SIOC 221A: HW4

Author: Victoria Boatwright

I acknowledge that I got some advice and thoughts from Turner Johnson, Grant Meiners, and Sophie Wynn.

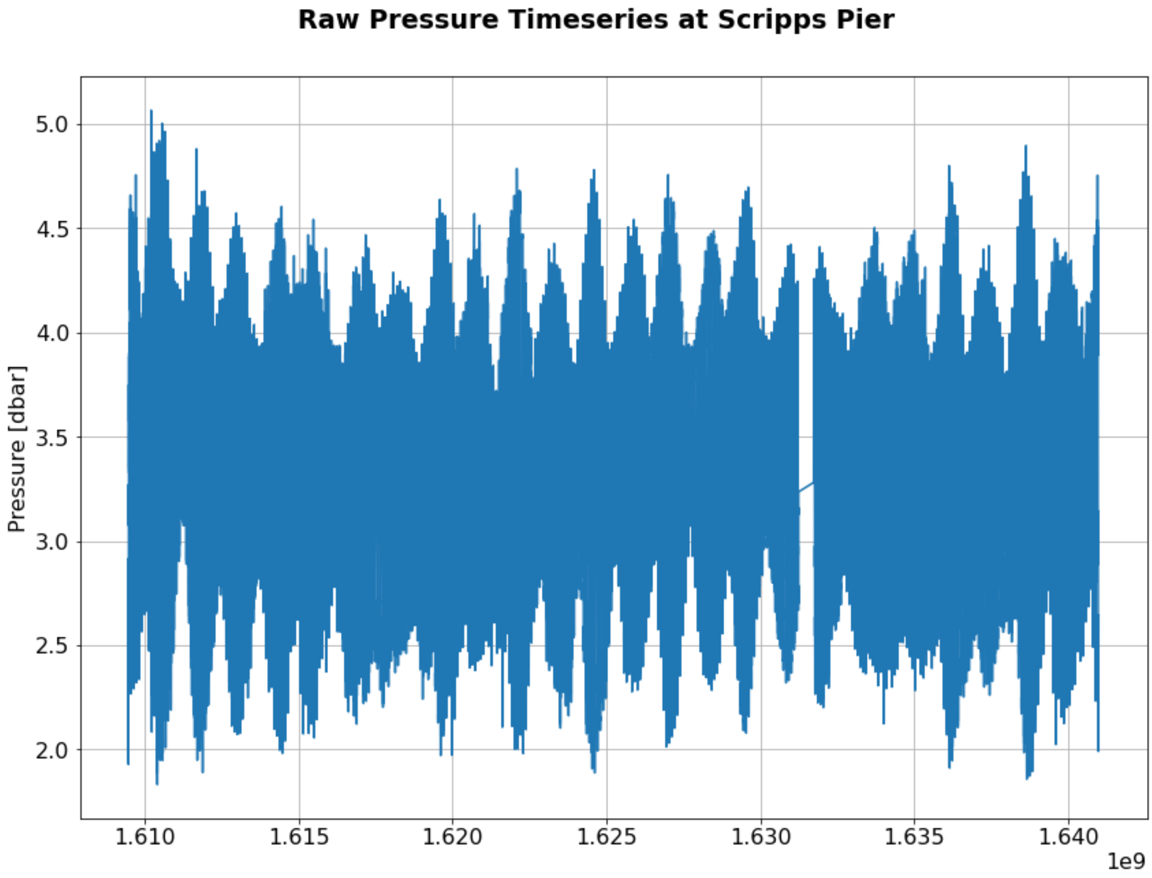
In [419…

```
import numpy as np
import matplotlib.pyplot as plt
import netCDF4
import datetime as dt
import matplotlib.dates as mdates
import pandas as pd
import xarray as xr
import scipy
import cmocean

plt.rcParams.update({'font.size': 16})
```

In [420…

```
# downloading scripps pier data

year_string = str(2021)
url_base = 'http://thredds.sccoos.org/thredds/dodsC/autoss/scripps_pier-
url = url_base+year_string+'.nc'

# read current file:
nc = netCDF4.Dataset(url)

time = nc['time'][:]
temp = nc['temperature'][:]
pressure = nc['pressure'][:]

# print(nc)
print(nc['time'].units)

fig,ax = plt.subplots(1,1,figsize=(14,10))
fig.suptitle('Raw Pressure Timeseries at Scripps Pier',y=0.95,fontweight=
ax.grid()
ax.plot(time,pressure)
ax.set_ylabel('Pressure [dbar]')
plt.show()

# show the gaps in the data
```

```
seconds since 1970-01-01 00:00:00 UTC
```


Raw Pressure Timeseries at Scripps Pier
Pressure [dbar]
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.610
1.615
1.620
1.625
1.630
1.635
1.640
1e9

