

Chapter #4 Interpolation and approximation (B)

1. For the given functions $f(x)$, let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(0.45)$, and find the absolute error.
 - a. $f(x) = \cos x$
 - b. $f(x) = \sqrt{1+x}$
 - c. $f(x) = \ln(x+1)$
 - d. $f(x) = \tan x$

2. Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:
 - a. $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$
 - b. $f(-\frac{1}{3})$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$
 - c. $f(0.25)$ if $f(0.1) = 0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$
 - d. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$

$$f(x) = x \sin x, \quad a = 0, \quad n = 4, \quad -1 \leq x \leq 1$$

- (i) Approximate f by a Taylor polynomial with degree n at the number a .
- (ii) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.
- (iii) Evaluate f and these polynomials at $x = 0.9$.