Traps for neutral atoms

Trapping of atoms

- Study of quantum collective behavior (BEC, Fermi gazes)
- Guiding of atoms : towards guided atoms interferometers. Gyroscope ?
- Traps for individual atoms : use as Qbits for quantum computation

Neutral atoms : small interaction with external fields.
$\longrightarrow$ Use of laser cooling technics to have cold sample : $T \simeq 10\ \mu$K

Interaction with electric field

- Static electric field. Induced dipole : $V = -\alpha \mathbf{E}^2/2$. Attraction towards high fields. No trap possible. Used in combination with other fields.
- Oscillating fields : CO_2 laser. Atoms confined in high intensity regions
- Laser light close to an atomic transition. Confinement towards high fields or low fields. Many possible geometries (standing wave etc ...) Higher confinement. But spontaneous emission.

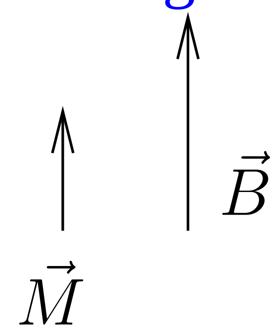
Interaction with magnetic field

- No population of excited states No spontaneous emission
- Large trapping volume
- Use of Radio-frequency fields : efficient evaporation

Magnetic trapping

Use interaction between a dipole moment and a magnetic field

$$\boxed{W = -\vec{M}.\vec{B}}$$



Order of magnitude : $M \simeq \mu_B = 1.4 MHz/G$

To have $W = 1K$, $B \simeq 1.5$ T requiered

Experiment made with H : Superconducting coils, permanent magnets

Trap depth of about 1 K

With laser cooling : temperature of a few 10 μK available

Field of a few G sufficient.

Zeeman effect

Neglecting the diamagnetic term, the magnetic interaction between an atom and a magnetic field writes

$$W = -\mathbf{M}.\mathbf{B},$$

where $\mathbf{M}$ is the magnetic dipole of the atom. $\mathbf{M}$ writes

$$\mathbf{M} = \mathbf{M_I} - \sum_i \frac{\mu_B}{\hbar}(\mathbf{L}_i + 2\mathbf{S}_i)$$

where $\mathbf{M_I}$ is the magnetic dipole of the nucleous and the sum is performed on each electron.
$\mu_B = \frac{\hbar|q|}{2m}$: Bohr magneton

For small magnetic field, W can be computed in each unperturbed energy level.
$\mathbf{M}$ is a dipolar operator $\Rightarrow$ $\mathbf{M}$ is proportionnal to $\mathbf{F}$,
where F is the total spin of the atom.
An atom of zero spin such as the the ground state of He or the alkali-earth do not interact with magnetic field.
Zeeman effect large for an alkali.

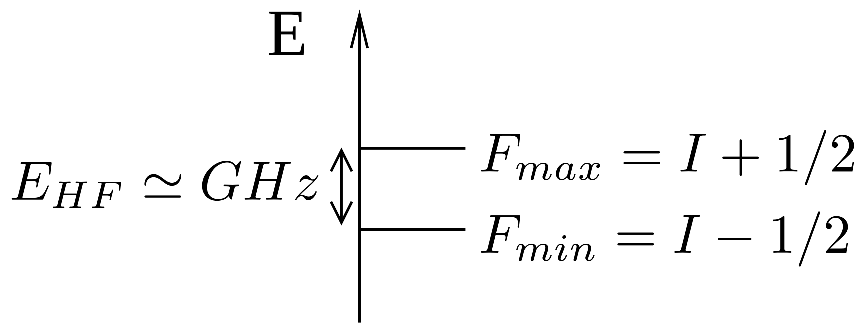
Zeeman effect : case of alkaline

Spin of the complete shells cancel. Ground state has $L = 0$.

$$\mathbf{M} = \mathbf{M_I} - \mathbf{2}\frac{\mu_\mathbf{B}}{\hbar}\mathbf{S},$$

where $\mathbf{M_I}$ is the nuclear spin. The gyromagnetic factor of the nucleus is much smaller than that of the electron (typically 10^3 smaller) as its mass is much larger. In the following we neglect it.

Ground state : hyperfine structure



2 cases : $\mu_B B \ll E_{HF}$

$\mu_B B \gg E_{HF}$

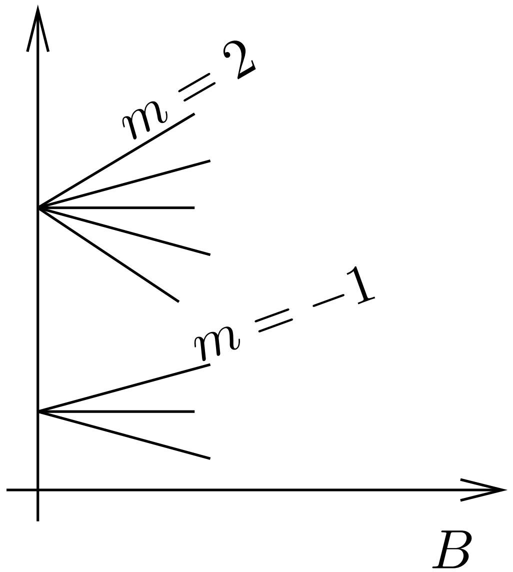
Zeeman effect : case of alkaline. 2

1. B small

Effect of W in each hyperfi ne state

B along z. Then Wigner Eckart gives $\langle m|\, W \,|m\rangle = \mu_B g_F m B$

Case of Rubidum 87



- $F = F_{max} : g_{F_{max}} = 1/F_{max}$.

  $|F_{max}, m = F_{max}\rangle = |m_I = I, m_S = 1/2\rangle$

  $F_{max}\mu_B g_{F_{max}} = \mu_B$

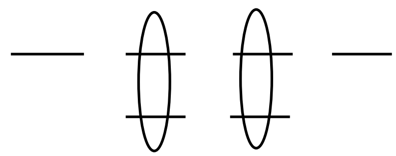
- $F = F_{min} \quad : g_{F_{min}} = -1/F_{max}$

Seen easily by developing
$|F_{max}, m = F_{max} - 1\rangle$ and $|F_{min}, m = F_{max} - 1\rangle$ on
$|m_I = I, m_S = -1/2\rangle$ and $|m_I = I - 1, m_S = 1/2\rangle$.

Zeeman effect : case of alkaline. 3

2. Second order zeeman effect

$E = \mu_B g_F m + \delta E^{(2)}$



$$\delta E^{(2)} = (-1)^F a_m \frac{(\mu_B B)^2}{E_{HF}}, \begin{cases} a_2 = a_{-2} = 0 \\ a_1 = a_{-1} = 3/4 \\ a_0 = 1 \end{cases}$$

2. B large

Good basis : m_S.

