Network-assisted Distributed Fairness-aware Interference Coordination for Device to Device Communication Underlaid Cellular networks

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Abstract

Device-to-device (D2D) communication underlaid cellular network is considered a key integration feature in future cellular network. However, without properly designed interference management, the interference from D2D transmission tends to degrade the performance of cellular users and D2D pairs. In this work, we proposed a Network-assisted distributed interference mitigation scheme to address this issue. Specifically, the base station (BS) acts as a control agent that coordinates the cross-tier interference from D2D transmission through a taxation scheme. The co-tier interference is controlled by non-cooperative game amongst D2D pairs. In general, the outcome of non-cooperative game is inefficient due to the selfishness of each player. In our game formulation, *reference user* who is the victim of co-tier interference is factored in to the payoff function of each player to obtain fair and efficient outcome. The existence, uniqueness of the Nash equilibrium(NE) and the convergence of the proposed algorithm are characterized using *Variational Inequality* theory. Finally, we provide simulation results to evaluate the efficiency of the proposed algorithm.

Keywords: `Device-to-device communications; Non-cooperative game; Interference mitigation; Variational inequality.`

1. Introduction

Device-to-device (D2D) communication underlaid cellular network is considered a key integration feature in the 3GPP LTE-A systems [1]. Contrary to traditional cellular network where cellular users (CUs) receive services from the base station, this feature allows two communicating CUs in proximity to reuse the radio resources of other CUs and transmit the signal directly without relaying through the BS [2]. Due to close transmission range and spectrum reuse, this kind of communication can enhance the spectral efficiency and resource utilization. However, D2D transmission introduces two set of interference in cellular networks; interference to CUs and interference amongst neighbouring

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D2D pairs on each subchannel. This can deteriorate the performance of the system if the interferences are not properly managed. Therefore, intelligent interference management needs to be considered in order to make the benefits of D2D communication available. Most of the related studies focused on mitigating the cross-tier interference from D2D pairs to CUs but co-tier interference amongst D2D pairs is not tackled.

In this work, we design a Network-assisted distributed interference coordination algorithm for D2D communication underlaid cellular networks to improve the data rate of D2D transmission while fulfilling the *Quality of serve* (QoS) requirements of CUs, we also consider the fairness among D2D pairs. The QoSs of CUs are ensured using a taxation scheme. Specifically, the BS maintains a *reuse price* which each D2D pair has to pay in order to reuse the radio resources. The interference from D2D transmission can be controlled by tuning this reuse price. Each D2D pair can reuse all radio resources of CUs and has to decide the transmit power on each resource, we modeled this process using non-cooperative game theory [3]. To obtain a fair and efficient outcome, we introduce the *reference D2D pair* into the payoff function of each D2D pair. This is encouraged by the reference line approach in digital subscriber line [4]. Roughly speaking, the reference D2D pair is the victim of co-tier interference and has the worst channel condition, by factoring the data rate of the reference D2D pair into the payoff function of each D2D pair as a taxation term, weak channel D2D pairs can be protected from excessive co-tier interference. To characterize the *Nash Equilibrium (NE)* of the formulated game, we borrow the convenience of *Variational Inequality (VI)* theory [5]. Sufficient conditions for the existence and uniqueness of the NE are given based on VI theory. In addition, condition for the convergence of the proposed algorithm is also given. The proposed scheme is Network-assisted distributed because it requires assistance from the BS to obtain information about the reference D2D pair. Finally, simulation results are given to evaluate the superiority of the proposed scheme. The main contribution of this work is summarized below

- A system model that permits multiple D2D pairs to utilize the resource of multiple CUs was designed in Section 3.

- We also proposed a Network-assisted distributed approach to interference coordination in Section 4 that requires no information passing amongst D2D pairs since information about the reference D2D pair is provided by BS [1] to address the problem of co-tier interference amongst D2D pairs using the concept of game theory.

- A cellular link performance management was also designed in Section 4 to address the cross-tier interference from D2D pairs to CUs.

- Finally, simulation conducted in Section 6 establishes the superiority of our proposed scheme.

2. Related Works

To date existing resource allocation schemes in this realm can be found in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. In [6], [7] the system throughput of a CU and a D2D pair is maximized considering the

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[1] we considered a network-assisted approach since the BS has the capability to provide information about the reference D2D pair.

target rate of CU. In [8], multiple CUs and a single D2D pair can coexist outside an interference restricted region. In [9], a centralized heuristic approach is proposed in which the resource of CU and D2D pair is matched based on the interference link gain from D2D transmitter to the BS, however the solution obtained can be far from optimal. In [10], with the aim of enhancing the system sum-rate of machine-type D2D links a joint mode selection and resource assignment scheme was designed by the author. The authors of [11] enhance the system capacity of multiple D2D pairs and CUs while fulfilling the QoSs of CUs and D2D pairs concurrently. However, the works in [6] - [11] are centralized which requires full knowledge of Link State Information (CSI), resulting in excessive information passing for most practical networks. In addition, the centralized optimization problem can be non-convex and associates with complex computation that increases exponentially with the size of the input. In practical design, distributed resource allocations with less information passing and low computational complexity are more preferred. A distributed algorithm based on coalition game approach for multiple D2D pairs and CUs was studied in [12] under the assumption that D2D pairs and CUs can cooperate to improve the sum-rate. An incentive compatible pricing approach was adopted in [13] where the BS suppress the interference from D2D transmission via pricing scheme. Nevertheless, this scheme can be improved significantly if the interference amongst D2D pairs is accounted for. An interference pricing scheme was design in [14] to manage interference from D2D communication, however, this work can be improved if interference amongst D2D pairs is managed. Distributed schemes in [15] and [16] are proposed based on Stackelberg and auction games respectively. However, in these works [6, 7, 8, 9, 11, 12, 13, 14, 15, 16], at most one resource of CU can be reused by a D2D pair, which may not lead to high resource utilization.

