A6Q1: Static Asset Allocation

Bodhi Nguyen

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1 Problem a)

Assume the Utility function is $U(x) = x - \frac{\alpha x^2}{2}$. Assuming x is Normal is mean μ and variance σ^2, calculate expected utility, certainty equivalent value x_{CE}, Absolute risk premium

$$E[U(x)] = \mu - \frac{\alpha}{2} * (\sigma^2 + \mu^2) \tag{1}$$

We have

$$U^{-1}(y) = \frac{1 \pm \sqrt{1 - 4\frac{\alpha}{2}(y)}}{\alpha} \tag{2}$$

$$x_{CE} = \frac{1 \pm \sqrt{1 - 4\frac{\alpha}{2}(\mu - \frac{\alpha}{2}(\sigma^2 + \mu^2))}}{\alpha} \tag{3}$$

And the absolute risk premium is $E[X] - x_{CE} = \mu - \frac{1 \pm \sqrt{1 - 4\frac{\alpha}{2}(\mu - \frac{\alpha}{2}(\sigma^2 + \mu^2))}}{\alpha}$

Assume you have a million dollars to invest. Choose z dollars to go into an investment with return x distributed Normal with mean μ, variance σ^2. The rest goes into a asset with a fixed annual return r. What is the optimal value of z?

We have that our wealth is $(1000000 - z)(1 + r) + z(1 + x)$, and so our wealth W is distributed Normal with mean $1000000 + r + z(\mu - r)$ and has variance $z^2\sigma^2$ as a result of the transformation of normal variables. We wish to maximize our expected utility, which according to our utility function above is thus

$$E[U(W)] = 1000000 + r + z(\mu - r) - \frac{\alpha}{2}(z^2\sigma^2 + (1000000 + r + z(\mu - r))^2) \tag{4}$$

$$\frac{dE[U(W)]}{dz} = (\mu - r) - \sigma^2 + 2 * (1000000 + r + z(\mu - r)) * (\mu - r)) \tag{5}$$

+ 2)

This gives us the solution

$$z^* = \frac{-(\mu - r) - 2(1000000 + r)(\mu - r)}{-\alpha\sigma^2 + 2(\mu - r)^2} \tag{6}$$