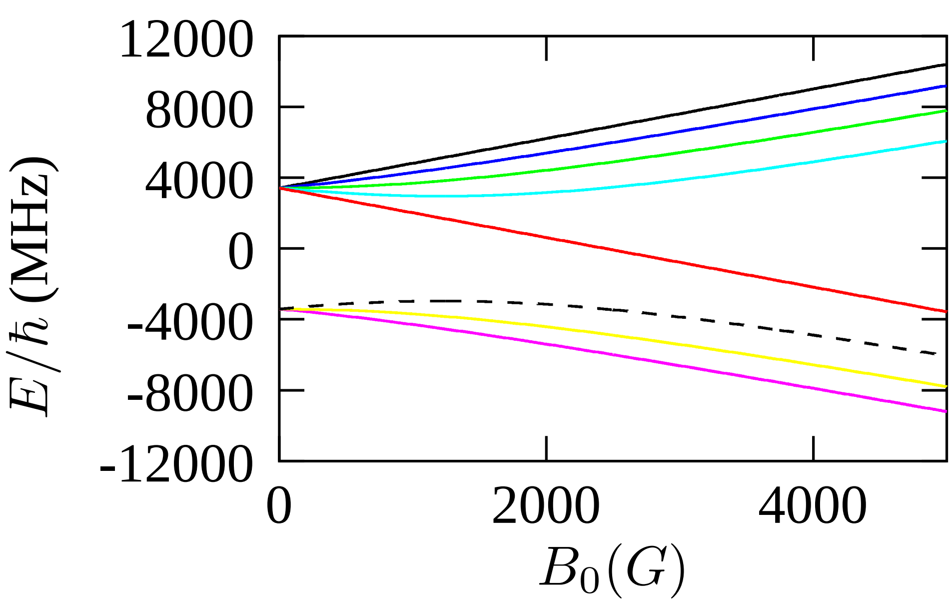
Zeeman energy : $W = \pm\mu_B B$

For $\mu_B B \gg \omega_{HF}$:

for Rb : $B \gg 0.4$ T

3. General case

Case of Rb^{87}



Field used : e few G

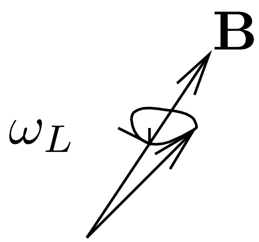
$\longrightarrow$ frist order zeeman effect sufficient

Not sufficient for :

- High fields
- Spectroscopy

Magnetic trap

Trap for atom requieres a non homogeneous magnetic field. It a priori depends on the projection of the spin on the local direction of the magnetic field. However, a simplification can be made for large enough magnetic field : the adiabatic following condition



2 time scales : $\omega_L \simeq \mu_B B$

ω_{rot} : rotation of $\mathbf{B}$

If $\omega_L \gg \omega_{rot}$, then the spin follows adiabatically the direction of $\mathbf{B}$.
The potential seen by the atom reduces to

$$V = mg\mu_B|\mathbf{B}|$$

We will come back later on this condition.

Wing theorem

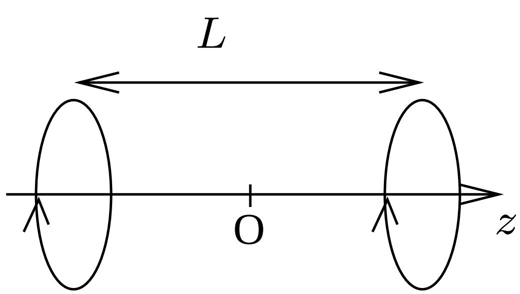
To realize a trap : extremum of $|\mathbf{B}|$ required.
Wing theorem : a maximum of $|\mathbf{B}|$ is impossible in free space

Let us suppose $|\mathbf{B}|$ has a maximum in O and let us take $\mathbf{B}(0) = B_0\mathbf{z}^0$.

$$|\mathbf{B}|^2 = B_0^2 + |\delta\mathbf{B}|^2 + 2B_0\delta B_z$$

where $\mathbf{B} = \mathbf{B}(0) + \delta\mathbf{B}$.
Because $|\delta\mathbf{B}|^2 \geq 0$, if $|\mathbf{B}|^2$ is maximum, then δB_z should be a minimum. But Maxwell equation gives $\Delta\delta B_z = 0$ and a function f that fulfills $\Delta f = 0$ has no extremum.

Field of a magnetic bottle :

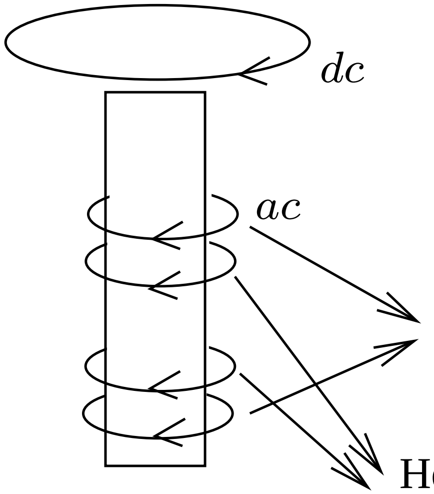


$$\begin{cases} B_z = b_0 + b''(z^2 - \frac{\rho^2}{2}) \\ B_\rho = -b''\rho z \end{cases}$$

Trap for atoms : minimum of $|\mathbf{B}|$.
Trapped states : low-field seeker states
Disadvantage : large internal energy $\Rightarrow$ unstable against spin flip transitions, two body inelastic collisions

Trapping high field seekers : time varying field

Static trap for high field seeker impossible. ac magnetic trap possible



Produces B_0

Produce $B_z = b_0 + b''(z^2 - \rho^2/2)$

Helmoltz : cancel b_0

$$V = -\mu_B|\mathbf{B}| = -\mu_B\sqrt{(B_0 + b''(z^2 - \rho^2/2))^2 + b''^2\rho^2} \simeq -\mu_B B_0 - \mu_B b''(z^2 - \rho^2/2)$$

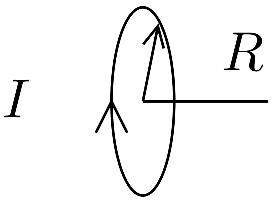
Atoms confined radially, expelled longitudinaly

Oscillating b'' : Regimes of stable motion of the atoms (like Paul trap)

Shallow trap (μ K), small confinement ($\omega_{osc} \simeq 5$ Hz)

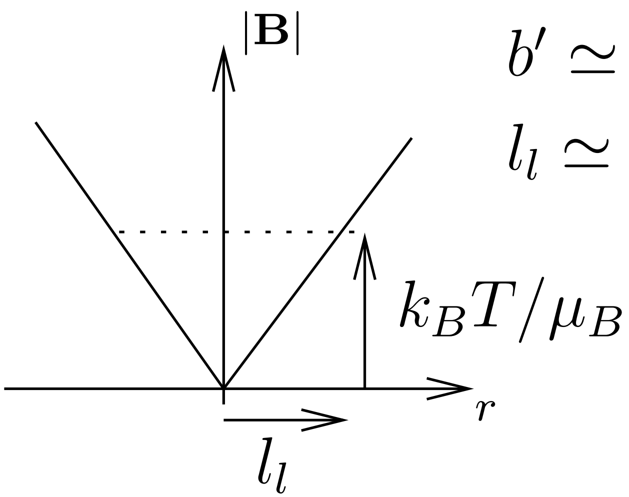
Linear traps

Goal : Reach a high confinement. Then linear tarps are a better solution than higher order traps.