Recently, some works have been done allowing each D2D pair reuses multiple radio resources of CUs [17] - [22]. In this scenario, the author in [17] used a merge-and-split technique to design an overlapping coalition game with the aim of maximizing the overall sum-rate. However, the performance is crippled by high signaling amongst D2D pairs. An efficient resource assignment scheme for multiple D2D pairs with the objective of maximizing the total system capacity was studied in [18]. Nevertheless, interference amongst D2D pairs was not discussed. The authors in [19] show that the average data rate of the D2D communication can be significantly improved, however a single D2D pair is considered and the co-tier interference between D2D pairs is neglected. In [20], minimum data rates of CUs and D2D pairs are guaranteed using a distributed resource allocation scheme, however this method is valid only for a suitable small data rate requirements of D2D pairs which may result in low spectral efficiency. In [21], the authors propose a distributed scheme using the convenience of non-cooperative Stackelberg game which results in the reduction of signaling overhead with compromised system performance. In addition, the selfishness of each D2D pair in the formulated game may lead to inefficient outcome. In [22], a distributed resource allocation algorithm to enhance the sum data rate of D2D pairs is designed, however this scheme is only optimal with a high *Signal-to-Interference-plus-Noise Ratio* (SINR) approximation, also, the fairness amongst D2D pairs is not considered.

3. System Model and Problem Formulation

In this work, we consider a single cell Orthogonal Frequency Division Multiple Access (OFDMA) which consists of N subchannels and each D2D pair can reuse all the subchannels for communication. The system is made up of two tiers: the cellular tier and D2D tier, $\mathcal{D} = \{1, ..., D\}$ and $C = \{1, ..., C\}$ are the sets of D2D pairs and CUs respectively. All transmitters and receivers are equipped with one antenna. In the cellular tier, both spectrum assignment and power control decisions for CUs are handled by the base station. The subchannels are orthogonally assigned to the CUs, hence there is no co-tier interference amongst CUs. In D2D tier, the competition for reusing the resources is handled in a decentralized fashion using non-cooperative game. Due to the overlapping of the resources of D2D pairs, co-tier interference amongst D2D pairs exist and should be mitigated. Additionally, D2D pairs coexist on the same subchannels with CUs so cross-tier interference between CUs and D2D pairs do exist and should also be mitigated.
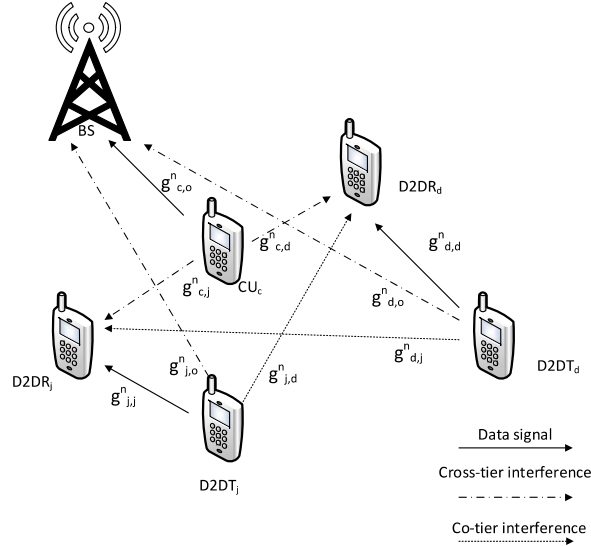

Figure 1: Co-tier and cross-tier interference in D2D transmission reusing the uplink cellular resources.

We focus on uplink transmission since in this case, the victim of cross-tier interference is the BS, which has the capability of handling the interference more efficiently than mobile users, e.g., using smart antenna techniques [23]. We assume that the channel conditions on all subchannels are fixed during the time of interest (e.g., in slow mobility scenario). During the uplink phase the BS is subjected to cross-tier interference from D2D pairs transmitting on the same subchannel with a CU. Let $\mathtt{CU}_c$ be CU c, $\mathtt{D2D}_d$ be D2D pair d, $\mathtt{D2DT}_d$ and $\mathtt{D2DR}_d$ are the transmitter and receiver of $\mathtt{D2D}_d$ respectively. The SINR of $\mathtt{CU}_c$ on subchannel n is formulated as

Table 1: Summary of used notations

Variables	Definition
$\mathcal{D}$	set of D2D pairs
C	set of cellular users
N	number of subchannels
$\mathcal{N}(c)$	set of subchannels assigned to a cellular user
τ	iteration index
$L^{\max}$	maximum number of iterations
$R_{d,ref}$	data of reference D2D pair
$r_c^{\min}$	rate requirement of cellular users
u_d	reward of selfish D2D pairs
U_d	payoff function of socially-aware D2D pairs
$\mathcal{P}_d$	strategy set of D2D pairs
p_d^n	transmit power of D2D pairs d on subchannel n
p_c^n	transmit power of cellular users c on subchannel n
p_{ref}^n	transmit power of reference D2D pair ref_d^n on subchannel n
$p_{d,\max}^n$	allowable transmit power of D2D pairs d on subchannel n
$p_{c,\max}^n$	allowable transmit power of cellular users c on subchannel n
$P_{d,\max}$	transmit power budget of D2D pairs
$P_{c,\max}$	transmit power budget of cellular users
$\mathbf{p}_d$	power allocation vector for D2D pairs
$\mathbf{p}_c$	power allocation vector for cellular users
$\boldsymbol{\theta}$	cross-tier interference price
$\mathbf{t}_d$	co-tier interference taxation term
J	Jain's fairness measure
σ^2	noise variance
$\bar{\sigma}_d^2$	normalized noise power
$\bar{g}_{j,d}^n$	normalized link gain from D2DT$_j$ to D2DR$_d$
$\bar{g}_{r,d}^n$	normalized link gain from D2DT$_r$ to D2DR$_d$

$$\text{SINR}_c^n = \frac{p_c^n g_{c,o}^n}{\text{I}_c^n + \sigma^2},$$

where $\text{I}_c^n = \sum_{d \in \mathcal{D}} p_d^n g_{d,o}^n$ represents the total interference from D2DTs to CU_c, $g_{d,o}^n$ and $g_{c,o}^n$ are the link gains from D2DT_d and CU_c to the BS respectively (Fig. 1). p_c^n and p_d^n are the transmit powers of CU_c and D2D_d on subchannel n respectively, σ^2 is the noise variance. D2DR_d on subchannel n also suffers interference from CU_c on subchannel n and other D2D_j sharing the same subchannel. The SINR of D2D_d pair on subchannel n is formulated as

$$\text{SINR}_d^n = \frac{p_d^n g_{d,d}^n}{p_c^n g_{c,d}^n + \text{I}_d^n + \sigma^2},$$

where $\text{I}_d^n = \sum_{j \in \mathcal{D}, j \neq d} p_j^n g_{j,d}^n$ represents interference from neighbouring D2D pairs, $g_{j,d}^n$ and $g_{c,d}^n$ are the link gains between D2DT_j to D2DR_d and CU_c to D2DR_d on subchannel n respectively. p_d^n and p_j^n are the transmit powers of D2DT_d and D2DT_j on subchannel n respectively. The achievable data rates of CU_c and D2D_d on subchannel n are given respectively as

$$R_c = \sum_{n=1}^{\mathcal{N}(c)} r_c^n,$$

$$R_d = \sum_{n=1}^{N} r_d^n,$$

where $r_c^n = \log_2\left(1 + \text{SINR}_c^n\right)$ and $r_d^n = \log_2\left(1 + \text{SINR}_d^n\right)$ are the data rates of CU_c and D2D_d on subchannel n respectively, $\mathcal{N}(c)$ is the subchannel set assigned to CU_c.

