HESKAY

OLABISI ONABANJO UNIVERSITY, AGO-IWOYE
DEPARTMENT OF MATHEMATICAL SCIENCES
2017/2018 RAIN SEMESTER EXAMINATION
MAT202 / ELEMENTARY DIFFERENTIAL EUQATIONS I

TIME ALLOWED: 2HOURS **INSTRUCTION: ANSWER ANY FOUR QUESTIONS**

1 (a) Define the following with suitable examples
- i. A differential equation
- ii. Order of a differential equation
- iii. Degree of a differential equation

(b) Obtain a differential equations from the following primitive equations.
(i) $y = x^2[A\ln(x) + B]$ (ii) $y = A\sin 3x + B\cos 3x$

(c) Show that $y'' - 6y' + 13y = 0$ has a general solution given by $y = e^{3x}(A\cos 2x + B\sin 2x)$ where A and B are constants

2 (a) Solve the following differential equations
(i) $\frac{dy}{dx} = (1+x)(1+y)$ (ii) $\frac{dy}{dx} + 3y = 3x^2e^{-3x}$

(b) Show that the expression $g(x,y) = \frac{x^2+y^2}{xy}$ is homogeneous, state the degree and solve the equation $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$.

(c) Determine whether the differential equation $(2y^2x - 2y^3)dx + (4y^2 - 6y^2x + 2x^2y)dy = 0$ is exact. Hence solve the differential equation.

3 (a) Solve the following homogeneous differential equations
(i) $y'' - 2y' + 10y = 0$ (ii) $y'' + 3y' - 40y = 0$

(b) Obtain the general solution of the equation: $y'' + 3y + 2y = \cos 2x$

(c) The population of a city doubles in 15 years. In how many years would the population triple if the rate of increase in the population is proportional to the number of inhabitants at a given time.

4 (a) Solve the differential equation $y'' - 6y' + 9y = 0, y(0) = 2, y'(0) = 4$

(b) Use the method of undermined coefficient to find the complete solution of $y'' - 3y' + 2y = e^x + x$

(c) Solve $y'' + y' - 2y = 3e^x$ by methods of variation of parameters

5 (a) Solve the equation: $y'' - 3y' + 2y = 0$, given that $y(0) = 3$ and $y'(0) = 4$.

(b) Verify that the function $y(x) = x^2e^x$ satisfies the initial value problem $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2e^x$

(c) The rate at which a bird disease spreads in a poultry farm is proportional to the number of birds already infected. If initially 50 birds were infected and after 7 days, the number of birds infected rose to 100. How many birds would be infected after 30 days?

6 (a) Find the Laplace transforms of the following
(i) $F(t) = \cos 3t$ (ii) $F(t) = \frac{3}{4}e^{2t}$

(b) Find the inverse Laplace transform of the following
(i) $L^{-1}\left[\frac{4}{s^2+9} - \frac{6}{s^2-9}\right]$ (ii) $L^{-1}\left[\frac{8x-4}{s^2+9}\right]$

(c) Using Laplace transform method, solve the equation $y'' - y' - 2y = 3e^{2x}$, given that $y(0) = 0$ and $y'(0) = -2$