In [482…

```
# downloading scripps pier data

year_string = str(2021)
url_base = 'http://thredds.sccoos.org/thredds/dodsC/autoss/scripps_pier-
url = url_base+year_string+'.nc'

# read current file:
nc = netCDF4.Dataset(url)

time = nc['time'][:]
temp = nc['temperature'][:]
pressure = nc['pressure'][:]

# given a time domain of interest

t0 = 70521
t1 = 88190

# and also gave us values where we have gaps

given_idx = [2420, 4952, 7496, 15042]


# fill missing

# knowing that each dt should be 4 min
timestep = 4*60 # 4 min in seconds (because data stored in seconds since
N = int((time[t1]-time[t0])/timestep)
for nn in np.arange(0,N):
    filled_time[ii] = time[t0]+nn*timestep

filled_pressure = np.zeros((len(filled_time))); filled_pressure[:] = np.
subpressure = pressure[t0:t1]

idx1 = given_idx[0]
# idx-1 will be up to existing data (in python)
filled_pressure[0:idx1-1] = subpressure[0:idx1-1]
# idx = the point to be filled
filled_pressure[idx1] = np.nanmean(subpressure[idx1-3:idx1+1]) # fill wi

# idx1+1 --> next idx + 1 (because newly added point! )
idx2 = given_idx[1]
filled_pressure[idx1+1:idx2+1] = subpressure[idx1:idx2]
# idx = the point to be filled --> new array = idx2 + 2
filled_pressure[idx2+2] = np.nanmean(subpressure[idx2-3:idx2+1]) # fill 

# new array: idx2 + 2 --> next idx3 + 2
# old array: idx2 --> idx3 -1
idx3 = given_idx[2]
filled_pressure[idx2+3:idx3+3] = subpressure[idx2:idx3]
# idx = the point to be filled
filled_pressure[idx3+4] = np.nanmean(subpressure[idx3-3:idx3+1]) # fill 

# new array: idx2 + 2 --> next idx3 + 2
# old array: idx2 --> idx3 -1
idx4 = given_idx[3]
filled_pressure[idx3+5:idx4+5] = subpressure[idx3:idx4]
# idx = the point to be filled
filled_pressure[idx4+6] = np.nanmean(subpressure[idx4-3:idx4+1]) # fill 

filled_pressure[idx4+7:] = subpressure[idx4:]
```

```
# check sizes

print(f'new pressure length: {len(filled_pressure)}')
print(f'compared to initial pressure length: {len(pressure)}')

sub_time = filled_time
sub_p = filled_pressure


print(f'subset pressure length: {len(sub_p)}')
print(f'subset time length: {len(sub_time)}')
```

```
new pressure length: 17676
compared to initial pressure length: 129244
subset pressure length: 17676
subset time length: 17676
```

In [496…

```
year_string = str(2021)
url_base = 'http://thredds.sccoos.org/thredds/dodsC/autoss/scripps_pier-
url = url_base+year_string+'.nc'

# read current file:
nc = netCDF4.Dataset(url)

time = nc['time'][:]
temp = nc['temperature'][:]
pressure = nc['pressure'][:]

# Matthew gave us a time domain of interest

t0 = 70521
t1 = 88190

# and also gave us values to deem as unworthy
# (I could actually search and find myself the missing data ... but I wo

# filter missing

given_idx = [2420, 4952, 7496, 15042]

for ii in given_idx:
    # make a running mean based on nearest +/- 2 values
    near_mean = np.nanmean(pressure[2-ii:ii+2])
    pressure[ii] = near_mean


sub_time = time[t0:t1]
sub_temp = temp[t0:t1]
sub_p = pressure[t0:t1]
```

In [498…

```
# convert to datetimes based on units of 'seconds since 1970-01-01'
s0 = dt.datetime(1970,1,1)
sub_dates = [s0+dt.timedelta(seconds=float(tt)) for tt in sub_time]
```

In [499…

```
# convert subset time units to days
sub_time = sub_time/3600/24
```

