S14/5113/14

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FACULTY OF SOCIAL AND MANAGEMENT SCIENCES
DEPARTMENT OF ECONOMICS
2014/2015 HARMATTAN SEMESTER EXAMINATION

COURSE CODE/TITLE: ECO205 – INTRODUCTORY MATHEMATICS FOR ECONOMISTS

INSTRUCTIONS: ANSWER ALL QUESTIONS IN SECTION A AND ANY OTHER TWO FROM SECTION B

$\int(7e^{-x}+\frac{2}{x})\partial x$

$\int(7e^{x}+2^{-x})$

TIME ALLOWED: 2HRS

Young theorem holds when $Z_{xy}=Z_{yx}$

SECTION A

$z=6y^3+6x^2+8xy^2+10$

$Z_y=18y^2+16xy$

$Z_x=12x+8y^2$

$Z_{xy}=16y$

$Z_{yx}=16y$ young theorem holds. $Z_{xx}\neq Z_{yy}$

$Z_{yy}=36y+16x$

$Z_{xx}=12$

It does not hold when $Z_{xx}=Z_{yy}$

1a. Differentiate $y = e^{x^2+5x+9}\ 6x^2$

b. Derive from first principle, $\frac{\partial y}{\partial x}$ when $y = 3x^{-2}$

c. Evaluate $\int(7e^{-x} + \frac{2}{x})\, dx$

d. Evaluate $\int_1^3 6x^2(x+1)dx$

e. If $Z = 6y^3 + 6x^2 + 8xy^2 + 10$, find Zy, Zx, Zxy, Zyx, Zyy, Zxx and ascertain Young's theorem

f. Given a firm's marginal cost function as $MC = Q^2 + 2Q + 4$
Find the total cost function, if the fixed cost is 100.

g. Given a Savings function as; $S = 0.02y^2 - y + 100$, find the marginal propensity to save (MPS) and the marginal propensity to consume (MPC) when $y = 40$

h. Given $P = -Q^2 - 10Q + 150$, find the price elasticity, when $Q = 4$

$-\frac{Q_d}{P}\cdot\frac{P}{Q}$

i. Given $U = x_1^{1/4}x_2^{3/4}$ find the marginal rate of commodity substitution in terms of x_1 and x_2, when $x_1 = 100$ and $x_2 = 200$

(30 MARKS)

SECTION B

2(a) Given a Cobb-Douglas production function, $Q = Al^{\alpha}K^{\beta}$, where Q = output, L = labour, K = capital, A = efficiency parameter, α, β = input shares.
(i) Use Euler's theorem to verify that the function is linearly homogenous and
(ii) Determine the marginal rate of technical substitution

(b)If a firms total cost function is given as TC = 50,000 + 20Q and it charges different prices P_1 and P_2 at two different markets with output Q_1 and Q_2. If the demand equation in each of the markets respectively is given by:

$P_1 + Q_1 = 500$

$2P_2 + 3Q_2 = 720$

$MRCS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{0.4}{0.62} = 0.67$

Find the prices in each market, that maximize profit. What is the maximum profit?

(20marks)

when $U=x_1^{1/4}x_2^{3/4}$ find MRCS when $x_1=100$ and $x_2=200$.

$\frac{3}{4}-\frac{1}{1}=\frac{3-4}{4}=\frac{-1}{4}$

$-3/4\ x_2^{3/4}$

$\frac{\partial U}{\partial x_1}=\frac{3}{4}x_1^{1/4}x_2^{3/4-1}$

0.75

3(a) The Demand and Total cost function of a commodity is given by

$4P + Q - 16 = 0$ and Total cost, $TC = 4 + 2Q - \frac{3Q^2}{10} + \frac{Q^3}{20}$.

Determine

(i) the value of Q, that maximizes profit

(ii) Show that MR = MC at this point

(b)Given a demand function $Q = 20 - 5P^2$, estimate the elasticity of demand at P = 2 and P = 3, and state the nature of the elasticity.

(20marks)

4(a) Given the utility function of an individual as $U = 2X_1X_2 + X_2$ where U is total utility, X_1 and X_2 are the quantities of the two commodities consumed, find

(i) the marginal utilities of the two commodities

(ii) the value of the marginal utility of the first commodity, when three (3) units of each commodity is consumed.

(b)Given a demand function, $Q = 100 - 2P_1 + P_2 + 0.2Y$, where $P_1 = 10$, $P_2 = 12$ and $Y = 2000$.

Find the (i) Price elasticity of demand (ii) Cross Price elasticity of demand and (iii) Income elasticity of demand.

(20marks)

5(a) A perfectly competitive firm produces two commodities G_1 and G_2 at #1000 and #1800 each respectively. If the total cost function, $TC = 2Q_1^2 + 2Q_1Q_2 + Q_2^2$, where Q_1 and Q_2 are the output levels of G_1 and G_2, Find (i) the values of Q_1 and Q_2 that maximises this profit (ii) the maximum profit at these values.

(b)The demand function for a firm in two separate markets is given by

$P_1 = 50 - 5Q_1$

$P_2 = 30 - 4Q_2$

If the total cost of the firm is $TC = 10 + 10Q$, find the prices P_1 and P_2 that maximises profit of the firm, and what is the maximum profit?

(20marks)