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DEPARTMENT OF MATHEMATICAL SCIENCES

2007/2008 HARMATTAN SEMESTER EXAMINATION

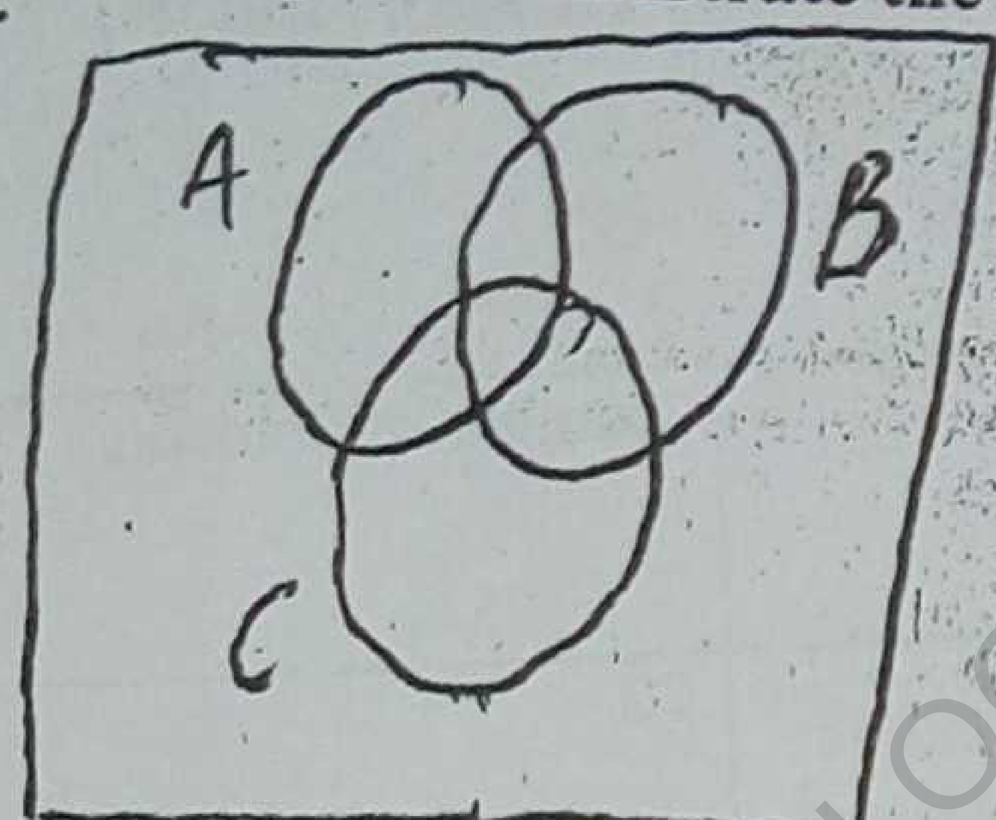
COURSE CODE: MAT 101

COURSE TITLE: ELEMENTARY MATHEMATICS 1

Time: 2hr

INSTRUCTION: ANSWER ALL QUESTIONS

1. Draw Venn diagram to illustrate the set $(A \cap B) - (A \cap B)$



$(A \cap B) - (A \cap B) = (A \cap B) \cap (A \cap C)'$

2. Simplify $(A \cap B' \cap C') \cup (A \cap B' \cap C)$

$(A \cap B' \cap C') \cup (A \cap B' \cap C)$

$[A \cap (B' \cap C)] \cup [A \cap (B' \cap C)]$

$A \cap [(B \cup C) \cup (B \cap C)]$

$A \cap [(B \cup C)' \cup (B' \cap C)]$

3. For what values of x is $3x^2 - x + 1 < x + 2$?

$3x^2 - x + 1 < x + 2$

14

$3x^2 - x - x + 1 - 2 < 0$

$3x^2 - 2x - 1 < 0$

$3x^2 - 3x + x - 1 < 0$

$3x(x-1) + 1(x-1) < 0$

$(3x+1)(x-1) < 0$

$3x+1 < 0$ or $x-1 < 0$

$x < -\frac{1}{3}$ or 1

4. **simplify $3\sqrt{2/5}$ - $\sqrt{5/2}$ + $2\sqrt{40}$**

$3\sqrt{\frac{2}{5}} - \sqrt{\frac{5}{2}} + 2\sqrt{40}$

$\frac{3\sqrt{2}}{\sqrt{5}} - \frac{\sqrt{5}}{\sqrt{2}} + 2\sqrt{10 \times 4}$

$\frac{3\sqrt{2}}{\sqrt{5}} - \frac{\sqrt{5}}{\sqrt{2}} + 4\sqrt{10}$

$\frac{3\sqrt{2}\cdot\sqrt{2} - \sqrt{5}\cdot\sqrt{5} + 4\sqrt{10}\cdot\sqrt{10}}{\sqrt{10}}$

