Numerical Mathematics ENG 2148 WS 2023

## Chapter #6 Practice questions

### Question 1

Use Euler's method to approximate the solutions for each of the following initial-value problems.

**a.** $y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0, \text{ with } h = 0.5$

**b.** $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1, \text{ with } h = 0.5$

**c.** $y' = 1 + y/t, \quad 1 \le t \le 2, \quad y(1) = 2, \text{ with } h = 0.25$

**d.** $y' = \cos 2t + \sin 3t, \quad 0 \le t \le 1, \quad y(0) = 1, \text{ with } h = 0.25$

### Question 2

Use the Runge-Kutta method to approximate the solutions for each of the following initial-value problems:

**a.** $y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0, \text{ with } h = 0.5$

**b.** $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1, \text{ with } h = 0.5$

**c.** $y' = 1 + y/t, \quad 1 \le t \le 2, \quad y(1) = 2, \text{ with } h = 0.25$

**d.** $y' = \cos 2t + \sin 3t, \quad 0 \le t \le 1, \quad y(0) = 1, \text{ with } h = 0.25$

## Question 3

A restaurant installs a kitchen ventilation system to control the amount of grease in the air due to cooking. The ventilation system reduces the amount of grease in the air by 90 percent every hour. Let $Q(t)$ be the amount in grams of grease in the air $t$ hours after the ventilation is activated. Then $Q$ satisfies the differential equation $\frac{dQ}{dt} = 2e^{-5t} - \frac{9}{10}Q$,

$$\frac{dQ}{dt} = 4e^{-2t} - \frac{8}{10}Q$$

where $4e^{-2t}$ is the rate at which the kitchen produces grease in grams per hour at time $t$. Use Euler's method to estimate the values of the solution curve $Q(t)$ at $Q(1)$ through $(0, 0.4)$ with $h = 1/3$.

## Question 2

A box is dropped from an airplane. The downward velocity $v(t)$ of the box, once its parachute opens, satisfies the differential equation

$$\frac{dv}{dt} = 12 - \frac{1}{10}(1 + e^{-t})v^2$$

Suppose the parachute opens when the velocity of the box is $10\ m/s$. Use Euler's method with three steps to approximate the velocity of the box one second after the parachute opens.