OLABISI ONABANJO UNIVERSITY, AGO-IWOYE

FACULTY OF SOCIAL AND MANAGEMENT SCIENCES

DEPARTMENT OF ECONOMICS

2014/2015 HARMATTAN SEMESTER EXAMINATION

COURSE CODE/TITLE: ECO303 – MATHEMATICS FOR ECONOMISTS

INSTRUCTIONS: ANSWER ALL QUESTIONS IN SECTION A AND ANY OTHER TWO FROM SECTION B

TIME ALLOWED: 2HRS 15MINUTES

SECTION A

i. Verify Euler's theorem for the function $Z = x^{-3}\log\frac{y}{x}$

ii. Find the total derivatives, $\frac{df}{dx}$ for $F = \frac{9x-7y}{2x+5y}$, where $y = 3x - 4$

iii. Evaluate $\int \frac{e^{3x}}{e^{3x}-1}\,dx$

Solve the following differential equations

iv. $(4y + 8t^2)dy + (16yt - 3)dt = 0$ v. $(12y^2t^3 + 10y)dy + (8y^3t)dt = 0$

Specify the order and the degree of the following differential equations

vi. $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{d^4y}{dx^4} - 75 = 0$ vii. $\left(\frac{d^2y}{dx^2}\right)^6 + \frac{d^3y}{dx^3} = 4 - y$

viii. Solve for the values of x and y in the matrix equation $\begin{vmatrix} 5 \\ 7 \end{vmatrix}(x) = \begin{vmatrix} 15 \\ y \end{vmatrix}$

ix. Given that $\frac{dy}{dt} + 4y = 12$, find $y(t)$ and establish the dynamic stability of $y(t)$

x. Use a general solution to solve this equation, $\frac{dy}{dt} + 4yt = 6t$

SECTION B

2(a) Given the demand function, $P = -Q_d^2 - 4Q_d + 68$ and the supply function, $P = Q_s^2 - 2Q_s + 12$. Calculate the (i) Consumer's Surplus (ii) Producer's Surplus

(b) Find the demand function $Q = f(P)$, if own price elasticity of demand, $\varepsilon = \frac{5p^2 + 2p}{q}$ and $Q = 500$, when $p = 10$

(c) The marginal revenue function of a monopolistic producer is $MR = 10 - 4Q$. Find (i) the total revenue function and (ii) the corresponding demand equation

3(a) Maximise the production function of a given firm: $Q = AL^{\alpha}K^{\beta}$ subject to a cost outlay $C = wl + rk$ where, l = labour, k = capital, A = efficiency parameter, α, β = input share parameters, w and r are the prices of labour and capital.

(b) A monopolistic producer of two goods G1 and G2 has a total function $TC = 5Q_1 + 10Q_2$, where Q_1 and Q_2 are quantities; G_1 and G_2 respectively. If P_1 and P_2 are the corresponding prices, then the demand equations are

$P_1 = 50 - Q_1 - Q_2$

$P_2 = 100 - Q_1 - 4Q_2$

Find the maximum profit, if the firms total costs are fixed at 100

4(a) Minimise a firm's total cost $C = 45K^2 + 90KL + 90L^2$ when the firm has to meet the production quota equal to $2K + 2L = 60$, by finding the quantity of capital and labour, using the Bordered Hessian Matrix, test if the cost is really minimised.

(b) Use Langrangiers multipliers method to find expressions for x_1 and x_2, which maximise the utility function $U = x_1^{1/2} + x_2^{1/2}$ subject to the general budgetary constraint $P_1X_1 + P_2X_2 = M$

5(a) The demand function for a commodity takes the form $Qd = a + bp + c/p$ for some constants a,b,c. When $p=1$, the quantity demanded is 60, when $p = 2$, it is 40, and when $p = 4$ it is 15. Use the technique of Gaussian elimination or matrix reduction to find the value for the constants a, b, and c.

(b) Solve the system of equations

$4x_1 + x_2 + 3x_3 = 8$

$-2x_1 + 5x_2 + x_3 = 4$

$3x_1 + 2x_2 + 4x_3 = 9$

Using Cramer's rule, find x_1, x_2 and x_3

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$