$\frac{6-5+40}{\sqrt{10}}$

$\frac{41}{\sqrt{10}}$

$\frac{41}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{41\sqrt{10}}{10}$

5. **Express $\frac{2x^2 - 4x + 1}{(x-1)^2(x-2)}$ as partial fractions**

$\cancel{2x^2-4x+1}$

$\frac{x^2-4x+1}{(x-1)^2(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)}$

$x^2-4x+1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$

$= A(x^2-3x+2) + B(x-2) + C(x^2-2x+1)$

$= x^2(A+C) + x(-3A+B-2C) + (2A-2B+C)$

Equate co-efficient

$x^2 \Rightarrow A + C = 2$ --- (i)

$x \Rightarrow -3A + B - 2C = -4$ --- (ii)

Constant $\Rightarrow 2A - 2B + C = 1$ --- (iii)

From eqn (i)

$A = 2 - C$ --- (iv)

Substitute $A = 2-C$ in e[q]

$-3(2-C) + B - 2C = -4$

$-6 + 3C + B - 2C = -4$

$B + C = -4 + 6 =$

$B + C = 2$ ---

Substitute $A = 2-C$ in eq

$2(2-C) - 2B + C = 1$

$4 - 2C - 2B + C = 1$

$2B + 2 = 3$ --- (

Solve eqn (v) and (vi)

$$B + C = 2$$
$$2B + C = 3$$
$$-B = 1$$
$$B = -1$$

Substitute $B = -1$ in eqn (5)

$$1 + C = 2$$
$$C = 1$$

Substitute $C=1$ in

$$A = 2 - 1 = 1$$

$$\frac{2x^2-4x+1}{(x-1)^2(x-2)} \equiv \frac{1}{(x-1)} + \frac{1}{(x-\ldots}$$

6. Evaluate in terms of In2. $\sum^{100}_{n=1} \ln 2^n$

$$\sum^{100}_{n=1} \ln 2^n$$
$$= \ln 2^1 + \ln 2^2 + \ln 2^3 + \ln 2^4 + \ldots + \ln 2^{100}$$
$$= \ln 2 + 2\ln 2 + 3\ln 2 + 4\ln 2 + \ldots + 100\ln 2$$
$$= \ln 2(1+2+3+4+\ldots+100)$$
$$= \ln 2(5050)$$

$1-20=210$

7. A sequence is denied by the rule $U_1 = 0$, $U_2 = 2$ and $U_r = U_{r-1} - U_{r-2}$ fo
sum of the first six terms in the sequence.

$$U_1 = 0, \quad U_2 = 2$$
$$U_3 = U_2 - U_1$$
$$= 2 - 0 = 2$$
$$U_4 = U_3 - U_2 = 2 - 2 = 0$$
$$U_5 = U_4 - U_3 = 0 - 2 = -2$$
$$U_6 = U_5 - U_4 = -2 - 0 = -2$$

$$U_1 + U_2 +$$
$$= 0 + 2 +$$
$$= 0$$

hemistry. Find

three times the other.

8. The sum of the first eight terms of an A.P is 60 and the sum of the next six term is 108. Find the first term.

$S_n = 60$; $S_{14} - S_8 = 108$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_8 = \frac{8}{2}\left[2a + (8-1)d\right]$

$S_8 = 4(2a + 7d) = 60$

$8a + 28d = 60$

$2a + 7d = 15$ ---(i)

$S_{14} = 7(2a + 13d)$

$S_{14} - S_8 = 7(2a + 13d) - (2a + 7d) = 108$

$= 14a + \overset{21}{\cancel{91}}d - 8a - 28d = 108$

$6a - 63d = 108$

$2a + 21d = 36$ ---(ii)