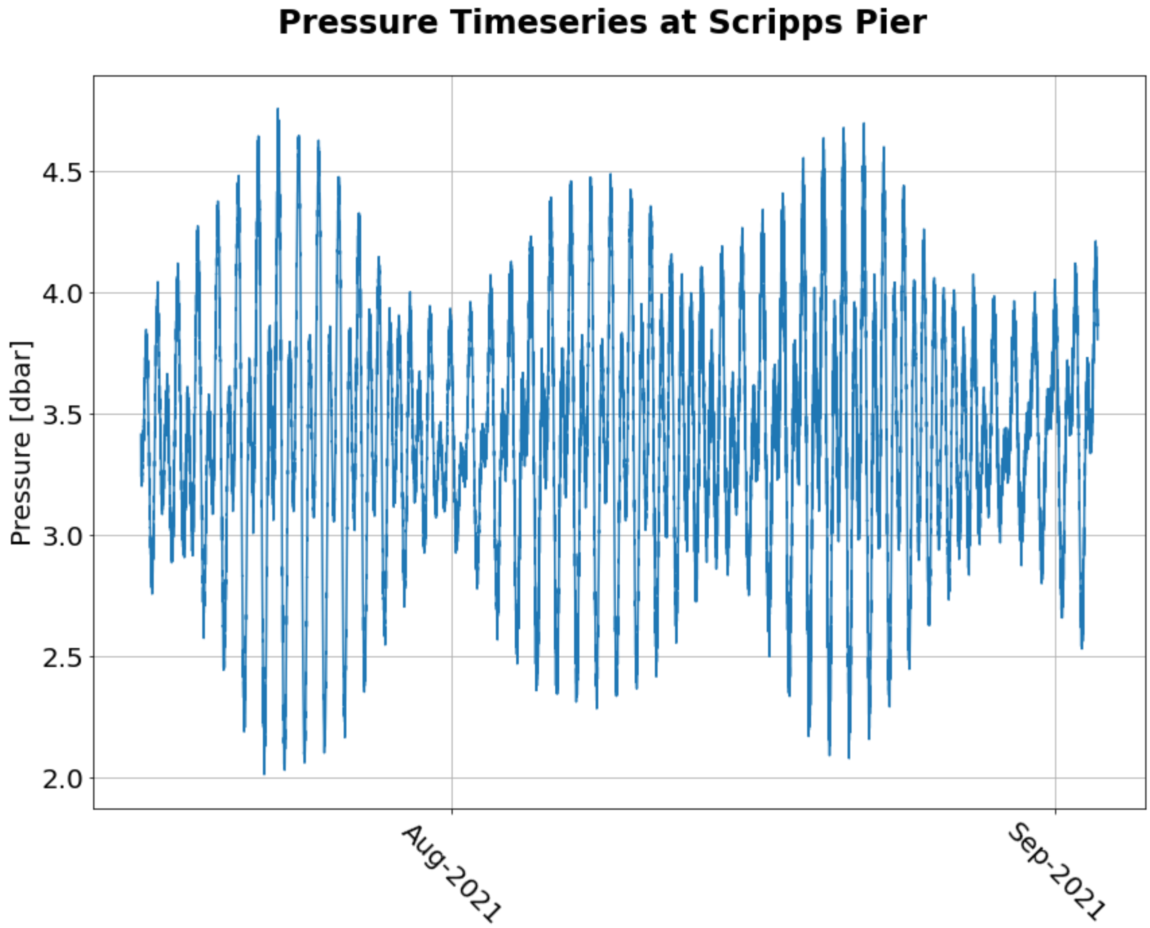
Visual Evaluation

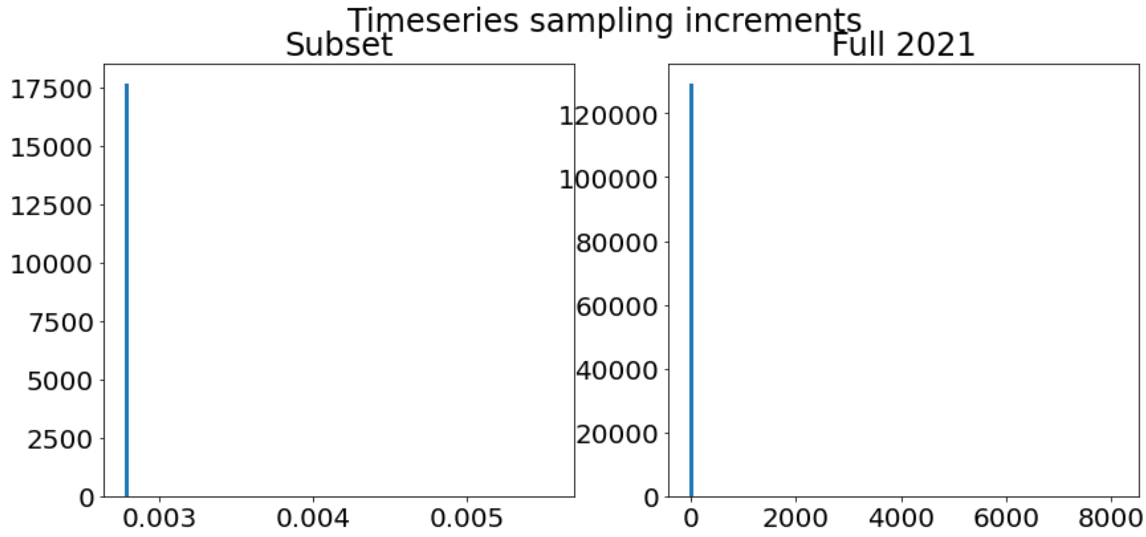
Plot the time series of pressure data from 2021, and examine the time increments between adjacent measurements. (You can do this in Matlab using the “diff” command or in python using “numpy.diff”, for example.) Are the data always uniformly spaced? What is the increment between measurements? How long is the time record? Are there other portions of the 2021 record that also have reliable uniform spacing?

1. Least-squares fit. Least-squares fit a mean and 3 major tidal constituents to your data. Plot the raw and model solution. What is the mean, and what are the total amplitudes of the tidal constituents? (Total amplitude should be determined from the square root of the sum of the squares of the sine and cosine amplitudes.)

