OLABISI ONABANJO UNIVERSITY, AGO-IWOYE
DEPARTMENT OF MATHEMATICAL SCIENCES
2017/2018 RAIN SEMESTER EXAMINATION

COURSE CODE: STA412 TIME ALLOWED: 2HRS
COURSE TITLE: STOCHASTIC PROCESSES
INSTRUCTION(S): ANSWER ANY FOUR QUESTIONS

1a) Define and explain the concept of stochastic processes and give three areas of application.

b Define a generating function A(x), and what do you understand by the following terms:

i. Moment generating function (ii) Characteristic function
iii. Probability generating function

2(a) Obtain the general form of the Chapman- Kolmogorov (C - K) equation.

b) Given X_i , i = 1, 2 be two independent poisson variable with parameter λ_1 and λ_2. Find the probability generating function and probability mass function of $Z = X_1 + X_2$.

c) Suppose x is the waiting time to obtain an head in an honest coin

$$P(x=1)=\tfrac{1}{2}, P(x=2)=(1/2)^2 \ldots P(x=k)=1/2^k$$

Obtain its probability generating function, hence find its variance.

3(a) Which of the following matrices are stochastic matrix, with reason(s):

$$A=\begin{pmatrix}0 & 1 & 0\\ 1/2 & 1/4 & 1/4\end{pmatrix} \quad B=\begin{pmatrix}1/4 & 3/4\\ 1/3 & 1/3\end{pmatrix} \quad C=\begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/2 & 0 & 1/2\end{pmatrix}$$

$$D=\begin{pmatrix}1/3 & 0 & 2/3\\ 3/4 & 1/2 & -1/4\\ 1/3 & 1/3 & 1/3\end{pmatrix} \quad E=\begin{pmatrix}0 & 1 & 0\\ 1/2 & 1/6 & 1/3\\ 1/3 & 2/3 & 0\end{pmatrix}$$

$\frac{1}{3} \times 35 = \frac{35}{3}$

11.33

b) Distinguish between the following terms

i) Markov process and Markov Chain
ii) Absorbing state and Transient state
iii) One-step transition probability matrix and n-step transition probability matrix

4(a) A ball is drawn with replacement from a bag which contain 9 white and 11 red balls, if the ball is white you win #1, but if the ball is red, you lose #1. Assuming you begin with #20 and your opponent begins with #10 and you play until one is ruined.

i) Find the probability that you are the one who is ruined.

ii) What is the probability that the gambler will be ultimately be ruined when P = 1/3 and the total amount of fund available for the tournament is #50 and the gambler has only #35.

b) If a man leaves home too late to catch his bus to work on any day, the probability that he is late the following day is 1/3, where as if he leaves in time to catch it on any day, the probability that he is late in the following day is 3/5. The man catches his bus on Tuesday and leaves for work each day.

i) Write down the transition probability matrix

ii) Calculate the probability that he catches his bus on the following Friday.

iii) Show that over a long period, the probability that he will catch his bus to work is 10/19.

c) Consider a judicial system. Demonstrate that it is queuing system by describing its component.

5a) Using limiting behaviour of Homogeneous chain, find the steady state probabilities of chain given by the transition matrix.

$$P = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$$

b) In a hypothetical market there are only two brands A and B customer buys A with probability 0.7, if his last purchase was A and buys brand B with probability 0.4 if his last purchase was B. Assuming Markov chain model, obtain

i) One step transition probability matrix ii) n- step transition probability matrix

iii) The stationary distribution. Hence highlight the proportion of customers who would buy brand A and brand B in the long run.

6(a) Consider a Markov chain with two states and the transition probability matrix give as

$$P = \begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix}$$

Obtain P^n using the method of generating functions

b) What do you understand by poisson process? State its governing rules.

c) Write down the forward Kolmogorov differential equation for the simple birth process.

Poisson process if F(t) is a given event in interval (0,t) of the possible probability controlling the occurrence of the given

governing.

[t, t+Δt] ∩ [Δ+t]

Pr{0 occurrence of [t, t+Δt] ∩ [Δt+t]t

Pr{1 — — — [t+ΔΔt] ∩ [ΔΔt+t]+Δ

" {2 — — — [At+ΔΔt]