## Activities

In working directory.

- > git init
- > git pull git@github.com:brittlundgren/py-astro-stat.git master

Create a directory for each week so far.

Your activity will be located in the week 5 directory.

The data you need for the activity will be in the 'data' directory.

# Python Astro Statistics

Statistics, Data Mining, and Machine Learning in Astronomy Chapter 4½

June 17<sup>th</sup>, 2014
Ben Tofflemire & Elijah Bernstein-Cooper

# Today's Lesson

#### Lecture

- 1. Bootstrapping
- 2. Comparing Distributions
- 3. Selection Effects & The Luminosity Function
- 4. Histogram Bins and Errors
- 5. Some helpful Python tips

Group Activities

Group Presentations

# 1. Bootstrapping Motivation

I want to estimate the uncertainty on a measurement.

### Problem:

I have no idea what the underlying error distribution is. I cannot simply use the analytical formulae Karen gave me last week!

#### Solution:

Who cares what the underlying distribution is, lets bootstrap it!

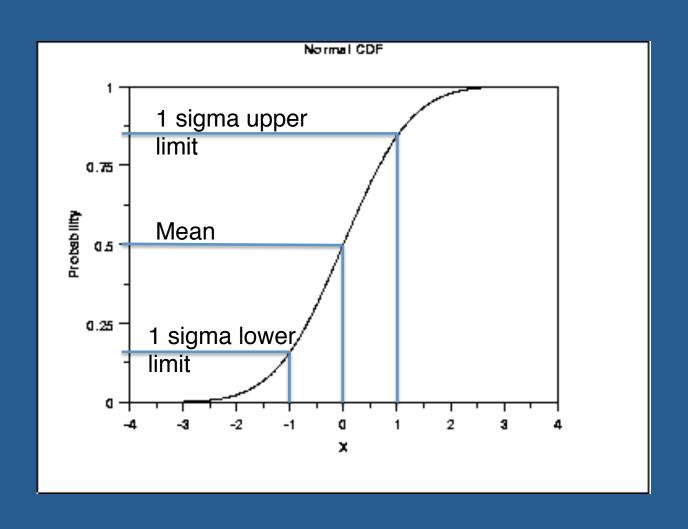
# **Bootstrapping Method**

Example: Error in the Mean

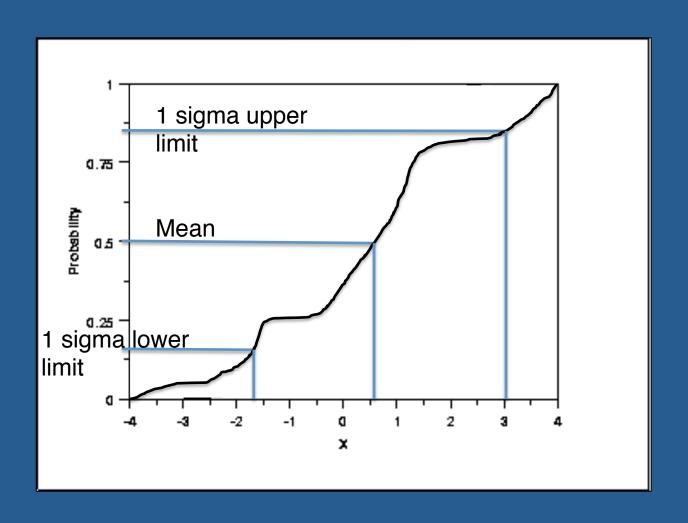
- 1) From your original sample, create N new samples by randomly drawing data from the original with replacement.
  - Make sure it is a <u>UNIFORM</u> random distribution!

- 2) Calculate the Mean for each resampling
- 3) Create a CDF of the resampled means.
  - This is your error distribution!

# **Bootstrapping Method**



# Bootstrapping Method





# Jackknife Alternative



# 2. Comparing Distributions

#### Motivation:

Is my distribution Gaussian?

 Can I use the analytical formulae Karen gave me last week?

Are two samples drawn from the same distribution?

#### Solution:

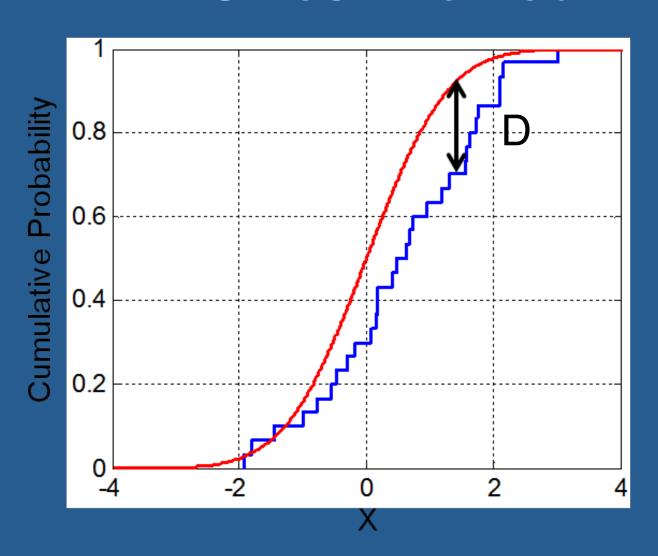
Kolmogorov-Smirnov (K-S) Test to the rescue!