Solve eqn (i) & (ii) simultaneously

$2a + 7d = 15$

$2a + 21d = 36$

$-14d = -21$

$d = 3/2$

$2a + 7\left(\frac{3}{2}\right) = 15$

$a = 9/4$

9. The first term of a geometric progression is two thirds the sum to infinity. Find the common ratio.

$a = \frac{2}{3}\left(\frac{a}{1-r}\right)$

$a = \frac{2a}{3-3r}$

$-3ar = 2a - 3a$

$-3ar = 2a - 3a$

$-3ar = -a$

$r = \frac{-a}{\ldots} = \frac{1}{\ldots}$

10. Expand $(1 + x + x^2)^3$ in powers of x

$$[(1+x) + x^2]^3 = (1+x)^3 + 3(1+x)^2x^2 + 3(1+x)(x^2)^2 + (x^2)^3$$

$$= (1+x)^3 + 3x^2(1+x)^2 + 3x^4(1+x) + x^6$$

$$(1+x)^3 \Rightarrow 1 + 3x + 3x^2 + x^3$$

$$(1+x)^2 \Rightarrow 1 + 2x + x^2$$

$$\therefore [(1+x) + x^2]^3$$

$$\Rightarrow 1 + 3x + 3x^2 + x^3 + 3(1 + 2x + x^2) + 3x^4(1+x) + x^6$$

$$= 1 + 3x + 3x^2 + x^3 + 3x^2 + 6x^3 + 3x^4 + 3x^5 + x^6$$

$$= 1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6$$

11. The roots of the equation $px^2 + qx + 4 = 0$ are α and β. Find in terms of p and the values of $\alpha^2 + \beta^2$

$$px^2 + qx + 4 = 0$$

$$\alpha + \beta = -q/p$$

$$\alpha\beta = \frac{4}{p}$$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= \left[\frac{-q}{p}\right]^2 - 2\left[\frac{4}{p}\right]$$

$$= \frac{q^2}{p^2} - \frac{8}{p}$$

$$= \frac{q^2 - 8p}{p^2}$$

15. Find the modulus and amplitude of $\frac{(2-i)^2(3-i)}{3+i}$

$\frac{(2-i)^2(3-i)}{3+i} = \frac{(2-i)(2-i)(3-i)}{3+i}$

$= \frac{(4-2i-2i+i^2)(3-i)}{3+i}$

$= \frac{(4-4i+i^2)(3-i)}{3+i}$

Modulus is $\underline{5}$ and Amplitude is $\underline{0}$

$i^2 = -1$

$\frac{(2-i)^2(3-i)}{3+i} = \frac{(4-1-4i)(3-i)}{3+i}$

$= \frac{(3-4i)(3-i)}{3+i}$

$= \frac{9-3i-12i+4i^2}{3+i}$

$= \frac{9-15i-4}{3+i} =$

$\frac{5-15i}{3+i}$.

$\frac{5-15i}{3+i} = \frac{5-15i}{3+i} \times \frac{3-i}{3-i}$

$= 5i$

$z = 0-5i$

$r = \sqrt{x^2+y^2} = \sqrt{0^2+(-5)^2} =$

16. The first two terms in the expansion of $(2 + ax)^n$ are $1024 + 15360x$. find a and n.

$(2+ax)^n = 2^n + {}^nC_1\, ax2^{n-1} + {}^nC_2\, x2^{n-2}$

first term $\Rightarrow 2^n = 1024$

$2^n = 2^{10}$

$n = 10$

Second term $\Rightarrow {}^nC_1\, ax2^{n-1} = 15360x$

$anx2^{n-1} = 15360x$

$an2^{n-1} = 15360$

$n = 10$

$\Rightarrow a \times 10 \times 2^{10-1} = 15360$

$10a \times 2^9 = 15360$

$5120a = 15360$

$a = \frac{15360}{512}$

$a = \underline{3}$

$a = 3, \quad n = \underline{10}$

17. Express 3cosx- 2sinx in the form rcos(x +α), giving the values of r and α

$3\cos x - 2\sin x$

$\sin x = r\cos(x-\alpha)$ --- (1)

$\sqrt{a^2+b^2}$

than

$\tan\alpha = \frac{-r\sin\alpha}{r\cos\alpha}$

$-b$

$\alpha = \tan^{-1}(-0.6666)$

$\alpha = -33^0 41'$

$\therefore 3\cos x - 2\sin x = \sqrt{13}\cos(x-(-