Magnetic field produced by similar coils

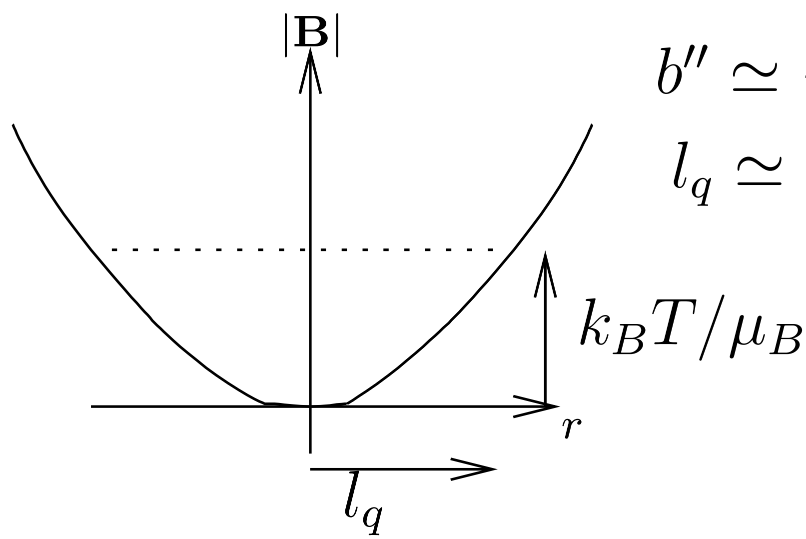
Linear traps



$$b' \simeq \frac{\mu_0 I}{R^2}$$

$$l_l \simeq \frac{k_B T R^2}{\mu_0 \mu_B I}$$

Quadratic trap



$$b'' \simeq \frac{\mu_0 I}{R^3}$$

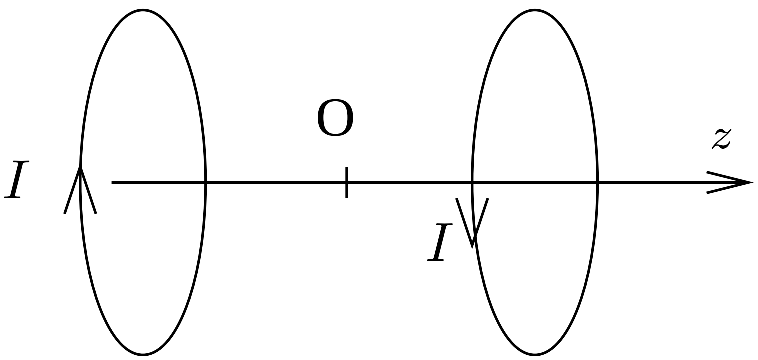
$$l_q \simeq \sqrt{\frac{k_B T \mu_B R^3}{\mu_0 I}}$$

$$\frac{l_l}{l_q} \simeq \sqrt{\frac{k_B T R}{\mu_0 \mu_B I}} \simeq \sqrt{\frac{l_l}{R}}$$

$$\boxed{l_l \ll R \Rightarrow l_l \ll l_q}$$

Realisation of linear trap

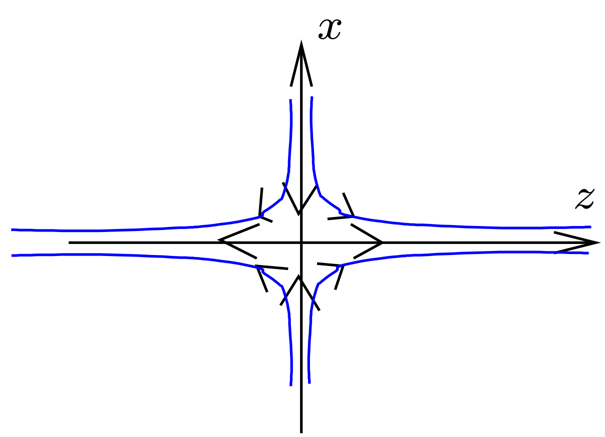
Linear trap : $\mathbf{B} = \mathbf{0}$ at the center. ($\mathbf{B}$ has no singularity)
Most simple realsisation : Anti-Helmoltz coils



On the z axis , $\mathbf{B} \| \mathbf{z^0}$

By symmetry : $\mathbf{B}(O) = \mathbf{0}$

$$\left\{ \begin{array}{l} B_z = G B_z \\ B_x = -G/2x \\ B_y = -G/2y \end{array} \right.$$

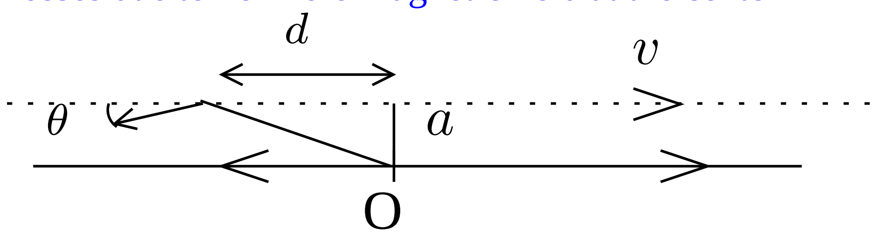


First trap for cold atoms $\left\{ \begin{array}{l} 2.7\text{cm} \\ 1900 At \end{array} \right.$ $\rightarrow$ depth of 17 mK
loaded from a zeeman slower (1985)

