Numerical Mathematics ENG 2148 WS 2023

Chapter #5 Practice questions

Question 1

(A) Use the forward, backward and central difference formulas to approximate the first derivativeof the function $f(x) = e^x$ at $x = 2$, take $h = 0.1$

(B) **For the function $f(x) = x\ ln\ x$, approximate $f''(x)$ at $x = 1$, using the second order central difference formula, with $h = 0.1$**

a. $f(x) = \ln x, x_0 = 1.0, h = 0.4$

b. $f(x) = x + e^x, x_0 = 0.0, h = 0.4$

c. $f(x) = 2^x \sin x, x_0 = 1.05, h = 0.4$

d. $f(x) = x^3 \cos x, x_0 = 2.3, h = 0.4$

Question

Approximate the following integrals using trapezoidal and Simpson's rules, n=2

a. $\int_{0.5}^{1} x^4\, dx$

b. $\int_{1}^{1.5} x^2 \ln x\, dx$

c. $\int_{0}^{1} x^2 e^{-x}\, dx$

d. $\int_{1}^{1.6} \frac{2x}{x^2 - 4}\, dx$

Question 2

a. Use the Trapezoidal rule with step size $\Delta x = 1$ to appoximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

b.

Consider the integral

$$\int_0^2 (2x+3)\,dx.$$

(a) Evaluate this integral exactly.
(b) Using the Trapezoidal Rule with $n = 4$ find an approximation to the integral.
(c) Explain your answer in part (b).**Hint:**Consider the error.

Question 3

Approximate the following integrals using trapezoidal and Simpson's rules, n=4

a. $\int_{0.5}^{1} x^4\,dx$

b. $\int_{1}^{1.5} x^2 \ln x\,dx$

c. $\int_{0}^{1} x^2 e^{-x}\,dx$

d. $\int_{1}^{1.6} \frac{2x}{x^2-4}\,dx$

Question 4

Use Simpson's rule with step size $\Delta x = 1$ to appoximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

Question 5

Recall that the error E_T in the trapezoidal rule for approximating $\int_a^b f(x)dx$ satisfies

$$\left| \int_a^b f(x)dx - T_n \right| = |E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{whenever } |f''(x)| \leq K \text{ for all } a \leq x \leq b.$$

Use the above error bound to determine a value of n for which the trapezoidal approximation to

$\ln 3 = \int_1^3 \frac{1}{x}dx$ has an error $|E_T| \leq \frac{1}{3}10^{-4}$.

Question 6

Determine the values of n and h required to approximate

$$\int_0^2 \frac{1}{x+4}\,dx$$

to within 10^{-5} and compute the approximation. Use

a. Composite Trapezoidal rule. **b.** Composite Simpson's rule.

Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)\,dx$.

Use the trapezoidal rule with step size $\Delta x = 2$ to approximate the integral $\int_0^4 f(x)dx$ where the graph of the function $f(x)$ is given below.