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2017

MCM/ICM

Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Human civilization on Earth has increasingly been plagued with income inequality, unproductive education systems, and gender-based discrimination. In the United States, the majority of society's wealth is held by the richest individuals, public education is underfunded and inefficient, and women are still underrepresented in certain sectors of the workforce. These societal failures result in suppressed opportunities for most individuals. However, with recent technological advances, Mars may soon be within reach; humans may now be presented with the opportunity to construct an ideal society from nothing, a utopia that will allow people equity in both income and wellbeing.

With an understanding of the complex nature of this problem, our model focuses on improving these three shortcomings. In the income model, our goal was to reduce income inequality by implementing the Nordic model of society. From this, we derived a minimum wage of \$20,000 as well as a progressive tax rate that increases with income until a cap of 50%. We measured inequality with the Gini coefficient, reducing it from .58 to .33, which approaches Nordic levels. In the education model, we used a weighted decision matrix, ranking different academic disciplines with how much they contribute towards our society's espoused values and fields of economic application. In the social equality model, we used opportunity cost analysis to simulate the retention rate of men and women in the workforce based on the cost of childcare and other demographic data. From this, we determined a standard childcare cost of \$7,500 as well as equal lengths of maternity and paternity leave. We incorporated the education and social equality models with the income and tax model to verify that our policies are economically viable.

After establishing our models, we generated a representative population to study the long-term dynamics of their demographic composition on Mars. We discovered that:

1. Human capital will initially decrease as Population Zero dies off, but begins to increase in the long run due to returns on our educational system
2. The demographics of Population Zero and subsequent migrants greatly affect the stability of our ideal utopian society, as undesired distributions could offset the equality our policies seek to establish
3. There exist oscillations in the age distribution of the population due to the nature of scheduled migrations from Earth to Mars

In attempting to establish a utopian society, we also consider the limitations of our model with respect to its scope. The colony on Mars is unprecedented and may exhibit different characteristics from the societies we know of on Earth. Specifically, it may prove difficult or even impossible to maintain true equilibrium in our economy, given that it is mostly isolated from other civilizations. In selecting our migrants, we assume that people are capable of both setting aside their earthly stereotypes and acting in accordance with their best long-term interests of preserving the utopian ideals.

Ultimately, we are not just approaching a new interplanetary frontier; we are exploring a new model for the ideal society.

DATED: 23 January 2017

FOR: Director, LIFE

SUBJECT: Societal Policy Recommendations

Our policy modeling team has established the included report to provide justification for certain policies that we believe will enable the Laboratory of Interstellar Financial & Exploration Policy (LIFE) to present the International Coalition on Mars with an ideal design for an economic-workforce-education system in the colony of 2100. Through our mathematical and computational models, we can draw recommendations on how to implement policies among Population Zero that will hold throughout the life of a continuously populated Martian colony.

We believe our policies prove to be a) scalable across consistent increases in population (10,000 migrants per 26 months), b) representative of a wide span of conceptual and mathematical models of the socio-economy, and c) last throughout the end of the 22nd century provided there are no large disturbances in migration.

With regard to income, we emphasize the importance of production within the state and happiness of the workforce. This leads us to choose certain extreme policy measures that allow for the sustainable welfare of the Martian population.

Gov-Corp: We propose a model where the government is also the sole firm in the economy. Such a Gov-Corp would be able to implement utopian policies not possible in a free market economy. However, with other migrations over time, many Gov-Corps will emerge, forming a network of firms that will behave differently.

Contingent Minimum Wage: Gov-Corp shall provide a minimum wage of \$20,000 to all of its working citizens, even if they contribute less production to the economy. This means that citizens working low-end jobs as well as those participating in work-learn programs will be guaranteed a salary at which they can live with fundamental necessities and comforts.

Progressive Income Tax: In order to promote equity our welfare state, Gov-Corp will need to tax its citizenry progressively. The tax, outlined in detail in our proposal, will still allow high-contributing individuals to succeed, but at a more reasonable rate of wealth accumulation than what we see on Earth.

Although we propose an ideal Population Zero, we recognize that the pioneers will eventually die off. Therefore, education is of the upmost importance in continuing the civ-

ilization throughout the century. Lifelong Learning Requirement: Gov-Corps will provide its adult citizenry with free education funded by revenues from the progressive tax, so long as they are engaged in one of three production options: working while learning part-time, working part-time while learning, or researching while learning.

Curriculum: The educational system will consist of a well-managed program of integrative sciences as well as applied humanities. This will provide technological advancement as well as intellectual progression for members of society.

With regard to equality and equity, we distinguish important differences between the two. We define equality as equal outcomes for everyone in society; we define equity as fair outcomes based on equal treatment of everyone in society.

Occupational Sectors: We propose that Population Zero represent men and women equally in various occupational sectors. This will allow women to be represented in fields they have been discriminated against. The establishment of equal gender distributions early on will allow for the elimination of traditional gender roles in the workforce.

Childcare: We recommend in our proposal that Gov-Corp set a \$7,500 childcare cost and equal length and full pay for maternity and paternity leave. From our calculations, this will result in 5-6% of the population in the childcare sector. If the cost of childcare is greater than an individual's opportunity cost, then a parent will leave the standard workforce and enter the childcare services sector of the economy for the contingent minimum wage. This equalizes opportunity costs for men and women and would eliminate divergence in male and female retention rates.

Note that our recommendations are sensitive to the composition of Population Zero, but not as much to the size. For example, a population that is not equally representative in each sector for men and women will result in unequal income distributions and gender disparity. Additionally, if we do not choose a fairly progression initial population, then the voyagers will bring with them existing stereotypes and contaminate future Martian generations with Earthly biases. Based on our simulation and analysis, we expect to achieve a balanced budget that supports the economic success of citizens and society, an effective educational system that contributes to human capital development, and a socially equal workforce that provides an equitable opportunity for advancement.

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A. HARNESSING COMPLEXITY: 10 YEAR VISION

Despite presenting massive technical and sociological challenges, the colonization of Mars also provides the possibility to redesign economic and societal incentive structures to achieve heights of civilization not possible on Earth. As such, the UTOPIA: 2100 project hopes to rise to this opportunity and create a new Martian society founded on egalitarian principles. Such a society would be driven of a smart city concept, in which every citizen is a node in an immensely connected network and government can cater to the needs of its populace like never before. However, rather than study the technological aspects of this utopian vision, we have been tasked to study the sociological, demographic and economic systems necessary to sustainably maintain a both a high productivity and increase citizens' wellbeing.

Taking inspiration from the ancient Greek concept of a *polis* (a term which describes not only the city-state but also the cohesive population that makes up its citizenry), we envision the initial colonization of Mars as the establishment of an ideal city government and community with Population Zero. To the ancient Greeks, the *polis* was not only the center of political life as the primary means of governance, but also a central social concept (a source of pride, identity and belonging) and an economic forum (a market for goods and resources to be pooled for the benefit of the citizenry) [1]. This design would feature

a each have an administration that essentially interacts as a government-firm, which will infrequently be referred to a Gov-Corps. Although radical, this structure allows for employers that are not profit-driven and capable of supporting massive welfare and education efforts, which we will discuss in later sections.