Now : $\simeq$ 100 G/cm, dissipation $\simeq$ 100 Watt

Losses in a quadrupole trap

Losses due to non zero magnetic field at the center



Adiabatic condition : $\dot{\theta} \ll \omega_l$

$\Delta\theta \simeq \pi/2$ for $d \simeq a$

$\rightarrow \dot{\theta} \simeq \frac{v}{a}$

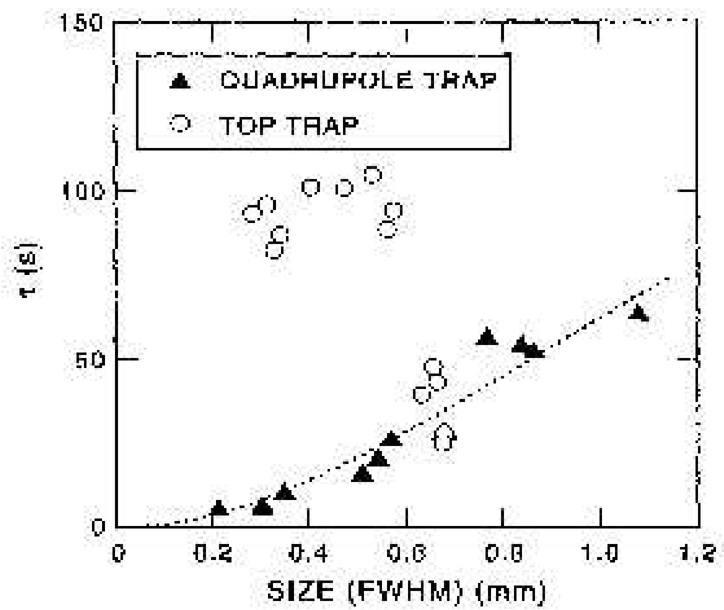
Larmor frequency : $\omega_l \simeq G\mu_B a$

The atom is lost if $a < \sqrt{\hbar v / G\mu_B}$.
The loss rate is the flux of atoms going inside the sphere of radius a surrounding O

$$dN/dt \simeq 4\pi a^2 nv \simeq n\frac{\hbar v^2}{G\mu_B}$$

Using $mv^2 \simeq k_B T$ and
$n \simeq N(\mu_B G/k_B T)^3$ we obtain

$$\boxed{dN/dt = -N\hbar\frac{(G\mu_B)^2}{m(k_B T)^2} = -N\frac{\hbar}{m}\frac{1}{l^2}}$$

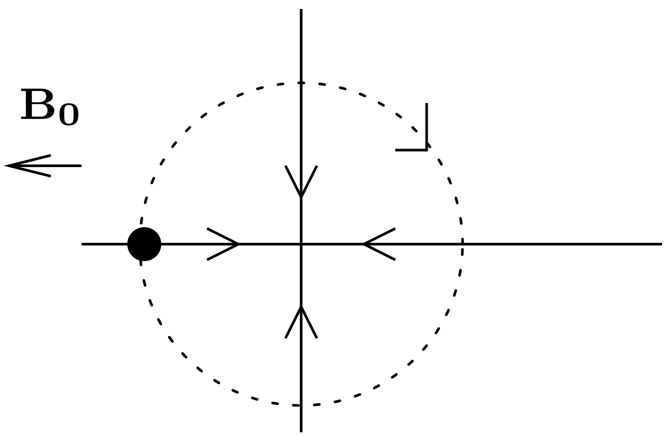


Solution : TOP trap

To remedy the Majorana losses problem, two solutions were proposed : the Time Orbital Potential and the addition of an optical plug.

- Time Orbital Potential Trap (TOP trap)

The idea is to displace the zero of the trap using an external field bfB_0. If the direction of $\mathbf{B_0}$ is turned, the zero of the field moves around a circle. If this rotation is fast enough, the atoms do not have time to follow and experience a time average potential.



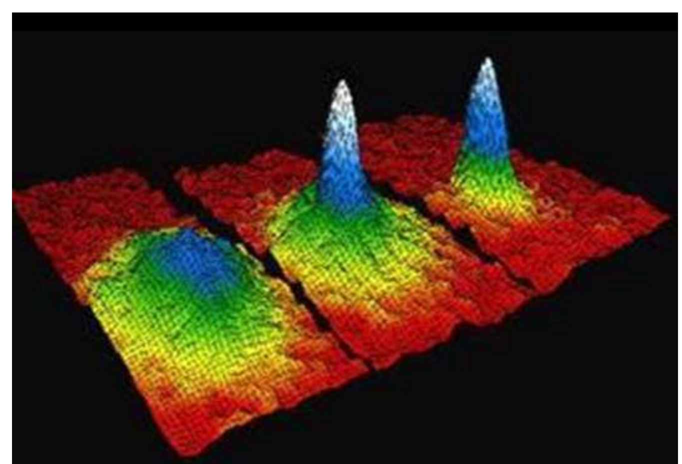
Instantaneous potential seen by the atoms

$$
\begin{aligned}
U &\simeq G\sqrt{4z^2 + (x - r_0\cos(\omega t))^2 + (y - r_0\sin(\omega t))^2} \\
&\simeq \mu_B G r_0 (1 - x/r_0\cos(\omega t) - y/r_0\sin(\omega t) + \frac{x^2+y^2}{4r_0^2} + 2z^2/r_0^2)
\end{aligned}
$$

Here we used $\sqrt{1+\epsilon} = 1 + \epsilon/2 - 1/8\epsilon^2$.
After average over a period

$$
U_{TOP} = \mu_B G r_0 + \frac{\mu_B G}{4r_0}(x^2 + y^2 + 8z^2)
$$

.



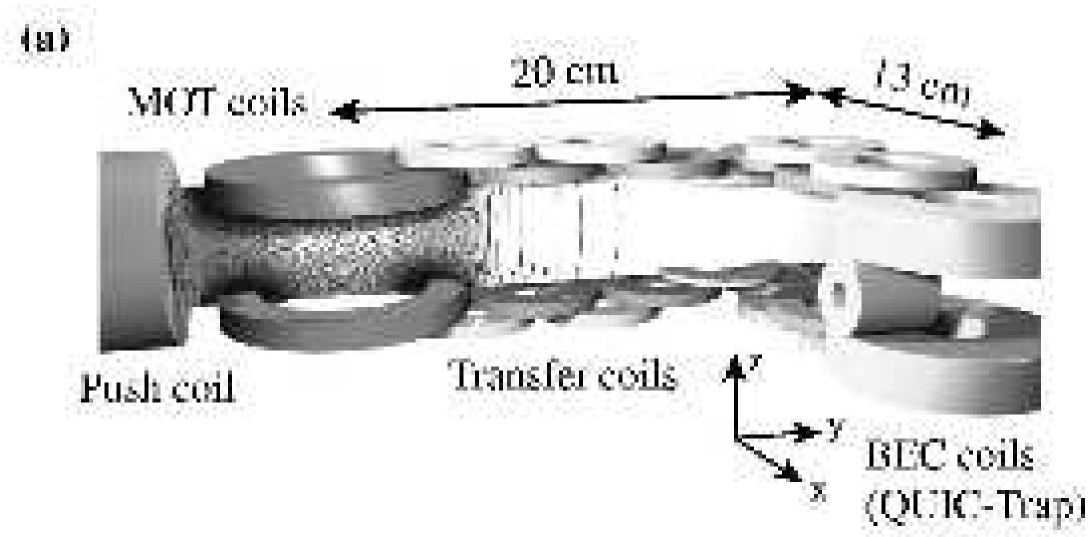
Condition of validity :
Adiabatic following of the spins : $\omega \ll \mu_B G r_0 = \mu_B B_0$
Atom motion do not follow : $\omega \gg \omega_{vib} = \sqrt{\mu_B G/2mr_0}$
FIRST BEC obtained in such a trap in 1995. $\omega = 7.5$kHz, $\omega_{vib} = 25$ Hz

Quadrupole trap used for transportation

Quadrupole trap used for transportation of “hot” clouds.