The main aim of this paper is to develop a fairness-aware distributed inteference management scheme for D2D communications underlaid cellular networks while guaranteeing the QoS requirement of CUs. To fulfil these tasks, a taxation scheme is developed at BS to manage the interference resulting from D2D communication while the transmit power of D2DTs are modeled as a non-cooperative power control game. Specifically, the BS charges each D2D_d a price, $\boldsymbol{\theta} = (\theta^n)_{n=1}^N$, for reusing the subchannel n. When the D2D pairs receive the price on each subchannel, they compete for the reuse of the subchannels in a non-cooperative game fashion, we now introduce the game formulation as follow.

Generally, game theory is used to analyze complex decision problems amongst decision makers. One can consider the simplest game model $\mathcal{G}$ given by $\mathcal{G} = \{\mathcal{D}, (\mathcal{P}_d)_{d \in D}, (u_d)_{d \in D}\}$, where the set of players is $\mathcal{D}$, u_d is the reward (the achievable data rate minus the cost paid to the BS) D2D_d pair receives. Each D2D pair selfishly solves the optimization problem below

$$\underset{\mathbf{p}_d \in \mathcal{P}_d}{\text{Maximize}}\ u_d = R_d - \sum_{n=1}^{N} \theta^n p_d^n g_{d,o}^n, \tag{1}$$

where the feasible set of strategies is :

$$\mathcal{P}_d = \left\{ \mathbf{p}_d = [(p_d^n)_{n=1}^N] : \sum_{n=1}^{N} p_d^n \leq P_{d,\max},\ 0 \leq p_d^n \leq p_{d,\max}^n \right\}, \tag{2}$$

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and $P_{d,\max}$ and $p^n_{d,\max}$ are the power budget and maximum transmit power on subchannel n respectively. In the above game formulation, there is no need for exchange of information amongst D2D pairs. However, the D2D pairs performance is sacrificed owing to the rational behaviour of the D2D pairs. To obtain a fair and efficient outcome, we introduced the idea of *reference D2D pair* which represents the typical victim of co-tier interference on each subchannel n [4]. Specifically, the data rate of the reference D2D pair is factored in to the reward which each D2D$_d$ can receive to mitigate the co-tier interference cause to other D2D pairs as a result of the selfishness of D2D$_d$. The reference D2D pair is chosen such that

$$\text{ref}^n_d = \arg \min_{d \in \mathcal{D}} g^n_{d,d}. \tag{3}$$

From each D2D pairs perspective, the data rate of the reference D2D pair is given as

$$R_{d,ref} = \sum_{n=1}^{N} \log_2 \left(1 + \frac{p^n_{\text{ref}^n_d} g^n_{\text{ref}^n_d}}{p^n_c g^n_{c,\text{ref}^n_d} + p^n_d g^n_{d,\text{ref}^n_d} + \sigma^2} \right). \tag{4}$$

Now with the concept of reference D2D pair in hand, unless otherwise specified, we consider the game $\mathcal{G} = \{\mathcal{D}, (\mathcal{P}_d)_{d \in D}, (U_d)_{d \in D}\}$ with the payoff function U_d is given by

$$U_d = R_d + R_{d,ref} - \sum_{n=1}^{N} \theta^n p^n_d g^n_{d,o}. \tag{5}$$

Roughly speaking, $R_{d,ref}$ can be interpreted as a taxation term in each D2D pair optimization problem to adjust its rational behaviour to a more social behaviour. Now each D2D pair aims at solving the maximization problem below

$$\underset{\mathbf{p}_d \in \mathcal{P}_d}{\text{Maximize}} \ U_d. \tag{6}$$

4. D2D Resource Allocation and Cellular Interference Management

In this section, we show that the solution of the game $\mathcal{G}$ and the cellular interference management.

4.1. D2D Resource Allocation

First notice that problem (6) is non-convex owing to the inclusion of the reference D2D pair in the payoff function, fortunately based on the asymptotic result in [4] that the duality gap approaches zero as the number of subchannels increases to infinity. Exploiting this fact, the *best response* (the optimal solution of (6)) for each D2DT$_d$ can be obtained based on the KKT systems [24] and given in *water-filling* form as follow

$$p^n_d = \text{WF}(\mathbf{p}_{-d}; \boldsymbol{\theta})^N_{n=1} \triangleq \left[\frac{1}{\left(\lambda_d + \theta^n g^n_{d,o} \right) \ln 2 + t^n_d} - \frac{p^n_c g^n_{c,d} + \mathrm{I}^n_d + \sigma^2}{g^n_{d,d}} \right]^{p^n_{d,max}}_0, \forall n \in \{1, ..., N\}, \tag{7}$$

where

$$t^n_d = \frac{g^n_{d,\text{ref}^n_d} \text{SINR}^n_{\text{ref}^n_d}}{\left(p^n_c g^n_{c,\text{ref}^n_d} + p^n_d g^n_{d,\text{ref}^n_d} + \sigma^2 + p^n_{\text{ref}^n_d} g^n_{\text{ref}^n_d} \right) \ln 2}, \forall d \in \mathcal{D}, n \in N, \tag{8}$$

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1: **Initialize parameters** $\boldsymbol{p}_d \in \mathcal{P}_d$; $\boldsymbol{\theta} = 0$; and $t_d^n \geq 0$;

2: Reference D2D pair selection according to (3);

3: Set the counter $\tau = 0$ and $L^{\max}$

4: **while** $\tau < L^{\max}$ **do**

5:     **for** $n \in N$; $d \in \mathcal{D}$ **do**

6:         Update $p_d^n(\tau + 1)$ according to (7) ;

7:         Update $t_d^n(\tau + 1)$ according (8);

8:     **end for**

9: **end while**

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and $[b]_x^y = \min\{y, \{\max\{b, x\}\}\}$, $\boldsymbol{t}_d = \left(t_d^n\right)_{n=1}^{N}$ represents the effect of interference D2D$_d$ causes to the reference D2D pair, $\mathbf{p}_{-d}$ is the power vector of all D2Ds except D2D$_d$. The dual variable λ_d is associated with the constraint $\sum_{n=1}^{N} p_d^n \leq P_d$. The optimal dual variables λ_d^* can be obtained such that $\sum_{n=1}^{N} p_d^n = P_d$. The proposed resource allocation algorithm for D2D pairs is summarized in Table 2.

