1.求下列函数的拉氏变换

(1) $(3+2t)e^{-2t}$

解 $\mathcal{L}(3+2t)=\frac{3}{s}+\frac{2}{s^2}$

$\mathcal{L}[(3+2t)e^{-2t}]=\frac{3}{s+2}+\frac{2}{(s+2)^2}$

(2) $te^{-(t-3)}u(t-1)$

解 $\mathcal{L}[te^{-(t-3)}u(t-1)]$

$=e^3\mathcal{L}[tu(t-1)e^{-t}]$

$\mathcal{L}[tu(t-1)]=\mathcal{L}[(t-1)u(t-1)+u(t-1)]$

$=(\frac{1}{s^2}+\frac{1}{s})e^{-s}$

$\therefore$ 原式 $=e^3\cdot[\frac{1}{(s+1)^2}+\frac{1}{s+1}]\cdot e^{-(s+1)}$

$=[\frac{1}{(s+1)^2}+\frac{1}{s+1}]\cdot e^{2-s}$

(3) $e^{-2t}\sin 3t$

解 $\mathcal{L}(\sin 3t)=\frac{3}{s^2+9}$

$\therefore\mathcal{L}(e^{-2t}\sin 3t)=\frac{3}{(s+2)^2+9}$

(4) t^2+2t

解: $\mathcal{L}(t^2+2t)=\frac{2!}{s^3}+2\frac{1}{s^2}=2(\frac{1}{s^3}+\frac{1}{s^2})$

(5) $t[u(t)-u(t-1)]$

解: $\mathcal{L}[t\,u(t)]=\frac{1}{s^2}$

$\mathcal{L}[t\,u(t-1)]=\mathcal{L}[(t-1)u(t-1)]+\mathcal{L}[u(t-1)]$

$=(\frac{1}{s^2}+\frac{1}{s})e^{-s}$

原式 $=\mathcal{L}[t\,u(t)]-\mathcal{L}[t\,u(t-1)]$

$=\frac{1}{s^2}+(\frac{1}{s^2}+\frac{1}{s})e^{-s}$

(6) $e^{-(t-2)}u(t-3)$

解: 原式 $=e^2\cdot\mathcal{L}[e^{-t}u(t-3)]$

$\mathcal{L}[u(t-3)]=\frac{1}{s}e^{-3s}$

$\Rightarrow\mathcal{L}[e^{-t}u(t-3)]=\frac{1}{s+1}e^{-3(s+1)}$

$\therefore$ 原式 $=\frac{1}{s+1}e^{-3s-1}$

2. 求下列函数的拉氏反变换

(1) $\frac{3S+2}{S^2+5S+4}$

解：原式 $=\frac{K_1}{S+4}+\frac{K_2}{S+1}$

$K_1=\left.\frac{3S+2}{S+1}\right|_{S=-4}=\frac{10}{3}$

$K_2=\left.\frac{3S+2}{S+4}\right|_{S=-1}=-\frac{1}{3}$

$\therefore \mathcal{L}^{-1}\left[\frac{3S+2}{S^2+5S+4}\right]$

$=\frac{10}{3}\mathcal{L}^{-1}\left(\frac{1}{S+4}\right)-\frac{1}{3}\mathcal{L}^{-1}\left(\frac{1}{S+1}\right)$

$=\left(\frac{10}{3}e^{-4t}-\frac{1}{3}e^{-t}\right)u(t)$

(2) $\frac{S^2+8}{S(S^2+2S+10)}$

解：原式 $=\frac{K_1}{S}+\frac{K_2}{S+1+3i}+\frac{K_2^*}{S+1-3i}$

$K_1=\left.\frac{S^2+8}{S^2+2S+10}\right|_{S=0}=\frac{4}{5}$

$K_2=\left.\frac{S^2+8}{S(S+1-3i)}\right|_{S=-1-3i}=\frac{(1+3i)^2+8}{(-1-3i)\cdot(-6i)}=\frac{1}{10}(1-3i)$

$\mathcal{L}^{-1}\left[\frac{S^2+8}{S(S^2+2S+10)}\right]=K_1\mathcal{L}^{-1}\left(\frac{1}{S}\right)+\mathcal{L}^{-1}\left(\frac{K_2}{S+1+3i}+\frac{K_2^*}{S+1-3i}\right)$