Is my distribution Gaussian?

- From your sample create a CDF.
- Find the maximum vertical difference between your
   CDF and a Gaussian CDF ('sup' = Supreme).

$$D_n = \sup_{x} |F_n(x) - F(x)|$$

If your sample is Gaussian, D -> 0 as n -> infinity.



• The D value corresponds to a probability  $(\alpha)$  that the two samples were drawn from the same distribution (depends on number of data points in the sample).

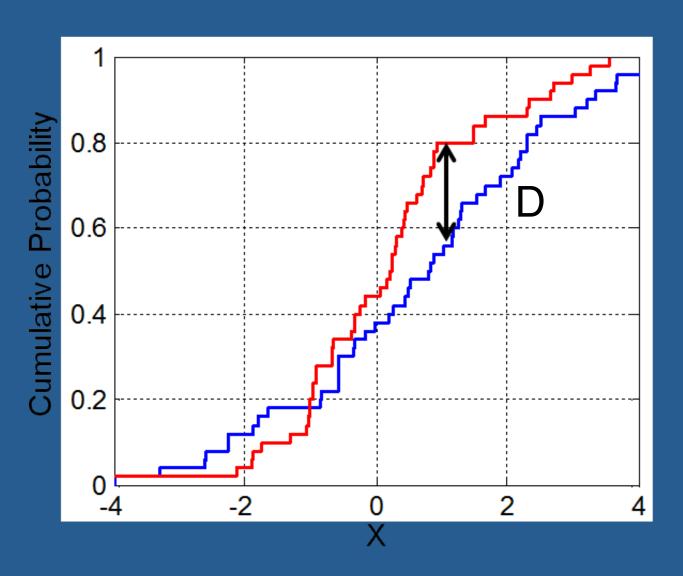
#### Caveat:

If  $\alpha$  is above some limit it does <u>not</u> explicitly tell you your distribution *is* Gaussian, it tells it is *indistinguishable* from a Gaussian distribution.

### **Upside:**

Extremely flexible and simple.

Does not require well sampled data in principle.



# 2. Comparing Distributions

There a plethora of other tools besides the K-S test to compare a sample with a Gaussian distribution.

### A few options are:

- Anderson Darling
- Shapiro-Wilk
- Z<sub>1</sub>
- Z<sub>2</sub>

All have their own strengths and weaknesses detailed in Chapter 4.7

#### Problem:

Often an observed distribution (f(x)) will be biased from the true distribution (h(x)) by some selection function (S(x)).

In General:

$$h(x) = f(x) / S(x)$$

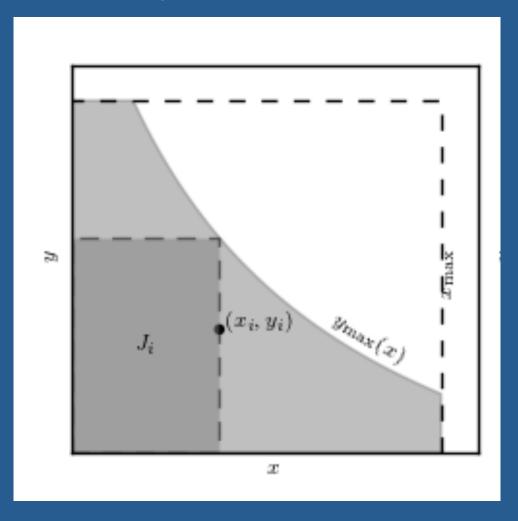
**Example: The Luminosity Function** 

The number of galaxies per unit luminosity (or absolute magnitude M+dM) per unit volume.

Analytical from from Schechter (1979).

$$\phi(M) = \frac{\ln 10}{2.5} \phi^* 10^{0.4(\alpha+1)(M-M^*)} exp \left[ -10^{0.4(M-M^*)} \right]$$

**Example: The Luminosity Function** 



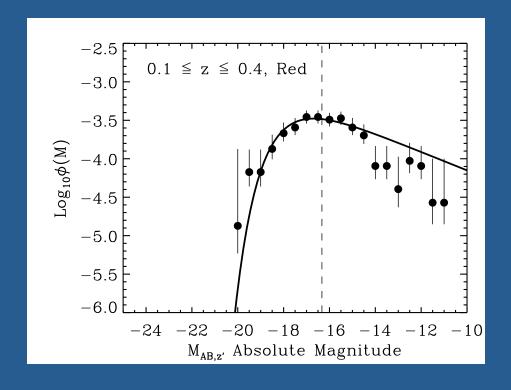
**Example: The Luminosity Function** 

$$\phi(M) = \frac{\ln 10}{2.5} \phi^* 10^{0.4(\alpha+1)(M-M^*)} exp \left[ -10^{0.4(M-M^*)} \right]$$