Remark 1. *To determine the reference D2D pair, first each D2D pair reports its own link gain to the BS to choose the reference pair based on* (3). *After this, the* ref_d^n *has to reports the following additional information to the BS:*

- *The interference link gain from other pairs (i.e., $g_{d,ref_d^n}^n$) in* (8).

- *The $SINR_{ref_d^n}^n$ in* (8).

Also the D2DT$_d$ needs to reports its transmit power to the BS, then the BS is able to derive the interference plus noise (i.e. $p_c^n g_{c,ref_d^n}^n + p_d^n g_{d,ref_d^n}^n + \sigma^2$) in (8) and broadcasts all these required information on its control channel to all D2D pairs to update their transmit powers. Note that in this procedure, the burden of collecting and disseminating the information is shifted at the BS side.

Remark 2. *The main difference between Fair Iterative Water-filling Algorithm (FIWA) and Iterative Water-filling Algorithm (IWA) in [25] is the incorporation of the reference D2D pair taxation term t_d^n which controls the effect of interference to reference D2D pair. When reference D2D pair taxation term is not considered, $t_d^n = 0$, FIWA is reduced to IWA. By including $t_d^n > 0$ each D2D pair acts in a more social manner taking into consideration the interference to the reference D2D pair. The complexity of the FIWA for a given reuse price $\boldsymbol{\theta}$ is $\mathcal{O}(N \log_2 N + ND)$ since obtaining the solution of the water-filling system in (7) requires a computational burden $\mathcal{O}(N \log_2 N)$ and selecting the reference D2D pair on each subchannel in (3) has a complexity $\mathcal{O}(ND)$. The convergence properties of FIWA are characterized in Section 5.*

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Table 3: Cellular Link Performance Management (CLPM)

1: **Initialize parameters** $\theta = 0$

2: **while** Any $R_c \left(p_c^n (\tau) \right) < r_c^{\min}, \quad c \in C$ **do**

3:     p_c^n is updated according to (11) $\forall n \in N; c \in C$.

4:     **if** Any $R_c \left(p_c^n (\tau) \right) < r_c^{\min}$ **then**

5:         Update price $\theta^n (\tau + 1) = \theta^n (\tau) + \triangle\theta \; \forall n \in \mathcal{N}(c)$.

6:         $\tau = \tau + 1$;

7:     **end if**

8: **end while**

4.2. Cellular Link Performance Management

The CUs being the primary users in the network who needs protection from the cross-tier interference resulting from D2D communication. The QoS of CU_c is given as

$$R_c \geq r_c^{\min}, \forall c \in C, \tag{9}$$

where $r_c^{\min}$ is the rate requirement of CU_c. Now to perform this task, we first update the transmit power of each $CU_c, \forall c \in C$ first, if it fails to guarantee (9) then the price on the corresponding subchannels in $\mathcal{N}(c)$ will be adjusted until the QoS is met, i.e., each CU_c first solve the following optimization problem

$$\begin{aligned}
\underset{0 \leq \mathbf{p}_c \leq \mathbf{p}_c^{\max}}{\text{Maximize}} \; & R_c \\
\text{s.t.} \sum_{n \in \mathcal{N}(c)} & p_c^n \leq P_{c,\max},
\end{aligned} \tag{10}$$

where $\mathbf{p}_c = [(p_c^n)_{n \in \mathcal{N}(c)}]$, $\mathbf{p}_c^{\max} = [(p_{c,\max}^n)_{n \in \mathbb{N}(c)}]^T$ and $p_{c,\max}^n$ is the highest transmit power allowed on subchannel n, $P_{c,\max}$ is the total transmit power budget. Since each subchannel is exclusively assigned to one CU at the cellular tier, optimization problem (10) is convex, the solution is easily obtained using KKT conditions and is given in water-filling form as

$$p_c^n = \left[\frac{1}{\ln 2\beta} - \frac{I_c^n + \sigma^2}{g_{c,o}^n} \right]_0^{p_{c,\max}^n}, \tag{11}$$

where β is the Lagrange multiplier chosen to meet $\sum_{n \in \mathcal{N}(c)} p_c^n = P_{c,\max}$. At this point, if $R_c \geq r_c^{\min}$, then the cross-tier interference price will not be adjusted, otherwise the BS will adjust the price by

$$\theta^n(\tau + 1) = \theta^n(\tau) + \triangle\theta \quad, \forall n \in \mathcal{N}(c),$$

where $\triangle\theta$ is a positive step size. The Cellular Link Performance Management (CLPM) is summarized in Table 3.

5. Convergence analysis

We now characterize the convergence of the FIWA. We first recall the definition of *Nash Equilibrium (NE)* [3].

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Definition 1. *A power profile* $(\mathbf{p}_d^*, \mathbf{p}_{-d}^*)$ *is a NE of the game* $\mathcal{G}$ *if it is a fixed point of the best response for each D2D$_d$ in* (7)*, i.e.,*

$$U_d(\mathbf{p}_d^*, \mathbf{p}_{-d}^*) \geq U_d(\mathbf{p}_d', \mathbf{p}_{-d}^*), \ \ \forall \mathbf{p}_d' \in \mathcal{P}_d, \forall d \in \mathcal{D}.$$

Intuitively, this means that at the NE, no D2D pair can obtain higher payoff by unilaterally adjusting its transmit power given the transmit powers of other D2DTs remain fixed. In general, it is hard to analyse the existence, uniqueness of the NE of $\mathcal{G}$ since the fixed point of the water-filling system in (7) is not easy to characterize. So we turn to *Variational Inequality (VI)* theory [5], a powerful mathematical tool which is able to address these problems. Let us recall the definition of VI problem as follow.