Another important overarching concept central to both the design of the Martian *poleis* and our modeling approach is complexity. Complexity is a term for ideas, problems and concepts that are difficult to define; in a mathematical sense, complex problems are typically characterized by a large number of variables, significant interdependencies best captured through network models, the interplay of stochastic and deterministic elements, and major components of randomness and regularity (similar to fractal phenomenon) [2]. Studying and designing complex systems obviously has huge ramifications for our effort to start a new society from scratch in a completely inhospitable environment. In order to account for the full complexity of this problem, we eschewed a simplistic model that would fully capture the variables of our study and instead opted for several multi-modal, interdependent models that are tied together by our conceptual understanding the ideal Martian workforce. The three aspects of the dynamic, demographic network we will focus on are income distribution, education policy, and gender equality.

B. DEFINING UTOPIA: METRICS OF SUCCESS AND KEY PARAMETERS

Two of the three targets of this study are focused on questions of balancing social equity and economic productivity. With this research we benefit from the work of many sociologists and economists who studied the causes of these factors on Earth and created metrics for the comparative analysis. For income distribution we will analyze income inequality through the Gini Index. The Gini Index is a commonly used metric that compares the actual cumulative income curve (called the Lorenz curve) to a the cumulative income curve of a perfectly equal society (a straight line from 0% of GDP to 100% of GDP with all earners receiving an equal percentage). The percent area difference between this ideal curve and the Lorenz curve is the Gini Index itself.

When studying gender we will look at two factors regarding the retention of women in the workforce: first, the number of mothers and fathers who take full maternity and paternity leave, and second, the number of men and women who leave the workforce after multiple children. In our research we found that much of the economic inequity that women face in the work was resulting from unequal childcare policies and, in some fields, historic,

low female participation rate [18]. This second root of inequity will be addressed by our initial demographic selections in Population Zero.

Success in education depends largely on meeting the needs for the future of the Martian colony. To do this our education program needs to ensure that not only are the intellectual requirements of population zero are met in subsequent generations, but as the technological advancement of society continues that citizens are prepared for Life-Long Learning (LLL). The prevailing literature on the future of education agrees that moving from a structure of large upfront educational gains (ie. the collegiate system) to continuous educational model is necessary to keep workers up to speed on modern technology and practices [11]. Furthermore, an LLL program not only serves as means of increasing productivity, but also a means to increase satisfaction, personal wellbeing, and societal cohesion [12]. Thus an LLL program is a major foundational piece of our vision of a Utopian workforce.

A primary variable used throughout our analysis is **Human Capital**, represented by H . Human Capital is the level of skills, knowledge, and intelligence possessed by an individual that allow them to make significant contributions beyond the standard worker. Throughout our model we assume that people accumulate human capital through both work and traditional schooling. Although academic study builds human capital at a faster rate initial, it suffers from decreasing marginal returns; whereas the benefit from work is more static.

C. POPULATION ZERO: DEMOGRAPHICS FOR SUSTAINABILITY

In order to study Population Zero, we decided to do selective Monte Carlo simulations of demographic fields based on our ideal demographic distributions. This simulated technique provides two vital aspects of our complex modeling approach: the first is variability, which provides a means to test robustness and base the generated population off real world data whenever possible, and the second is control, which allows us to manipulate the distributions and variables in order to create equitable conditions whenever possible. The simulated demographics formed a basis of our further analysis, and is crucial to our later assumptions.

With regards to economic productivity and income distribution, we used real world data from the US Census Bureau to create our initial economic productivity distribution. The reason for this is that any over idealized assumptions would reduce the complex realism

of a demographic model, in addition to concerns over sustainability. Even if a hyper-productive workforce could be recruited for Population Zero, this could not be replicated in subsequent migrations or in subsequent generations.

With education and social distributions, we took on more idealized assumptions when constructing Population Zero. We determined, through discussion and intuition based on the developmental needs for our Utopian society, that the idea distribution would consist of the following levels: 10% PhD equivalent, 30% MS equivalent, 40% BS equivalent, and 20% AS equivalent.

We translated education level and time in workforce into human capital, H , by accumulating through the following calculations. We also propose, via an analysis of how much work constitutes similar levels of human capital as our degree-based educational categories, the following human capital thresholds for each category: 30 units for a child of 10 years, 70 units for a graduate of the government mandated AS program, 100 units for a BS equivalent, 125 for a MS equivalent, and 145 for a PhD equivalent. This effectively captures the idea of diminishing marginal returns on formal degree designations for human capital development. In addition, individuals earned $1H$ for each year in the labor force (after age 25). The resulting distribution of human capital can be found in Figure 1.

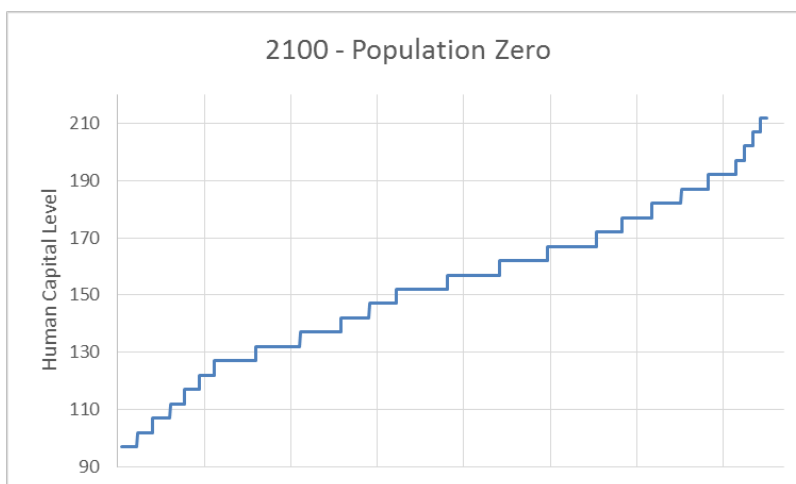


Figure 1. Distribution of Human Capital within Human Capital

In selecting gender and ethnic distributions, we assume an equitable distribution in each occupation to begin with. For gender these means we assume that the male and female income distribution curves are identical. The reason for this is that unequal conditions cannot be corrected for by equal policies and, as experience on Earth has shown unequal

policies, even if they are correcting unequal conditions, lead to societal tension. The long term viability of these assumptions will be discussed later in the paper.

When selecting an ideal age distribution we considered two factors: sustainability, a sufficient number of each generation to maintain population with low population growth in the long run, and practicality, as Population Zero must survive the 4 month journey to Mars. To do this we first calculated sustainable death rates based on a life expectancy of 80 years (a reasonable expectation in the year 2100) and an average death rate of 0.0086, a figure which is matched by our birth rate [5]. From these sustainable death rates, we calculated the survival rate, which was discretely integrated through to obtain a population distribution by age group. (A more thorough explanation of these calculations can be found in the Appendices).

Once we had a sustainable demographic distribution, we confined the population between 5 and 65 due to rigors of space flight. Rather than simply truncating the ends of our distribution, we recalculated a new distribution to maintain sustainable ratios. This final distribution is pictured below.