In [502…

```
fig,ax = plt.subplots(1,1,figsize=(14,10))
fig.suptitle('Pressure Timeseries at Scripps Pier',y=0.95,fontweight='bo
ax.grid()
ax.plot(sub_dates,sub_p)
ax.set_ylabel('Pressure [dbar]')
ax.xaxis.set_major_formatter(mdates.DateFormatter('%b-%Y'))
ax.xaxis.set_major_locator(mdates.MonthLocator())
ax.tick_params(axis='x', labelrotation = -45)
plt.show()

# space between measurements:

deltat = np.diff(time)/60 # time in s --> convert to min
sub_dt = np.diff(sub_time) # time in days

fig,axes = plt.subplots(1,2,figsize=(14,6))
fig.suptitle('Timeseries sampling increments')
axes[0].hist(sub_dt,100)
axes[0].set_title('Subset')
axes[1].hist(deltat,100)
axes[1].set_title('Full 2021')
plt.show()

print(f'Appears to be sampling (in the small Aug-Sep window) every {np.na
print(f'Length of time record = {(sub_time[-1]-sub_time[0]): 5.2f} days'

print('Compare with full 2021 record: ')
print(f'Appears to be sampling (across all 2021) every {np.nanmean(delta
print(f'Length of time record = {(time[-1]-time[0])/3600/24: 5.2f} days'
```






```
Appears to be sampling (in the small Aug-Sep window) every  0.002778 day
s!
Length of time record =  49.09 days
Compare with full 2021 record:
Appears to be sampling (across all 2021) every  4.1 minutes!
Length of time record =  365.00 days
```

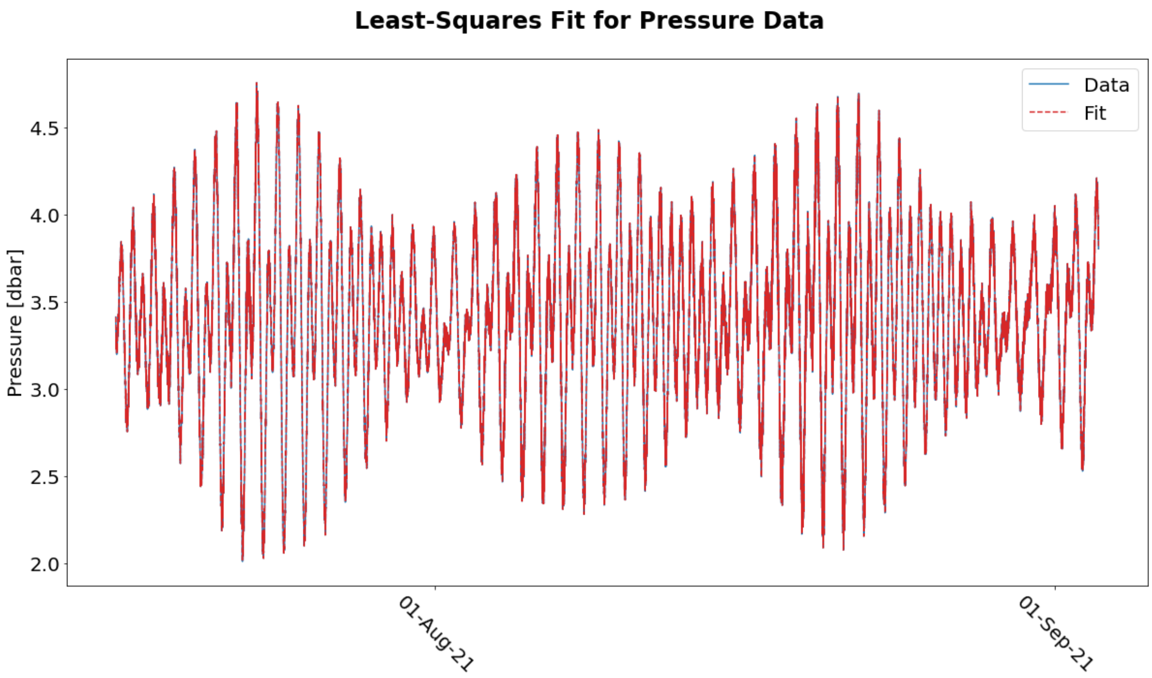
Least-squares fit.

Least-squares fit a mean and 3 major tidal constituents to your data. Plot the raw and model solution. What is the mean, and what are the total amplitudes of the tidal constituents? (Total amplitude should be determined from the square root of the sum of the squares of the sine and cosine amplitudes.)