Definition 2. *Given a closed and convex set* $\mathcal{K} \subseteq \Re^n$ *and a mapping* $\mathbf{F} : \mathcal{K} \to \Re^n$, *the VI problem* $VI(\mathcal{K}, \mathbf{F})$ *is to find a vector* $x^* \in \mathcal{K}$ *(called the solution of the VI) such that [5]:*

$$(\mathbf{y} - \mathbf{x}^*)^T \mathbf{F}(\mathbf{x}^*) \geq 0 \ \forall y \in \mathcal{K}. \tag{12}$$

The similarity between the game $\mathcal{G}$ and VI problem is given in the following Proposition.

Proposition 1. *The NE of* $\mathcal{G}$ *is equivalent to the solution of* $VI(\hat{\mathcal{P}}, \mathbf{F})$*, where* $\hat{\mathcal{P}} \triangleq \prod_d^D \mathcal{P}_d$ *and the mapping* $\mathbf{F}(\mathbf{P}) = [(\mathbf{F}_d(\mathbf{P}))_{d \in \mathcal{D}}]^T$ *is given by*

$$\mathbf{F}_d(\mathbf{P}) = -\nabla U_d(\mathbf{P}), \tag{13}$$

where $\mathbf{P} = (\mathbf{p}_d)_{d=1}^D$.

Proof. If $\mathbf{P}^*$ is the NE of $\mathcal{G}$, then $\mathbf{p}_d^*, \forall d \in \mathcal{D}$ is the optimal solution of the optimization problem (6) and it has to satisfy the maximum principle $(\mathbf{p}_d - \mathbf{p}_d^*)^T \mathbf{F}_d(\mathbf{P}) \geq 0, \forall \mathbf{p}_d \in \mathcal{P}_d$. Summing this condition over all $d \in \mathcal{D}$ and considering the Cartesian product of $\hat{\mathcal{P}}$ leads to a connection between the NE of $\mathcal{G}$ and the solution of $VI(\hat{\mathcal{P}}, \mathbf{F})$. $\square$

We now proceed to introduction of the main Theorems.

Theorem 1. *Given* $\theta \geq 0$ *and* $\mathbf{t}_d$ *is fixed,* $\mathcal{G}$ *always admits a NE.*

Proof. From the result of Proposition 1, the NE of $\mathcal{G}$ is similar to the solution of $VI(\hat{\mathcal{P}}, \mathbf{F})$, it all remains to show that the $VI(\hat{\mathcal{P}}, \mathbf{F})$ admits at least one solution. Based on the result in [5], the $VI(\hat{\mathcal{P}}, \mathbf{F})$ has at least one solution if $\mathbf{F}$ is continuous and $\hat{\mathcal{P}}$ is compact and convex. From definitions (2) and the payoff functions in (5), (12) and (13), it is easy to show that $\mathbf{F}$ is continuous and $\hat{\mathcal{P}}$ is compact and convex, this completes the proof. $\square$

Theorem 2. *Given* $\theta \geq 0$ *and* $\mathbf{t}_d$ *fixed,* $\mathcal{G}$ *admits a unique NE if* $\mathbf{A}$ *is positive definite(PD) where* $\mathbf{A}$ *is defined as*

$$[\mathbf{A}]_{d,j} = \begin{cases} 1, & \text{if } j = d, \\ -\max_{1 \leq n \leq N} \left\{ \bar{g}_{j,d}^n \zeta_{j,d}^n \right\}, & \text{if } j \neq d, \end{cases} \tag{14}$$

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where $\zeta_{j,d}^n$ is given as

$$\zeta_{j,d}^n = \frac{p_{j,\max}^n + \left(\sum\limits_{j=1, r \neq j}^{D} p_{r,\max}^n \bar{g}_{r,d}^n + \bar{\sigma}_j^n \right)}{\bar{\sigma}_d^n},$$

where $\bar{\sigma}_d^n \triangleq \frac{p_c^n g_{c,d}^n + \sigma^2}{g_{d,d}^n}$ and $\bar{g}_{r,d}^n \triangleq \frac{g_{r,d}^n}{g_{d,d}^n}$.

Proof. See the appendix. □

Theorem 3. *The convergence of the FIWA is guaranteed when*

$$\rho\left(\mathbf{H}_{\max}\right) < 1, \tag{15}$$

where $\rho\left(\mathbf{H}_{\max}\right)$ is the spectral radius of $\mathbf{H}_{\max}$ and $\mathbf{H}_{\max}$ is a $D \times D$ matrix defined as

$$[\mathbf{H}_{\max}]_{d,j} = \begin{cases} \max_{1 \leq n \leq N} \bar{g}_{j,d}^n \frac{p_{j,\max}^n}{p_{d,\max}^n}, & \text{if } j \neq d, \\ 0, & \text{otherwise}, \end{cases}$$

where $\bar{g}_{j,d}^n \triangleq \frac{g_{j,d}^n}{g_{d,d}^n}$.

Proof. The solution of a VI($\hat{\mathcal{P}}, \mathbf{F}$) can be seen as euclidean projection of a proper map onto the convex set $\mathbf{P}$ [5]. Therefore, based on the fact that the projection in (7) is non-expansion [5] and the condition $\rho(\mathbf{H}^{max}) \leq 1$ we have for each $d \in \mathcal{D}$

$$\|\text{WF}(\mathbf{p}_{-d}(t), \boldsymbol{\theta}) - \text{WF}(\mathbf{p}_{-d}(t+1), \boldsymbol{\theta})\| \leq \|\mathbf{H}_{\max}(\mathbf{p}_d(t) - \mathbf{p}_d(t+1))\|$$

$$\leq \|\mathbf{H}_{\max}\| \, \|\mathbf{p}_d(t) - \mathbf{p}_d(t+1)\|$$

$$\leq \|\mathbf{p}_d(t) - (\mathbf{p}_d(t+1)\|.$$

This means that the water-filling mapping in (7) is a contraction one, following the Banach Contraction Theorem, the convergence of the FIWA is verified. □

