously over the federated RDF system in Fig. 1. It is easy to identify some common substructures over these three queries, which suggests the possibility of some sharing computation. This motivates us to revisit the classical problem of multiple query optimization in the context of federated RDF systems. Specifically,  $Q_1$  is introduced in Example 1 and find out all news about Canada;  $Q_2$  retrieves all people who graduated from Canadian universities; and  $Q_3$  is to retrieve all semantic web-related workshops held in Canada. All the three queries will be used as running examples throughout this paper.

#### A. Challenges & Our Solution

Although multiple query optimization have been well studied in distributed relational databases [14], some techniques commonly referred to as data movement and data/query shipping [12] are not easily applicable in federated RDF systems. For example, we cannot require one source to send intermediate results directly to another source [9]. Moreover, moving data from one source to another one for join processing [14] is also infeasible.

To the best of our knowledge, only *one* proposal about multiple SPARQL query optimization exists in literature [13], but *only in the centralized environment*, where all RDF datasets are collected in one physical database. Given a batch of SPARQL queries, it identifies all maximal common edge subgraphs (MCES) among query graphs. Each MCES leads to a possible MCES-based rewritten query. A cost model is proposed to select one MCES-based rewriting with the minimum cost. Since MCES identification algorithm has exponential time complexity, the approach is not scalable with respect to the size and the number of query graphs. Furthermore, there is no data shipment in the centralized environment. Rewriting multiple queries into some rewritten queries as [13] can generate many intermediate results in federated RDF systems and result in large cost for data shipment.

Our method has two novel characteristics:

- 1) As we rewrite multiple queries with commonalities, we consider both “OPTIONAL” and “FILTER” clauses of SPARQL. This allows us significant rewrite opportunities.
- 2) We propose a cost model-driven greedy solution for multiple query rewriting; our cost model considers the cost for both query evaluation and data shipment, while our solution considers both the common substructures and the selectivity of queries. Also, the linear time complexity of our greedy algorithm guarantees the scalability of our rewriting strategy with regard to the sizes and numbers of queries that are posed simultaneously.

In addition, we also study partial match joins in federated RDF systems, which do not arise in the centralized counterpart. We optimize partial match joins during multiple query processing in federated RDF systems. Experiments show that the optimized join approach can improve query performance by 10% compared with the naive join techniques.

To the best of our knowledge, this is the first study of multiple SPARQL query optimization over federated RDF systems, with the objective to reduce the query response time and the number of remote requests.

## II. BACKGROUND

### A. RDF and Federated RDF System

**Definition 1: (RDF Graph).** An RDF graph is denoted as  $G = \{V, E, L\}$ , where  $V(G)$  is a set of vertices that correspond to all subjects and objects in RDF data;  $E \subseteq V \times V$  is a set of directed edges that correspond to all triples in RDF data;  $L$  is a set of edge labels. For each edge  $e \in E$ , its edge label is its property.

In the context of federated RDF systems,  $G$  is a combination of many RDF graphs located at different source sites.

**Definition 2: (Federated RDF System)** A *federated RDF system* is defined as  $W = (S, g, d)$ , where (1)  $S$  is a set of source sites that can be obtained by looking up URIs in an implementation of  $W$ ; (2)  $g : S \rightarrow 2^{E(G)}$  is a mapping that associates each source with a subgraph of RDF graph  $G$ ; and (3)  $d : V(G) \rightarrow S$  is a partial, surjective mapping which models the fact that looking up URI of vertex  $u$  results in the retrieval of the source represented by  $d(u) \in S$ .  $d(u)$  is called the *host* source of  $u$ , and is unique for a given URL of vertex  $u$ .

Obviously, RDF graph  $G$  is formed by collecting all subgraphs at different sources, i.e.,  $\bigcup_{s \in S} g(s) = G$ . Consider the RDF graph  $G$  distributed among six different sources in Fig. 3. Given a vertex “ $g:v2$ ”,  $d(“g:v2”)=\text{GeoNames}$ , where “ $g$ ” is abbreviation of “GeoNames”. This means that vertex “ $g:v2$ ” is dereferenced by the host GeoNames.

Since there might be multiple RDF sources that describe an entity identified by vertex  $u$ , vertex  $u$  may be contained in multiple sources. However,  $u$  is only dereferenced by the host source  $d(u)$ . In Fig. 3, the vertices at their host sources are denoted as the grey circles and they connect to the corresponding mirrors at other sources by dashed lines. Let us still consider the vertex “ $g:v2$ ”; it is distributed among two

<sup>3</sup><http://aksw.github.io/LSQ/>

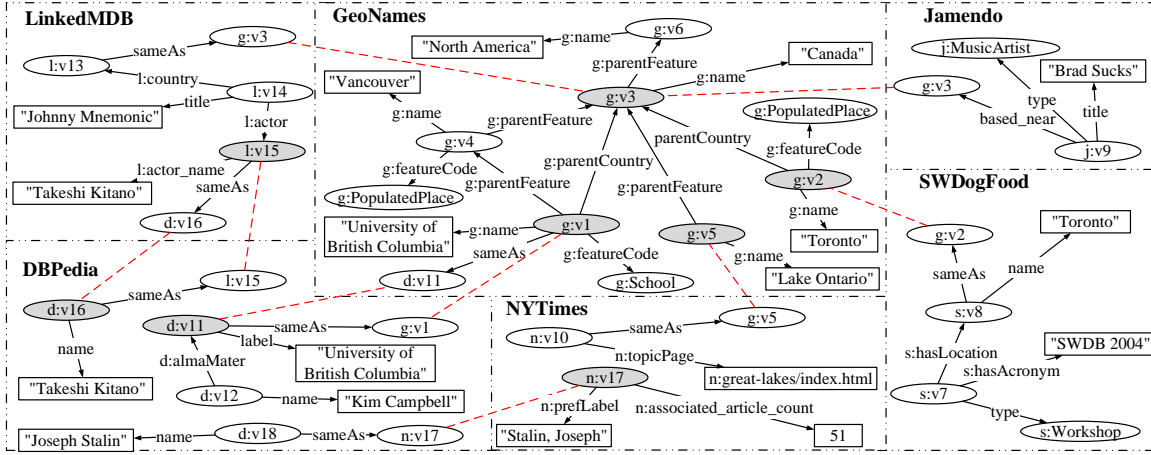


Fig. 3. Example Federated RDF System

sources, GeoNames and SWDogFood, and only GeoNames is its host source.

### B. SPARQL

SPARQL is a structured query language over RDF where primary building block is the basic graph pattern (BGP).

**Definition 3: (Basic Graph Pattern)** A basic graph pattern is denoted as  $Q = \{V(Q), E(Q), L\}$ , where  $V(Q) \subseteq V(G) \cup V_{Var}$  is a set of vertices, where  $V(G)$  denotes vertices in RDF graph  $G$  and  $V_{Var}$  is a set of variables;  $E(Q) \subseteq V(Q) \times V(Q)$  is a set of edges in  $Q$ ; each edge  $e$  in  $E(Q)$  either has an edge label in  $L$  (i.e., property) or the edge label is a variable.

In federated RDF systems, a match of BGP  $Q$  may span over different sources. Specifically, a match distributed over a set of sources  $S' \subseteq S$  is a function  $\mu$  from vertices in  $V(Q)$  to vertices in  $\bigcup_{s \in S'} g(s)$ .

**Definition 4: (BGP Match over Federated RDF System)** Consider an RDF graph  $G$ , a federated RDF system  $W = (S, g, d)$  and a BGP  $Q$  that has  $n$  vertices  $\{v_1, \dots, v_n\}$ . For  $S' \subseteq S$ , a subgraph  $M$  of  $\bigcup_{s \in S'} g(s)$  with  $n$  vertices  $\{u_1, \dots, u_n\}$  is said to be a *match* of  $Q$  if and only if there exists a function  $\mu$  from  $\{v_1, \dots, v_n\}$  to  $\{u_1, \dots, u_n\}$ , where the following conditions hold: (1) if  $v_i$  is not a variable,  $\mu(v_i)$  and  $v_i$  have the same URI or literal value ( $1 \leq i \leq n$ ); (2) if  $v_i$  is a variable, there is no constraint over  $\mu(v_i)$  except that  $\mu(v_i) \in \{u_1, \dots, u_n\}$ ; (3) if there exists an edge  $\overrightarrow{v_i v_j}$  in  $Q$ , there also exists an edge  $\overrightarrow{\mu(v_i) \mu(v_j)}$  in  $\bigcup_{s \in S'} g(s)$ ; furthermore,  $\overrightarrow{\mu(v_i) \mu(v_j)}$  has the same property as  $\overrightarrow{v_i v_j}$  unless the label of  $\overrightarrow{v_i v_j}$  is a variable.

The set of matches for  $Q$  over  $S'$  is denoted as  $\llbracket Q \rrbracket_{S'}$ .

**Definition 5: (Compatibility)** Given two BGP queries  $Q_1$  and  $Q_2$  over a set of sources  $S'$ ,  $\mu_1$  and  $\mu_2$  define two matching functions  $V(Q_1) \rightarrow V$  and  $V(Q_2) \rightarrow V$ , respectively.  $\mu_1$  and  $\mu_2$  are *compatible* when for all  $x \in V(Q_1) \cap V(Q_2)$ ,  $\mu_1(x) = \mu_2(x)$  (denoted as  $\mu_1 \sim \mu_2$ ); otherwise, they are incompatible, denoted as  $\mu_1 \not\sim \mu_2$ .

Our notion of a SPARQL query can be defined recursively as follows by combining BGPs using the following standard SPARQL algebra operations [17].

**Definition 6: (SPARQL Query)** Any BGP is a SPARQL query. If  $Q_1$  and  $Q_2$  are SPARQL queries, then expressions ( $Q_1$  AND  $Q_2$ ), ( $Q_1$  UNION  $Q_2$ ), ( $Q_1$  OPT  $Q_2$ ) and ( $Q_1$  FILTER  $F$ ) are also SPARQL queries.

The results of a query  $Q$  over sources  $S'$  are defined as follows.