$=\left\{\frac{4}{5}+2e^{-t}\left[\frac{1}{10}\cos(3t)-\frac{3}{10}\sin(3t)\right]\right\}u(t)$

$=\left[\frac{4}{5}+\frac{1}{5}e^{-t}\cos(3t)-\frac{3}{5}e^{-t}\sin(3t)\right]u(t)$

(3) $\frac{(s+2)(1+e^{-s})}{s^2+4s+3}$

解：$\mathcal{L}^{-1}\left[\frac{s+2}{(s+3)(s+1)}\right]=\mathcal{L}^{-1}\left[\frac{\frac{s+2}{s+1}\Big|_{s=-3}}{s+3}+\frac{\frac{s+2}{s+3}\Big|_{s=-1}}{s+1}\right]$

$=\frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s+3}+\frac{1}{s+1}\right)$

$=\frac{1}{2}(e^{-3t}+e^{-t})u(t)$

$\therefore \mathcal{L}^{-1}\left[\frac{(s+2)(1+e^{-s})}{s^2+4s+3}\right]$

$=\frac{1}{2}(e^{-3t}+e^{-t})u(t)+\frac{1}{2}(e^{-3t+3}+e^{-t+1})u(t-1)$

(4) $\frac{s^2+4s+5}{s^2+3s+2}$

解：$\mathcal{L}^{-1}\left(\frac{s^2+4s+5}{s^2+3s+2}\right)=\mathcal{L}^{-1}\left[1+\frac{s+3}{(s+2)(s+1)}\right]$

$=\mathcal{L}^{-1}\left[1+\frac{\frac{s+3}{s+1}\Big|_{s=-2}}{s+2}+\frac{\frac{s+3}{s+2}\Big|_{s=-1}}{s+1}\right]$

$=\mathcal{L}^{-1}\left(1-\frac{1}{s+2}+\frac{2}{s+1}\right)$

$=\delta(t)+(2e^{-t}-e^{-2t})u(t)$

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3. 求下列函数逆变换的初值和终值

(1) $\frac{2s+3}{s(s^2+4s+3)}$

解：$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2s+3}{s^2+4s+3} = 0$

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{2s+3}{s^2+4s+3} = 1$

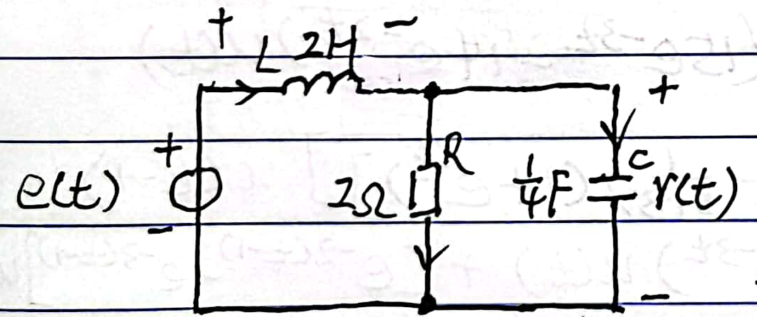
(2) $\frac{2s+3}{s^2+4}$

解：$\frac{2s+3}{s^2+4}$ 在 $j\omega$ 轴上存在极点 $s=\pm 2j$

∴ 终值不存在

$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2s^2+3s}{s^2+4} = 2$

4. 如图所示电路，求其系统函数，并写出微分方程



$$\begin{cases} L\frac{di_L(t)}{dt} + r(t) = e(t) \\ i_L(t) = \frac{r(t)}{R} + C\frac{dr(t)}{dt} \end{cases}$$

$\therefore LC\frac{d^2r(t)}{dt^2} + \frac{L}{R}\frac{dr(t)}{dt} + r(t) = e(t)$

微分方程：$\frac{1}{2}\frac{d^2r(t)}{dt^2} + \frac{dr(t)}{dt} + r(t) = e(t)$

$\Rightarrow \frac{1}{2}s^2R(s) + sR(s) + R(s) = E(s) \Rightarrow (\frac{1}{2}s^2+s+1)R(s) = E(s)$