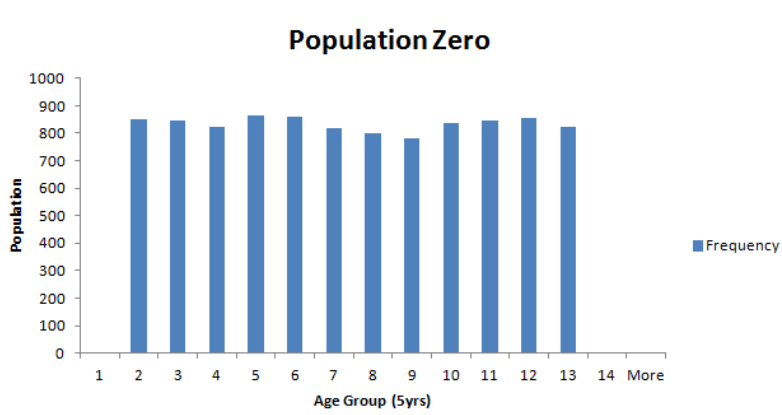


Figure 2. Demographic Distribution of Population Zero

D. ADDRESSING INCOME INEQUALITY

The income distribution is a crucial aspect of a utopian society which balance productivity and wellbeing. are among the most important aspects Before we introduce our model of income and policies to promote equality, we must make a few assumptions:

1. Wages are paid based on the productivity of a worker, and he/she only earns as much output as he put in the economy. Since Gov Corps is the sole firm in the

- economy (for now), it should be able to give workers the wages they deserve, wage laws notwithstanding.
2. People are rational agents in the economy, which means they will strive to produce as much output as they can to earn the highest wages. In addition, they will only spend as much as they earn, and will not fail financially and become bankrupt.
 3. Minors under the age of 15 will be members of the Martian Academy or in child-care, during which they do not directly work and contribute to the output of Marsopolis.
 4. The only labor source in our economy is the GovCorp, which manages all industries. Any income from other sources is ignored.

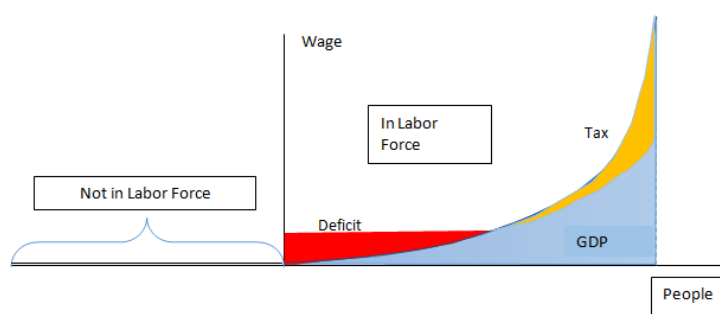


Figure 3. Income Distribution and Policy Implementation

Such assumptions led us to create the model of GDP shown above: the minors, elderly, students, etc. people not in the labor force receive welfare from the government. This may take the form of free elementary education, support for research in our Lifetime Learning Program, free healthcare for the sick and elderly, free child support for newborns and infants, etc. For people earning below our minimum wage, we provide bonuses to match the minimum wage, so that these people can support their families and maintain a sufficiently high quality of life. Lastly, the people earning above minimum wage will bear the tax burden incurred from the previous two groups, with a progressive tax implemented so that the richer people contribute a higher proportion to government welfare. (This is conceptually visualized in Figure 1.)

With these guidelines in mind, we set off to collect data concerning the specific parameters in our model. For data on the percentage of people in the labor force, we looked at the Nordic model, Norway specifically. We found that labor force participation rate was 71.2% for people over 16, and 18.1% of people were under 16 [4]. To simplify the

data in our model, we deemed the labor force participation rate 70% for Population Zero and that 20% were under 16. This meant that the labor force was $70\%(1-20\%)=56\%$ of our total population, and 44% were not working or earning a salary. Given that Population Zero has 10,000 people, this translates to 5,600 workers.

We gathered our work force data using a representative sample of collected from the working population of Connecticut. We used this set of data because it was reasonably balanced and was small enough (27,000 people) to work with [5]. We determined that an inverse model fitted the actual wage-over-person curve best, and used a solver in Excel to generate an equation. After scaling down this curve to 5,600 people, we determined the equation:

$$W(x) = \frac{62205676}{5670 - x} - 970$$
$$\text{for } 0 \leq x \leq 5600$$

Where x is the ranking of a person income, 0 being poorest and 5600 being the richest, and $W(x)$ is the wage of that person, pre-tax. (Curve of Actual wages, modeled wages, model-fit analysis in appendix).

Next, we generated a minimum wage and calculated the welfare required to sustain such a baseline. For this, we again turned to the Nordic model to determine how much this should be. Though Norway has no legal minimum wage, trade unions and conventions mean wages are usually above USD \$15/hour [6]. Translating to yearly wages, this is around \$24,000. Since Norway has higher prices than average, we decided our colony to have a minimum wage of \$20,000. Given our calculations from the equation, we found that approximately \$16.4 million had to be paid to ensure this population earned minimum wage and could their standard of living.

In addition, we calculated total public education and government operation costs (healthcare was excluded from this analysis, so its burden on the public was ignored). Yet again we looked at the Nordic model, which gave an average of \$20,000 for postsecondary education [7]. Given our Martian Academy curriculum is so advanced, we assumed it had similar costs per pupil. Since we assumed 20%, or 2,000 of our residents were minors in school, this brings education costs to $2000 \times \$20000 = \40 mil. In addition, the government must have money to fund infrastructure, research, etc. public operations in order to maintain a high standard of living for all citizens. To estimate these costs, we again turned to the Nordic model [8]. The operational costs of the Norwegian government for fiscal year

2016 was 109 million kr, roughly equivalent to \$2407 per capita. For our 10,000 people Marsopolis, we decided on an overall spending of \$20 mil, for simplicity. Total government spending, then, equals $40+20=\$60$ million. Total tax revenue collected should be $60+16.4=\$76.4$ million.

Finding this value, we then set about designing a tax scheme to collect this amount. We proposed a simple progressive tax, where only earnings above minimum wage were taxed. Further, the percentage taxed would be proportional to the earnings above minimum wage, so the richer were taxed a higher percentage. This would achieve our goal of promoting income equality. Using computer software, our equation for taxing was determined:

$$T(x) = W(x)1.56 * 10^{-6}(W(x) - 20000)$$

$$x \geq 20000$$

Where $W(x)$ is the wage of a person, and $T(x)$ is the total tax.

However, it was also found that the highest earners would receive an unreasonably high tax rate. We therefore altered the equation and defined the maximum tax value as 50%, while keeping total tax revenue constant. Our new taxing equation, then, became:

$$T(x) = \begin{cases} \left(\frac{62205676}{5670-x} - 970\right) * 1.56 * 10^{-6} * \left(\frac{62205676}{5670-x} - 20970\right) & 2703 \leq x \leq 4925 \\ 0.5 * \left(\frac{62205676}{5670-x} - 970\right) & 4925 \leq x \leq 5600 \\ 0 & \text{other} \end{cases}$$

Using this equation, income equality was noticeably reduced. To quantify this change, we used the Gini coefficient, a common measure of income inequality that ranges from 0 to 1, 0 being perfect equality and 1 being perfect inequality [9]. We calculated the Gini coefficient pre-tax to be 0.58 (Appendices), whereas post-tax Gini coefficient was reduced to 0.36 [10].

E. EDUCATION: LIVING AND LEARNING ON MARS

As part of training highly skilled workers for the utopian economy, gov-corp has a responsibility to provide a free, consistent, and highly dynamic education to all children in the polis. This means constantly staffing a workforce of highly qualified educators for the government mandated school system as well as advanced education options for all citizens

willing to participate. We leverage two separate models of education in order to determine the aspects of an ideal education policy for Martian colonization: the first is a weighted decision matrix to determine which academic disciplines will be weighted most heavily in the curriculum, and second a human capital model to assess how well the educational system replenishes the distribution of critical skills following the decline of Population Zero.