M\*:



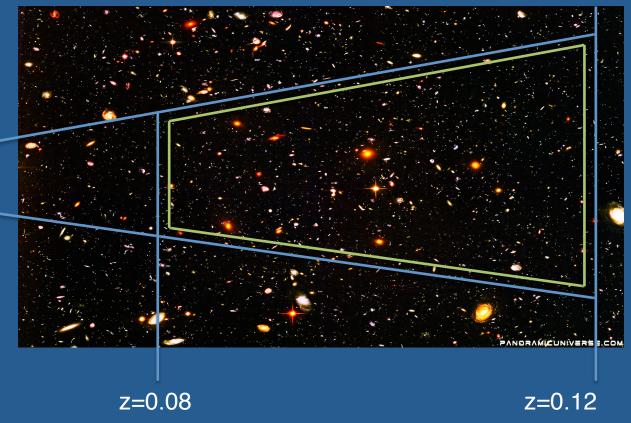


Example: The Luminosity Function Say we care to probe galaxies with -11 < M < -21 in the volume 0.08 < z < 0.12.

Telescope z = 0.08z=0.12

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M = 17



Telescope

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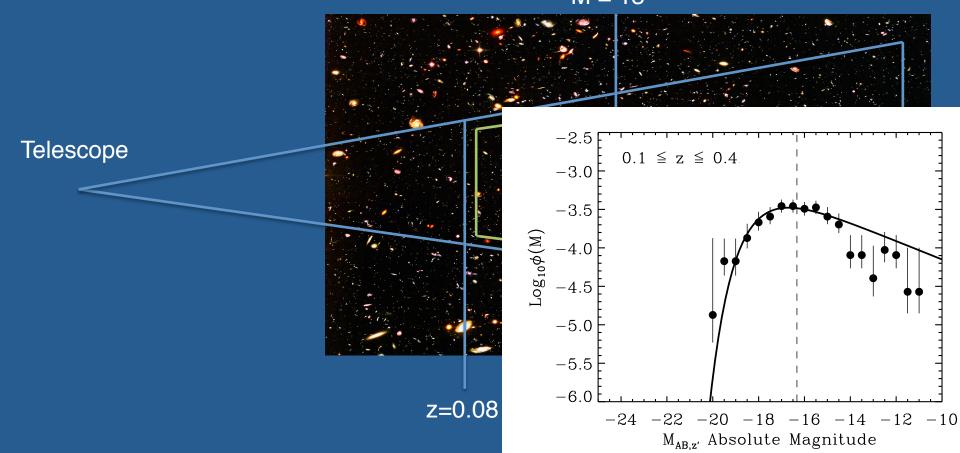
Telescope

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Telescope

Example: The Luminosity Function Say we care to probe galaxies with -11 < M < -21 in the volume 0.08 < z < 0.12. M = 13



Example: The Luminosity Function

1/V<sub>max</sub> Method:

Advantage:

Conceptually simple and easy to implement.

Disadvantage:

Assumes the Luminosity function is constant with increasing distance.

#### Alternative:

Lynden-Bell C- method

- Does not assume LF is constant
- Does not make sense to me, but hey, it's in chapter 4.9

# 4. Histogram Bins and Errors

When making a histogram:

How big do you make your bins?

How do you calculate the error on that bin?

# 4. Histogram Bins and Errors

### When making a histogram:

- How big do you make your bins?
  - Scott:  $\Delta_{\rm b} = 3.5 \, {\rm o} / {\rm N}^{1/3}$
  - Freedman-Diaconis:  $\Delta_b = 2 (q_{75} q_{25}) / N1/3 = 2.7\sigma_G / N^{1/3}$
- How do you calculate the error on that bin?
  - If the number of data point is a bin is large,
    - $\sigma = \sqrt{N}$  (Gaussian)
  - If the number is data points per bin is below ~15
    - Have to use a Poisson formalism.

# 5. Helpful Python Tips

Docstrings are descriptive text for a function.

To read descriptions use the help() function. For lpython: '?' after any variable, function, class, or module

#### Example:

>>> import numpy as np

>>> help(np.argmax)

Written by " text " after a function definition

Help on function argmax in module numpy.core.fromnumeric:

argmax(a, axis=None)

Indices of the maximum values along an axis.

#### Parameters

```
a : array_like
    Input array.
    axis : int, optional
    By default, the index is into the flattened array, otherwise along the specified axis.
```

#### Returns

```
index_array : ndarray of ints
    Array of indices into the array. It has the same shape as `a.shape`
    with the dimension along `axis` removed.
```

#### See Also

```
ndarray.argmax, argmin
amax : The maximum value along a given axis.
unravel_index : Convert a flat index into an index tuple.
```

#### Notes

In case of multiple occurrences of the maximum values, the indices corresponding to the first occurrence are returned.

#### Examples

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