$\Rightarrow H(s) = \frac{R(s)}{E(s)} = \frac{2}{s^2+2s+2}$

5、已知因果系统的微分方程为 $\frac{d^2}{dt^2}r(t)+5\frac{d}{dt}r(t)+6r(t)=e(t)+e(t-1)$，$r(0_-)=2$，$r'(0_-)=1$，$e(t)=u(t)$，试用拉氏变换法求系统的完全响应，零输入响应，零状态响应

解：$s^2R(s)-sr(0_-)-r'(0_-)+5[sR(s)-r(0_-)]+6R(s)=E(s)(1+e^{-s})$

$$(s^2+5s+6)R(s)=sr(0_-)+r'(0_-)+5r(0_-)+E(s)(1+e^{-s})$$

$$R(s)=\frac{2s+11}{s^2+5s+6}+E(s)\frac{1+e^{-s}}{s^2+5s+6}$$

又 $E(s)=\mathcal{L}[u(t)]=\frac{1}{s}$

$$\therefore R(s)=\left(\frac{7}{s+2}-\frac{5}{s+3}\right)+\left[\frac{1}{6s}-\frac{1}{2(s+2)}+\frac{1}{3(s+3)}\right](1+e^{-s})$$

$$r_{zi}(t)=\mathcal{L}^{-1}\left(\frac{7}{s+2}-\frac{5}{s+3}\right)$$

$$=(7e^{-2t}-5e^{-3t})u(t)$$

$$r_{zs}(t)=\mathcal{L}^{-1}\left\{\left[\frac{1}{6s}-\frac{1}{2(s+2)}+\frac{1}{3(s+3)}\right](1+e^{-s})\right\}$$

$$=\left(\frac{1}{6}-\frac{1}{2}e^{-2t}+\frac{1}{3}e^{-3t}\right)u(t)$$

$$+\left[\frac{1}{6}-\frac{1}{2}e^{-2(t-1)}+\frac{1}{3}e^{-3(t-1)}\right]u(t-1)$$

完全响应：$r(t)=r_{zi}(t)+r_{zs}(t)$

$$=\left(\frac{1}{6}+\frac{13}{2}e^{-2t}-\frac{14}{3}e^{-3t}\right)u(t)$$

$$+\left[\frac{1}{6}-\frac{1}{2}e^{-2(t-1)}+\frac{1}{3}e^{-3(t-1)}\right]u(t-1)$$

6. 已知因果系统的微分方程为 $\frac{d^2}{dt^2}r(t)+7\frac{d}{dt}r(t)+10r(t)=\frac{d}{dt}e(t)+3e(t)$, $r(0_-)=1$, $r'(0_-)=1$, $e(t)=e^{-2t}u(t)$

(1) 求系统的完全响应，零输入响应，零状态响应

解：$s^2R(s)-sr(0_-)-r'(0_-)+7[sR(s)-r(0_-)]+10R(s)$

$=sE(s)-e(0_-)+3E(s)$

$$\left.\begin{aligned}&\therefore (s^2+7s+10)R(s)=s+8+(s+3)E(s)\\&E(s)=\mathcal{L}[e^{-2t}u(t)]=\frac{1}{s+2}\end{aligned}\right\}$$

$$\Rightarrow R(s)=\frac{s+8}{s^2+7s+10}+\frac{s+3}{(s+2)(s^2+7s+10)}$$

$$=\left(\frac{2}{s+2}-\frac{1}{s+5}\right)+\left[\frac{K_1}{(s+2)^2}+\frac{K_2}{s+2}+\frac{K_3}{s+5}\right]$$

$$K_1=\left.\frac{s+3}{s+5}\right|_{s=-2}=\frac{1}{3}$$

$$K_3=\left.\frac{s+3}{(s+2)^2}\right|_{s=-5}=-\frac{2}{9}$$

$$K_2=\left.\frac{d}{ds}\left(\frac{s+3}{s+5}\right)\right|_{s=-2}=\left.\frac{2}{(s+5)^2}\right|_{s=-2}=\frac{2}{9}$$

$$\Rightarrow R(s)=\left(\frac{2}{s+2}-\frac{1}{s+5}\right)+\frac{1}{9}\left[\frac{3}{(s+2)^2}+\frac{2}{s+2}-\frac{2}{s+5}\right]$$