For curriculum design and prioritization we began by looking at academic disciplines and ideal societal outcomes. Whereas the academic disciplines were derived from the standard breakdown readily available on Wikipedia, educational outcomes were given much consideration. We ultimately decided to analyze the ability of disciplines to both create skills directly applicable to the economic sectors of a Martian colony as well as inculcate values that would promote innovation, problem solving and societal cohesion. A breakdown can be seen in the Appendices.

Applying a comparative approach, we determined the relative weight of values to 30% and economic applications to be 70%. Within each of these outcome classes, we rank ordered them in importance for the next 10 years and weighted them accordingly. Following this, we evaluated each academic discipline's ability to support the various objectives and gave them points for supporting highly weighted goals. (A full version of this matrix and model can be seen in the Appendices.) **We determined the six most important educational priorities to be: Computer Science, Systems Engineering, Chemical Engineering, Biology, Chemistry, and Hydrology.**

In addition to curriculum development, we used a discrete dynamical system to model the accumulation of H over time. within our lifelong learning program, citizens have three production options after they graduate from the school system at age 20: working while learning part-time (W/PL), working part-time while learning (PW/L), and researching while learning (R/L). Each of the production options corresponds to a certain distribution of working and learning: 80% to 20%, 30% to 70%, and 5% to 95%, respectively. As we continue to model the citizenry over the long run based on 5 year time steps, we can calculate the addition to a citizens human capital based on which production option they choose over any period.

The following human capital model was derived from experimentation with the marginal returns on work and learning. After said experimentation, we concluded that human capital accumulation can be approximated by the following marginal returns: 1 additional unit of human capital per year of work and 5 additional units of human capital per

year of learning. Therefore, based on our stated distributions above, the marginal benefits on human capital of the three production options are as follows: 9 units for W/PL, 19 units for PW/L, and 24 for R/L. Running these distributions through the simulated Population Zero through the year 2110, we found a distribution of human capital as displayed in Figure 5. A full analysis of the long run changes in the human capital distribution will be discussed with the 100 year plan.

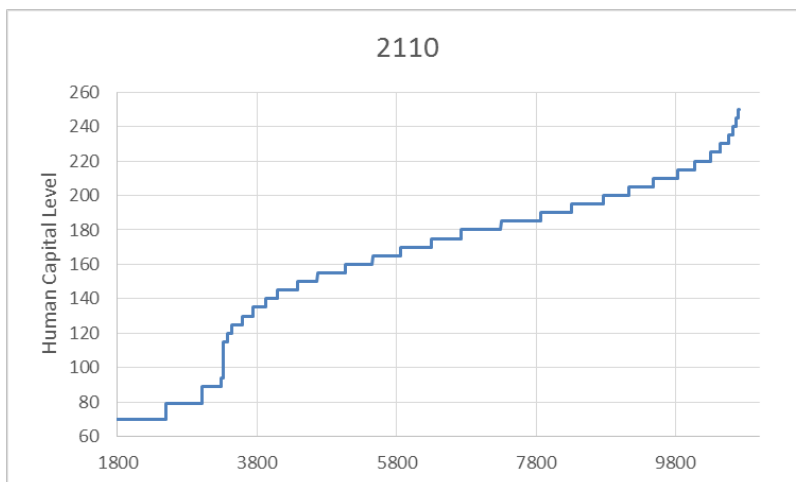


Figure 4. Human Capital Distribution at 2110

F. SOCIAL EQUITY: AN ECONOMIC MODEL

If the workforce is the primary driver of progress in our Martian utopia, we must emphasize the importance of retention rate in the labor market. Turnover between occupations and excessive absences from the workforce result in diminished human capital. Therefore, it is in society's best interest to provide incentives for parents of newly born children to retain their positions while parenting pre-school children. This leads us to choose the retention rate as the metric we will use to evaluate a society's equity efficacy, as Earth is marked by historically large spreads between men and women in that regard.

In order to equalize the retention rate gap, the government must provide for, and mandate, equal length of maternity and paternity leave, defined now as parental leave. Parental leave would allow parents to remain in the workforce as their job would await them after the leave ended. The point of ensuring every parent takes the same amount of leave stems from the estimation that human capital depreciates by around 1% for each

month of parental leave, while human capital accrues via on-the-job experience at a rate of only 0.2%. [3]

Given that we have established an equal leave strategy for gender equality in the workforce, the main issue in retention becomes childcare. If the financial costs of childcare services outweigh the opportunity costs of working, then it makes sense for one parent to exit the workforce until his or her child enters the educational system at age 5. Here we will assume both parents do not decide to exit simultaneously under any conditions, as one parent can successfully remove the need for childcare services. We will also make the stipulation that parents exiting the workforce will enter the gov-corp sponsored public childcare sector, receiving the minimum wage.

On an individual basis, parameters for a couples exit decision include number of children, the mothers salary, and the fathers salary. These are then integrated with our chosen policy constants to determine whether an individual couple will choose to have only one working parent. An analysis of each of the constants will be included in Appendix UL1. The model is represented here, where a true statement constitutes parental exit from the workforce:

Now that an individual model has been established, we can integrate further demographic constants in order to analyze the population as a whole. An analysis of each of the constants will also be included in Appendix UL1.

$$Cost_{childcare} \geq Cost_{opportunity}$$

$$Cost_{childcare} = Children * Cost_{services} * Age_{School} - MAX(Leave_{maternity}, Leave_{paternity}) + Salary_{min}$$

$$Cost_{opportunity} = MIN(Salary_{mother}, Salary_{father}) * Age_{School} - Leave_{MinSalary} * (1 - Fraction_{PaidLeave})$$

Monte Carlo simulation was used to generate 5,000 random couples, representing those with children and those without. Note that a couple with zero children could either be married without children or simply two single people that have never met. This model cares only about children as inputs to an exit decision. Within the simulation, random incomes were generated based on the inverse distribution curve provided in our previous income model.

After running the simulation, we generate a sample population from which we see the ratio of the total population out of the workforce due to children, as well as the ratios of males and females out of the workforce. Sample findings can be seen in Figure 1 of Appendix UL1. Through running multiple iterations of the model, we can tell that it is

most sensitive to the cost of childcare, how many women choose to bear children along with how many children they choose to have, and minimum wage. Additionally, the ratios increase across the first five years due to the lack of children aged 0-5 within Population Zero in 2100, but stabilize in the following years up until 2110. Note that the gender ratios are practically the same; this is due to the fair income distribution we designed in a previous model that does not discriminate based on gender.

Since the cost of childcare services makes the most difference in a couples exit decision according to our analysis, emphasis for our equity policy should go towards choosing the optimal cost for state-run childcare. We thus recommend a \$7,500 per child childcare cost, as it will result in approximately 5-6% of the population as childcare providers. Such a childcare system will incentivize all adults to stay in the workforce, with the only exceptions being those working part-time as they seek out further education and of course the retired elderly. Parents that exit their jobs to care for children will do so in an additional capacity in the childcare sector that will bring them back into the immediate workforce. This will result in an overall more productive society, but also one that cares closely for the early development of its children.

G. MODELING SUBGROUPS

As demonstrated previously, our global model optimizes the minimum wage, salary distribution, age distribution, etc. many factors to create an efficient, balanced, highly functioning society with a high standard of living of everyone. Nevertheless, different classes and subgroups still form amongst the unity, and it is crucial that we examine these different subgroups to make sure our society advances as a whole, and no group is left behind.

To divide the population into subgroups, we used two objective criteria: salary and marriage/kids status. We determined three salary ranges: low (at or below minimum wage), medium (above minimum wage with variable tax), and high (wage has 50% tax). This, combined with the status of kids (yes or no), gives us six subgroups. We shall examine each groups specific prioritized need, and observe how these desires coincide with or conflict with the values of other groups.