**Definition 7: (SPARQL Result over Federated RDF System)** Given a federated RDF system  $W = (S, g, d)$ , the result of a SPARQL query  $Q$  over a set of sources  $S' \subseteq S$ , denoted as  $\llbracket Q \rrbracket$ , is defined recursively as follows:

- 1) If  $Q$  is a BGP,  $\llbracket Q \rrbracket_{S'}$  is defined in Definition 4.
- 2) If  $Q = Q_1$  AND  $Q_2$ , then  $\llbracket Q \rrbracket_{S'} = \llbracket Q_1 \rrbracket_{S'} \bowtie \llbracket Q_2 \rrbracket_{S'} = \{\mu_1 \cup \mu_2 \mid \mu_1 \in \llbracket Q_1 \rrbracket_{S'}, \mu_2 \in \llbracket Q_2 \rrbracket_{S'}, \mu_1 \sim \mu_2\}$
- 3) If  $Q = Q_1$  UNION  $Q_2$ , then  $\llbracket Q \rrbracket_{S'} = \llbracket Q_1 \rrbracket_{S'} \cup \llbracket Q_2 \rrbracket_{S'} = \{\mu \mid \mu \in \llbracket Q_1 \rrbracket_{S'} \vee \mu \in \llbracket Q_2 \rrbracket_{S'}\}$
- 4) If  $Q = Q_1$  OPT  $Q_2$ , then  $\llbracket Q \rrbracket_{S'} = (\llbracket Q_1 \rrbracket_{S'} \bowtie \llbracket Q_2 \rrbracket_{S'}) \cup (\llbracket Q_1 \rrbracket_{S'} \setminus \llbracket Q_2 \rrbracket_{S'}) = \{\mu_1 \cup \mu_2 \mid \mu_1 \in \llbracket Q_1 \rrbracket_{S'}, \mu_2 \in \llbracket Q_2 \rrbracket_{S'}, \mu_1 \not\sim \mu_2\}$
- 5) If  $Q = Q_1$  FILTER  $F$ , then  $\llbracket Q \rrbracket_{S'} = \Theta_F(\llbracket Q_1 \rrbracket_{S'}) = \{\mu_1 \mid \mu_1 \in \llbracket Q_1 \rrbracket_{S'}, \mu_1 \text{ satisfies } F\}$

If  $S' = S$ , i.e., the whole federated RDF system  $W$ , we call  $\llbracket Q \rrbracket_S$  the results of  $Q$  over federated RDF system  $W$ .

The problem to be studied in this paper is defined as follows:

*Given a set of SPARQL queries and a federated RDF system  $W = (S, g, d)$ , our problem is to find the results of each query in  $Q$  over  $W$ .*

### III. FRAMEWORK

The federated RDF system consists of a control site as well as some RDF sources. The control site is amenable to receive a SPARQL query  $Q$  and decompose it into several subqueries that are sent to relevant sources. Assume that several subqueries that are sent to the same source share common substructures. In order to improve the query performance, the control site will rewrite them as a single SPARQL that is sent to the relevant source. Results of decomposed subqueries are returned to the control site and joined together to form complete results that are presented to users.



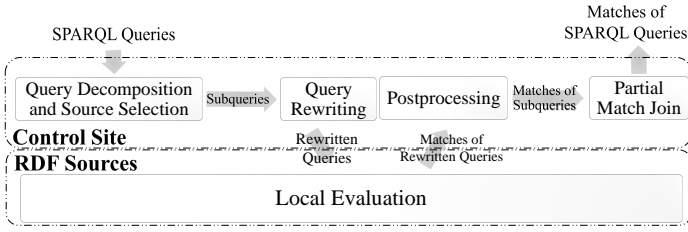


Fig. 4. Scheme for Federated SPARQL Query Processing

Generally speaking, our framework consists of five steps: *query decomposition and source selection*, *query rewriting*, *local evaluation*, *postprocessing* and *partial match join* (see Fig. 4). We briefly review the five steps before we discuss them in details in upcoming sections. Note that only *local evaluation* is conducted over the remote sources and the other four steps work at the control site.

**Query Decomposition and Source Selection.** We first decompose a query  $Q$  into a set of subqueries expressed over relevant sources. Most existing solutions [19], [7], [8], [18], [20] use select relevant sources based on properties of triple patterns. The query decomposition and source selection are not the focus of our paper. We utilize existing query decomposition and source selection techniques in this work.

Specifically, given a batch of SPARQL queries  $\{Q_1, \dots, Q_n\}$ , we obtain subqueries  $\mathcal{Q} = \{q_1^1 @ S(q_1^1), \dots, q_1^{m_1} @ S(q_1^{m_1}); q_2^1 @ S(q_2^1), \dots, q_2^{m_2} @ S(q_2^{m_2}); \dots; q_n^1 @ S(q_n^1), \dots, q_n^{m_n} @ S(q_n^{m_n})\}$ , where  $\{q_i^1 @ S(q_i^1), \dots, q_i^{m_i} @ S(q_i^{m_i})\}$  come from original SPARQL query  $Q_i$  and  $S(q_i^j)$  is the set of relevant sources for subquery  $q_i^j$ . For example, each of queries  $Q_1$ ,  $Q_2$  and  $Q_3$  in Fig. 2 is decomposed into three subqueries, as shown in Fig. 5.

**Query Rewriting.** After the first step, we have multiple subqueries. The set of subqueries that are planned to be sent to the same source provides an opportunity for multiple query optimization. The control site uses FILTER and OPTIONAL operators to rewrite these subqueries as a single query that will be sent to the relevant source. Our query rewriting technique is based on a cost model that reduces both the response time and the number of remote accesses while considering the time for both local evaluation and data shipment.

Assume that source  $s$  is assigned a set of subqueries  $\mathcal{Q}_s = \{q_s^1, \dots, q_s^n\}$ . After query rewriting, we obtain a set of rewritten queries  $\hat{\mathcal{Q}}_s$  ( $|\hat{\mathcal{Q}}_s| \leq |\mathcal{Q}_s|$ ) that will be sent to source  $s$ . Each rewritten query comes from a subset of  $\mathcal{Q}_s$ . We will discuss this in Section V.

**Local Evaluation.** After query rewriting, we rewrite subqueries into a set of rewritten queries  $\hat{\mathcal{Q}}_s$ . We send the set of rewritten queries to their relevant sources and evaluate them there. Local evaluation results will be return back to the control site.

**Postporcessing.** The local evaluation results from evaluating  $\hat{\mathcal{Q}}_s$  over the RDF sources are a superset of evaluating the original subqueries  $\mathcal{Q}_s$ . Therefore, the control site necessitates a postporcessing step to check each local evaluation result against each query in  $\mathcal{Q}_s$ . In this paper, we propose a postporcessing method which only requires a linear scan on

the local evaluation results of the rewritten queries. We will discuss the details In Section VI.

**Partial Match Join.** For each subquery  $q_i^j$  in  $\mathcal{Q}$ , collecting the matches at each relevant source in  $S(q_i^j)$ , we obtain all its matches. Assume that an original query  $Q_i$  ( $i = 1, \dots, n$ ) is decomposed into a set of subqueries  $\{q_i^1 @ S(q_i^1), \dots, q_i^{m_i} @ S(q_i^{m_i})\}$ , we obtain query results  $\llbracket Q_i \rrbracket_s$  by joining  $\llbracket q_i^1 \rrbracket_{S(q_i^1)}, \dots, \llbracket q_i^{m_i} \rrbracket_{S(q_i^{m_i})}$ . Considering the context of multiple SPARQL queries over a federated RDF system, we propose an optimized solution to avoid duplicate computation in join processing (in Section VII).

#### IV. QUERY DECOMPOSITION AND SOURCE SELECTION

Given a SPARQL query, each triple pattern corresponds to a subquery. Each triple pattern maps to a set of relevant sources based on the values of its subject, property and object [22]. Consider triple pattern “?y sameAs ?x” in  $Q_1$  of Fig. 1. Since RDF sources “GeoNames”, “NYTimes”, “DBPedia”, “SWDogFood” and “LinkedMDB” contain the property “sameAs”, this triple pattern has five relevant sources. However, triple pattern “?x g:parentFeature ?l” in  $Q_1$  only maps to source “GeoNames”, because that is the only source which has property “g:parentFeature”.

One-triple patterns can be combined together to form a larger subquery if a source is exclusively selected for a set of connected triple patterns. For example, both “?x g:parentFeature ?l” and “?l g:name ‘Canada’” of  $Q_1$  correspond to the same source, i.e., “GeoNames”. Thus, these two triple patterns can be combined to form a larger subquery  $q_1^1$  as shown in Fig. 5. Note that, if a group of triple patterns shares exactly the same set of multiple RDF sources, they cannot be combined together. This is because that combining them together may miss some matches crossing different RDF sources.

For queries  $Q_1$ ,  $Q_2$  and  $Q_3$  in Fig. 2, our solution decomposes each of them into three subqueries, as shown in Fig. 5. In particular,  $q_1^1$  has a single relevant source “GeoNames”;  $q_2^1$  has five sources except for “Jamendo” and the relevant source  $q_3^1$  is only “NYTimes”.

#### V. QUERY REWRITING

As mentioned in Section I, when multiple SPARQL queries are posed simultaneously, there is room for sharing computation when executing these queries. Assume that multiple decomposed subqueries that are expected to sent to the same source share some common substructures. A possible optimization is to rewrite them (at the control site) into a single SPARQL query and then send it to relevant sources, which can save both the number of remote accesses and query response time. Obviously, different query rewritings may lead to different performance; thus, this section proposes a cost-driven query rewriting scheme. To simplify presentation, we first assume that the SPARQL query originally issued at the control site is a BGP. Our solution is easily to extended to handle general SPARQL queries, which is discussed in Section VII.