$r_{zi}(t)=(2e^{-2t}-e^{-5t})u(t)$

$r_{zs}(t)=\frac{1}{9}(3te^{-2t}+2e^{-2t}-2e^{-5t})u(t)$

$r(t)=r_{zi}(t)+r_{zs}(t)$

$=\left(\frac{1}{3}te^{-2t}+\frac{20}{9}e^{-2t}-\frac{11}{9}e^{-5t}\right)u(t)$

(2) 求系统函数和单位冲激响应，并画出零极点图

解：$E(s) = \mathcal{L}[e^{-2t}u(t)] = \frac{1}{s+2}$

$$H(s) = \frac{R(s)}{E(s)} = (s+2)\left[\frac{s+8}{s^2+7s+10} + \frac{s+3}{(s+2)(s^2+7s+10)}\right]$$

$$= (s+2)\frac{(s+8)(s+2)+s+3}{(s+2)^2(s+5)} = \frac{s^2+11s+19}{(s+2)(s+5)}$$

$$= \frac{\left(s+\frac{11-3\sqrt{5}}{2}\right)\left(s+\frac{11+3\sqrt{5}}{2}\right)}{(s+2)\ (s+5)}$$

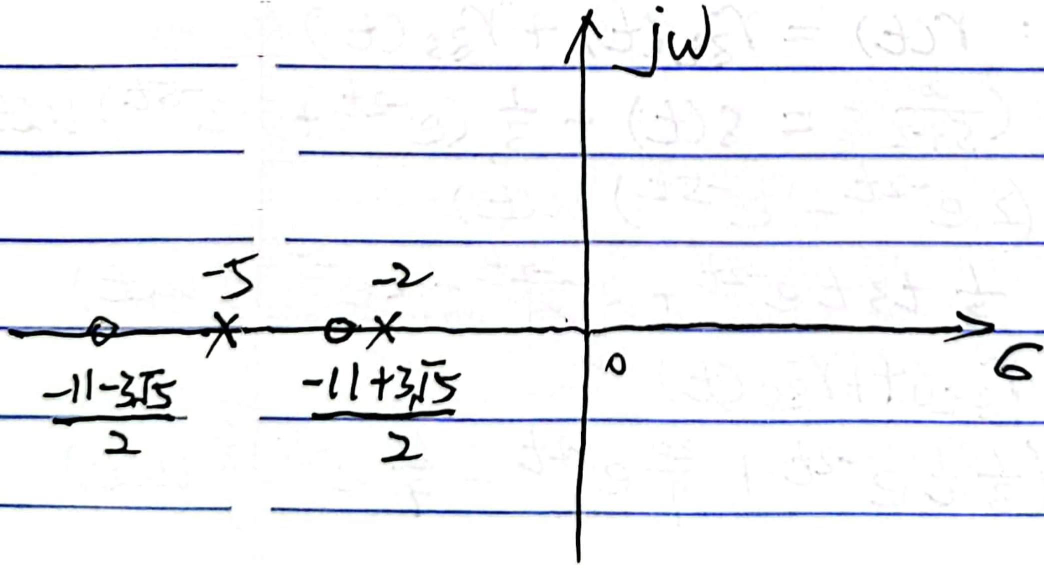
$$h(t) = \mathcal{L}^{-1}[H(s)]$$

$$= \mathcal{L}^{-1}\left[\frac{s+8}{s+5} + \frac{s+3}{s^2+7s+10}\right]$$

$$= \mathcal{L}^{-1}\left[1+\frac{3}{s+5} + \frac{\frac{1}{3}}{s+2} + \frac{\frac{2}{3}}{s+5}\right]$$

$$= \mathcal{L}^{-1}\left(1+\frac{1}{3}\cdot\frac{1}{s+2} + \frac{11}{3}\cdot\frac{1}{s+5}\right)$$