Corollary 1. *The sufficient condition (14) of Theorem 2 for the game $\mathcal{G}$ to admit a unique NE is more binding than the convergence condition (15) of Theorem 3.*

Proof. From Theorem 2, $\mathcal{G}$ admits a unique NE when the matrix $\mathbf{A}$ is PD, since PD matrix is also P-matrix, we have $\rho(\mathbf{I} - \mathbf{A}) < 1$ [25] where $\mathbf{I}$ denote an unit matrix. Therefore $\lim_{l \to \infty} (\mathbf{I} - \mathbf{A})^l = 0$ [26], since $\zeta_d^n > \frac{p_{j,\max}^n}{p_{d,\max}^n}$, then $\lim_{l \to \infty} (\mathbf{H}_{\max})^l = 0$. Because $\rho(\mathbf{H}_{\max}) < 1$ if and only if $\lim_{l \to \infty} (\mathbf{H}_{\max})^l = 0$ [25], we end the proof by concluding that condition (14) in Theorem 2 is more binding compared to the condition (15) in Theorem 3. □

Corollary 1 means that, provided that the condition in (14) of Theorem 2 is met, the uniqueness of the NE is fulfilled and the FIWA globally converges to that NE. The sufficient condition for Theorem 2 and Theorem 3 can be made explicit as follow, based on [27] and [28, theorem 5.6.9] we have

$$\rho(\mathbf{H}_{\max}) = \rho\left((\mathbf{I} - \mathbf{A})^T\right) \leq \|\mathbf{I} - \mathbf{A}\|, \tag{16}$$

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where $\|.\|$ is some matrix norms [28]. Hence, a sufficient condition for (15) is $\|\mathbf{H}_{\max}\|_\infty^\mu \le 1$, where $\|.\|_\infty^\mu$ represents the weighted block maximum norm, given as [28]

$$\|\mathbf{H}_{\max}\|_\infty^\mu \triangleq \max_{d \in \mathcal{D}} \frac{1}{\mu_d} \sum_{j \ne d} \mu_j [\mathbf{I} - \mathbf{A}]_{d,j}, \tag{17}$$

where $\boldsymbol{\mu} \triangleq [\mu_1, ...\mu_D]^T > 0$.

Based on (16) and (17), the necessary conditions for Theorem 2 are given as

$$\max_d \frac{1}{\mu_d} \sum_{j \ne d} \mu_j \max_{1 \le n \le N} \left\{ \bar{g}_{j,d}^n \zeta_{j,d}^n \right\} < 1, \tag{18}$$

and

$$\max_j \frac{1}{\mu_j} \sum_{d \ne j} \mu_d \max_{1 \le n \le N} \left\{ \bar{g}_{j,d}^n \zeta_{j,d}^n \right\} < 1. \tag{19}$$

Also, the conditions in Theorem 3 are

$$\max_d \frac{1}{\mu_d} \sum_{j \ne d} \mu_j \max_{1 \le n \le N} \left\{ \bar{g}_{j,d}^n \frac{p_{j,\max}^n}{p_{d,\max}^n} \right\} < 1, \tag{20}$$

and

$$\max_j \frac{1}{\mu_j} \sum_{d \ne j} \mu_d \max_{1 \le n \le N} \left\{ \bar{g}_{j,d}^n \frac{p_{j,\max}^n}{p_{d,\max}^n} \right\} < 1. \tag{21}$$

Remark: From (18)-(21), the convergence of FIWA and uniqueness of NE in $\mathcal{G}$ are guaranteed if the co-tier interference is not too high [2]. The achievement of NE is independent of the cross-tier interference price vector and cross-tier interference. The above condition can be considered as admission control for a pair to communicate in D2D mode.

6. Simulation Results

In this section, we conduct simulations to access the performance of the CLPM and FIWA. A single cell scenario is considered with radius of 500 m, and the BS is situated at the centre of the cell. CUs and D2Ds are randomly located in the cell at a distance of at least 1m away from the BS to avoid the case where the distance between BS and mobile users is zero. In order for theorem 2 and theorem 3 to be met, the minimum separation amongst any two different D2Ds (D2D$_d$ and D2D$_j$) is set to at least 50 m. An snapshot of the topology is given in Fig. 2.

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[2] Note that this condition may not satisfied in practice when there are too many close by D2D pairs transmit at the same time. In such case, admission control is required to satisfy this condition, e.g., setting a high target SINR in the discovery phase of a D2D pair.
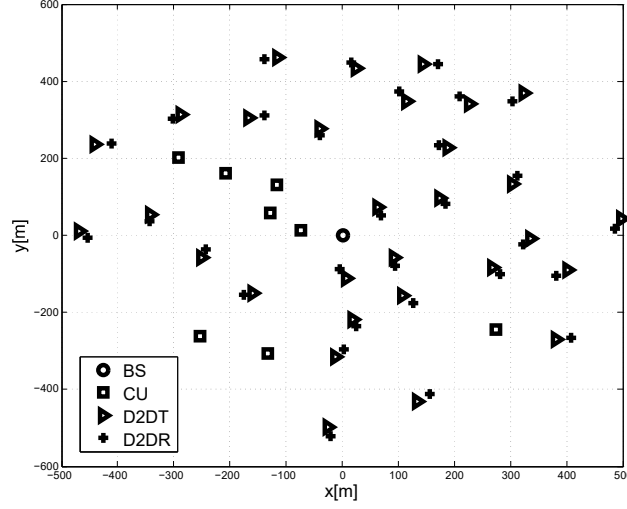
Figure 2: An example of topology.

