OLABISI ONABANJO UNIVERSITY, AGO-IWOYE FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICAL SCIENCES

COURSE CODE: STA 462

COURSE TITLE: DECISION THEORY

DATE: JULY 2018 TIME ALLOWED: 2 HOURS CREDITS: 3

Instruction: Answer any FOUR QUESTIONS(GRAPH PAPERS SUPPLIED)

1(a) Give two axioms involving preference relations among mixtures of rewards. Show that (with equivalence relation $\longleftrightarrow\bullet$)

$$[r_1 , r_0]_q \longleftrightarrow\bullet [[r_1, r_0]_p , r_0]_{q/p}$$

(b) A contractor has the opportunity to bid for one of two jobs A and B. The details of the cost to him for preparing for the bids, his estimated probabilities for being successful, and the profits for the three possible states of good weather, moderate weather and bad weather are given in the following table:

DETAILS OF JOBS

ITEM	JOB A	JOB B
Cost of preparing for bid	$ 2, 500	$ 5,000
Probability of success	.6	.5
PROFIT:		
Good weather	$25,000	$35,000
Moderate weather	$20,000	$30,000
Bad weather	$15,000	$25,000

Which bid should he prepare if his utility for money, relative to his capital, is given by the following table?

when of state transition are of only 2 types birth which increase of state variable by one & death, which increase of state by one

MONEY AND UTILITIES

AMOUNT	UTILITY	AMOUNT	UTILITY
-$5,000	0	$25,000	.9
-$2,500	.05	$30,000	.95
$15,000	.7	$35,000	1
$20,000	.75		

Take the probability of good weather to be .5, moderate to be .2 and bad to be .3.

2(a) A random variable Z has possible values 0, 1, 2, 3… with corresponding probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$… That is, $\Pr(Z=k)= \frac{1}{2^{k+1}}$, for any non-negative integer k. Determine the following:

(i) Pr(Z is an even number) (ii) Pr(Z is a odd number) (iii) $\Pr(Z \leq 4)$

(iv) $\Pr(Z>4)$.

(b) A gambler has in his pocket a fair coin with head and a tail as well as a two-headed coin. He dips his hand into his pocket and selects one of the coins at random and when he flips it, it shows heads. (i) What is the probability that it is the fair coin? (ii) Suppose he flips the same coin a second time and again it shows heads. What is the now the probability that it is the fair coin? (c) Suppose he flips the same coin a third time and it shows tails. What is now the probability that it is the fair coin?

© A certain network of caves has n compartments each of which has independent probability .01 of being flooded through some crevices after a rainy session. (i) What is the probability of at least one of the compartments will be flooded after a rainy session. (ii) What is the probability of at least one of the compartments will be flooded after two consecutive but independent rainy sessions?.

3(a) The Bayes theorem for the posterior distribution $P(A_k/B)$ from a prior distribution

$Pr(A=k)=P(A_k)$ for k= 1,2,..N is given by

$$P(A_k/B=i) = \frac{P(B=i/A_k)P(A_k)}{\sum_{j=1}^{N} P(B=i/A_j)P(A_j)}.$$

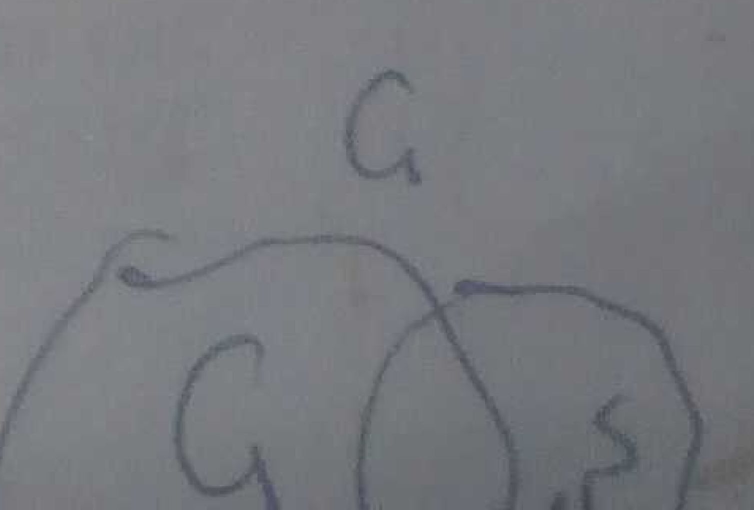
Write out the appropriate expression for posterior probabilities $P(A_k/B=i)$, k=0,1,2,3,

i=0,1,2 in a situation where prior is binomial $(3, \frac{1}{4})$ and $P(B=i/A_j)$ is binomial $(2, \frac{1}{3})$.

