

Chapter #4 Interpolation and approximation

Question 1

- a) A teacher is monitoring the progress of students using a computer-based revision course. The improvement in performance, y marks, is recorded for each student along with the time, x hours, that the student spent using the revision course. The results for a random sample of 10 students are recorded below.

x hours	1.0	3.5	4.0	1.5	1.3	0.5	1.8	2.5	2.3	3.0
y marks	5	30	27	10	-3	-5	7	15	-10	20

Use the method of least squares to fit the straight line $y = a + bx$ to the data given.

- b) Use the method of least squares to fit the straight line $y = a + bx$ to the data given below

x	0	1	2	3
y	2	5	8	11

Use the linear model you found in part (b) to predict the life expectancy in the year

- i. 1.5 ii.
3.5

iii. Comment your prediction in (I and ii)

c) Find the least squares line through the points (1, 1), (3, 3.5), (5, 4) and (8, 7.5)

Question 2

The data below shows results of a group of students on Mathematics and Science test.

The results of a group of students on Mathematics and Science tests are compared.

Student	1	2	3	4	5	6	7	8	9	10
Maths test (M)	64	67	69	70	73	74	77	82	84	85
Science test (S)	68	73	68	75	78	73	77	84	86	89

- Calculate the equation of the least-squares line of best fit for
- Use the equation to predict the (average) score in science of students who score 80 in Mathematics.
- Use linear spline method to predict the score in science of a student who scored 80 in Mathematics.
- Use quadratic spline method to predict the score in science of a student who scored 80 in Mathematics.

Question 3

Let $f(x) = 4 + \sin(2x)$.

- Find the linear ($L(x)$) and quadratic ($Q(x)$) approximations to the function $f(x)$ near $x = 1$.
- Approximate the values of $f(x)$ for $x = 0.5, 0.75, 1, 1.25$ and 1.5 using the functions $L(x)$ and $Q(x)$. Compare the values of your approximations to the actual values of $f(x)$ at these values. What do you notice?

Question 4

(A)

- a. Find the Taylor polynomial of degree 4 for $f(x) = x^{-2}$ about the point $x_0 = 1$.
 - (ii) Use the result of (i) to approximate $f(0.9)$ and $f(1.1)$.
 - (iii) Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations made in part (ii). Which of the two approximations in part (ii) do you expect to be closer to the correct value?
- (i)

Question 5

Find the cubic spline interpolation for the data:

$x:$	1	2	3	4	5
$f(x):$	1	0	1	0	1

Question 6

The following data define the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

$T^{\circ}\text{C}$	0	8	16	24	32	40
$o, \text{mg/L}$	14.621	11.843	9.870	8.418	7.305	6.413

Estimate $o(27)$ using

- (a) linear regression interpolation,
- (b) Quadratic interpolation
- (b) Quadratic spline,
- (c) cubic splines.

Note that the exact result is 7.986 mg/L