A Brief Introduction to Bayesian Inference

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Two Schools of Thought

- Classical (Frequentist)
  - Focused on computing some quantity (mean, std dev, confidence interval, etc.) from a set of observations.
- Bayesian
  - Focused on estimating the probability of some parameter (observed or from a model) given some prior knowledge of the domain.

Some Terminology

- Probability Density (distribution) Function
  - Usually just called the pdf. It gives the probability of obtaining a value X. Written P(X).
- Conditional Probability
  - The probability of obtaining some value X given that you have obtained some value Y. Written as P(X|Y).
- Joint Probability
  - The probability of A and B happening together. It has the definition P(AB) = P(A|B)P(B)

An Identity

- Since A and B are arbitrary, then it follows that P(AB) = P(BA)
- Applying our definition of joint probability, we find

$$P(AB) = P(A|B)\,P(B) = P(B|A)\,P(A) = P(BA)$$

$$=> P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Bayes' Theorem

Comparing my observed data, D, to some model, M.

likelihood

My initial belief in my model; the *prior*.

Having accounted for my data, this is my updated belief in my model; the *posterior*.

$$P(M|D) = \underbrace{\frac{P(D|M)}{P(D)}}\,P(M)$$

The support that the data provides for the model.

Using Bayes' Theorem

- Developing Bayes' Theorem was nearly trivial, but employing it in real calculations can be a very daunting task.
- There are several steps which must be done to utilize Bayes' Theorem correctly.

Using Bayes' Theorem (cont.)

- Formulate the data likelihood function P(D|M)
- Choose a prior
- Determine the posterior P(M|D)
- Search the model space to find the maximum a posteriori estimate.
- Quantify uncertainty using credible regions (these are much like confidence intervals).