(b) The families in a community are divided into 3 income groups G_1, G_2, G_3 consisting of N_1, N_2, N_3 families respectively. Each group is composite in the sense that it consists of Urhobos, Itsokos, Ijaws, and Itshekiris, their respective proportion for the group G_j being

$q_{j1}, q_{j2}, q_{j3}, q_{j4}$, with $\sum_{k=1}^{4} q_{jk} = 1, j = 1,2,3$. A family is chosen at random and it is found to be Itshekiri. What is the probability that it belongs to the Group G_3?

(c) A proportion 0.75 of a population of children have a certain disease. An apparatus has been designed to detect if each child has the disease. As it happens the apparatus is not infallible. With only a probability .92 it can give a positive response if the child has the disease. With a probability 0.07 it can also give a positive response even when the child does not have the disease. What is the chance that a child that gives a positive response has actually got the disease?



4(a) Distinguish between pure and mixed prospects. A mixed bet is given by $B \sim^* [a_1, [a_2, a_3, a_4]_{r,s,t}]_p$ such that r,s,t and p are probabilities and r+s+t= 1. Show that the pay-off U for the bet is given as $U = pa_1 + (1-p)(r(a_2 - a_4) + s(a_3 - a_4) + a_4)$.

(b) Consider a group of 120 car owners having an 80% chance of no accidents in a year, a 20% chance of being in a single accident in a year, and no chance of being in more than one accident in a year. Assume that there is a 50% probability that after the accident the car will need repairs costing N500, a 40% probability that the repairs will cost N5,000 and a 10% probability that the car will need to be replaced, which will cost N15,000. What is the group car owner's expected claim payments ?

(c) In addition to the information in 2b above on the group of 100 car owners suppose the policy provides for a N250 deductible and N12,250.00 benefit limits. What would the expected claim payments be?

5(a) What is an admissible action?

In what sense would you say all Bayes solutions are usually admissible?

(b) The risk function in a statistical decision problem is given below.

State	d_1	d_2	d_3	d_4	d_5	d_6	d_7
θ_1	5	8	5	4	2	7.5	5.5
θ_2	8	3	4	4	8	4.5	2.5

(i) Plot a convex hall of the risk functions

(ii) What rule or rules can we ignore for good decisions

(iii) Find the minimax rule for the statistician and the minimax risk

(iv) Find a Bayes rule with respect to the prior probabilities $\left(\frac{1}{5}, \frac{4}{5}\right)$ and $\left(\frac{4}{5}, \frac{1}{5}\right)$

(v) Find the prior distribution(s) with respect to which the rules d_2, d_4, d_6 are Bayes.

6(a)(i) What is utility function? Give the properties.

(ii) Let V_1, V_2, V_3 be utility functions on IR. Show that any convex combination W of the form $W = \alpha^2 V_1 + 2\alpha(1-\alpha)V_2 + (1-\alpha)^2 V_3$

where $0<\alpha<1$, also satisfies the properties of the utility function.

(b) Consider the following utility function

$$u(M) = \begin{cases} M & \text{if} \quad 0<M\leq 15 \\ 5+M/2 & \text{if} \quad 15<M\leq 25 \\ 15 & \text{if} \quad M>25 \end{cases}$$

(i) What is the probability p of winning for a fair bet with initial money $M_o=10$ and with the possibility of ending up with $M_l = 0$ or $M_w = 25$

(ii) Is the fair bet in b(i) worth taking?

(iii) How large a probability of winning should a person with the given u(M) have so that he *is just indifferent to the bet?*