In [505…

```
# using given frequency data

# calculate least squares fit with mean & 3 major constituents
data = sub_p
time_array = sub_time


# periods in days:
O1 = 25.82/24 # principal lunar diurnal
K1 = 23.93/24 # luni-solar diurnal period
M2 = 12.42/24 # principal lunar period

'''
AA = np.array([np.ones(len(sub_time)),
               sub_time,
               np.sin(2*np.pi*time_array/O1), np.cos(2*np.pi*time_array/(
               np.sin(2*np.pi*time_array/K1), np.cos(2*np.pi*time_array/I
               np.sin(2*np.pi*time_array/M2), np.cos(2*np.pi*time_array/I
'''



AA = np.array([np.ones(len(sub_time)),
               np.sin(2*np.pi*time_array/O1), np.cos(2*np.pi*time_array/(
               np.sin(2*np.pi*time_array/K1), np.cos(2*np.pi*time_array/I
               np.sin(2*np.pi*time_array/M2), np.cos(2*np.pi*time_array/I


x = np.dot(np.linalg.inv(np.dot(AA.T, AA)), np.dot(AA.T, data))
fit = np.dot(AA, x)
o1_amp = np.sqrt(x[1]**2 + x[2]**2 )
k1_amp = np.sqrt(x[3]**2 + x[4]**2 )
m2_amp = np.sqrt(x[5]**2 + x[6]**2 )
total_amplitude = np.sqrt(x[1]**2 + x[2]**2 + x[3]**2 + x[4]**2 + x[5]**2

ls_mean = x[0];
print(f'Pressure mean fit: {ls_mean} dbar')
print('\n')
print(f'Total amplitudes of tidal constituents: \nO1 Component Amplitude

fig,ax = plt.subplots(1,1,figsize=(20,10))
fig.suptitle('Least-Squares Fit for Pressure Data',fontweight='bold',y=0
ax.plot(sub_dates,data,color='tab:blue',label='Data')
ax.plot(sub_dates,data,color='tab:red',linestyle='--',label='Fit')
ax.xaxis.set_major_formatter(mdates.DateFormatter('%d-%b-%y'))
ax.xaxis.set_major_locator(mdates.MonthLocator())
ax.tick_params(axis='x', labelrotation = -45)
ax.set_ylabel('Pressure [dbar]')
ax.legend(loc='upper right')
plt.show()
```

```
Pressure mean fit: 3.4096088387882575 dbar


Total amplitudes of tidal constituents:
O1 Component Amplitude: 0.23330962985680348
K1 Component Amplitude: 0.3661783487040946
M2 Component Amplitude: 0.48337009755967403
All Constituents Amplitude: 0.6497435014360858
```



In [493…

```
x
```

Out[493]:

```
array([nan, nan, nan, nan, nan, nan, nan])
```

Fourier Transform

a) Plot the real and imaginary parts of the Fourier transform. Find the peaks.

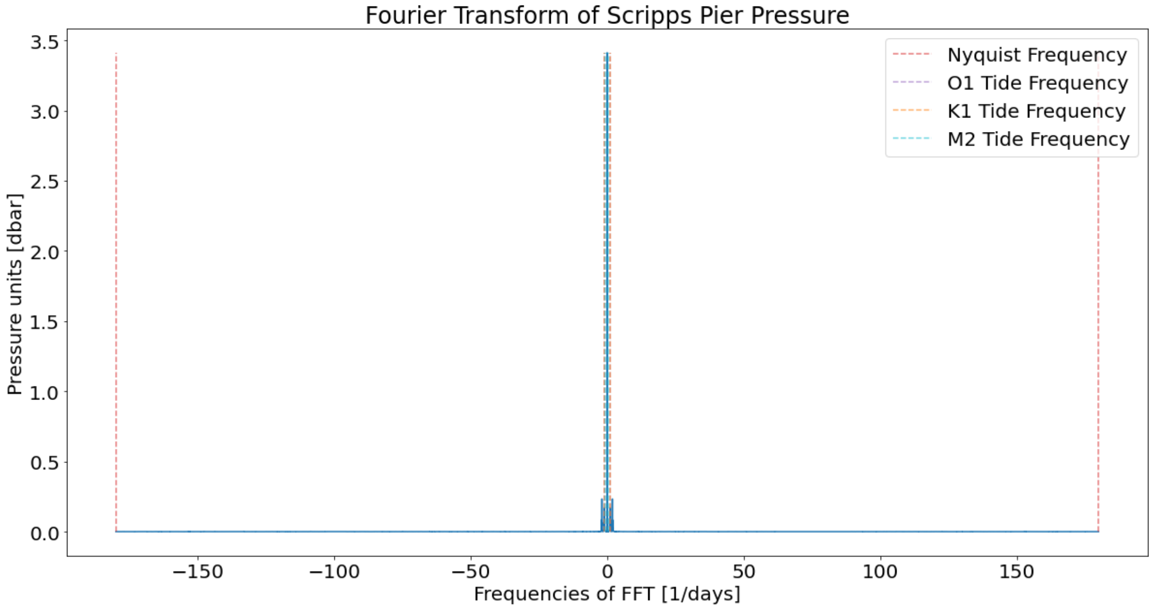
What frequencies correspond to these peaks? Are they what you'd expect based on the known tidal frequencies?