##### A. Intuition

We first discuss how to rewrite multiple queries with the common substructure into a single SPARQL query. Specifically, we utilize “OPTIONAL” and “FILTER” operators to

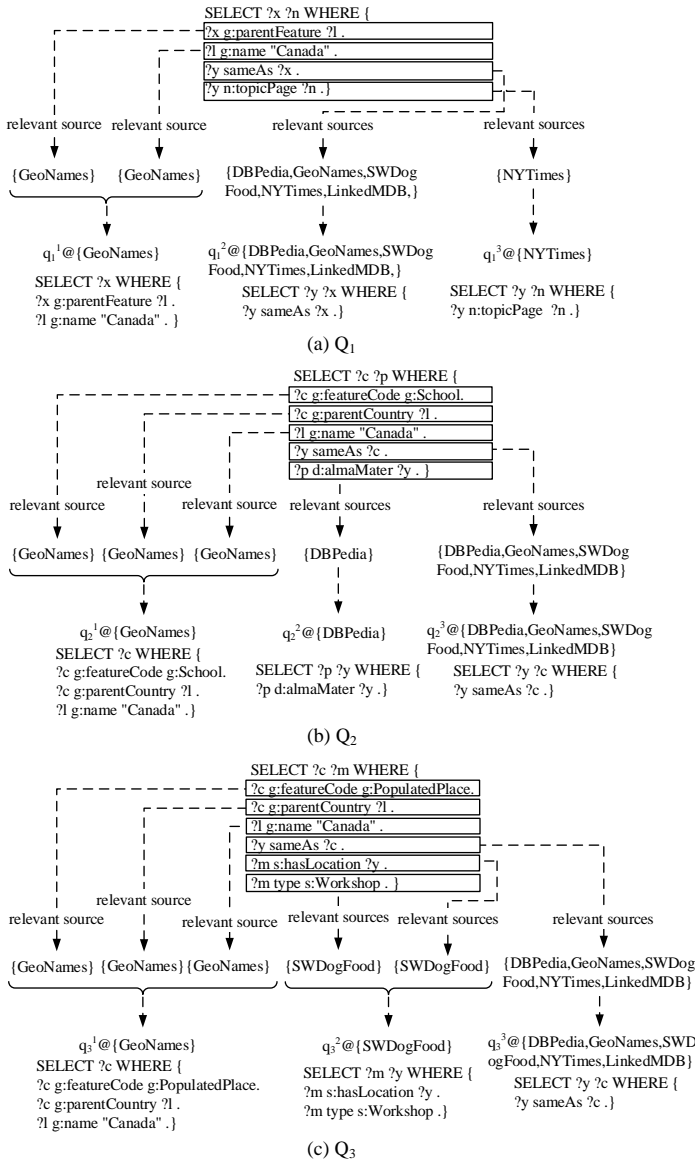


Fig. 5. Basic Query Decomposition and Source Selection Result for  $Q_1$

make use of common structures among different queries for rewriting.

**1) OPTIONAL-based Rewriting:** Given a set of subqueries with a common substructure, we can rewrite them as query  $\hat{q}$  with multiple OPTIONAL clauses, where the main graph pattern of  $\hat{q}$  is the common substructure among these subqueries. Obviously, the rewriting can reduce the number of remote requests.

Formally, given a set of subqueries  $\{q_1@{s}, q_2@{s}, \dots, q_n@{s}\}$  over a source  $s$ , if  $p$  is the common subgraph among  $q_1, \dots, q_n$ , we rewrite these subqueries as a query with OPTIONAL operators as follows.

$$\hat{q}@{s} = p \text{ OPT } (q_1 - p) \text{ OPT } (q_2 - p) \dots \text{ OPT } (q_n - p)@{s}$$

Let us consider two subqueries over GeoNames,  $q_1^1@{GeoNames}$  and  $q_2^1@{GeoNames}$ , as shown in Fig. 6. The two subqueries are decomposed from  $Q_1$  and  $Q_2$ . They share a common substructure, i.e., triple pattern “ $?l \text{ g:name}$

“Canada””. Therefore, they can be rewritten to a single query, where “ $?l \text{ g:name}$  “Canada”” maps to the main pattern. The subgraphs that  $q_1^1$  and  $q_2^1$  minus “ $?l \text{ g:name}$  “Canada”” map to two OPTIONAL clauses, respectively. The query rewriting is illustrated in Fig. 6. The rewritten query can avoid one remote request for GeoNames.

Because existing RDF stores implement OPTIONAL operators using left-joins, the result cardinality of a SPARQL query with OPTIONAL operator is upper bounded by result cardinality of its main graph pattern. Thus, the cardinality of the rewritten query does not increase much. We also introduce a cost model to measure the benefit of the rewriting rule in Section V-B.

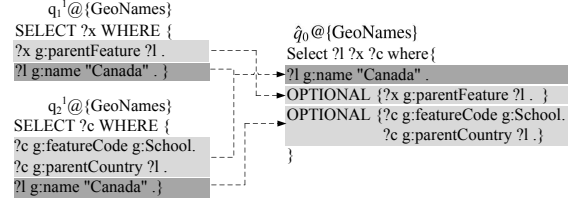


Fig. 6. Rewriting Queries using OPTIONAL Operators

Although the intermediate results increase little, using OPTIONAL operators adds at least one extra left-joins for each optional clause. This results in larger local evaluation cost. Thus, this need a trade off between many queries being rewritten together and many optional clauses. We will consider the number of OPTIONALS into the cost model as discussed in Section V-B.

**2) FILTER-based Rewriting:** Let us consider two subqueries  $q_2^1@{GeoNames}$  and  $q_3^1@{GeoNames}$  in Fig. 7, which are decomposed from  $Q_2$  and  $Q_3$ . Although they distinguish from each other at the first triple pattern, the only difference is constant bounded to objects in the first triple pattern. We can rewrite the two queries using FILTER, as shown in Fig. 7. In other words, if some subqueries issued at the same source have the common structure except the constants on some vertices (subject or object positions), they can be rewritten as a single query with FILTER.

Formally, if a set of subqueries  $\{q_1@{s}, q_2@{s}, \dots, q_n@{s}\}$  employ the same query structure  $p$  except for some vertex labels (i.e., constants on vertices), we rewrite them as follows.

$$\hat{q}@{s} = p \text{ FILTER } \left( \bigvee_{1 \leq i \leq n} \left( \bigwedge_{v \in V(q_i)} f_i(v) = v \right) \right) @{s}$$

where  $f_i$  is a bijective isomorphism function from  $q_i$  to  $p$ .

Based on FILTER operator, we can perform selection over main pattern results. Thus, the result cardinality of SPARQL with FILTER operator is also upper bounded by result cardinality of its main graph pattern.

In addition, unlike OPTIONAL operators adding extra columns, FILTER operators do not introduce extra joins and only generate a little more partial matches. Hence, the cost of data shipment will increase little.

**3) Hybrid Rewriting:** A hybrid rewriting strategy is also feasible by using both OPTIONAL and FILTER. Let us consider the three subqueries issued at the same source GeoNames. Fig. 8 illustrates a hybrid rewriting strategy, using OPTIONAL followed by FILTER.

```

q21@{GeoNames}
SELECT ?c WHERE {
?c g:featureCode g:School .
?c g:parentCountry ?l .
?l g:name "Canada" . }

q11@{GeoNames}
SELECT ?c WHERE {
?c g:featureCode g:PopulatedPlace .
?c g:parentCountry ?l .
?l g:name "Canada" . }

q11@{GeoNames}
SELECT ?l ?c ?f where {
?c g:featureCode ?f .
?c g:parentCountry ?l .
?l g:name "Canada" .
Filter(?f = g:School || ?f = g:PopulatedPlace)
}

```

Fig. 7. Rewriting Queries using FILTER Operator

```

q11@{GeoNames}
SELECT ?x WHERE {
?x g:parentFeature ?l .
?l g:name "Canada" . }

q21@{GeoNames}
SELECT ?c WHERE {
?c g:featureCode g:School .
?c g:parentCountry ?l .
?l g:name "Canada" . }

q31@{GeoNames}
SELECT ?c WHERE {
?c g:featureCode g:PopulatedPlace .
?c g:parentCountry ?l .
?l g:name "Canada" . }

q22@{GeoNames}
Select ?l ?x ?c ?f where {
?l g:name "Canada" .
OPTIONAL { ?x g:parentFeature ?l . }
OPTIONAL { ?c g:featureCode ?f .
?c g:parentCountry ?l .
Filter(?f = g:School || ?f = g:PopulatedPlace)
}
}

```

Fig. 8. Rewriting Queries using OPTIONAL and FILTER Operators

### B. Cost Model

The cost of a rewriting strategy can be expressed with respect to the total time. The total time is the sum of all time (also referred to as cost) components. There are two time components for evaluating a rewriting strategy: time for local evaluation and time for data shipment. We discuss the two components respectively in the following sections.

1) *Cost Model for BGPs*: As mentioned in [13], selective triple patterns in BGP have higher priorities in evaluation. We verify the principle in two real and synthetic RDF repositories, DBpedia and WatDiv. For DBpedia, we download real SPARQL workload that records more than 8 millions SPARQL queries issued in 14 days of 2012<sup>4</sup> and randomly sample 10000 queries to verify the principle; for WatDiv, to verify the principle, we generate 12500 queries from 125 templates provided in [2]. Given a triple pattern  $e$ , its selectivity is defined as  $sel(e) = \frac{||e||}{|E(G)|}$ , where  $||e||$  denotes the number of matches of  $e$  and  $|E(G)|$  denotes the number of edges in RDF graph  $G$ . The experimental results are shown in Fig. 9. As the experimental results show, the cardinality of a query is positively associated with the selectivity of the most selective triple pattern for both real and synthetic datasets.