- Moving coils
  Motorized translation

- Transfer from coil to coil



Static traps of non vanishing magnetic fi eld : IOFFE confi guration

Traps with non vanishing $\mathbf{B}$ at the center : $\mathbf{B}(O) = B_0\mathbf{z^0}$

$$|\mathbf{B}|^2 = B_0^2 + 2B_0\delta B_z + |\delta\mathbf{B}|^2$$

$|\delta\mathbf{B}|^2$ at most of second order because $\delta\mathbf{B}$ at most of first order.

$|\mathbf{B}|$ minimum $\Rightarrow \nabla|\mathbf{B}|^2 = 0 \Rightarrow \nabla\delta B_z = 0$. Thus δB_z is at least of second order.

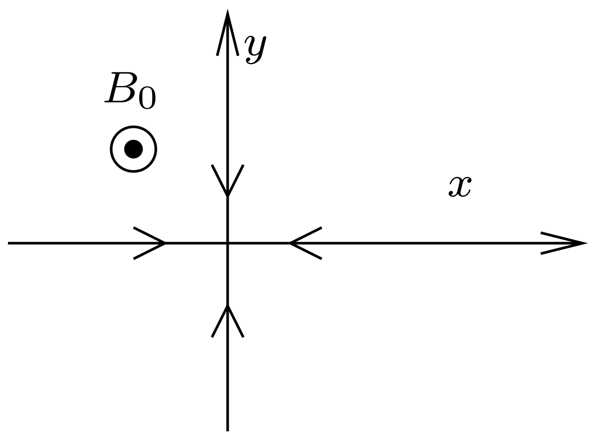
First order expansion of $\mathbf{B}$

$\nabla\mathbf{B} = 0 \Rightarrow \partial B_x/\partial x = -\partial B_y/\partial y$
$\nabla\wedge\mathbf{B} = 0 \Rightarrow \partial B_x/\partial y = \partial B_y/\partial x$

$$\Rightarrow B_\perp = \begin{pmatrix} G & G' \\ G' & -G \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Up to a rotation in the (xy) plane,

$$\begin{aligned} B_z &= B_0 \\ B_x &= b'x \\ B_y &= -b'y \end{aligned}$$



2D quadrupole : responsible for transverse confinement.

Longitudinal confinement

2^{nd} order term in δB_z : $b''z^2$. Maxwell equations requieres that it does not come alone !
Assuming invariance by rotation along z,

$$\begin{aligned} B_z^{(2)} &= b''(z^2 - \tfrac{\rho^2}{2}) \\ B_\rho^{(2)} &= -b''\rho z \end{aligned}$$

IOFFE confi guration

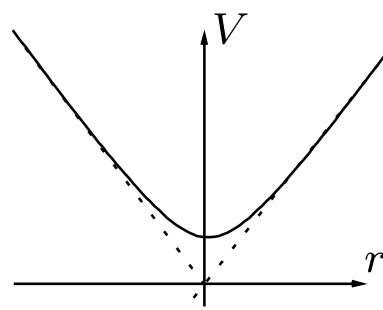
Magnetic field modulus developed to second order

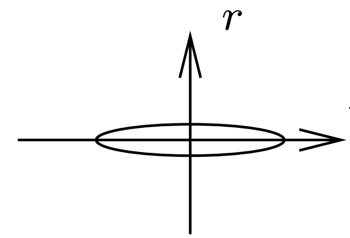
$$|\mathbf{B}| = B_0 + b''z^2 + r^2\left(\frac{b'^2}{2B_0} - b''/2\right).$$

Potential $V = \mu|\mathbf{B}| = \frac{1}{2}M\omega_z^2 z^2 + \frac{1}{2}M\omega_\rho^2\rho^2$, $\omega_\rho = \sqrt{\mu(b'^2/B_0 - b''/2)/M}$, $\omega_z = \sqrt{2\mu b''/M}$

B_0 chosen small so that $b'' \ll b'^2/B_0$

$\Rightarrow$ cigar shape trap





Validity of harmonic expansion : $r \ll B_0/b'$.

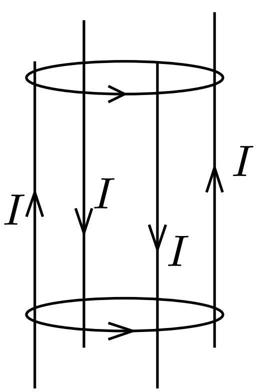
At $r_\perp$ large, linear trapping domain.

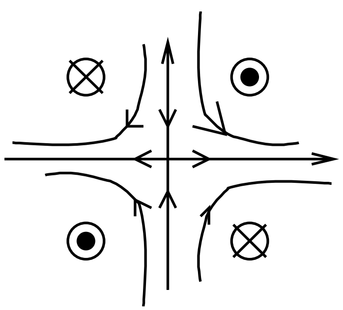
Realization

4 current carrying bars $\longrightarrow$ 2 dimensional quadrupole field.

Longitudinal confinement realized with two coils running parallel current. (more separated than in the Helmholtz configuration)

Two coils in Helmholtz configuration : decrease B_0 in order to increase the transverse confinement.





BEC obtained in such a trap.

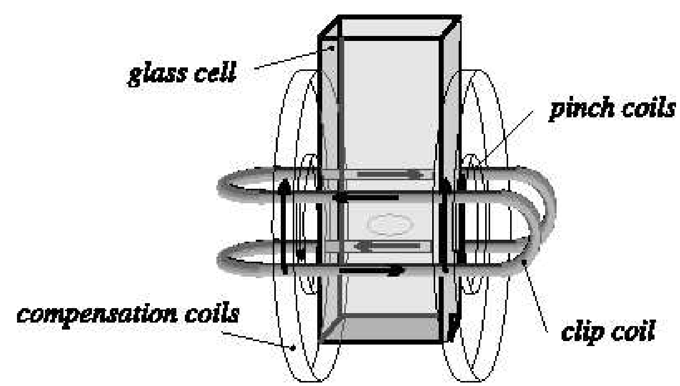
$I \simeq 6 \times 575A$. $b' = 275$ G/cm.

