# Assignment 12

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## 1 Problem 3

We want to prove that the MC Error can be represented as the sum of discounted TD errors. It's easier to start with the sum of discounted TD errors, as this is a telescoping sum.

$$\sum_{u=t}^{T-1} \gamma^{u-t}(R_{u+1} + \gamma V(S_{u+1}) - V(S_u)) \tag{1}$$

$$= (R_{t+1} + \gamma V(S_{t+1}) - V(S_t)) + \gamma^1(R_{t+2} + \gamma V(S_{t+2}) - V(S_{t+1})) + ... + \gamma^{T-1-t}(R_T + \gamma V(S_T) - V(S_{T-1})) \tag{2}$$

$$= R_{t+1} + \gamma V(S_{t+1}) - V(S_t) + \gamma^1 R_{t+2} + \gamma^2 V(S_{t+2}) - \gamma V(S_{t+1}) + ... + \gamma^{T-1-t} R_T + \gamma^{T-t} V(S_T) - \gamma^{T-t-1} V(S_{T-1}) \tag{3}$$

If we gather the $R$ terms, we recognize that most of the $V$ terms will cancel out as a telescoping sum, and so the above is equal to

$$(\sum_{u=t}^{T-1} \gamma^{u-t}(R_{u+1})) - V(S_t) = G_t - V(S_t) \tag{4}$$

which is exactly the MC error.