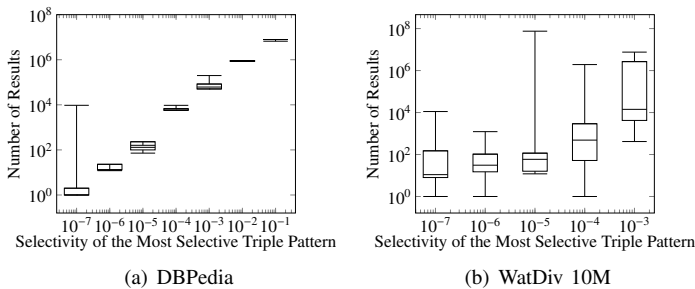


Fig. 9. Experiment Results of the Relationship Between the Cardinality and the Most Selective Triple Pattern

Based on the above observation, we define the cardinality

of evaluating a basic graph pattern  $Q$  as follows.

$$card(Q) = \min_{e \in E(Q)} \{sel(e)\}$$

where  $sel(e)$  is the selectivity of triple pattern  $e$  in  $Q$ .

As mentioned before, all results of queries are sent to the control site to join for the final results after local evaluation. Hence, the time for data shipment can be defined based on the number of results. Given a BGP  $Q$ , its cost of data shipment is defined as follows.

$$cost_{DS}(Q) = card(Q) \times T_{MSG} = \min_{e \in E(Q)} \{sel(e)\} \times T_{MSG}$$

where  $T_{MSG}$  is the unit time to transmit a data unit.

Meanwhile, the time for evaluating a query is proportional to the size of the results. Hence, given a BGP  $Q$ , its cost of local evaluation is defined as follows.

$$cost_{LE}(Q) = card(Q) \times T_{CPU} = \min_{e \in E(Q)} \{sel(e)\} \times T_{CPU}$$

where  $T_{CPU}$  is the CPU unit time to construct a result.

In real applications, for estimating the selectivity of triple pattern, we can employ the heuristics introduced by [23] that can estimate the selectivity without pre-computed statistics about the RDF source. Simply speaking, the selectivity of a triple pattern is computed according to the type and number of unbound components and is characterized by the ranking  $sel(S) < sel(O) < sel(P)$ , i.e. subjects are more selective than objects and objects more selective than predicates [23]. Furthermore, many RDF sources also provide VOID (Vocabulary of Interlinked Datasets)<sup>5</sup> that expresses metadata about RDF datasets and can also be utilized to implement the heuristic with pre-computed statistics in [23] to improve the estimation accuracy.

The relative coefficients,  $T_{CPU}$  and  $T_{MSG}$ , in the cost model are greatly influenced by the resources of each RDF sources and the network topology of the federated RDF system. They should be estimated in the offline as the metadata.

2) *Cost Model for General SPARQLs*: Then, we extend the cost model to handle general SPARQLs. The design of our cost model is motivated by the way in which a SPARQL query is evaluated on popular RDF stores. This includes a well-justified principle that the graph patterns in OPTIONAL clauses and the expressions in FILTER operators are evaluated on the results of the main pattern (for the fact that the graph pattern in the OPTIONAL clause is a left-join and the FILTER operator is selection) [13]. This suggests that a good optimization should keep the result cardinality from the common subgraph as small as possible for two reasons: 1) The result cardinality of a SPARQL query with the OPTIONAL operators and FILTER operators is upper bounded by result cardinality of its main graph pattern clause since graph patterns in the OPTIONAL clause are simply left-joins and FILTER expressions are simply selection; 2) Intermediate result from evaluating the main graph pattern is not well indexed, which means that more efforts are needed for processing the rewriting graph patterns in the OPTIONAL clause and FILTER expressions if a main graph pattern is not selective.

According to the above reasons, we assume that the OPTIONAL clause and the FILTER expressions are evaluated

<sup>4</sup><http://aksw.org/Projects/DBPSB.html>

<sup>5</sup><http://www.w3.org/TR/void/>

on the results of the main pattern. Hence, given a SPARQL  $Q$ , its cardinality  $card(Q)$  is defined as follows.

$$card(Q) = \begin{cases} \min_{e \in E(Q)} \{sel(e)\} & \text{if } Q \text{ is a BGP;} \\ \min\{card(Q_1), card(Q_2)\} & \text{if } Q = Q_1 \text{ AND } Q_2; \\ card(Q_1) + card(Q_2) & \text{if } Q = Q_1 \text{ UNION } Q_2; \\ card(Q_1) + \Delta_1 & \text{if } Q = Q_1 \text{ OPT } Q_2; \\ card(Q_1) + \Delta_2 & \text{if } Q = Q_1 \text{ FILTER } F; \end{cases} \quad (1)$$

where  $\Delta_1$  and  $\Delta_2$  are empirically trivial values [13].

Then, given a set of subqueries  $Q = \{q_1@s, q_2@s, \dots, q_n@s\}$  over a source  $S$  using OPTIONAL and FILTER, if  $p$  is their common subgraph among  $q_1, \dots, q_n$ , we rewrite them into a SPARQL query  $\hat{q}$ . The cost of evaluating  $\hat{q}$  is defined as follows.

$$cost_{DS}(\hat{q}) = card(\hat{q}) \times T_{MSG} = (\min_{e \in p} \{sel(e)\} + \Delta_1 + \Delta_2) \times T_{MSG}$$

Here, because  $\Delta_1$  and  $\Delta_2$  are trivial values and  $card(\hat{q})$  is mostly credited to the cardinality of  $p$ , we ignore the trivial variables  $\Delta_1$  and  $\Delta_2$  while assuming that the term  $cost(p)$  is much larger. Hence, we have the following cost function of data shipment.

$$cost_{DS}(\hat{q}) = \min_{e \in p} \{sel(e)\} \times T_{MSG}$$

On the other hand, since the OPTIONAL and FILTER operators are evaluated on the result of main pattern on popular RDF stores, the time for local evaluation is also based on the cardinality of the main pattern.

Furthermore, since each OPTIONAL operator adds simply a left-join with the results of the main pattern on popular RDF stores, the time for local evaluation of multiple OPTIONAL operators is approximately equal to the time of multiple execution of main pattern.

We verify the above principle by using WatDiv 10M on two popular RDF stores, Jena 2.13.0<sup>6</sup> and Sesame 2.7<sup>7</sup>. We generate some queries with multiple OPTIONAL operators by using the query rewriting algorithm proposed in Section V-C. The results in Fig. 10 show that the query response time of a query with multiple OPTIONALS is proportional to the number of OPTIONALS on both Jena and Sesame.

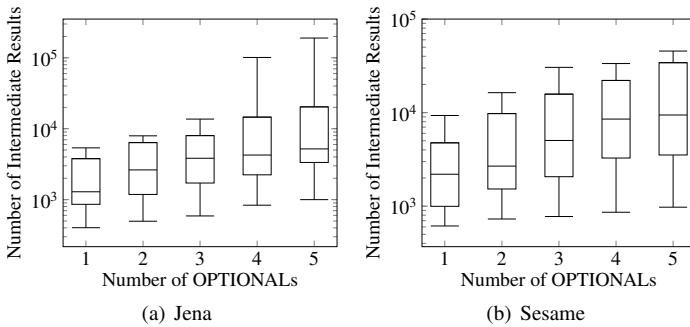


Fig. 10. Experiment Results of the Relationship Between the Local Evaluation Cost and the Number of OPTIONALS

Thus, given a SPARQL  $Q$ , its time for local evaluation can be estimated as follows.

$$cost_{LE}(Q) = \begin{cases} \min_{e \in E(Q)} \{sel(e)\} \times T_{CPU} & \text{if } Q \text{ is a BGP;} \\ \min\{cost_{LE}(Q_1), cost_{LE}(Q_2)\} & \text{if } Q = Q_1 \text{ AND } Q_2; \\ cost_{LE}(Q_1) + cost_{LE}(Q_2) & \text{if } Q = Q_1 \text{ UNION } Q_2; \\ cost_{LE}(Q_1) \times (|OPT_{Q_2}| + 1) + \Delta_3 & \text{if } Q = Q_1 \text{ OPT } Q_2; \\ cost_{LE}(Q_1) + \Delta_4 & \text{if } Q = Q_1 \text{ FILTER } F; \end{cases} \quad (2)$$

where  $|OPT_{Q_2}|$  is the number of OPTIONALs in  $Q_2$ , and  $\Delta_3$  and  $\Delta_4$  are also empirically trivial values. Here, the definition of the local evaluation cost of a query containing OPTIONAL operators is recursive, which means that the local evaluation cost of a query is proportional to the number of OPTIONAL operators.

Thus, given a rewritten query  $\hat{q}$ . The local evaluation cost of evaluating  $\hat{q}$  is defined as follows.

$$cost_{LE}(\hat{q}) = (|OPT_{\hat{q}}| \times \min_{e \in p} \{sel(e)\} + \Delta_3 + \Delta_4) \times T_{CPU}$$

Similar to the cost function of data shipment, since  $\Delta_3$  and  $\Delta_4$  are trivial values can also be ignored, the cost function of local evaluation is as follows.

$$cost_{LE}(\hat{q}) = |OPT_{\hat{q}}| \times \min_{e \in p} \{sel(e)\} \times T_{CPU}$$

3) *Total Rewriting Cost*: In summary, for a rewritten query  $\hat{q}$  with the main pattern  $p$ , its total cost is defined as follows.

$$\begin{aligned} cost(\hat{q}) &= cost_{LE}(\hat{q}) + cost_{DS}(\hat{q}) \\ &= \min_{e \in p} \{sel(e)\} \times (|OPT_{\hat{q}}| \times T_{CPU} + T_{MSG}) \end{aligned} \quad (3)$$

where  $|OPT_{\hat{q}}|$  is the number of OPTIONAL operators in  $\hat{q}$ .

Obviously, given a set of subqueries  $Q$  over a source  $S$ , we may have multiple query rewriting strategies. Thus, we need to define the cost of a specific query rewriting. Formally, we define the rewriting cost as follows.

**Definition 8: (Rewriting Cost)** Given a set of subqueries  $Q = \{q_1@s, q_2@s, \dots, q_n@s\}$  on a source  $s$  using OPTIONAL and FILTER, if  $p$  is their common subgraph among  $q_1, \dots, q_n$ , we rewrite them into a SPARQL query  $\hat{q}$ . The cost of the rewriting is the cost of the rewritten query  $\hat{q}$  with main basic graph pattern  $p$  as shown in the following formula:

$$cost_{LE}(Q, \hat{q}) = cost_{LE}(\hat{q}) = \min_{e \in p} \{sel(e)\} \times (|OPT_{\hat{q}}| \times T_{CPU} + T_{MSG}) \quad (4)$$

where  $|OPT_{\hat{q}}|$  is the number of OPTIONAL operators in  $\hat{q}$ .