There are total of 16 subchannels, and each CU is allocated a set of subchannels. The subchannels are allocated to CUs in a Round-Robin manner, i.e., in the first round, one subchannel is allocated to the CU that has the best channel condition, then this CU is excluded in that particular round and the next subchannel is assigned to the next CU with the best link gain. This process is repeated until all subchannels are allocated to all CUs. The path loss is modeled as $37 + 30\log_{10}(d)$, where d (meters) represents the the transmitter - receiver distance [29]. The shadowing is modeled as lognormal random number with zero mean and standard deviation of 4dB. The noise variance is assumed as -120dBm [29]. $P_{c,\max}$ and $P_{d,\max}$ are normalized by the maximum transmit power as 1, $\forall c \in C$ and $\forall d \in \mathcal{D}$ respectively. The rate requirement of the CU_c is $r_c^{\min} = 8.37$ bits/s/Hz, $\forall c \in C$. Finally, we set $\Delta\theta = 3.5 \times 10^5$.
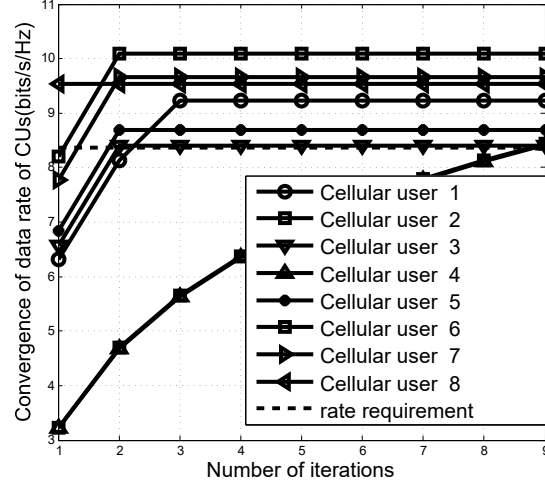
Figure 3: Convergence of the data rate of each CU, $D = 30$.
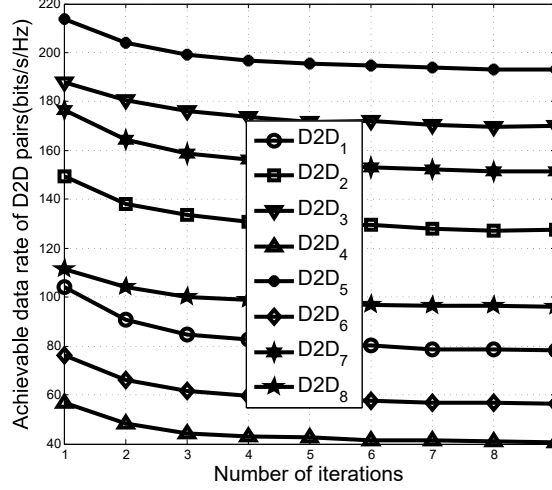

Figure 4: Convergence of the data rates of D2Ds, $C = 8$, $D = 8$.

We first investigate the convergence of the proposed scheme. Fig. 3 and 4 demonstrate the convergences of the CLPM and FIWA respectively for one channel realization. It is observed in Fig. 3 that during the initial iteration, the CUs obtained a low data rate because D2D pairs transmit at high power (note that at this point, the reuse price $\theta = 0$) causing severe interference to the cellular link. The BS senses the interference and updates the price on the corresponding subchannel forcing D2DTs to reduce their transmit powers, this results in the increment of the data rate of CU until it satisfies the target requirement. In Fig. 4, we plot the data rates of D2D pairs during the non-cooperative competition, it can be observed that the FIWA converges within 9 iterations.

In Fig.5 we compare the average data rate of D2D pair from the CLPM and FIWA with some existing algorithms. Note that the optimal solution is centralized and achieves maximum total reward u_d of all $D2D_d$, the optimal solution

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can be obtained by using the transformation $x_d^n = \ln p_d^n$ as in [30]. The Selfish version is the scheme in [21] where each D2D$_d$ rationally maximizes its own reward u_d given in (1). For the scheme in [20], each D2DT selfishly minimizes the cost for reusing the resources while fulfilling its data rate requirement. This suggests that our proposed algorithm obtains the best gain because it manages the co-tier interference amongst D2D pairs. Fig. 5 demonstrates that our proposed algorithm significantly outperform the selfish version, the scheme in [20] and the cellular mode. Note that in Fig. 5, for any D2D pair to operate in the cellular mode, when $C + D \geq N$, more subchannels are added to support D2D pairs to communicate in cellular mode.
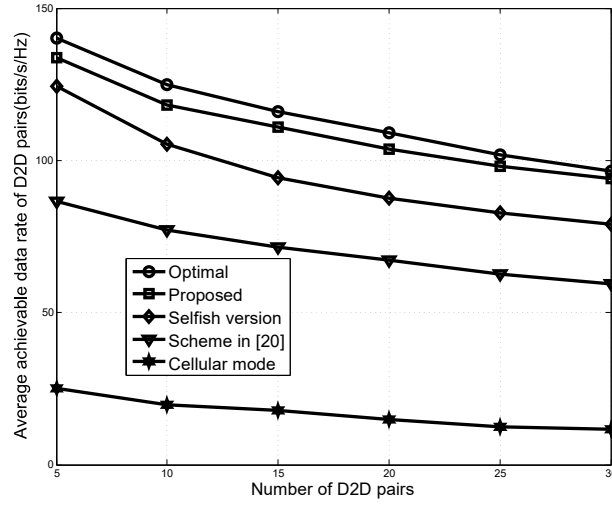

Figure 5: D2D data rates versus the number of D2D pairs, $C = 8$.
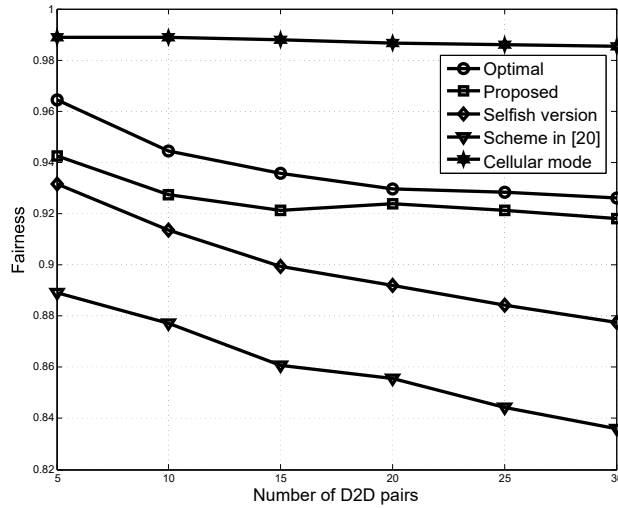

Figure 6: Fairness against the number of D2D pairs, $C = 8$.

