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FACULTY OF SOCIAL SCIENCES  
DEPARTMENT OF ECONOMICS  
2016/2017 Harmattan Semester Examination

Course Code: ECO303

Course Title: Mathematics for Economist II

Instructions: Attempt any TWO questions from EACH OF THE SECTIONS IN SEPARATE ANSWERS BOOKLETS

Time Allowed: 2Hrs:30Mins

SECTION A: (Time Allowed 1HR:15Mins)

Question 1

The total cost of producing  $q$  unit of a certain product is described by the function  $C=200000+3000q+0.4q^2$  and the demand function for  $q$  unit is given as  $400p = 200000 - q$ .

Required:

- Determine the number of units of  $q$  that should be produced in order to minimize average cost per unit.
- Use the relevant function(s) from the above to show that  $MC=AC$  at the minimum point of  $AC$ .
- Using the functions above, determine the units of output that should be produced to maximize profit and its maximum profit obtainable.
- Suppose per unit tax of 20 is imposed, what is the new profit?. Compare your result with (c) and state the implication of the tax imposed.

Question 2

- OOU Ventures is producing two goods  $X$  and  $Y$ . The profit function  $\Pi$  of OOU Venture is given as  $\Pi = 128x - 4x^2 + 8xy - 8y^2 + 64y - 28$ . Find: (i) the profit maximising level of output for each of the two commodities produced by OOU using cramer's rule. (ii) show that profit is actually maximized.
- SAB and KAYMARKS Nigeria Ltd put up a bonanza sales and parcelled together in a set types of their tentile materials  $X, Y$  and  $Z$ . The first tentile contains 3 bundles of type  $X$ , 4 bundles of type  $Y$  and 2 bundles of types  $Z$  were sold for #38000. The second set contains 5 bundles of type  $X$ , and 6 bundles of type  $Y$  which was sold for #41000, while the third set containing 2 bundles of type  $X$ , 3 bundles of  $Y$  and a bundle of type  $Z$  was sold for #24000. Represent the above information as a system of equation and use any method of your choice to determine the selling price of each of tentile materials.

Question 3

- A market contains firm A and firm B whose their price policies are different. The price of firm A is  $P_A = Q_A + 800$  and the price of firm B is  $P_B = 800 - 4Q_B$ ; and the market has limited quantity of 50units to be sold at a particular point in time, noting that the firms are constrained to equal units. Given that the cost of production in the market is  $TC = 1200 + 100Q$ , find (i) revenue functions of A and B; (ii) the output of each firm that maximizes profit level; (iii) the prices of the two firms.

- Solve the following system of equation

$$x_1 + x_2 + x_3 = 6$$

$$2x_1 - 3x_2 - 12x_3 = 5$$

$$x_1 - x_2 - 5x_3 - 3 = 0$$

Question 4

- Show whether or not the function  $Z = x^3 - 3xy^2$  satisfies the Laplace theorem.
- Explain Euler's theorem using relevant examples and it economic implication
- Verify  $f_{xy} = f_{yx}$  when  $f(x, y) = 2x^3 + 2y^3 - 6xy$



- (d) Find the relative extrema of the function  $y = x^3 - 3x^2 + 2$  and determine the nature of those points.
- (e) With illustration distinguished between the Jacobian and the Hessian determinant of matrix.
- (f) Find the first, second partials of the function, if  $q = 4K^{0.4}L^{0.6}$  and the degree of homogeneity of this function

## **SECTION B: (Time Allowed 1HR:15Mins)**

### **Question 1**

- (a) Given the total cost function as  $C = q^3 - 90q^2 + 550$  and the price function as  $P = 110 - 5q$ . Determine
- The output level that maximise profit
  - The output level that maximizes the net earnings
- (b) Given  $(5yt^2)dy + (5y^2t + 8t^2)dt = 0$ .
- Determine whether the differential equation is exact or not
  - Solve for  $F(t,y)$

### **Question 2**

- (a) Use the general equation to solve the differential equation:  $\frac{dy}{dt} - 4yt = 6t$ , hence establish the dynamic stability or otherwise of the ordinary differential equation
- (b) Determine the integrating factor in each of the following:
- $t^2dy + (3yt)dt = 0$
  - $(7y + 4t^2)dy + (4ty)dt = 0$

### **Question 3**

- (a) Given the own price elasticity of demand  $\epsilon = (6p + \frac{1}{2}p^2/q)$ . Find the demand function provided  $q = 250$ , when  $p=5$
- (b) Given the investment function as  $I(t) = 8(\sqrt[3]{t})$
- Derive the investment path equation
  - The capital formation between the beginning of the first year and the end of the eighth year
  - Period of time (t) it takes the investment to yield returns above #3000.

### **Question 4**

- (a) Evaluate (i)  $\int \frac{2x^3+1}{3x^4+2x} dx$  (ii)  $\int x^2 e^{x^3+3} dx$  (iii)  $\int_1^2 (2-4x)^9 dx$
- (b) Specify the order and the degree of the following differential equations
- $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} (x^2y) - 4y^3 = 0$
  - $(\frac{d^3y}{dx^3})^4 + (\frac{d^4y}{dx^4})^2 - 75y = 0$
  - $y^{III} + (y^{II})^3 - 4 = y$
  - $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = 12x$