### C. Query Rewriting Algorithm

The problem of query rewriting is that given a set  $Q$  of subqueries  $\{q_1, \dots, q_n\}$ , we compute a set  $\hat{Q}$  of rewritten queries  $\{\hat{q}_1, \dots, \hat{q}_m\}$  ( $m \leq n$ ) with the smallest cost (The cost function is defined in Equation 4). Note that each rewritten query  $\hat{q}_i$  ( $i = 1, \dots, m$ ) comes from rewriting a set of original subqueries in  $Q$ , where these subqueries share the same main pattern  $p_i$ .

The cost of the whole rewriting is formatted as follows:

$$cost(Q, \hat{Q}) = \sum_{\hat{q} \in \hat{Q}} cost(\hat{q}) \quad (5)$$

Generally speaking, we find the set of common patterns  $P$ , where each subquery contains at least one of patterns in  $P$ . Here, if a subquery  $q$  contains a pattern  $p \in P$ , we call that  $p$

<sup>6</sup><http://jena.apache.org/>

<sup>7</sup><http://rdf4j.org/>

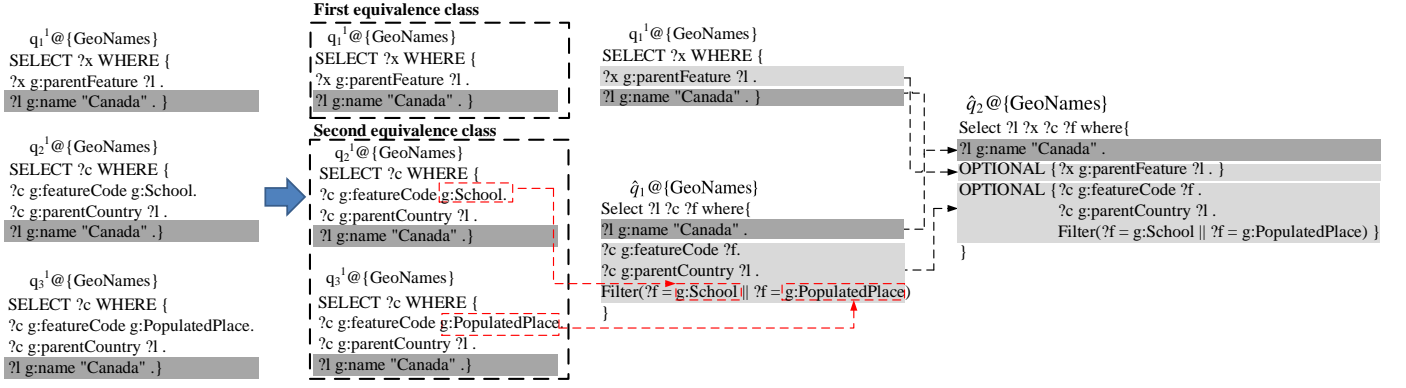


Fig. 11. Example of Rewriting Subqueries

hit  $q$ . According to Section V-A, if a set of subqueries can be rewritten as a rewritten query  $\hat{q}$ , they must share one common main pattern  $p$ . Therefore, we have the following equation.

$$\text{cost}(Q, \hat{Q}) = \sum_{p \in P} \min_{e \in p} \{ \text{sel}(e) \} \times (|OPT_{\hat{q}}| \times T_{CPU} + T_{MSG}) \quad (6)$$

where  $|OPT_p|$  is the number of OPTIONAL operators that the rewritten query constructed by  $p$  contains.

Given a set of original queries  $Q$ , Equation 6 is a set-function with respect to set  $P$ , i.e., a set of main patterns. Unfortunately, finding the optimal rewriting is a NP-complete problem as discussed in the following theorem.

**Theorem 1:** Given a set of subqueries  $Q$ , finding an optimal rewriting  $\hat{Q}$  to minimize the cost function in Equation 6 is a NP-complete problem.

**Proof:** We prove that by reducing the weighted set cover problem into the problem of selecting the optimal set of patterns. The weight set cover problem is defined as follows: given a set of elements  $U = \{1, 2, \dots, m\}$  (called the universe), a set  $S$  of  $n$  sets whose union equals the universe and a function  $w$  to mapping each set in  $S$  to a non-negative value, the set cover problem is to identify the least weighted subset of  $S$  whose union equals the universe.

To reduce the weighted set cover problem to the problem of selecting the optimal set of patterns, we map each set in  $S$  to a pattern and each element in the universe  $U$  to a subquery. A set  $s$  in  $S$  containing an element  $e$  in  $U$  maps to the pattern corresponding to  $s$  hitting the subquery corresponding to  $e$ . The weight of a set  $s$  in  $S$  is the cost of its corresponding pattern.

Hence, finding the smallest weight collection of sets from  $S$  whose union covers all elements in  $U$  is equivalent to the problem of selecting the optimal set of patterns. Since the weighted set cover problem is NP-complete [6], the problem of selecting the optimal set of patterns is also NP-complete.

Then, we propose a greedy algorithm that iteratively selects the locally optimal triple pattern in Algorithm 1. Algorithm 1 is a greedy algorithm. Let  $Q$  denote all original subqueries. At each iteration, we select a triple pattern  $e_{max}$  with the largest value  $\frac{\text{sel}(e_{max}) \times (|OPT_{\hat{q}}| \times T_{CPU} + T_{MSG})}{|Q'|}$ , where  $Q'$  denote all subqueries

hit by  $e_{max}$ , i.e, these queries containing  $e_{max}$ . We propose a hybrid strategy to rewrite all queries in  $Q'$ . We divide  $Q'$  into several equivalence classes, where each class contains subqueries with the same structure except for some constants on subject or object positions. Subqueries in the same equivalence class can be rewritten to a query pattern with FILTER operators as discussed in Section V-A. Furthermore, all queries in  $Q'$  can be rewritten into SPARQL  $\hat{q}$  with OPTIONAL operator using  $e_{max}$  as the main pattern. We remove queries in  $Q'$  from  $Q$  and iterate the above process until  $Q$  is empty.

---

#### Algorithm 1: Query Rewriting Algorithm

---

**Input:** A set of subqueries  $Q$ .

**Output:** A set of rewritten queries sets  $Q_{OPT}$ .

---

- 1 **while**  $Q \neq \emptyset$  **do**
  - 2     Select the triple pattern  $e_{max}$  with the largest value  $\frac{\text{sel}(e_{max}) \times (|OPT_{\hat{q}}| \times T_{CPU} + T_{MSG})}{|Q'|}$ , where  $Q'$  is the set of subqueries hit by  $e_{max}$ ;
  - 3     Extract the largest common pattern  $p$  of queries in  $Q'$ ;
  - 4     Build a rewritten query  $\hat{q}$ , where  $p$  is its main pattern;
  - 5     Divide  $Q'$  into a collection of equivalence classes  $C$ , where each class contains subqueries isomorphic to each other;
  - 6     **for each class**  $C \in C$  **do**
  - 7         Generalize a pattern  $p'$  isomorphic all patterns in  $C$ , where  $p'$  does not contain any constants;
  - 8         Build a query pattern with  $p'$ ;
  - 9         Add FILTER operators by mapping  $p'$  to patterns in  $C$ ;
  - 10        Add the pattern into  $\hat{q}$  as an OPTIONAL pattern;
  - 11     Add  $\hat{q}$  into  $Q_{OPT}$ ;
  - 12      $Q = Q - Q'$ ;
  - 13 **Return**  $Q_{OPT}$ ;
- 

Given subqueries  $q_1^1 @ \{GeoNames\}$ ,  $q_2^1 @ \{GeoNames\}$  and  $q_3^1 @ \{GeoNames\}$  in Fig. 11, we select the triple pattern “ $?l$  g:name “Canada”” in the first step. It hits the three subqueries. We divide them into two equivalence classes  $\{q_1^1\}$ ,  $\{q_2^1, q_3^1\}$  according to the query structure. Then, we rewrite  $\{q_2^1, q_3^1\}$  using FILTER operator. Finally, we rewrite the three queries using OPTIONAL operator using “ $?l$  g:name “Canada””.



**Theorem 2:** The total cost of patterns selected by using Algorithm 1 is no more than  $(1 + \ln |\cup_{q \in Q} E(q)|) \times \text{cost}_{opt}$ , where  $\cup_{q \in Q} E(q)$  is the set of triple patterns of all subqueries in  $Q$  and  $\text{cost}_{opt}$  denotes the smallest cost of patterns that hit all subqueries.

*Proof:* According to Equation 6, selecting patterns to hit subqueries is equivalent to selecting triple patterns to hit subqueries. Thus, although we only select the most beneficial triple pattern in Algorithm 1 (Line 2), it is equivalent to selecting the most beneficial pattern graph to hit subqueries. A result in [5] shows that the approximation ratio of the greedy algorithm to the optimal solution of the weighted set-cover problem is  $(1 + \ln |\cup_{q \in Q} E(q)|)$ . ■

## VI. LOCAL EVALUATION & POSTPROCESSING

A set of subqueries  $Q$  that will be sent to source  $s$  are rewritten as queries  $\hat{Q}$  and evaluated at source  $s$ . Let  $\llbracket \hat{q} \rrbracket_{\{s\}}$  denote the result set of  $\hat{q} \in \hat{Q}$  at source  $s$ ;  $\hat{q}$  is obtained by rewriting a set of original subqueries in  $Q$ . Thus,  $\llbracket \hat{q} \rrbracket_{\{s\}}$  is always the union of the results of the subqueries that are rewritten, and we track the mappings between the variables in the rewritten query and the variables in the original subqueries. The result of a rewritten query might have empty (null) columns corresponding to the variables from the OPTIONAL operators. Therefore, a result in  $\llbracket \hat{q} \rrbracket_{\{s\}}$  may not conform the description of every subquery in  $Q$ . We should identify the valid overlap between each result in  $\llbracket \hat{q} \rrbracket_{\{s\}}$  and each subquery in  $Q$ , and check whether a result in  $\llbracket \hat{q} \rrbracket_{\{s\}}$  belongs to the relevant sources of a subquery. We return to each query the result it is supposed to get.