Dipole coils :

$I = 6 \times 455A$, $b'' = 365$ G/cm^2.

Oscillation frequencies

$\omega_\perp = 2\pi \times 280$Hz, $\omega_z = 2\pi \times 24$Hz.



Value of B_0

In order to achieve a high transverse confinement, one want to decrease the minmum field B_0. But if B_0 is reduced too much, then spin flip losses may occur.

Naive approach

As the atoms move in the transverse direction, the magnetic field direction rotates. The rotation frequency is smaller than ω_{vib}. Then if $\omega_{vib} \ll \omega_l = \mu_B B_0/\hbar$, adiabaticity is fulfilled. How small is the spin flip rate ?

A quantum approach

Let us study here a very simplified model where we consider spin 1/2.

Assumptions :

•Magnetic field direction depends only on x, y

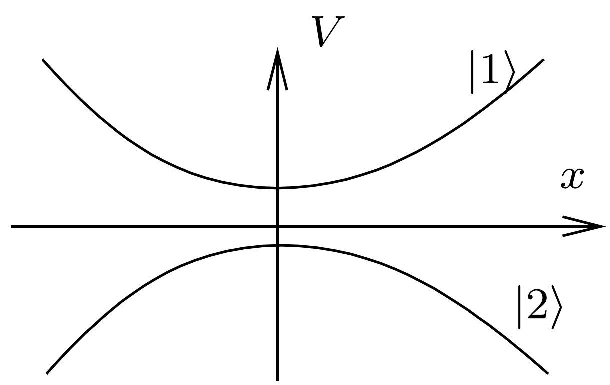
• Size of cloud $l \ll \frac{B_0}{b'}$. angles given to first order in x, y

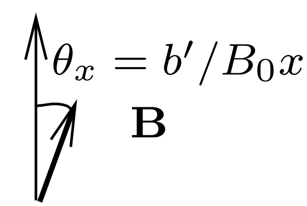
Case of ground state wave function : $l = \sqrt{\hbar/2M\omega_{vib}} \Rightarrow \mu B_0 \gg \hbar\omega_{vib}$, $B_0^3 \gg \frac{b'^2\hbar^2}{M\mu}$

Case of temperature T : $k_B T \ll \mu B_0$

Adiabatic states :

$$
\begin{aligned}
|1\rangle(x) &= e^{-i(\theta_x S_y + \theta_y S_x)}\,|\uparrow\rangle \\
&\simeq |\uparrow\rangle + (b'x/2B_0 - ib'y/2B_0)\,|\downarrow\rangle \\
|2\rangle(x) &= e^{-i(\theta_x S_y + \theta_y S_x)}\,|\downarrow\rangle \\
&\simeq |\downarrow\rangle + (-b'x/2B_0 - ib'y/2B_0)\,|\uparrow\rangle
\end{aligned}
$$





Kinetic energy operator : $\frac{P^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

$$
\frac{P^2}{2m} = -\frac{\hbar^2}{2m}\Delta(|1\rangle\langle 1| + |2\rangle\langle 2|) \underbrace{-\frac{\hbar^2}{2m}b'/B_0\left(-\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)|2\rangle\langle 1| - \frac{\hbar^2}{2m}b'/B_0\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)|1\rangle\langle 2|}_{W}
$$

Spin flip rate

W couple $|1\rangle$ to untrapped states $|2\rangle$.

Initial state : ground state of $|1\rangle = |1, \varphi_0\rangle = |1, n_x = 0, n_y = 0, n_z = 0\rangle$.

Final state : continuum
W acts only on x and y. $\rightarrow |f\rangle = |2, \mathbf{k_f}, n_z = 0\rangle$.
$\mathbf{k_f} = k_{fx}\mathbf{x^0} + k_{fy}\mathbf{y^0}$ (We will neglect the potential in the state $|2\rangle$.)

Fermi Golden Rule : $$\boxed{\Gamma = \frac{2\pi}{\hbar}\rho(E_f)|\langle 2, k_f|W|1, \varphi_0\rangle|^2.}$$

$$\langle 2, k_f|W|1, \varphi_0\rangle = \frac{\hbar^2}{2m}\frac{b'}{B_0}\langle \mathbf{k_f}|\partial_x + i\partial_y|\varphi_0\rangle = i\frac{\hbar^2}{2m}\frac{b'}{B_0}(k_x + ik_y)\langle \mathbf{k_f}|\varphi_0\rangle$$

$$|\langle 2, k_f|W|1, \varphi_0\rangle|^2 = \frac{\hbar^4}{4m^2}\frac{b'^2}{B_0^2}k_f^2\frac{1}{L^2}\left(\frac{\hbar}{2\pi m\omega}\right)e^{-\frac{\hbar k_f^2}{m\omega}}, \qquad \varphi_0 = \sqrt{\frac{\hbar}{2\pi m\omega}}e^{-\frac{\hbar k^2}{m\omega}}$$

density of states n 2D : $\rho_{2D} = \frac{mL^2}{2\pi\hbar^2}$

We use : $\frac{b'^2}{B_0} = m\omega^2/2\mu_B$ and $\hbar^2 k_f^2/(2m) \simeq \mu_B B_0$

$$\boxed{\Gamma = \frac{\omega}{2\pi}e^{-\frac{2\mu_B B_0}{\hbar\omega}}}$$

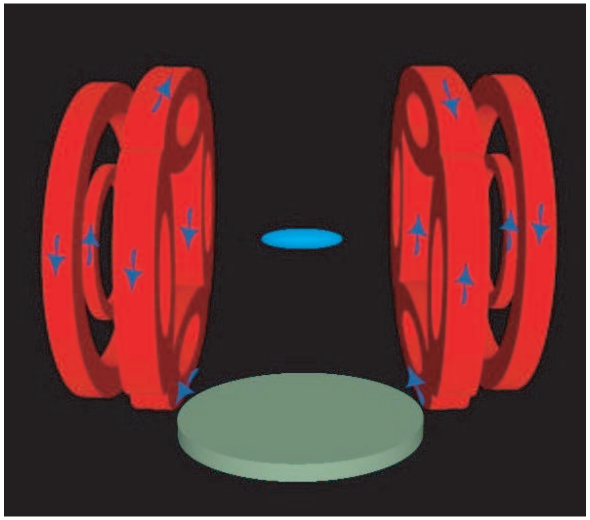
Thermal state
Similar calculation. Calculation of the probability to occupy the momentum k_f.