Lastly, in Fig. 6 we demonstrate the fairness of the resource sharing based on Jain's fairness measure which is

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derived as [31]

$$J_{(1,...,D)} = \frac{\left(\sum\limits_{d=1}^{D} R_d\right)^2}{D \cdot \sum\limits_{d=1}^{D} R_d^2}, \tag{22}$$

this measurement determines how fair the resources are allocated to D2D pairs, note that $J_{(1,...,D)}^{\max} = 1$ when $R_d = R_j, \forall j \neq d, j, d \in \mathcal{D}$. As expected, cellular mode achieves the highest fairness since the subchannels are orthogonally assigned to CUs using a Round Robin scheme, resulting in roughly similar average data rates for all CUs. We observe that increasing number of D2D pairs decreases the fairness measure of D2D data transmission under the proposed scheme. The proposed algorithm obtains nearly same fairness measure as the optimal solution from the centralized algorithm. Both the selfish version [21] and the scheme in [20] achieved lower fairness measure due to selfish actions of D2D pairs without considering the harm they cause to other D2D pairs.

7. Conclusion

In this work, we proposed a network-assisted distributed fairness-aware interference management scheme for D2D communications underlay cellular networks to improve the data rate of D2D transmission while fulfilling the QoSs of CUs. In order to meet the QoSs of CUs, a taxation scheme was developed at the BS. The transmit power control problem amongst D2D pairs is modeled using non-cooperative game approach. In order to obtain a fair and efficient outcome, reference D2D pair is factored into the payoff function of each D2D pair. Sufficient conditions for the existence and uniqueness of the NE as well as the convergence of the proposed algorithm to the NE are provided using VI theory. Our proposed algorithm is network-assisted distributed and only requires assistance from the network in updating the reuse price and broadcasting the reference D2D pair, hence it is potential for a practical design. Finally, simulation results are given to demonstrate the superiority of the proposed CLPM and FIWA.

Appendix

In this Appendix, we present the Proof of Theorem 2. As shown in [26], the $\text{VI}(\hat{\mathcal{P}}, \mathbf{F})$ has unique solution the mapping $\mathbf{F}$ has strong monotonicity on $\hat{\mathcal{P}}$. Let $\mathbf{P} = (\mathbf{p_d})_{d=1}^{D}$ and $\mathbf{P}' = \left(\mathbf{p'_d}\right)_{d=1}^{D}$, the mapping $\mathbf{F}$ is said to have strong monotonicity on $\hat{\mathcal{P}}$ if a constant $\alpha > 0$ exist such that

$$(\mathbf{p_d} - \mathbf{p'_d})^T (\mathbf{F}_d(\mathbf{P}) - \mathbf{F}_d(\mathbf{P}')) \geq \alpha \|\mathbf{p_d} - \mathbf{p'_d}\|^2. \tag{23}$$

We now define

$$\varphi_d^n \triangleq \sqrt{P_d^n + \sum\limits_{j=1, j \neq d}^{D} p_j^n \bar{g}_{j,d}^n + \bar{\sigma}_d^n} \times \sqrt{(P_d^n)' + \sum\limits_{j=1, j \neq d}^{D} (p_j^n)' \bar{g}_{j,d}^n + \bar{\sigma}_d^n}, \tag{24}$$

$$P_d^n \triangleq \frac{P_d^n - (p_j^n)'}{\varphi_d^n}, \tag{25}$$

From (23), we have the following derivation

$$(\mathbf{p_d} - \mathbf{p'_d})^T(\mathbf{F}_d(\mathbf{P}) - \mathbf{F}_d(\mathbf{P'})) = \sum_{d=1}^{D}\sum_{n=1}^{N} \frac{p_d^n - (p_j^n)'}{\varphi_d^n} \times \left(\frac{(p_d^n) - (p_j^n)'}{\varphi_d^n} + \frac{\sum_{j\neq d} \bar{g}_{j,d}^n (p_d^n - (p_j^n)')}{\varphi_d^n} \right)$$

$$\geq \sum_{d=1}^{D} \left(\sum_{n=1}^{N} (p_d^n)^2 - \sum_{j\neq d} \left| \sum_{n=1}^{N} p_d^n \frac{\bar{g}_{j,d}^n \varphi_j^n}{\varphi_d^n} p_j^n \right| \right)$$

$$\geq \sum_{d=1}^{D} \left(\sum_{n=1}^{N} (p_d^n)^2 - \sum_{j\neq d} \left(\hat{p}_d \max_{1\leq n\leq N} (\frac{\bar{g}_{j,d}^n \varphi_j^n}{\varphi_d^n}) \hat{p}_j \right) \right)$$ (26)

$$\geq \sum_{d=1}^{D} \left(\hat{p}_d \sum_{j=1}^{D} [\mathbf{A}]_{j,d} \hat{p}_j \right) = \hat{\mathbf{p}}^T \mathbf{A} \hat{\mathbf{p}}$$

with

$$\hat{p}_d = \left(\sum_{n=1}^{N} (P_d^n)^2 \right)^{\frac{1}{2}}, \hat{\mathbf{p}} = [\hat{p}_d]_{d=1}^{D},$$ (27)

where (26) follows Cauchy-Shwarz inequality. Since matrix $\mathbf{A}$ is PD and also a P-matrix [27], it follows the Theorem 3.3.4(b) in [32] and Lemma 2 in [27], that the constant $c(\mathbf{A}) \triangleq \min_{\|x\|_2=1} \{\max_{1\leq d\leq D} \{x_d(\mathbf{A}x)_d\}\}$ is positive. Also

$$\max_{1\leq d\leq D} x_d(\mathbf{A}x)_d \geq c(\mathbf{A}) \|x\|_2^2,$$ (28)

equation (43) holds for an arbitrary $x \in \mathrm{R}^D$. At this point, joining (27), (28) and summing over all d, we obtained

$$(\mathbf{P} - \mathbf{P'})^T(\mathbf{F}(\mathbf{P}) - \mathbf{F}(\mathbf{P'})) \geq \frac{\eta_{\min}(\mathbf{A})}{\max_{1\leq d\leq \mathcal{D}} \max_{1\leq n\leq N} \left(\varphi_{d,\max}^n \right)^2} \|\mathbf{P} - \mathbf{P'}\|_2^2,$$ (29)

where

$$\varphi_{d,\max}^n = p_{d,\max}^n + \left(\sum_{j=1,j\neq d}^{D} p_{d,\max}^n \bar{g}_{j,d}^n + \bar{\sigma}_d^n \right),$$ (30)

and $n_{\min}(\mathbf{A}) > 0$ representing the smallest eigenvalue in $\mathbf{A}$. From the last inequality (29) it can be verified that strong monotonicity property of $\mathbf{F}$ exist which ends the proof.

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