First, we analyze the poor, childless adults. This group actually encases two groups in the population: those working below minimum wage and those completely unemployed. For those already working, they would most likely seek higher paying jobs, which would in

turn require them to earn more human capital through work or further education. For those without a job, they would most likely prioritize getting a job, no matter the skill or pay, in order to receive the minimum wage. Though this group is often overlooked in real life [1], our program of lifelong learning actually ensures that they have a continuous opportunity to advance their careers and increase their productivity and wages with time [19].

This group constitutes poor couples that have at least one child. They are confronted by two objectives they most likely wish to achieve: the care, education, and healthy growth of their child or children, and the financial pressure to return to the workforce after mandatory maternity and paternity leave following the child. Our social model aims to keep both career paths open: while people can still choose to stay at home and take care of their children, they can still pursue a career, and their children will still be taken care of. With our mandatory maternity and paternity leave policies, we guarantee that couples do not lose their productivity after childbirth. Our childcare compensations take significant financial pressure off the couples shoulders, also allowing couples to adopt the stay-home mother/father model is they so wish too, without financial worries.

For the middle class, childless workers in our colony, their primary goal we are concerned with is the advancement of their careers. Unlike the lower-earning workers, these people incur a significant opportunity cost by attending further education after being in the workforce for a significant duration. Therefore, the goal of our society is to allow these workers to retain their human capital while absorbing new knowledge through our LLL program. Our program achieves this by spacing out learning, so workers have a chance to apply their acquired knowledge before further advancing their academic paths. The middle class couples, in addition to individual career ambitions mentioned above, would most likely be concerned with the quality of education their children are receiving. They want to ensure their children are prepared for the future workforce, and have the opportunities to pursue a career of their choice. Our Martian Academy accomplishes this by exposing students to work at an early age, so children learn to apply their knowledge, gain new information, and understand their career paths.

The high-salary single workers, we presume, will not be motivated to work or learn more for higher wages. After all, a steep 50% income tax necessitates that people would not be so keen to earn more. Therefore, these workers may have the true intellectual curiosity to conduct research full-time, regardless of its effect on human capital. The government supports this by providing research grants, funding these highly technical and creative projects which will (hopefully) lead to technological breakthrough. Though this incurs a burden on

the government, the benefits of technology can be said to outweigh the drawbacks.

Finally, the high-salary families, in addition to the activities mentioned above, would presumably want to transfer their wealth and knowledge to their children (since this is what well-off families in real life typically do). Unfortunately, this is rather difficult in Marsopolis. Since all jobs are organized by GovCorps, human capital will become the most influential factor for hiring; since the only school in the colony is the Martian Academy, there is no guarantee that these privileged children will end up having the most desirable jobs. Although this is desirable for the equality of the colony, this is clearly not favorable to the rich families who want to escort their children to success.

As can be seen, different subgroups of society value different aspects of it. The poorer value welfare, whereas the richer value economic freedom and educational growth. The poorer value education as a means towards achieving greater human capital and wages, whereas the richer value education more inherently. The childless pursue more ambitious career paths with job training, part-time work-research programs, whereas the families desire more time off, parental leave, and less intensive jobs. Overall, the Marsopolis we designed attempts to favor the socioeconomically disadvantaged, at the expense of high tax rates and decreased privileges for the wealthy.

H. IN THE LONG RUN: 100 YEAR VISION

We found in general that our 10-year plan was well-developed and robust in the face of scalability and long run trends. This may be because of our design preference towards sustainability which was incorporated from the beginning. Our overall dynamic demographic model, which began with the procedurally generated Population Zero, was continued into subsequent generations using the reasonable death rate distribution discussed earlier and a distributed birth rate based on current census data [5]. The resulting age distributions of 2150 and 2200 are shown below in figure6 and 7.

Though the population was sustained into the long run, complex oscillation emerged as time progressed. We believe that this behavior may be a result of the initial practicality limitation placed on the age distribution of Population Zero. We do not think this will present significant challenges to the holistic functioning of society. However, some significant changes must be made to the recommended policies.

When considering gender equality, subsequent populations would not be as evenly distributed across occupations. Our primary colonization efforts entail finding an equal

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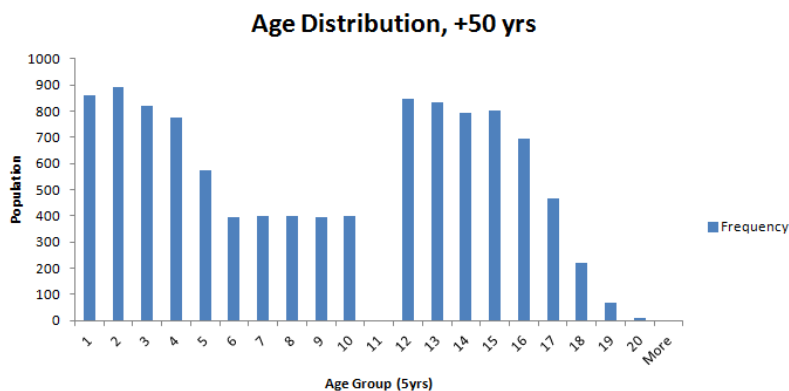


Figure 5. Age Distribution at 2150

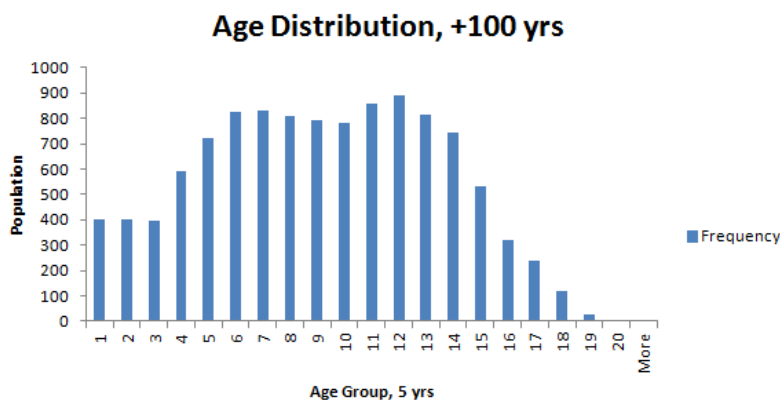


Figure 6. Age Distribution at 2200

ratio of men to women within each occupation, in order to curb traditional gender biases. However, the de facto population on Earth does not match our ideal population; after several migrations it would become increasingly hard to find highly qualified females in STEM sectors such as mechanical engineering, physics and astronomy, and electrical and computer hardware engineering, as well as all others. Currently, women make up half of the workforce with a college education in the United States, but only comprise 29% of science and engineering sectors, with only the following workforce shares for the previously mentioned specialties: 7.9%, 11.1%, and 10.7%, respectively [14]. We see similar patterns with ethnic and racial minority groups as well as low-income populations. Therefore, the 46 migrations that will occur over the next 100 years may exhaust the populations we are trying to tap for the ideal workforce of Mars. In the example of mechanical engineering, there were 277,500 jobs in 2014 according to the Bureau of Labor Statistics and 32,727

degrees in the field were awarded in the same year [15][16]. While 7.9% of this occupation on Earth would be enough to sustain subsequent migrations to Mars, the probability of getting similarly skilled mechanical engineers between men and women is low.