We find $$\boxed{\Gamma = \frac{\hbar\omega^2}{4\pi k_B T}e^{-\frac{\mu_B B_0}{k_B T}}}$$

Different coils confi gurations

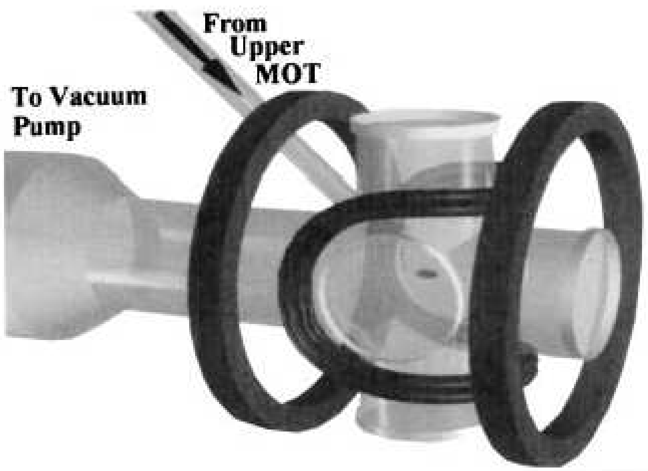
Many different coils configurations can lead to a IOFFE trap. We give here some examples.

- Cloverleaf configuration



Advantage : Good optical access.
typical parameters :
$b' = 80$ G/cm, $b'' = 25$ G/cm^2
Heat dissipation : $\simeq$ kW, water cooling with pressurised water (15 bar)

- Baseball configuration

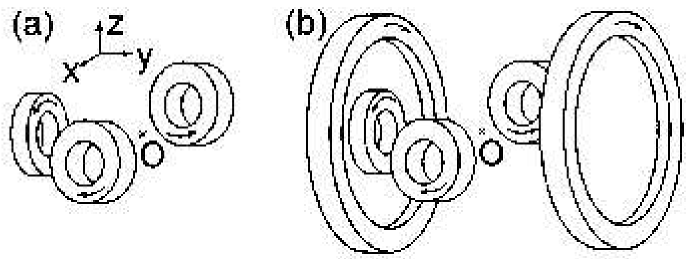


Advantage : Less heat dissipation
$b' = 300$ G/cm, $b'' = 30$ G/cm^2
For Rb^{87} in $|F = 2, m = 2\rangle$: $\omega_{perp} = 400$ Hz, $\omega_z = 10$ Hz

Different coils configurations

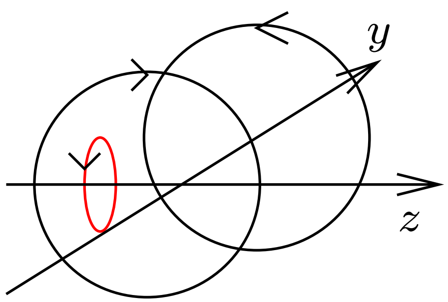
- 3 coils trap

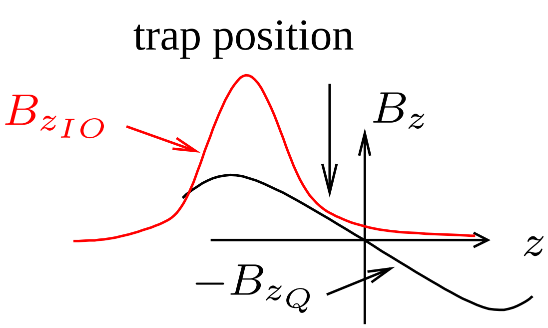
3 coils of same radius with same current.



Advantage : small heat dissipation, simpler than baseball configuration.
For a few hundred W dissipation :
$b' = 100$ G/cm
$b'' = 60$ G/cm^2

- Not symmetric 3 coils trap





QUIC trap
For same current in the three coils, smaller bias field.
No need of extra compensation coils.

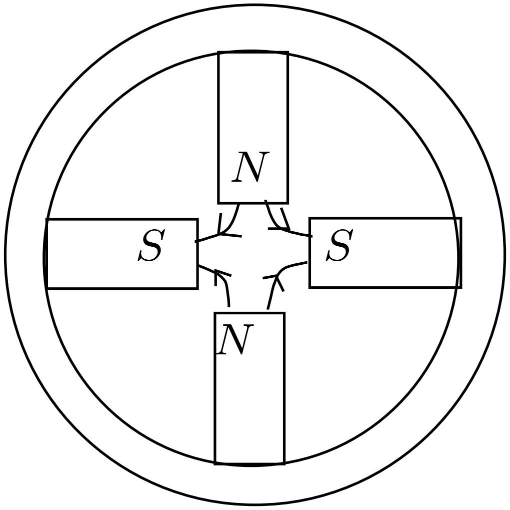
T. Esslinger et al. : $b' = 220$ G/cm, $b'' = 260$ G/cm^2.
For Rb^{87}, $B_0 = 2$ G : $\omega_\perp = 2\pi \times 200$ Hz, $\omega_z = 2\pi$ Hz

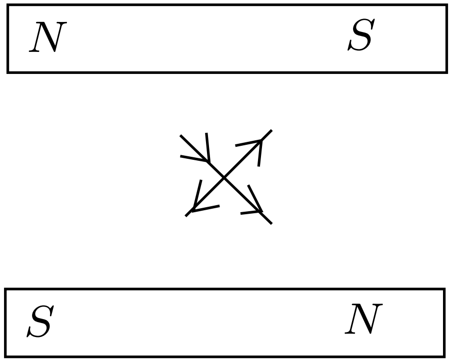
Loading : merging of two quadrupoles

Use of permanent magnet

Some experiments use permanent magnet to create the magnetic field.

Advantage : no heat dissipation, high gradiant





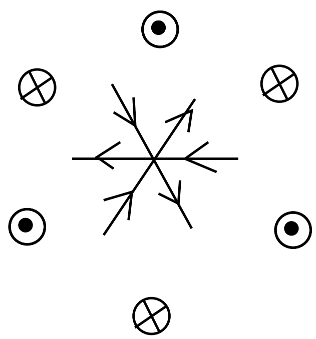
Cornell experiment : $b' = 450$ G/cm

Inconvenient : No possibility to switch off

Higher order fi elds

Transverse field : grows in ρ^n, $n > 1$.