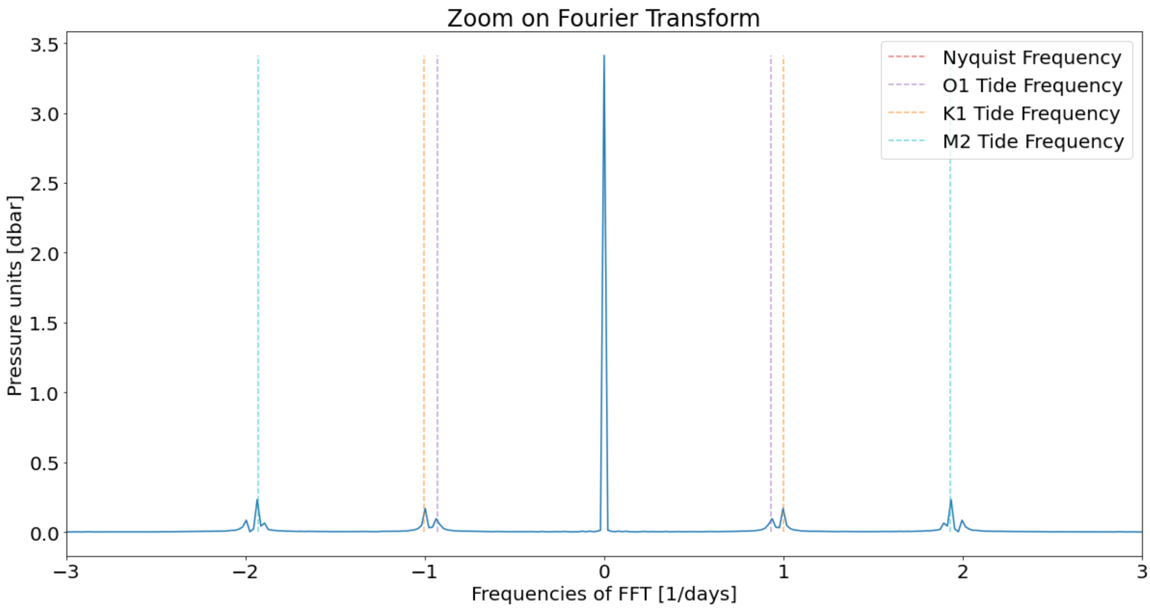
In [506…

```
O1 = 25.82/24 # principal lunar diurnal
K1 = 23.93/24 # luni-solar diurnal period
M2 = 12.42/24 # principil lunar period
```

In [507…

```
# fourier transform

# our sampling period is ~ 4min, shown previously in sub_dt (as days)
step = sub_dt # reusing sub_dt
step = np.nanmean(step)


# then we know Nyquist is 1/2*dt = 1/(2*4/60/24) = ~180
Nyq = 1/(2*step)

N = len(sub_time)

fftp = scipy.fft.fft(sub_p)
freq = scipy.fft.fftfreq(N,step)
freq = scipy.fft.fftshift(freq)
fftplot = scipy.fft.fftshift(fftp)
ifftp = scipy.fft.ifft(sub_p)


fig,ax = plt.subplots(figsize=(20,10))
# ax.plot(freq, fftplot)
ax.plot(freq, 1.0/N * np.abs(fftplot))
ax.vlines(x=[Nyq,-Nyq],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),li
ax.vlines(x=[O1,-O1],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),line
ax.vlines(x=[K1,-K1],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),line
ax.vlines(x=[M2,-M2],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),line
ax.set(ylabel='Pressure units [dbar]',xlabel='Frequencies of FFT [1/days
ax.legend()
plt.show()

fig,ax = plt.subplots(figsize=(20,10))
# ax.plot(freq, fftplot)
ax.plot(freq, 1.0/N * np.abs(fftplot))
ax.vlines(x=[Nyq,-Nyq],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),li

ax.vlines(x=[1/O1,-1/O1],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),
ax.vlines(x=[1/K1,-1/K1],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),
ax.vlines(x=[1/M2,-1/M2],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),

ax.set(ylabel='Pressure units [dbar]',xlabel='Frequencies of FFT [1/days
ax.set_xlim([-3,3])
ax.legend()
plt.show()
```





There is a spike at zero because I didn't remove the mean! So that peak is expected.

Otherwise, the peaks in the Fourier transform match with the frequency of each tidal constituent. We have one other peak at a slightly higher frequency for which we don't have a matching tidal constituent, which could be caused by some other basin perturbation.

b) Now use the Fourier coefficients to identify the mean pressure and the amplitudes of the major peaks.

In [508…

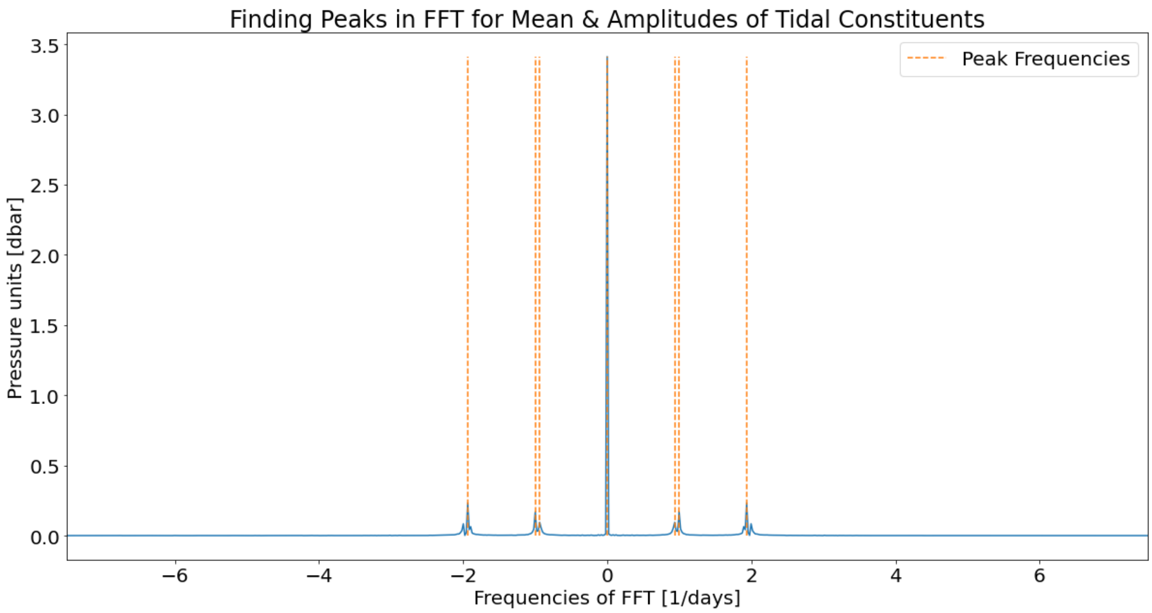
```
fftp = scipy.fft.fft(sub_p)
freq = scipy.fft.fftfreq(N,step)
freq = scipy.fft.fftshift(freq)
fftplot = scipy.fft.fftshift(fftp)
ifftp = scipy.fft.ifft(sub_p)

norm_fft[peaks]
```