Using the previously developed equity model, we can simulate some subsequent populations based on changing demographics. Although the income distributions for men and women were originally equal, we can now handicap the female salary distribution by a realistic amount. A simple way to illustrate this point will be to multiple the randomized female distribution by a ratio of the female median annual earnings in the United States (\$39,621 in 2014) and the male median annual earnings (\$50,383 in 2014). Using this simple handicap, although it does not account for decreasing wage gap over time for the twenty-second century, we can generate new populations that represent more Earthly gender inequality in terms of differently weighted workforce sectors and unfair pay. Though this modified simulation, we see that Earthly income inequality greatly affects the equity of the workforce on Mars (see Figure), as significantly more women quit their jobs to become childcare providers, almost double the percentage of men.

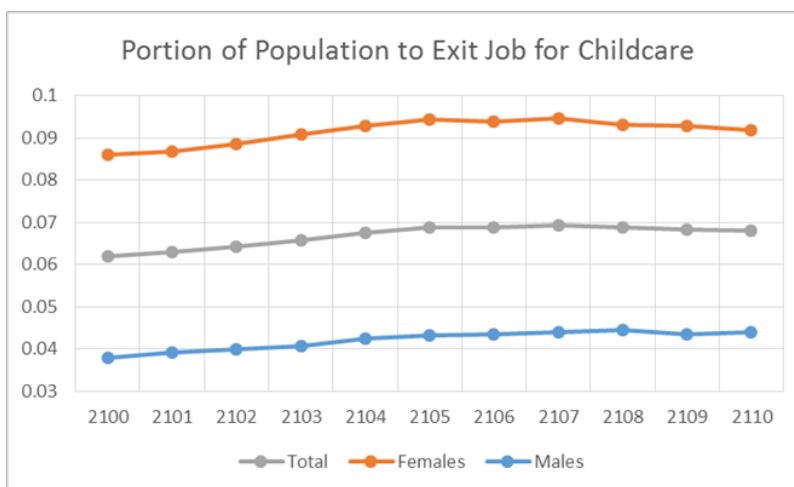


Figure 7.

Over time, as each migration creates a new polis, there will emerge interaction between poleis that mimic those between firms. Though we do not predict or design for a market economy, as each individual gov-corp is still primarily interaction with its citizens rather than trying to make significant profits on the citizens of other poleis, some specialization will naturally occur. Our goal is not to have incredible powerful and significant poleis that will dominate the colony, even though an earlier polis will naturally have more

time in development that newer poleis. Thus, we propose citizen migration from old poleis to new poleis in order to offset unequal distributions, as their initial populations will have had more consistent distributions across sectors and their new Martian generations will have eventually passed through our improved educational system (which churns our high human capital graduates, with even distribution with regard to gender and other diversity demographics).

With regards to educational curriculum, we modified our weighted decision matrix in order to reflect the greater importance of innovation, cohesive values, and managing greater complexities across society. **Prompting significant changes to our recommended curriculum, the highest valued academic disciplines in the long run are: Systems Engineering, Computer Science, Sociology, Art, Literature, and Mathematics.** Thus, while the 10 year educational vision is set mostly on technical development and sustaining the human population in a complex new environment, the 100 year long-term vision is more focused on humanities and higher-level systems thinking. This is outcome makes sense due to the fact that as technology becomes more developed, humans will be able to rely less and less on production and more on culture. A content populace over the long-run is one that finds purpose in existence and continues to develop intellectually rather than simply increasing its technical knowledge.

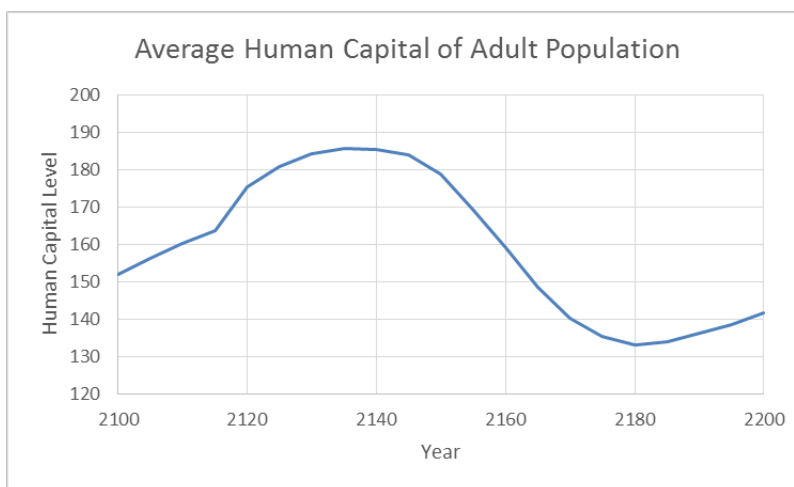


Figure 8. Average Societal Human Capital Over Time

Tracking human capital overtime results in more alarming trends. As time goes on, the average human capital throughout society increases until the well-educated Population Zero dies off in which a period of decline follows. However as the Martian LLL program

comes into effect, the trend reverses and H begins to rise again. This suggests that the program will be reach an equilibrium near Population Zero rates. Another point of note is the increasing inequality of human capital as the simulation continues as seen in Figure 10. Though this may seem problematic, it is likely beneficial for society as a whole given that individuals with extremely high H will likely function as Mars's innovator class, propelling another century of societal and technological revolutions.

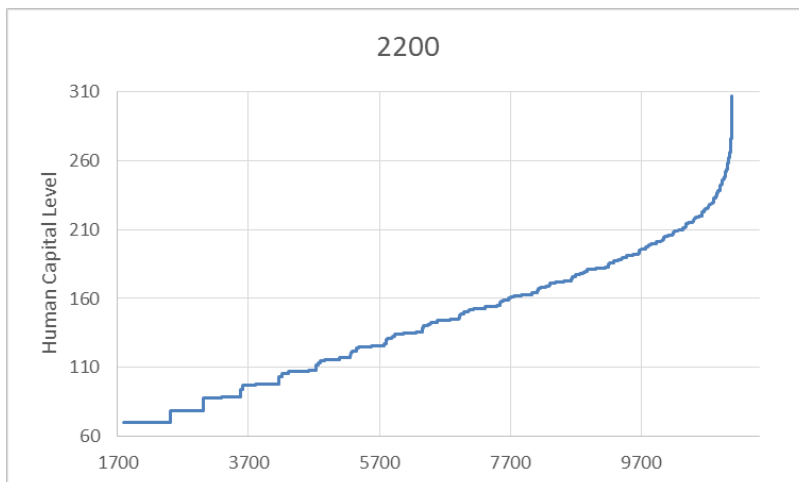


Figure 9. Human Capital Distribution at 2200

I. EXTERNAL SHOCK: EARTH EVACUATION

So far, we have seen Marsopolis adapt well when it is slowly expanding. When our polis expands rapidly, though, the situation is very different. For instance, if there was a massive migration to Mars in our model, even if it only doubles the population, the results would be catastrophic. For example, welfare provided by government would plummet, and standards of living would drastically decrease, if not plummet.

Several factors explain this incompatibility. First, the stability of our demographics would be compromised by the sudden influx of people. Our economy's tax rates are built considering a specific proportion of people would be in school, a specific proportion of people would be in the work force, and a specific progressive tax scheme set so that government revenue equals expenditures. A sudden influx of people could unbalance any of these factors and force welfare levels down. The equations below illustrate this point [17]:

$$G = T$$

$$C * n = k * Y$$

$$C * n = k * F(L, K, H, N)$$

$$G \propto n, T \propto A$$

Here, Line 1 states that government expenditures (G) should equal government revenue in taxes (T), as they always should. Line 2 assumes that expenditures are proportional to number of people (n) and cost per head (C), and assumes that taxes are proportional to average tax rate (k) and total output (Y). Line 3 further breaks down output into its factors of production: technology (A), labor (L), physical capital (K), human capital (H), and natural resources (N).