$$= \left(1+\frac{1}{3}e^{-2t} + \frac{11}{3}e^{-5t}\right)u(t)$$



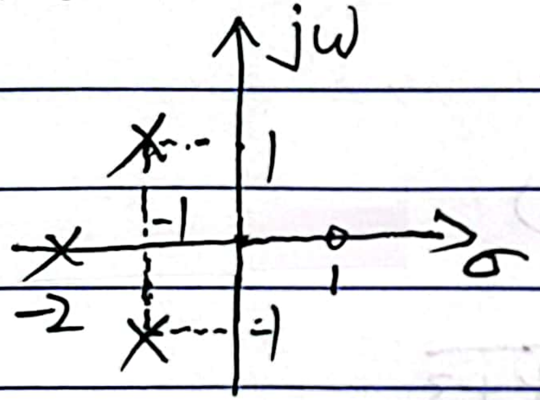
(3) 判断系统稳定性

解 因为极点 $p_1=-5$，$p_2=-2$ 均小于0

在 $j\omega$ 轴左侧

所以该系统稳定

7. 已知系统零极点图如图所示，$H(0)=-1$，求系统函数



设 $H(s)=K\dfrac{s-1}{(s+2)(s+1-j)(s+1+j)}$

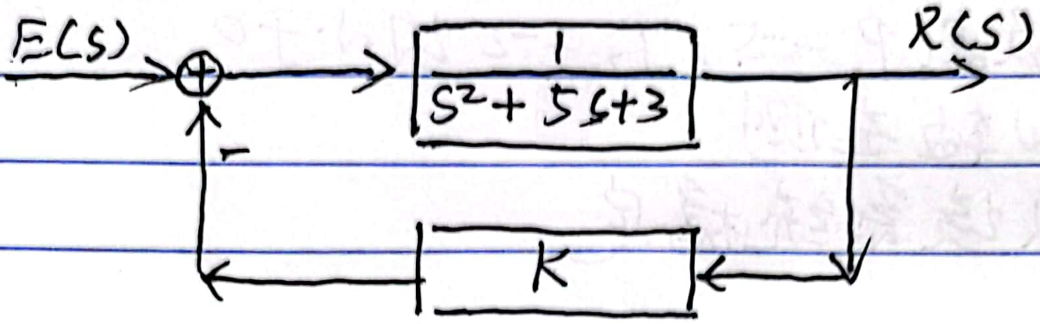
又 $H(0)=-1$

则 $K\dfrac{-2}{-j\cdot j}=-1$

$\therefore k=\dfrac{1}{2}$

$\therefore H(s)=\dfrac{s-1}{2(s+2)(s+1-j)(s+1+j)}$

8、已知连续时间系统的系统框图如图所示



(1) 求系统函数

解 $R(S)=\dfrac{E(S)-KR(S)}{S^2+5S+3}$

$\Rightarrow (S^2+5S+K+3)R(S)=E(S)$

$H(S)=\dfrac{R(S)}{E(S)}=\dfrac{1}{S^2+5S+K+3}$

(2) 为使系统稳定，实系数K应满足什么条件

解：H(S)的极点均在jw轴左侧

$\Delta=25-4(K+3)=-4K+13$

$\begin{cases}\Delta\geq 0\\ \dfrac{-5+\sqrt{\Delta}}{2}<0\end{cases}$ 或 $\begin{cases}\Delta<0\\ \dfrac{-5}{2}<0\end{cases}$

即 $\begin{cases}-4K+13\geq 0\\ -5+\sqrt{13-4K}<0\end{cases}$ 或 $13-4K<0$

$\Rightarrow K>-3$ 或 $K>\frac{13}{4}$

$\therefore K>-3$

(3) 在临界稳定条件下，求整个系统的单位冲激响应$h(t)$

解：$\begin{cases} \Delta \geq 0 \\ \frac{-5+\sqrt{\Delta}}{2} = 0 \end{cases}$ 或 $\begin{cases} \Delta < 0 \\ \frac{-5}{2} = 0 \end{cases}$

$$\Rightarrow \Delta = 13 - 4K = 25$$

$$\Rightarrow K = -3$$

$$\Rightarrow H(s) = \frac{1}{s^2+5s} = \frac{1}{5}\left(\frac{1}{s} - \frac{1}{s+5}\right)$$

$$\Rightarrow h(t) = \frac{1}{5}\left(1 - e^{-5t}\right)u(t)$$