To achieve the above objective, we perform an intersection between each result in  $\llbracket \hat{q} \rrbracket_{\{s\}}$  and each subquery. The algorithm distributes the corresponding part of this result to  $q@ \{s\}$  as one of its query results, if the result meets the following two conditions: 1) the columns of this result corresponding to those columns of a subquery  $q@ \{s\} \in Q$  are not null; and 2) the columns of the result meet the constraints in the FILTER operators rewritten from  $q$ . This step iterates over each row and each subquery in  $Q$ . The checking on  $\llbracket \hat{q} \rrbracket_{\{s\}}$  only requires a linear scan on  $\llbracket \hat{q} \rrbracket_{\{s\}}$ . Therefore, it can be done on-the-fly as the results of  $\hat{q}@ \{s\}$  is streamed out from the evaluation.

Fig. 12 illustrates how we distribute  $\llbracket \hat{q}_1 \rrbracket_{\{GeoNames\}}$  to  $q_1^1@ \{GeoNames\}$ ,  $q_2^1@ \{GeoNames\}$  and  $q_3^1@ \{GeoNames\}$ , after evaluating rewritten query  $\hat{q}_1$ . For the first row, we find that the columns of this result only correspond to those columns of a subquery  $q_1^1@ \{GeoNames\}$ , so we distribute the row to  $\llbracket q_1^1 \rrbracket_{\{GeoNames\}}$ . The second row in  $\llbracket \hat{q}_1 \rrbracket_{\{GeoNames\}}$  corresponds to the columns of both subqueries  $q_2^1@ \{GeoNames\}$  and  $q_3^1@ \{GeoNames\}$ , but it only meets the constraints in the FILTER operators rewritten from  $q_2^1$ . Hence, we distribute the row to  $\llbracket q_2^1 \rrbracket_{\{GeoNames\}}$ . We iterate this process until that all results are distributed.

## VII. JOINING PARTIAL MATCHES

Query evaluation over federated RDF systems requires joining local partial matches, which we discuss in this section.

The straightforward method to obtain results of all original queries is to join subquery matches for each original SPARQL query independently. For each subquery in  $Q$ , collecting the

$\llbracket \hat{q}_1 \rrbracket_{\{GeoNames\}}$					
?l	?x	?c	?f	?l	?x
g:v3	g:v5			g:v3	g:v5
g:v3		g:v1	g:School	g:v3	g:c
g:v3		g:v2	g:PopulatedPlace	g:v3	g:v1
g:v3		g:v4	g:PopulatedPlace	g:v3	g:v2
				g:v3	g:v4

Fig. 12. Example for Distributing Results

matches at each relevant source, we obtain all its matches. Assume that an original query  $Q_i$  ( $i = 1, \dots, n$ ) is decomposed into a set of subqueries  $\{q_i^1@ \{S(q_i^1)\}, \dots, q_i^{m_i}@ \{S(q_i^{m_i})\}\}$ . We need to obtain query result  $\llbracket Q_i \rrbracket$  by joining  $\llbracket q_i^1 \rrbracket_{S(q_i^1)}, \dots, \llbracket q_i^{m_i} \rrbracket_{S(q_i^{m_i})}$  together. In the following, for the sake of simplicity, we abbreviate  $\llbracket q_i^{m_i} \rrbracket_{S(q_i^{m_i})}$  to  $\llbracket q_i^{m_i} \rrbracket$ .

However, considering multiple queries, there may exist some common computation in joining partial matches. For example, let us consider the subqueries in Fig. 5. Fig. 13 shows their join graphs. We can observe that  $q_2^1$  is isomorphic to  $q_3^1$ , and  $q_2^2$  is isomorphic to  $q_3^2$ . Meanwhile, the join variable between  $q_2^1$  and  $q_2^2$  is the same to the join variable between  $q_3^1$  and  $q_3^2$ . Hence, we can merge  $\llbracket q_2^1 \rrbracket \bowtie \llbracket q_2^2 \rrbracket$  and  $\llbracket q_3^1 \rrbracket \bowtie \llbracket q_3^2 \rrbracket$  into  $(\llbracket q_2^1 \rrbracket \cup \llbracket q_3^1 \rrbracket) \bowtie (\llbracket q_2^2 \rrbracket \cup \llbracket q_3^2 \rrbracket)$ .

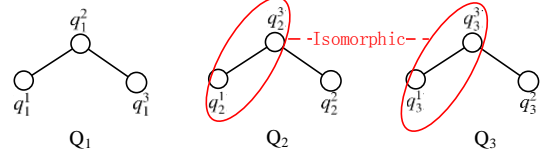


Fig. 13. Example Join Graphs

Taking advantage of these common join structures, we can speed up the query response time for multiple queries. Formally, given subqueries  $q_i^1$  and  $q_i^2$  for query  $Q_i$  and subqueries  $q_j^1$  and  $q_j^2$  for query  $Q_j$ , if  $q_i^1$  has the same structure to  $q_i^2$ ,  $q_j^1$  has the same structure to  $q_j^2$  and the join variables between  $q_i^1$  and  $q_i^2$  are the same to the join variables between  $q_j^1$  and  $q_j^2$ , then we can merge  $\llbracket q_i^1 \rrbracket \bowtie \llbracket q_i^2 \rrbracket$  and  $\llbracket q_j^1 \rrbracket \bowtie \llbracket q_j^2 \rrbracket$  into  $(\llbracket q_i^1 \rrbracket \cup \llbracket q_j^1 \rrbracket) \bowtie (\llbracket q_i^2 \rrbracket \cup \llbracket q_j^2 \rrbracket)$ .

The use of the above optimization technique is beneficial if the cost to merge the same two joins is less than the cost of executing two joins separately. To illustrate the potential benefit of the above optimization technique, let us compare the costs of the two alternatives:  $\llbracket q_i^1 \rrbracket \bowtie \llbracket q_i^2 \rrbracket$  and  $\llbracket q_j^1 \rrbracket \bowtie \llbracket q_j^2 \rrbracket$  versus  $(\llbracket q_i^1 \rrbracket \cup \llbracket q_j^1 \rrbracket) \bowtie (\llbracket q_i^2 \rrbracket \cup \llbracket q_j^2 \rrbracket)$ .

The cost of executing  $\llbracket q_i^1 \rrbracket \bowtie \llbracket q_i^2 \rrbracket$  and  $\llbracket q_j^1 \rrbracket \bowtie \llbracket q_j^2 \rrbracket$  separately is the sum of the costs of two joins. Thus,

$$\begin{aligned} & \text{cost}(\llbracket q_i^1 \rrbracket \bowtie \llbracket q_i^2 \rrbracket) + \text{cost}(\llbracket q_j^1 \rrbracket \bowtie \llbracket q_j^2 \rrbracket) \\ &= \min\{\text{card}(\llbracket q_i^1 \rrbracket), \text{card}(\llbracket q_i^2 \rrbracket)\} + \min\{\text{card}(\llbracket q_j^1 \rrbracket), \text{card}(\llbracket q_j^2 \rrbracket)\} \end{aligned} \quad (7)$$

On the other hand, the cost of executing  $(\llbracket q_i^1 \rrbracket \cup \llbracket q_j^1 \rrbracket) \bowtie (\llbracket q_i^2 \rrbracket \cup \llbracket q_j^2 \rrbracket)$  is as follows.

$$\begin{aligned} & \text{cost}((\llbracket q_i^1 \rrbracket \cup \llbracket q_j^1 \rrbracket) \bowtie (\llbracket q_i^2 \rrbracket \cup \llbracket q_j^2 \rrbracket)) \\ &= \min\{\text{card}(\llbracket q_i^1 \rrbracket \cup \llbracket q_j^1 \rrbracket), \text{card}(\llbracket q_i^2 \rrbracket \cup \llbracket q_j^2 \rrbracket)\} \end{aligned} \quad (8)$$

The our optimization technique is better if it acts as a sufficient reducer, that is, if  $\llbracket q_i^1 \rrbracket$  and  $\llbracket q_j^1 \rrbracket$  overlap a lot and  $\llbracket q_i^2 \rrbracket$  and  $\llbracket q_j^2 \rrbracket$  overlap a lot. Otherwise, we do two joins separately. It is important to note that neither approach is systematically the best; they should be considered as complementary.

It is easy to identify some common substructures between these join graphs. Obviously, we can do this part only once to avoid duplicate computation. We can find common substructures by using frequent subgraph mining technique [24]. Specifically, we can first find a common substructure among all join graphs, where vertices (i.e., the subqueries) in the common substructure have the largest benefit. We perform the join for this common substructure; and then iterate the above process. We do not discuss this tangential issue any further.

### VIII. HANDLING GENERAL SPARQL

So far, we only consider BGP (basic graph patterns) evaluation over federated RDF systems. In this section, we discuss how to extend our method to general SPARQLs with UNION, OPTIONAL and FILTER statements.

**Queries with UNION operators.** The query with UNION operators  $Q_1 \text{ UNION } Q_2$  can be directly decomposed into two BGPs  $Q_1$  and  $Q_2$ . Then, we can pass the batch of BGPs for multiple optimization. Finally, the result to the original query with UNION operators can be generated through the union of the results from the transformed BGPs after MQO.