This is where the inequality is shown: though maintenance costs in our colony are proportional to its population, total output is most likely not. It is unlikely that some revolutionary technology which doubles our productivity, is given to us as mass migration onto Mars occurs. It is possible that the labor available will grow proportionally with migration, yet the physical capital in our colony simply cannot accommodate all additional labor. There exists neither the resources nor the time to double our factories and carrying capacity among colonies to prepare for sudden migration. Most likely, our new migrants will not have the same skills as our graduates of the Martian Academy do, therefore they will possess less human capital and will not be able to work many jobs in the colony. Finally, the amount of land and natural resources will most likely not change: it is highly unlikely that Mars will double in size as our colony swells.

All these factors combined, mean that to prevent GovCorp from exhausting its funds, it must choose one of two choices: either increase tax rate K or reduce gov. spending per head (C). However, we quickly realize that the former choice is unrealistic as well: our maximum tax rate is already 50%, so must we increase it to 100% if the population doubles? What would incentive people to work, then? This means in the face of rapid migration, our only realistic option is to slash welfare, either by reducing spending as a whole or only providing extensive benefits (educational, medical, scientific) to our original inhabitants, giving our refugees only basic housing and food. Either way, the vision of Marsopolis as a utopia would quickly break down.

This economic unsustainability also gives way to the educational unsustainability in the event of a mass migration. Since there is no guarantee for continued funding in the education sectors, there will likely be less educators available, and therefore less pupils in the Martian Academy.

J. CONCLUSIONS

Strengths: After dealing with and evaluating our model of a Mars colony extensively, we believe there are several advantages to our approach. First, by approaching the problems of income distribution, education, and equality separately, we were able to optimize each sector, allowing for a more comprehensive view of our society. For example, we were able to determine parental leave policies while trying to achieve equality, determine taxes and minimum wage policies while analyzing income, and create an educational curriculum while solving the education problem, none of which compromise each other. Second, our model is relatively scalable and is designed to develop over time. We designed the population demographics so that a stable population could be reached after decades; similarly, the income and equality models are compatible with larger population sizes.

Weaknesses: Despite our research, there still exist some potential improvements in our model. One flaw is the lack of a unifying variable which connects all three of our models: we use wage in dollars in our income model, human capital while modeling equality, and the perceived needs of the colony when creating an education curriculum. We could, for example, have modeled income as a result of human capital, education as a factory producing it, and equality as a function based on it; however, we felt that some of these relations would be too arbitrary to model.

Another weakness is the inherent lack of data in this problem. We used a representative American population as the basis of our model, and utilized the Nordic model for our goals. In an isolated planet colony, neither may match reality closely. There has long been a historic lack of data on isolated societies, aka utopias, since they do not tend to last long. Given the very nature of the problem, therefore, we may never generate a truly accurate model for the alien environment of Mars.

Final Summary and Future Research: We attempted to model a Martian colony which promoted equality and general wellbeing using income, education, and work policies. Despite using an ordinary initial population, we were able to devise a model which was stable and produced desirable results. For future research, we could attempt a unified model with one key variable, such as the human capital model mentioned above. Also, we could use our generated demographic distribution as inputs to our models. Finally, we could design contingency plans for scenarios such as mass migration from Earth.

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		Applications						
	Agriculture	Colonial Expansion	Health	Entertainment	Education	Research	Manufacturing	Administration
0.7	0.16666667	0.02777778	0.2222	0.11111111	0.0833333	0.055556	0.13888889	0.19444444
0.6	0.02777778	0.19444444	0.1389	0.11111111	0.2222222	0.083333	0.05555556	0.16666667

Agriculture	Colonial Expansion	Health	Entertainment	Education	Research	Manufacturing	Administration
0	0	0	1	1	0	0	0
0	1	0	0	1	0	0	1
0	0	0	1	1	0	0	1
0	0	0	1	1	0	0	0
0	0	0	1	1	1	0	0
0	0	0	0	1	1	1	1
0	0	0	0	1	0	0	1
0	1	0	0	1	0	0	1
0	1	1	0	1	0	0	1
0	0	1	0	1	1	0	1
1	0	1	0	1	1	0	0
1	0	1	0	1	1	0	0
1	0	0	0	1	1	0	0
0	0	0	0	1	1	1	0
0	0	0	0	1	1	1	1
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1	1	1	1	1	1	1	1
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0	1	0	0	1	1	1	0
0	1	0	0	1	1	1	0
0	1	1	0	1	1	1	1
0	0	1	0	1	1	0	0
0	1	1	0	0	0	0	1

SR	LR
0.316111	0.56
0.333889	0.39
0.362222	0.5
0.316111	0.56
0.265	0.45
0.450556	0.356667
0.344444	0.393333
0.333889	0.39
0.459444	0.633333
0.418889	0.486667
0.519444	0.523333
0.519444	0.523333
0.363889	0.44
0.344444	0.456667
0.480556	0.556667
0.452222	0.453333
0.510556	0.486667
0.82	0.64
0.567222	0.423333
0.489444	0.456667
0.333889	0.373333
0.333889	0.373333
0.775556	0.796667
0.462778	0.506667
0.333889	0.39

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Appendix – Discussion of Variables in the Equity/ Childcare Model

Figure 1: Sample Equity Model Dashboard:

INPUTS:												
births per population rate	0.0086											
length of paid maternity leave, months	12											
length of paid paternity leave, months	12											
fraction of salary for leave	1											
cost of childcare	7500											
minimum wage	20000											
start age for primary schooling	60											
population size of males	5000											
population size of females	5000											
fraction of females at childbearing age	0.465											
fraction of women without children	0.71263											
fraction of women with 1 child	0.172											
fraction of women with 2 children	0.231											
fraction of women with 3 children	0.185											
fraction of women with 4+ children	0.03											
RESULTS:												
Year	2100	2101	2102	2103	2104	2105	2106	2107	2108	2109	2110	
Ratio of population out of workforce	0.0521	0.0531	0.0542	0.0554	0.0564	0.0583	0.0589	0.0591	0.059	0.0583	0.0581	
Ratio of females out of workforce	0.0536	0.0546	0.0556	0.0566	0.0578	0.0602	0.0606	0.0608	0.061	0.0602	0.0594	
Ratio of males out of workforce	0.0506	0.0516	0.0528	0.0542	0.055	0.0564	0.0572	0.0574	0.057	0.0564	0.0568	
Net new children in system	N/A	82	78	81	79	82	4	6	-7	-10		

Individual Model (Policy Factors):

- Cost of childcare – the mandated cost of government provided childcare per child, in dollars; this is the value that we care about adjusting the most to affect policy decisions; \$7,500 was chosen as it results in 5-6% of parents in the childcare services sector and thus keeps most of the population in the workforce
- Start age for primary schooling – the age at which children are introduced into the public primary school system, in months; 60 months was chosen
- Length of paid maternity leave – the time a mother may take off work without giving up her employment when bearing a child, in months; 12 was chosen as equal leave times result in greater income equality across genders
- Length of paid paternity leave – the time a father may take off work without giving up his employment when helping bear a child, in months; 12 was chosen as equal leave times result in greater income equality across genders
- Fraction of salary for leave – the portion of full employment pay that a mother or father is entitled to when taking parental leave; 1 was chosen for this model as it creates less hostility towards taking the mandated leave
- Minimum wage – the lowest amount earned by any working member of society, in dollars, in that anyone making less will make the difference in terms of a welfare benefit; \$20,000 was chosen, as discussed in the income model formulation