Out[508]:

```
array([0.23311523, 0.16988187, 0.09508423, 3.4118574 , 0.09508423,
       0.16988187, 0.23311523], dtype=float32)
```

In [509…

```
# find means & amplitudes from FFT
from scipy import signal

norm_fft = 1.0/N * np.abs(fftplot)
peaks, _ = scipy.signal.find_peaks(norm_fft,height=0.09) # threshold=(2,.


fig,ax = plt.subplots(figsize=(20,10))
ax.plot(freq, norm_fft)
# ax.vlines(x=[Nyq,-Nyq],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot)),
ax.vlines(x=[freq[peaks]],ymin=0,ymax=np.nanmax(1.0/N * np.abs(fftplot))
ax.set(ylabel='Pressure units [dbar]',xlabel='Frequencies of FFT [1/days
ax.legend()
ax.set_xlim([-7.5, 7.5])
plt.show()
```



Nice - found peaks! And we can see they align with the least squares fit. Now we can find the mean and amplitudes of tidal constituents and compare quantitatively with those from the least-squares fit:

In [510…

```
fft_mean = np.nanmax(norm_fft)
fft_peaks = norm_fft[peaks]

print(f'Mean from FFT is: {fft_mean: 4.4f} dbar')
print('Peak amplitudes from FFT include: ')
for pp in peaks:
    print(f'{norm_fft[pp]: 4.4f} dbar')
```

```
Mean from FFT is:  3.4119 dbar
Peak amplitudes from FFT include:
 0.2331 dbar
 0.1699 dbar
 0.0951 dbar
 3.4119 dbar
 0.0951 dbar
 0.1699 dbar
 0.2331 dbar
```

c) Do these spectral peaks align with the results from the least-squares fit? Is there anything you could do to further check your results?

In [511…

```
# know that peaks are ordered from - to +

M2_peaks = [peaks[0],peaks[6]]
K1_peaks = [peaks[1],peaks[5]]
O1_peaks = [peaks[2],peaks[4]]

M2_fft = np.sqrt(norm_fft[M2_peaks[0]]**2 + norm_fft[M2_peaks[1]]**2)
K1_fft = np.sqrt(norm_fft[K1_peaks[0]]**2 + norm_fft[K1_peaks[1]]**2)
O1_fft = np.sqrt(norm_fft[O1_peaks[0]]**2 + norm_fft[O1_peaks[1]]**2)

print(f'Mean from FFT: {fft_mean} dbar\ncompared with \nMean from Least-S

print('\n')
print(f'Total amplitudes of tidal constituents from FFT: \nO1 = {O1_fft}


print(f'Compare with total amplitudes from least-squares fit: \nO1 = {o1_
```

```
Mean from FFT: 3.4118573665618896 dbar
compared with
Mean from Least-Sqaures: 3.4096088387882575 dbar


Total amplitudes of tidal constituents from FFT:
O1 = 0.134469404270174 dbar;
K1 = 0.24024923802473988 dbar;
M2 = 0.32967471424779016 dbar
Compare with total amplitudes from least-squares fit:
O1 = 0.23330962985680348 dbar;
K1 = 0.3661783487040946 dbar;
M2 = 0.48337009755967403 dbar
```

