Department of Engineering

Mathematics 4 (Numerical Mathematics) ENG 2 148 WS 2024

Chapter #1 Practice questions

Question 1

a. Use Matlab algorithms to convert the number that is based on last 4 digits of your GUtech ID number (shown below) to base 2.

e.g., 19-0533 becomes 05.33: **convert 05.33 to base 2**

Question 2

a. Determine the machine representation in single precision on a 32-bit word-length computer for the following decimal numbers

i) -8×2^{-2}

Sign **Mantissa** **Exponent**

ii) 0.025

Sign **Mantissa** **Exponent**

iii) 492.125

Sign **Mantissa** **Exponent**

b. Identify the floating-point numbers corresponding to the following bit strings

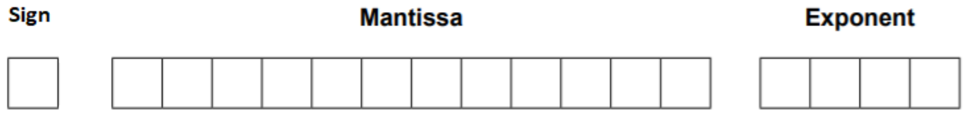
i) 1 11111111 00000000000000000000000

ii) 0 10000001 01100000000000000000000

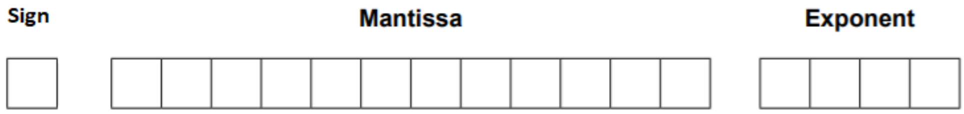
iii) 0 10001011 10111100100000000000000

c. Consider the following number presented in IEEE single precision 32 bits. Determine the sign σ, exponent e, significand/mantissa m and determine the value of x

   i) 01101011101101010000000000000000



   ii) 11001100101111100010000000000000



d. Use the 64-bit long real format to find the decimal equivalent of the following floating-point machine numbers

   i) 0 10000001010 10010011000000⋯0

   ii) 1 10000001010 01010011000000⋯0

Question 3

Write a Matlab programming code that converts any decimal number to its IEEE-754 64-bit floating point binary representation. Your function should accept a decimal number as input and should output the sign bit, the normalized mantissa and the biased exponent as binary strings of 1s and 0s without spaces in between. A sample output would look as follows:

```
Input: -103.45
IEEE-754, 64-bit floating point number representation:
Sign = 1
Mantissa  (52 bits) = 1001110111001100110011001100110011001100110011001101
Exponent (11 bits) = 10000000101
```

Test with based on your GUtech ID number (shown below) **on your GUtech ID number (shown below).**

e.g., 19-0533 becomes 5.33: **convert 19.533** to its IEEE-754 64-bit floating point binary representation

Question 4

a) Compute the absolute error and relative error in approximations of x by x^*.

i. $x = \pi$ and $x^* = 22/7$

ii. $x = e$ and $x^* = 2.718$

iii. $x = \sqrt{2}$ and $x^* = 1.414$

b) Perform the following calculations

i. exactly,

ii. using three-digit chopping arithmetic, and

iii. using three-digit rounding arithmetic.

iv. Compute the absolute error and relative error with the exact value determined to at least five digits

a) $\frac{4}{5} \times \frac{7}{3}$

b) $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

c) Use the formula $\dfrac{f(x+h)-f(x)}{h}$

to approximate the derivative of $f(x) = 3x^2$ at $x = 1$ using $h = 0.1$.

i. Compute absolute error

ii. Compute relative error

Question 5

a) Consider the function $f(x) = 2x - \sin x$

i. Evaluate $f(0.2)$ using 3 digit (decimal) rounding arithmetic.

ii. Compute the absolute error and relative error with the exact value determined to at least five digits

b) With how many digits approximates x∗ = 0.00017460 to x = 0.00017458?

c) With how many digits approximates x∗ = 10000, to x = 9999.999?

Question 6

a) Consider the function

$$f(x) = \frac{x^2}{x-2}$$

and suppose we want to evaluate it for some x

i. Let $x = 0$. Is the problem of evaluating $f(x)$, for $x = 0$, a well-conditioned one?

ii. Let $x = 2.2$. Is the problem of evaluating $f(x)$, for $x = 2.2$, a well-conditioned one?

b) Consider the function

$$f(x) = x^2 + x - 1975$$

and suppose we want to evaluate it for some x

i. Let $x = 20$. Is the problem of evaluating $f(x)$, for $x = 20$, a well-conditioned one?

ii. Let $x = 44$. Is the problem of evaluating $f(x)$, for $x = 44$, a well-conditioned one?

Question 7

Imagine an engineer measures a steel beam's length as 5.00 m, but the true length is 5.03. To evaluate the precision of measurements, calculate relative error and absolute error.