# Snakes and Ladders

Bodhi Nguyen

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## 1 State Space and Rules

In the Snakes and Ladders game, there are 100 states, represented by the board below:

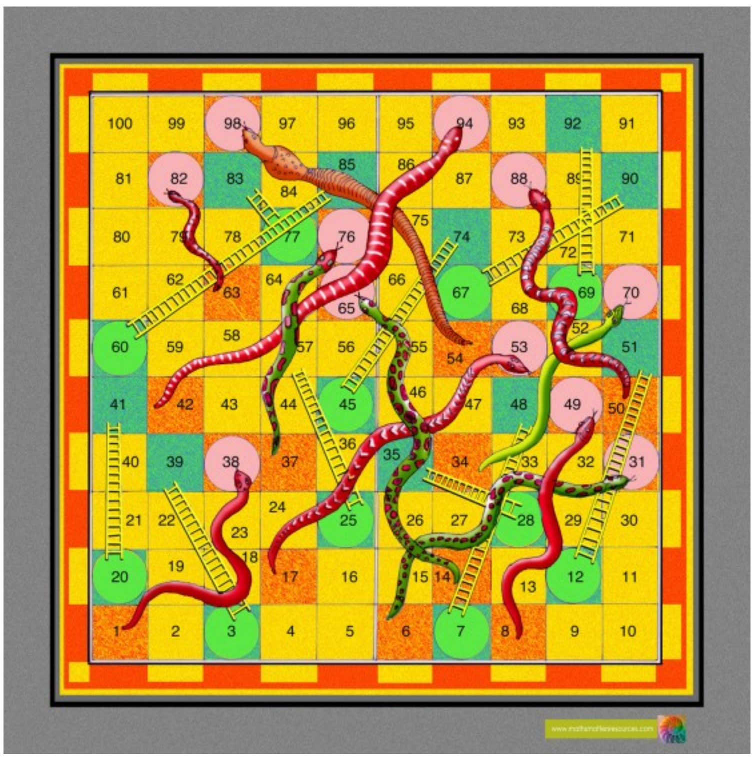


Figure 1: Snakes and Ladders board

There are two types of special states: snake states, which send you back to a state with a lower index, and ladder states, which send you to a state with a higher index. The game is played in the single player version by rolling a die on each move. The die roll determines the number of spaces a player moves, starting with the first roll determining which state 1-6 the player moves to. If landing on a special state, the player immediately moves to the lower/higher state specified by the snake/ladder.

To win, the player must get to state 100. Note the "bounce back" rule, in which if the player rolls and goes to a state higher than 100, they must "bounce back" the equivalent number of spaces. For example, if the player is on space 97 and rolls a 4. The player first goes to space 100, then back to space 99 (97 + 3 = 100, then 100 - (4-3) = 99).

## 2 Transition Probabilities

State 100 is an absorbing state. On a space not including the snake states and ladder states or the states 95, 96, 97, 98, or 99 (a "normal" state), the transition probabilities are, for current state $i$:

$$P(S_{t+1} = i + k | S_t = i) = \frac{1}{6}$$

for $k \in 1, 2, 3, 4, 5, 6$.

The snake and ladder states simply have the same transition probabilities of the state they point to; effectively, the snake and ladder states don't have their own unique transition probabilities since the player never stays on them. For example, the snake state 38 has the same transition probabilities as state 1, which has a probability of 1/6 to move to any state 2-7.

The bounce back states (95, 96, 97, 99) (which doesn't include 98 since 98 is a snake state) have the following transition probabilities created from the following process:

1. Initialize each $P(S_{t+1} = j | S_t = i) = 0$
2. For each $k \in 1, 2, 3, 4, 5, 6$, if the roll $\leq$ (100 - current state), add $\frac{1}{6}$ to $P(S_{t+1} = i + roll | S_t = i)$
3. If roll $>$ (100 - current state), add $\frac{1}{6}$ to $P(S_{t+1} = 100 - (i + roll - 100) | S_t = i)$