**Queries with OPTIONAL operators.** To handle queries with OPTIONAL operators, it requires a preprocessing step on the input queries. Specifically, by the definition of OPTIONAL, a query with a OPTIONAL operator  $Q_1 \text{ OPTIONAL } Q_2$  is rewritten into two BGPs, since our MQO algorithm only works on BGPs. The equivalent BGPs of a query  $Q_1 \text{ OPTIONAL } Q_2$  are several BGPs:  $Q_1$  and  $Q_1 \text{ AND } Q_2$ . For example, after applying the above preprocessing, we can transform the query with OPTIONAL operators in Fig. 14(a) to a group of two BGPs as in Fig. 14(b).

```
SELECT ?x ?n WHERE {
  ?x g:parentFeature ?l .
  ?l g:name "Canada" .
  OPTIONAL { ?x g:name ?n . }
```

(a) Query with OPTIONAL Operator

```
SELECT ?x WHERE {
  ?x g:parentFeature ?l .
  ?l g:name "Canada" . }
SELECT ?x ?n WHERE {
  ?x g:parentFeature ?l .
  ?l g:name "Canada" .
  ?x g:name ?n . }
```

(b) Equivalent BGPs

Fig. 14. Query with OPTIONAL Operator to Its Equivalent BGPs

**Queries with FILTER operators.** For queries with FILTER operators, during data localization, we move possible value constraints into the local queries to reduce the size of intermediate results as early as possible. For example, for the query with FILTER operators in Fig. 15(a), it is decomposed to two local queries as in Fig. 15(b).

In addition, when we use FILTER-based rewriting strategy to rewrite a local query, we merge the original FILTER expressions and the rewritten FILTER expressions by using the intersection operators. For example, the local query in Fig. 15(b) can be rewritten to the query in Fig. VIII.

### IX. EXPERIMENTAL EVALUATION

In this section, we evaluate our federated multiple query optimization method (FMQO) over both real (FedBench) and

```
SELECT ?x ?n WHERE {
  ?c g:featureCode g:School.
  ?c g:name ?n
  ?c g:parentCountry ?l .
  ?l g:name "Canada" .
  ?y sameAs ?c
  FILTER (regex(str(?n), "Toronto", "i"))}
```

(a) Query with FILTER Operator

```
SELECT ?x ?n WHERE {
  ?c g:featureCode g:School.
  ?c g:name ?n
  ?c g:parentCountry ?l .
  ?l g:name "Canada" .
  FILTER (regex(str(?n), "Toronto", "i"))}
```

(b) Local Queries

Fig. 15. Query with FILTER Operator to Its Local Queries

```
SELECT ?x ?n WHERE {
  ?c g:featureCode ?f.
  ?c g:name ?n
  ?c g:parentCountry ?l .
  ?l g:name "Canada" .
  FILTER (regex(str(?n), "Toronto", "i") && ?f = g:School)}
```

Fig. 16. Rewritten Query with FILTER Operators

synthetic datasets (WatDiv). Table 1 shows the statistics of datasets. In addition, we compare our system with two state-of-the-art federated SPARQL query engines: FedX [22] and SPLENDID [7].

#### A. Setting

**WatDiv.** WatDiv [2] is a benchmark that enables diversified stress testing of RDF data management systems. In WatDiv, instances of the same type can have the different sets of attributes. We generate three datasets varying sizes from 10 million to 100 millions triples. WatDiv provides its own workload generator which generates templates and instantiates them with actual RDF terms from the dataset. We directly use WatDiv's own workload generator of WatDiv to generate different workloads for testing.

**FedBench.** FedBench [21] is a comprehensive benchmark suite for testing and analyzing both the efficiency and effectiveness of federated RDF systems. It includes 6 real cross-domain RDF datasets and 4 real life science domain RDF datasets with 7 federated queries for each RDF dataset. To enable multiple query evaluation, we use these 14 queries as seeds and generate different kinds of workloads in our experiments. In particular, for each benchmark query, we remove all constants (strings and URIs) at subjects and objects and replace them with variables. By doing this, we extract a general representation of a SPARQL query as a template. Then, we instantiate these templates from benchmark queries with actual RDF terms from the dataset. By default, we generate 150 queries each for cross-domain and life science domain RDF datasets.

Collection	Dataset	Number of Triples	Domain
WatDiv	WatDiv10M	10,916,457	Synthetic
	WatDiv 50M	54,963,869	
	WatDiv100M	108,997,714	
Cross-Domain	DBPedia subset	42,855,253	Generic
	NY Times	337,563	News
	LinkedMDB	6,147,997	Movies
	Jamendo	1,049,647	Music
	GeoNames	107,950,085	Geography
	SW Dog Food	103,595	SW Conferences and Publications
Life Science	DBPedia subset	42,855,253	Generic
	KEGG	1,090,830	Chemicals
	Drugbank	766,920	Drugs
	ChEBI	7,325,744	Compounds

TABLE I. BASIC STATISTICS OF DATASETS

We conduct all experiments on a cluster of 10 machines

running Linux, each of which has one CPU with four cores of 3.06GHz. Each site has 16GB memory and 150GB disk storage. The prototype is implemented in Java. At each site, we install Sesame 2.7 to build up an RDF source. Each source can only communicate with the control site through HTTP requests and cannot communicate with each other.

For FedBench, we assume that each dataset is resident at a source site. For WatDiv, we partition them using METIS [10] into  $m$  parts located at different source sites, where parameter  $m$  varies from 4 to 16 in our experiment.

### B. Evaluation of Proposed Techniques

In this section, we use WatDiv 10M and a query workload of 150 queries to evaluate each proposed technique in this paper. In other words, 150 queries are posed simultaneously to the federated RDF systems storing WatDiv 10M.

**Effect of the Rewriting Strategies.** In this experiment, we compare our SPARQL query rewriting techniques using only OPTIONAL operators (denoted as OPT-only) and only FILTER operators (denoted as FIL-only). We also re-implement the rewriting strategies proposed in [13] (denoted as Le et al.) to rewrite subqueries. Our query rewriting technique is denoted as FMQO. Fig. 17 shows the query response time and the number of remote requests for a workload by using the four rewriting strategies.

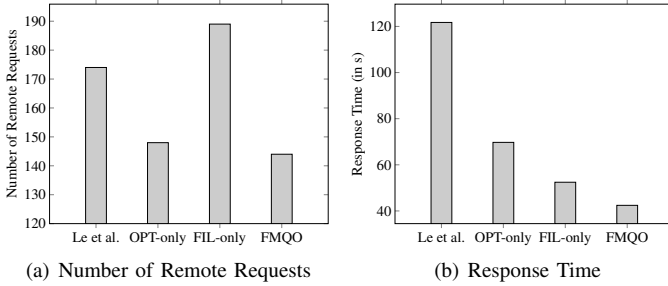


Fig. 17. Evaluating Different Rewriting Strategies

Given a workload, since the number of subqueries sharing common substructures is often more than the number of subqueries having the same structure, FIL-only leads to the largest number of rewritten queries, meaning it results in more remote requests than other rewriting strategies. Le et al. first cluster all subqueries into some groups, and then find the maximal common edge subgraphs of the group of subqueries for query rewriting. Thus, the number of rewritten queries generated by Le et al. is no less than the number of the groups. In contrast, OPT-only and FMQO use some triple patterns to hit subqueries. Hence, the number of rewritten queries generated by OPT-only and FMQO is the number of selected triple patterns. In real applications, most maximal common edge subgraphs found by Le et al. also contain most of our selected triple patterns. Hence, Le et al. generate more rewritten queries than OPT-only and FMQO, which means more remote requests. Finally, FMQO obtains the smallest number of rewritten queries.

Since OPT-only generates smaller number of rewritten queries and share more computation than Le et al., OPT-only can result in faster query response time. A query with OPTIONAL operators is slower than a query with FILTER operators, assuming they have the same main pattern, since the

former is based on left-join and the latter is based on selection. Hence, although more queries are generated by using FIL-only rewriting strategy, their query response times are faster than Le et al. and OPT-only. Generally speaking, FIL-only takes about half time of Le et al., as shown in Fig. 18(b) and two thirds of OPT-only. Furthermore, as shown in Fig. 18(b), our rewriting technique using both OPTIONAL and FILTER operators has the best performance. This is because it takes advantages of both the rewriting strategy only using OPTIONAL and the rewriting strategy only using FILTER.

**Evaluation of the Cost Model.** In this section, we evaluate the effectiveness of our cost model and cost-aware rewritten strategy in Section V-B. In Fig. 18, we analyze the effect of our cost function to measure the cost of rewriting. We design a baseline (FMQO-R) that does not select the locally optimal triple patterns as presented in Algorithm 1 but randomly select triple patterns to rewrite subqueries.

Compared to the baseline, we find out that cost-based selection causes fewer remote requests, as shown in Fig. 18(a). This is because the patterns with lower cost are shared by more subqueries, which results in fewer rewritten queries. In addition, in our cost-based rewriting strategy, we prefer selective query patterns, resulting in lower query response times, as shown in Fig. 18(b). Generally speaking, the cost model-based approach can provide speed up of twice.

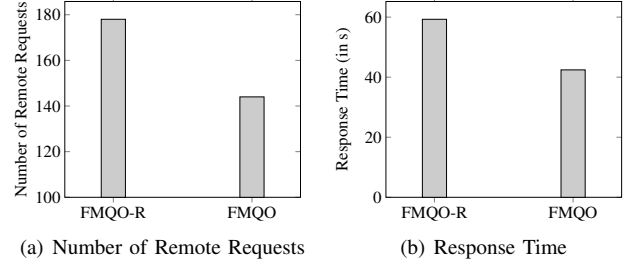


Fig. 18. Evaluating Cost Model

**Effect of Optimization Techniques for Joins.** We evaluate our optimized join strategy proposed in Section VII. We design a baseline that runs multiple federated queries with only rewriting strategies but not our optimization techniques for joins (denoted as FMQO-QR). Although this technique does not affect the number of remote requests, it reduces the join cost by making use of common join structures. In general, it reduces join processing time by 10%, as shown in Fig. 19.

