x	0	1	2	3
y	2	5	7	11

① linear

$$S_0(x) = 2 + \frac{5-2}{1-0}(x-0) \Rightarrow 2 + 3x \qquad x \in [0,1]$$

$$S_1(x) = 5 + \frac{7-5}{2-1}(x-1) \Rightarrow 5 + 2(x-1) = 5 + 2x - 2 =$$
$$2x + 3 \qquad x \in [1,2]$$

$$S_2(x) = 7 + \frac{11-7}{3-2}(x-2) \Rightarrow 7 + 4x - 8 = 4x - 1 \quad x \in [2,3]$$

$$S(x) = \begin{cases} 2 + 3x, & x \in [0,1] \\ 2x + 3, & x \in [1,2] \\ 4x - 1, & x \in [2,3] \end{cases}$$

② Quadratic

$$Q_i(x) = \frac{z_{i+1} - z_i}{2(t_{i+1} - t_i)}(x - t_i)^2 + z_i(x - t_i) + y_i$$

$$z_{i+1} = -z_i + 2\left(\frac{y_{i+1} - y_i}{t_{i+1} - t_i}\right)$$

$z_0 = 0$

$i = 0$

$z_1 = -(0) + 2\left(\dfrac{5-2}{1-0}\right) = 6$

$i = 1$

$z_2 = -(6) + 2\left(\dfrac{7-5}{2-1}\right) = -2$

$i = 2$

$z_3 = -(-2) + 2\left(\dfrac{11-7}{3-2}\right) = \cancel{6}10$

$z_0 = 0, \quad z_1 = 6, \quad z_2 = -2, \quad z_3 = 6$

$Q_0(x) = \dfrac{6-0}{2(1-0)}(x-0)^2 + 0(x-0) + 2 \implies 3x^2 + 2 \qquad x \in [0,1]$

$Q_1(x) = \dfrac{-2-6}{2(2-1)}(x-1)^2 + 6(x-1) + 5 \implies -4(x-1)^2 + 6(x-1) + 5$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x \in [1,2]$

$Q_2(x) = \dfrac{\cancel{6}-(-2)}{2(3-2)}(x-2)^2 - 2(x-2) + 7 \implies \overset{6}{\cancel{4}}(x-2)^2 - 2(x-2) + 7$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x \in [2,3]$

$$Q(x) = \begin{cases} 3x^2 + 2, & x \in [0,1] \\[2mm] -4(x-1)^2 + 6(x-1) + 5, & x \in [1,2] \\[2mm] \overset{6}{\cancel{4}}(x-2)^2 - 2(x-2) + 7, & x \in [2,3] \end{cases}$$

③ Cubic

$$f(x) = \frac{(x_{i+1} - x)^3}{6h} M_i + \frac{(x - x_i)^3}{6h} M_{i+1} + \frac{(x_{i+1} - x)}{h}\left(y_i - \frac{h^2}{6} M_i\right)$$

$$+ \frac{(x - x_i)}{h}\left(y_{i+1} - \frac{h^2}{6} M_{i+1}\right)$$

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right)$$

$$M_0, M_1, M_2, M_3$$

$$M_0, M_3 = 0$$

$i = 1$

$$M_0 + 4M_{\cancel{1}1} + M_{\cancel{3}2} = \frac{6}{(1)^2}\left(2 - 2(5) + 7\right)$$

$$M_0 + 4M_1 + M_2 = -6$$

$$4M_1 + M_2 = -6 \qquad M_1 = \frac{-6 - M_2}{4}$$

$i = 2$

$$M_1 + 4M_2 + M_3 = 6\left(5 - 2(7) + 11\right)$$

$$M_1 + 4M_2 \qquad = 12 \qquad\qquad M_2 = \frac{18}{5}$$

$$-\frac{6}{4} - \frac{1}{4}M_2 + 4M_2 = 12 \qquad\qquad M_1 + 4\left(\frac{18}{5}\right) = 12$$

$$\frac{15}{4}M_2 = \frac{27}{2} \qquad\qquad M_1 = -\frac{12}{5}$$

$M_{0r} =$

$M_0 = 0, \quad M_1 = -12/5, \quad M_2 = 18/5, \quad M_3 = 0$

$i = 0$

$$f(x) = \frac{(1-x)^3}{6}(0) + \frac{(x-0)}{6}\left(-\frac{12}{5}\right) + \frac{(1-x)}{1}\left(2 - \frac{1^2}{6}(0)\right)$$

$$+ \frac{(x-0)}{1}\left(5 - \frac{1^2}{6}\left(-\frac{12}{5}\right)\right)$$

$$f_0(x) = -\frac{2}{5}x + 2(1-x) + \frac{27}{5}x$$

$i = 1$

$$f(x) = \frac{(2-x)^3}{6}\left(-\frac{12}{5}\right) + \frac{(x-1)^3}{6}\left(\frac{18}{5}\right) + \frac{(2-x)}{1}\left(5 - \frac{1}{6}\left(-\frac{12}{5}\right)\right)$$

$$+ \frac{(x-1)}{1}\left(7 - \frac{1}{6}\left(\frac{18}{5}\right)\right)$$

$$= -\frac{2}{5}(2-x)^3 + \frac{3}{5}(x-1)^3 + \frac{27}{5}(2-x) + \frac{32}{5}(x-1)$$

$i = 2$

$$f(x) = \frac{(3-x)^3}{6}\left(\frac{18}{5}\right) + \frac{\cancel{(x-2)^3}}{\cancel{6}}(0) + (3-x)\left(7 - \frac{1}{6}\left(\frac{18}{5}\right)\right)$$

$$+ \frac{(x-2)}{1}\left(11 - \frac{1}{6}(0)\right)$$

$$f(x) = \frac{3}{5}(3-x)^3 + \frac{32}{5}(3-x) + 11(x-2)$$

$$f(x) = \begin{cases} -\dfrac{2}{5}x + 2(1-x) + \dfrac{27}{5}x, & x \in [0,1] \\[4mm] -\dfrac{2}{5}(2-x)^3 + \dfrac{3}{5}(x-1)^3 + \dfrac{27}{5}(2-x) + \dfrac{32}{5}(x-1), & \\ & x \in [1,2] \\[4mm] \dfrac{3}{5}(3-x)^3 + \dfrac{32}{5}(3-x) + 11(x-2), & \\ & x \in [2,3] \end{cases}$$