Assignment 10

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1 Problem 1: A-S Formulation

We consider a special case of the Optimal Market-Making problem we covered in class (AvellandeaStoikov formulation) where the market-maker has a cash amount of $W \in R$ at time 0 and an inventory of shares equal to $I \in Z$ (note: this could be positive or negative), but is not going to be market-making until time T. The market maker's Value Function at time t (with $0 \leq t \leq T$) is given by the Expected Utility at time T (conditional on the time t and the OB Mid Price S_t at time t). We know $dS_t = \sigma dz_t$ which mean that $S_{t2} = N(S_{t1}, \sigma^2 * (t_2 - t_1))$

We want to evaluate the conditional expectation $E[-e^{\gamma(W+IS_t)}|(t, S_t)]$ which is just the value function. Pulling terms out of the conditional, and using the fact that S_t is distributed normally (so we can use the MGF of the normal distribution to evaluate the expectation), we get that this is equal to

$$-e^{\gamma(W)}E[e^{\gamma(IS_t)}|(t, S_t)] = -e^{\gamma(W)} * e^{\gamma(IS)} * e^{\gamma^2(\frac{\sigma^2 I^2 (T-t)}{2})}$$

To solve for the reservation bid and ask prices, we use the fact that

$$V(t, St, W - Q^{(b)}(t, St, I), I + 1) = V(t, St, W, I)$$
$$V(t, St, W + Q^{(a)}(t, St, I), I - 1) = V(t, St, W, I)$$

and the closed form expression we just solved for. Setting the terms of the exponentials equal across both sides of the equation, for the reservation bid we get that

$$-\gamma(W - Q^b) + \gamma(I+1)(S) + \gamma^2 \frac{\sigma^2(I+1)I^2(T-t)}{2} = -\gamma(W) + \gamma(I)(S) + \gamma^2 \frac{\sigma^2(I)I^2(T-t)}{2}$$

Canceling common terms and dividing by γ get us the indifference price:

$$Q^b = s + \gamma \frac{\sigma^2(I)(T-t)(-1-2I)}{2}$$

The same exact process for the ask price yields

$$Q^a = s + \gamma \frac{\sigma^2(I)(T-t)(1-2I)}{2}$$