Figure 2: Sample Individual Model Execution:

		INDIVIDUAL MODEL:					
Inputs:							
children	2						
female salary	32000						
male salary	43000						
Cost of Childcare:							
	Care cost		Opportunity				
If	720000	>=	1940000		then one parent will exit workforce		
Else					both parents stay in workforce		
Output:							
Exit?	0						

Aggregate Model (Demographic Factors):

- Births per population rate – the number of births per year over the whole population; 0.0086 was chosen based on analysis of population demographics
- Fraction of females at childbearing age – the portion of women between 15 and 49 that are capable of healthily bearing children; 0.465 was selected based on U.S. census dataⁱ
- Distribution of women with 0-4 number of children: the portion of women that at any point in time have 0 through 4 children; 0.713, 0.172, 0.231, 0.185, and 0.03 were chosen to represent those respective categories; we determined these numbers based on U.S. census data, extrapolating slightly to get one additional category for 4+ childrenⁱⁱ

Figure 3: Sample Aggregate Model Execution (partial):

		MONTE CARLO:							100						
		YEAR 2100		AVGS:	0.1056	0.0506	0.055		YEAR 2101		AVGS:	0.1084	0.052	0.0564	
	rand	children	female	male	exit?	f-exit?	m-exit?	rand2	children	female	male	exit?	f-exit?	m-exit?	
1	0.490525095	0	38389.01	13113.57	0	0	0	0.917442	0	38389.01	13113.57	0	0	0	
2	0.640113452	0	500048.3	27982.12	0	0	0	0.699195	0	500048.3	27982.12	0	0	0	
3	0.011296523	0	15812.01	18155.67	0	0	0	0.178788	0	15812.01	18155.67	0	0	0	
4	0.779384808	1	28205.31	11921.87	0	0	0	0.229907	1	28205.31	11921.87	0	0	0	
5	0.608527773	0	10937.18	11062.37	0	0	0	0.690093	0	10937.18	11062.37	0	0	0	
6	0.713449368	1	11590.05	30060.98	0	0	0	0.829924	1	11590.05	30060.98	0	0	0	
7	0.812224228	2	89134.19	20845.49	0	0	0	0.007377	3	89134.19	20845.49	0	0	0	
8	0.952569358	3	40841.11	339950.8	0	0	0	0.673303	3	40841.11	339950.8	0	0	0	
9	0.828582225	2	28986.03	10247.77	1	0	1	0.527396	2	28986.03	10247.77	1	0	1	
10	0.467046067	0	12136.59	13326.21	0	0	0	0.337508	0	12136.59	13326.21	0	0	0	
11	0.996058546	4	11069.36	33061.88	1	1	0	0.761438	4	11069.36	33061.88	1	1	0	
12	0.24177474	0	11026.5	33602.72	0	0	0	0.385541	0	11026.5	33602.72	0	0	0	
13	0.808745792	2	52774.09	12544.84	0	0	0	0.569919	2	52774.09	12544.84	0	0	0	
14	0.218208294	0	12195.53	19253.37	0	0	0	0.642798	0	12195.53	19253.37	0	0	0	
15	0.346919167	0	370858.3	12889.6	0	0	0	0.987244	0	370858.3	12889.6	0	0	0	

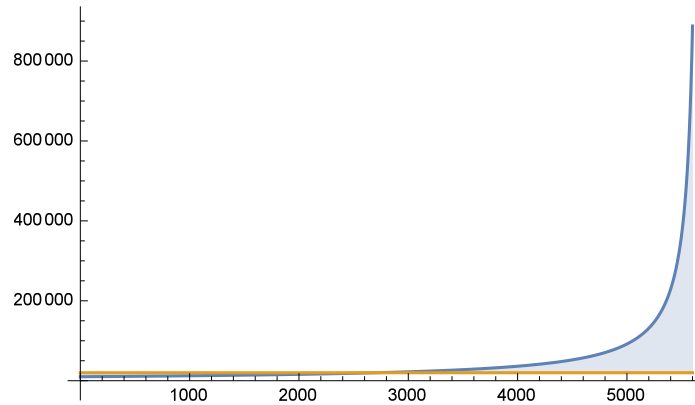
-full simulation spans 5000 couples and years 2100-2110

ⁱ United States. Census Bureau. *Population by Age and Sex: 2012*. Current Population Survey, Annual Social and Economic Supplement, 2012. Accessed January 22, 2017.

ⁱⁱ United States. Census Bureau. *Fertility of Women in the United States: 2012*. By Lindsay M. Monte and Renee R. Ellis. Population Characteristics. 20-575.

$$f[x]:=62205676/(5670-x)-970$$

Plot[{f[x], 20000}, {x, 0, 5600}, Filling -> {1 -> {2}}, PlotRange -> All]



NSolve[62205676/(5670-x)-970==20000,x]//N

{{x -> 2703.59}}

Integrate[20000-62205676/(5670-x)+970,{x,0,2703}]/N

1.63952×10^7

Integrate[62205676/(5670-x)-20970,{x,2703,5600}]/N

1.72323×10^8

$(1.63952 \times 10^7 + 6 \times 10^7) / 1.72323 \times 10^8$

0.443326

NSolve[Integrate[(62205676/(5670-x)-970)*k(62205676/(5670-x)-20970),{x,2703,5600}]==1.63952{k}]

{{k -> 1.5616236465059038^-6}}

.5/1.5616236465059038^-6

320180.

$\text{Integrate}[(62205676/(5670 - x) - 970) * 1.5616236465059038^{*-6(62205676/(5670 - x) - 20970)}, \{x, 2703, 5600\}]$

7.63952×10^7

$t[x]:=(62205676/(5670 - x) - 970) * 1.5616236465059038^{*-6(62205676/(5670 - x) - 20970)}$

$\text{NSolve}[t[x]==f[x]/2, x]$

$\{\{x \rightarrow -58459.6\}, \{x \rightarrow 5487.66\}\}$

$f[5485]//N$

335277.

$t[5485]//N$

165072.

$\text{Integrate}[(62205676/(5670 - x) - 970) * 1.5616236465059038^{*-6(62205676/(5670 - x) - 20970)}, \{x, 2703, 4925\}]$

$\text{Integrate}[(62205676/(5670 - x) - 970) * .5, \{x, 4925, 5600\}]$

7.64271×10^7

$(62205676/(5670 - 4925) - 970)//N$

82527.6

$p[x]:=\text{Piecewise}[\{\{f[x] - 20000, x < 2703\}, \{t[x], 2703 < x < 4926\}, \{.5 * f[x], x > 4926\}\}]$

$\text{Integrate}[p[x], \{x, 0, 5600\}]/N$

5.99987×10^7

$1/5600 * \left(5601 - 2 * \frac{\sum_{i=1}^{5600} ((5601-i)*(f[i]-p[i]))}{\sum_{i=1}^{5600} (f[i]-p[i])} \right) // N$

0.364641

$$1/5600 * \left(5601 - 2 * \frac{\sum_{i=1}^{5600} ((5601-i) * (f[i]))}{\sum_{i=1}^{5600} (f[i])} \right) // N$$

0.581979