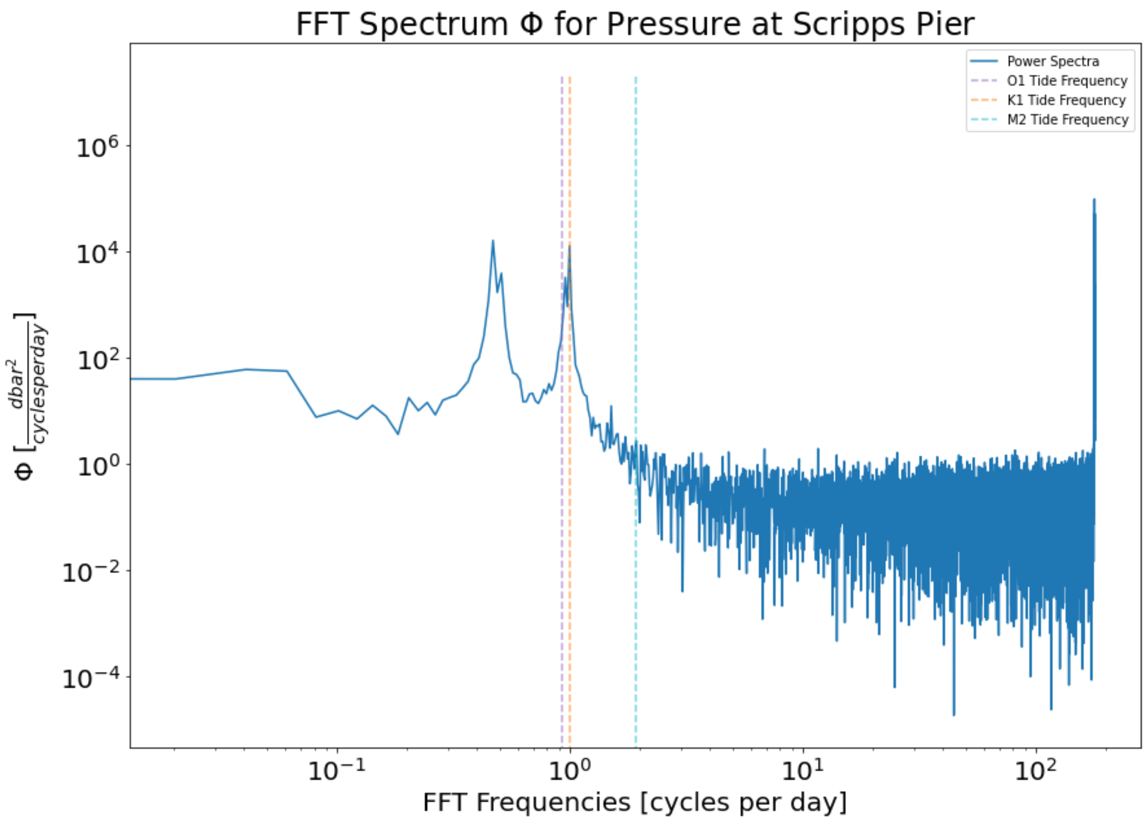
We can see that the means align in magnitude. On the other hand, some of the amplitudes don't match perfectly - all of the FFT amplitudes are smaller than a matching frequency for the least-squares amplitudes. For example, the FFT O1 tide total amplitude is less than the least-squares O1 tide total amplitude. Overall though, they have a similar order of magnitude.

To further check my results, I could check the standard error of the mean from the data and compare the misfit of each of the fitting methods (FFT and least-squares) to tell which ones might be overfitting or underfitting.

d) Now plot the spectral energy (based on the squared magnitude of the Fourier coefficients) as a function of frequency. Is the spectrum red, white, or blue?

In [514…

```
# fftp v norm_fft
even_idx = np.arange(0,N,2)
amp = abs(fftp[even_idx])**2 # even N

# corresponding frequencies
period = step*N
df = 1/period
fN = Nyq # from before: Nyq = 1/(2*step)
freqs = np.arange(0,fN,df) # frequency vector, in [1/day], goes from 0 t



# spectrum:
amp = amp/(N^2) # correct normalization
amp = amp*2 # account for discarded redundant complex FFT coefficients
amp = amp/df

plt.rcParams.update({'font.size': 20})
# plot
fig,ax = plt.subplots(figsize=(14,10))
ax.plot(freqs,amp,label='Power Spectra')
ax.vlines(x=[1/O1,-1/O1],ymin=0,ymax=np.nanmax(amp),linestyle='--',color=
ax.vlines(x=[1/K1,-1/K1],ymin=0,ymax=np.nanmax(amp),linestyle='--',color=
ax.vlines(x=[1/M2,-1/M2],ymin=0,ymax=np.nanmax(amp),linestyle='--',color=

ax.set_xscale('log'); ax.set_yscale('log')
ax.set(xlabel=r'FFT Frequencies [cycles per day]',
       ylabel=r'$\Phi$ [$ \frac{dbar^2}{ cycles per day } $]',
       title=r'FFT Spectrum $\Phi$ for Pressure at Scripps Pier')
ax.legend(fontsize=10)
plt.show()

# versus already normalized ?
```



When the spectra has fourier coefficients all around the same amplitude, this is a white spectrum. When the lower frequencies are weighted more than high frequencies, it is a red spectrum. When higher frequencies are weighted more than lower frequencies, it is a blue spectrum.

Looking at our spectrum, there are higher amplitudes at higher frequencies (aka, the higher frequencies are weighted more than the lower frequencies); this means that we have a blue spectrum!

Notes from "office hours", end of classes, and numpy documentation/other resources on FFT (for me!):

different kinds of spectra:

- spectra where all fourier coefficients are the same amplitude --> white spectrum because all visible frequencies are the same
- lower frequencies are higher (weighted more) than high frequencies: red spectrum
- higher frequencies are higher (weighted more) than the lower frequencies: blue spectrum

The function rfft calculates the FFT of a real sequence and outputs the complex FFT coefficients for only half of the frequency range. The remaining negative frequency components are implied by the Hermitian symmetry of the FFT for a real input (y[n] = conj(y[-n])). In case of N being even: ; in case of N being odd . The terms shown explicitly as are restricted to be purely real since, by the hermitian property, they are their own complex conjugate.