Assignment 8

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1 Problem 1

In summary, you are in charge of a bank where you need to decide how much to invest on any given day, with reserve requirement restrictions and the goal to also make enough cash available to your customers for possible withdrawals.

1. You can borrow $y > 0$ from another bank at the end of each day with constant daily interest rate R
2. You can invest in a risky asset S_t at the end of each day, the amount of which can be changed each day with no transactions cost
3. Your cash in reserve is c at the start of the day, which must be greater than or equal to a reserve requirement C, otherwise you pay a penalty $K * cot(\frac{\pi c}{2C})$ for some fixed constants C and K
4. Borrowing and investing is constrained so that at the end of each day there is positive cash and the penalty can be paid if needed
5. Deposit rates are ignored, the first half of the day is for deposits, the second half of the day for withdrawals
6. Withdrawals that cannot be fulfilled will be asked again the next day
7. All Quantities are continuous
8. Goal: Maximize Expected Utility of Assets - Liabilities at end of T-day horizon conditional on current state

From this we define a state as a tuple (c, p, s) which represents the amount of total cash, the amount of pending withdrawals, and lastly the price of the risky asset that is being used for investment. Since this market is frictionless we can assume the bank takes all the money out of the risky asset and decides everyday how to reinvest their cash on hand. An action is a tuple (x, y) that represents the quantity of the risky asset to invest in (x) and the amount to borrow each day (y).

State transitions from the end of one day to the end of another day are:

For s: s follows some known price process of the assets following some differential equation.

For cash: The amount borrowed in time t minus the amount to be paid back the next day and the amount invested in the stock minus the possible penalty, adding net deposits/withdrawals. This is capped at 0 cash on the downside. Then, investing gains are added at the end.

$$c_{t+1} = max(y_t - y_t(1+R) - q_t s_t - K * cot(\frac{min(C, y_t - y_t(1+R) - q_t s_t}{2C}) + (d_{t+1} - w_{t+1} - p_t), 0) + q_t s_{t+1} \quad (1)$$

The pending withdrawals evolve as the max of the current withdrawals + previous pending withdrawals minus the available cash after investment and borrowing as well as deposits, compared with 0. This only becomes zero when there is enough cash on hand during that day to pay off all previous pending withdrawals as well as the current day's withdrawals.

$$p_{t+1} = max(w_{t+1} + p_t - (y_t - y_t(1+R) - q_t s_t - K * cot(\frac{min(C, y_t - y_t(1+R) - q_t s_t}{2C}) + d_{t+1}), 0) \quad (2)$$

The reward is defined at the net worth at the end of time T, which is the utility on total available cash - pending withdrawals, so using some utility function $U()$ this is $U(c_T - p_T)$. This problem can be solved using an ADP method like approximate value iteration, since the state space and action space are both continuous and cannot be represented precisely.