Numerical Mathematics ENG 2148 WS 2024

Chapter #2 Solution of non-linear equations: Practice questions

Question 1

a) Use Bisection method to find n_3 for $f(x) = \sqrt{x} - cos\, x$ on $[0, 1]$.

b) Use Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval.

i. $[0, 1]$

ii. $[1, 3.2]$

iii. $[3.2, 4]$

c) Use Bisection method to find solutions accurate to within 10^{-5} for

$$x \cos x - 2x^2 + 3x - 1 = 0 \quad \text{for } 0.2 \le x \le 0.3$$

Question 2

a. Approximate the zero(s) of the given functions. Use Newton's Method and continue the process until two successive approximations differ by less than 0.001.

i. $f(x) = x^3 + 4,\ [-2, 1]$

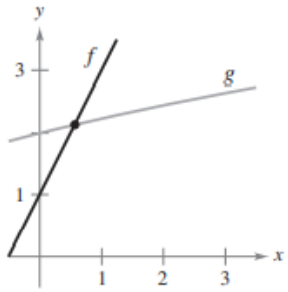
ii. $f(x) = x^3 + x - 1,\ [-1, 1]$

iii. $f(x) = 1 - x + \sin x,\ [-2, 2.5]$

b. Apply Newton's Method to approximate the value(s) of the indicated point(s) of intersection of the two graphs. Continue the process until two successive approximations differ by less than 0.001.

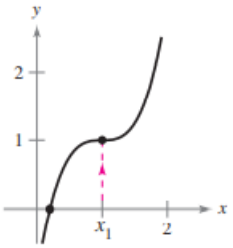
$f(x) = 2x + 1$

$g(x) = \sqrt{x + 4}$



c. Apply Newton's Method using the given initial guess and explain why the method fails.

$y = 2x^3 - 6x^2 + 6x - 1, \quad x_1 = 1$



Question 3

a. Use Secant method to find n_3 for $f(x) = x^2 - 6$. With $x_0 = 3$ and $x_1 = 2$.

b. Use Secant method to find solutions accurate to within 10^{-4} for the following problems:

i. $x^3 - 2x^2 - 5 = 0, \quad [1, 4]$

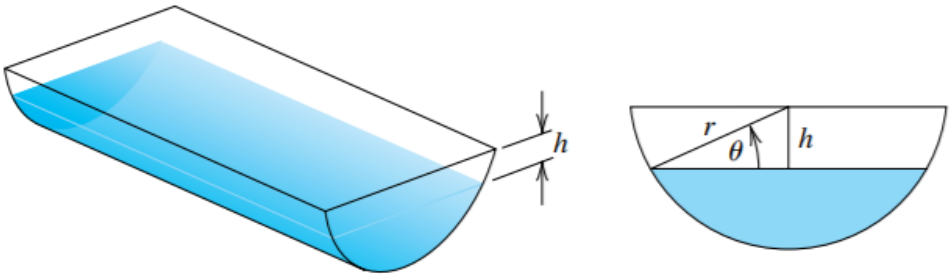
ii. $x^3 + 3x^2 - 1 = 0, \quad [-3, -2]$

iii. $x - \cos x = 0, \quad [0, \frac{\pi}{2}]$

Question 4

A trough of length L has a cross section in the shape of a semicircle with radius r (see the accompanying figure). When filled with water to within a distance h of the top, the volume V of water is

$$V = L\left[0.5\pi r^2 - r^2 \arcsin\left(\frac{h}{r}\right) - h(r^2 - h^2)^{1/2}\right].$$



Suppose $L = 10$ ft, $r = 1$ ft, and $V = 12.4$ ft^3. Find the depth of water in the trough to within 0.01 ft.

Question 5

Problem: In mechanical engineering, consider a beam subjected to a certain load, and the deflection of the beam can be modeled by the following nonlinear equation:

$$f(x) = x^3 - 4x^2 + 3 = 0$$

You need to find the root of this equation, which gives you the point where the deflection of the beam is zero within the interval [1, 3]. Solve this using the **bisection method**.

Question 6

Problem:

An environmental engineer is designing a filtration system for a wastewater treatment plant. The design requires the system to maintain a water flow rate of 500 liters per minute through a pipe. The flow rate Q (in liters per minute) through the pipe is governed by the following equation based on the Darcy-Weisbach equation for head loss:

$$Q = \frac{A}{\sqrt{R(f)}}$$

Where:

- A is a constant (in this case, $A = 10^4$).
- $R(f)$ is a resistance function that depends on the friction factor f.
- The equation for $R(f)$ is given by:

$$R(f) = 100 + 2f^2 - 20f$$

The goal is to find the friction factor f such that the flow rate is exactly 500 liters per minute. Using the bisection method, find the friction factor f that satisfies the flow rate condition.

Question 7

Use any programming for all the three methods discussed in this chapter to find solution accurate to within 10^{-5} of this problem.

$\boldsymbol{2x + 3cos\, x - e^x = 0}$ for $\boldsymbol{-2 \leq x \leq 2}$

Compare these methods for their:

i) Timing/how long has taken to reach desired output,
ii) No of iteration to reach desired output.