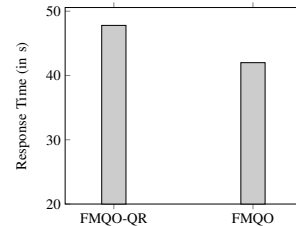


Fig. 19. Effect of Optimization Techniques for Joins

### C. Evaluating Scalability

In this subsection, using WatDiv, we test the scalability of our method in four aspects: varying the number of queries, varying the number of query templates, varying the dataset

sizes and varying the number of sources. We design a baseline that runs multiple federated queries sequentially (denoted as No-FMQO). This baseline uses the existing techniques for data localization without using the source topology graph and does not employ any optimizations for multiple queries. We also compare our method with FedX and SPLENDID. By default, the dataset is WatDiv 10M, the number of sources is 4, the number of queries is 150 and the number of templates is 10.

**Varying Number of Queries.** We study the impact of the size of the query set, which we vary from 100 to 250 queries, in increments of 50. Fig. 20 shows the experimental results.

Due to query rewriting, FMQO can merge many subqueries into fewer rewritten queries, which results in smaller number of remote requests, as shown in Fig. 20(a). FMQO can reduce the number of remote accesses by 1/2-2/3, compared with No-FMQO. Since FedX and SPLENDID do not provide their numbers of remote requests, we do not compare FMQO with FedX and SPLENDID in Fig. 20(a).

In terms of evaluation times (Fig. 20(b)), since the baseline method without any optimization, No-FMQO, does not share any replicate computation, it takes a third more time than FMQO approach. In addition, FedX and SPLENDID always employ a semijoin algorithm to join intermediate results. Since almost all partial matches of subqueries participate in the join in WatDiv, the semijoin algorithm is not always efficient for this dataset. Hence, No-FMQO and FMQO are also twice faster than FedX and SPLENDID.

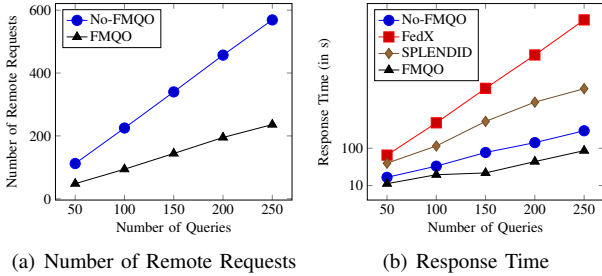


Fig. 20. Varying Number of Queries

**Varying Number of Query Templates.** We study the impact of the number of templates. We vary the number of templates from 5 to 25, in increments of 5. The results are shown in Fig. 21. As before, we do not compare FMQO with FedX and SPLENDID in Fig. 21(a).

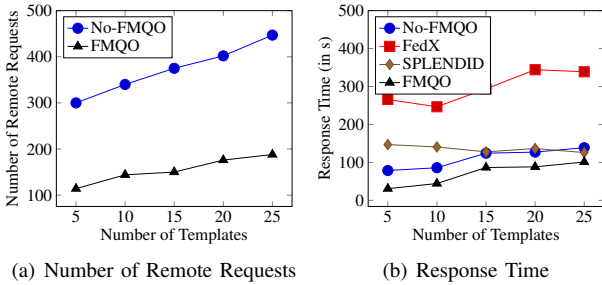


Fig. 21. Varying Number of Query Templates

Fig. 21(a) shows that the number of remote requests increases with the number of templates. This is because, as

the number of queries is kept constant, more templates mean that fewer queries have the common substructures. Since our rewriting techniques use the common substructures to rewrite queries, fewer queries containing the common substructures result in more number of rewritten queries. Therefore, the number of the remote requests increases by 50% (from about 100 to about 150). More rewritten queries mean that less computation is shared by different queries, so the performance of FMQO becomes worse as shown in Fig. 21(b). However, the response time of FMQO increases slowly, so it is still 50% less than No-FMQO and SPLENDID, and two thirds less than FedX.

**Varying Size of Datasets.** Here, we investigate the impact of dataset size on the optimization results. We generate three WatDiv datasets varying the from 10 million to 100 million triples. Fig. 22 shows the results.

While this does not affect the number of remote requests, it clearly affects evaluation times, as shown in Fig. 22. As the size of RDF datasets gets larger, the response time of all three methods increases. However, the rate of increase for FMQO is smaller than other competitors. The response time of FMQO decreases from 60% of No-FMQO to 50% of No-FMQO, while the response time of FMQO always is 30% of SPLENDID to 20% of FedX.

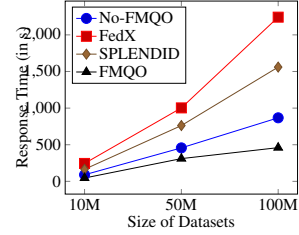


Fig. 22. Varying Size of Datasets

**Varying Number of Sources.** In this experiment, we vary the number of sources from 4 to 16. Fig. 23 presents the scalability of our solution adapting to different number of RDF sources. As the number of sources increases, a query may be relevant to more sources and it is decomposed into more subqueries. Thus, more rewritten queries are generated to evaluate the input queries. However, FMQO grows much slower than No-FMQO in both the number of remote accesses and query response time. It confirms that FMQO has better scalability with the number of sources.

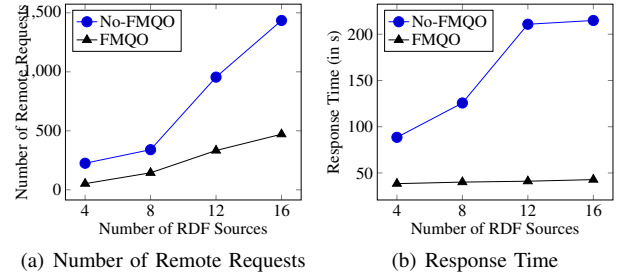


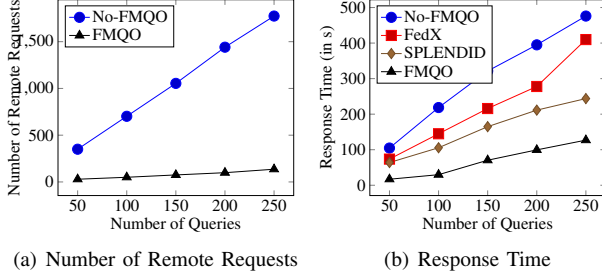
Fig. 23. Varying Number of RDF Sources

#### D. Performance over Real Dataset

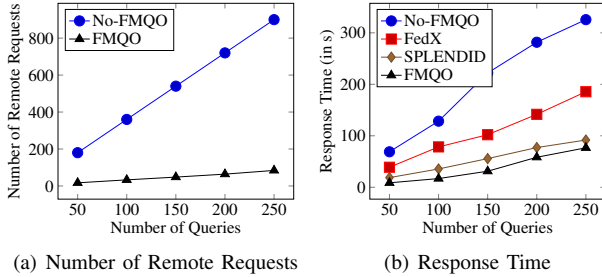
In this experiment, we test FMQO using the real RDF dataset, FedBench. Since real datasets do not allow changing



data sizes and the number of sources, we only test the methods by varying the number of queries in Figs. 24 and 25. FMQO Experiments confirm that FMQO lead to fewest remote requests for FedBench and best query performance, since FMQO often rewrite multiple queries into a single SPARQL, which results in fewer remote requests. Furthermore, the cost-driven rewriting strategy in our method guarantees that rewritten queries are always faster than evaluating them sequentially.



(a) Number of Remote Requests  
Fig. 24. FedBench (Cross Domain)



(a) Number of Remote Requests  
Fig. 25. FedBench (Life Science)

## X. RELATED WORK

There are two threads of related work: SPARQL query processing in federated RDF systems and multi-query optimization.

**Federated Query Processing.** Many approaches [19], [22], [7], [8], [18], [20] have been proposed for federated SPARQL query processing. Since RDF sources in federated RDF systems are autonomous, they cannot be interrupted during query execution. Therefore, the major challenge is query decomposition and source selection. The challenge is the main differences among existing approaches.

For data localization, most papers propose the metadata-assisted methods. They find the relevant RDF sources for a query by simply matching all triple patterns based on the metadata. In particular, the metadata in DARQ [19] is named service descriptions, which describes the data available from a data source in form of capabilities. SPLENDID [7] uses Vocabulary of Interlinked Datasets (VOID) as the metadata. QTree [8], [18] is another kind of metadata. It is a variant of RTree, and its leaf stores a set of source identifiers, including one for each source of a triple approximated by the node. HiBISCuS [20] relies on capabilities to compute the metadata. For each source, HiBISCuS defines a set of capabilities which map the properties to their subject and object authorities. UPSP [16] extends HiBISCuS by adding the indices named unique predicate types about all distinct predicates at each RDF source. A unique predicate is a property that is not involved in some special kinds of joins.

Besides the metadata-assisted methods, there are still a few papers that do not require the metadata. In FedX [22], the source selection is performed by using ASK queries. FedX sends ASK queries for each triple pattern to the RDF sources. Based on the results, it annotates each pattern in the query with its relevant sources.

**Multiple SPARQL Queries Optimization.** Le et al. [13] first discuss how to optimize multiple SPARQL queries evaluation, but only in a centralized environment. It first finds out all maximal common edge subgraphs (MCES) among a group of query graphs, and then rewrites the set queries into a query with OPTIONAL operators. In the rewritten queries, the MCES constitutes the main pattern, while the remaining subquery of each individual query generates an OPTIONAL clause. Konstantinidis et al. [11] discuss how to optimize multiple SPARQL queries evaluation over multiple views. They first find out some atomic join operations among multiple queries. Then, they map each atomic join operation to a view one and compute it just once to avoid redundant work.

There also have been a few papers on multi-query processing and optimization [15], [3] on Hadoop. HadoopSPARQL [15] discuss how to translate a set of join operators into one Hadoop job to share the computation of multiple SPARQL queries. Anyanwu et al. [3] extend the “multi starjoin” processing in multiple SPARQL queries to “multi-OPTIONAL” processing, which reduces the number of MapReduce cycles.

## XI. CONCLUSION

In this paper, we study the problem of multiple query optimization over federated RDF systems. Our optimization framework, which integrates a novel algorithm to identify common subqueries with a cost model, rewrites queries into equivalent queries that are more efficient to evaluate. We also discuss how to efficiently join intermediate results. Extensive experiments show that our optimizations are effective.

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