$n = 2$: octopole field



$$|\mathbf{B}| = b''\rho^2$$

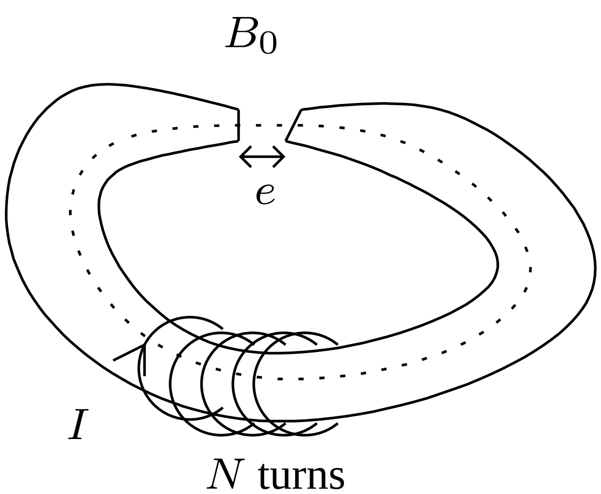
$$B_x = b''(x^2 - y^2)$$
$$B_y = -2b''xy$$

Steeper potential.

Towards smaller structures

Gradiant and curvature proportionnal to $1/R^2$ and $1/R^3$ respectively. Thus small coils are preferred. But coils with their water cooling system takes a lot of room.
Size of the system : compromize between optical access and magnetic confinement.
To overcome this difficulty, one solution is to use magnetic material.

Use of magnetic material



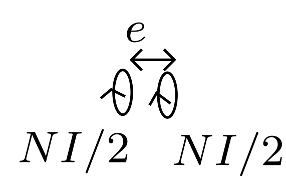
$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$ Magnetic material : $\mathbf{H} = \frac{\mathbf{B}}{\mu_r \mu_0}$, $\mu_r \simeq 10^4$ for iron

Ampere law : $\nabla \wedge \mathbf{H} = \mathbf{j_{ext}}$

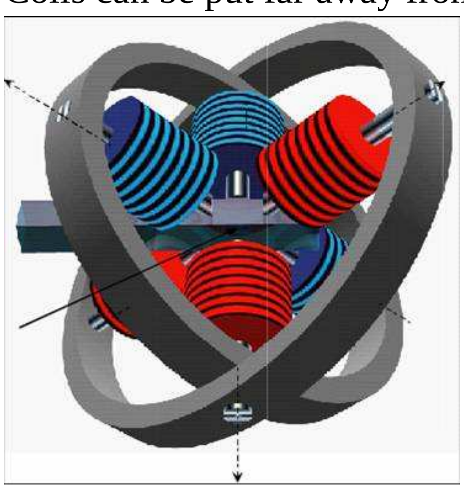
$$NI = \int_C \mathbf{H}.\mathbf{dl} = \int_C \frac{B}{\mu_r \mu_0} dl$$

Neglecting the contribution of the part inside the material gives

$$B_0 \simeq \mu_0 NI/e$$



Magnetic field similar to that produced by coils distant by e.
Coils can be put far away from the interesting region. Can be larger. Less heat dissipation.

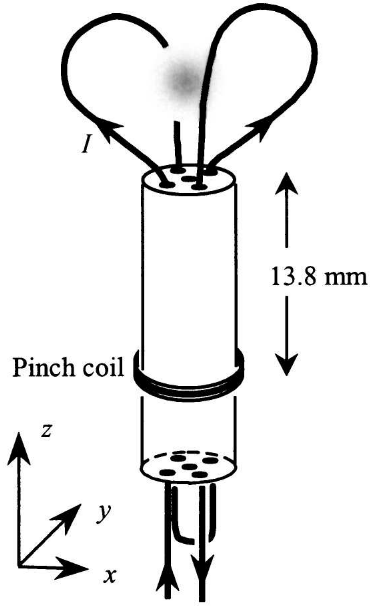


Heat dissipation : $\simeq 200$ W

Confinement : $\omega_{perp} = 750$ Hz .
$\omega_z = 5$ Hz

Use of micro-structures

2-D quadrupole field produced with wires of radius
$R = 260\ \mu$m
Gradiant : $b' = 5000$ G/cm

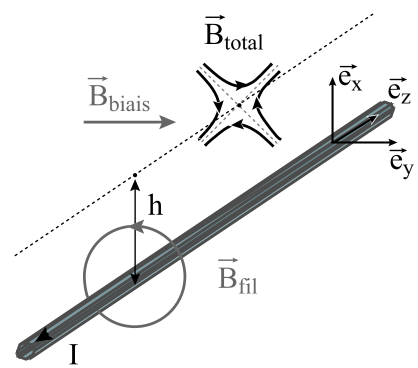


Guiding atoms using a single wire and an homogeneous magnetic field
Single current carrying wire + homogeneous magnetic field

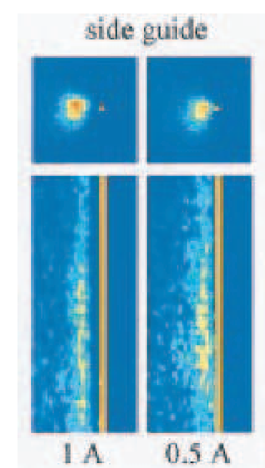
2 D quadrupole field : $$h = \frac{\mu_0 I}{2\pi B_{bais}}$$

$$b' = \frac{\mu_0 I}{2\pi h^2}$$

Realised with 50 μm wire.
$h = 300\ \mu$m
$b' = 1000$ G/cm



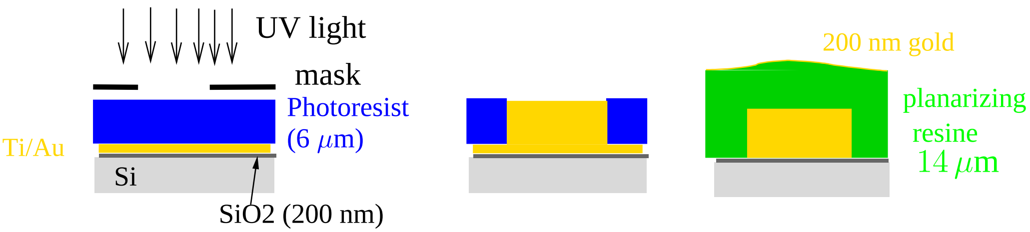
(a)



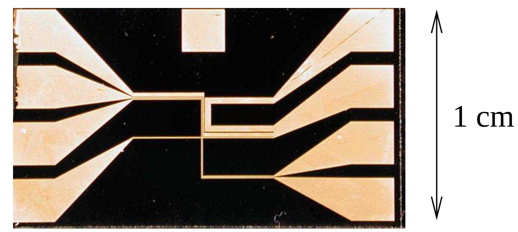
Chip mounted micro-structures

Wires mounted on a chip :
$\longrightarrow$ Good heat dissipation
$\longrightarrow$ Mechanical stability

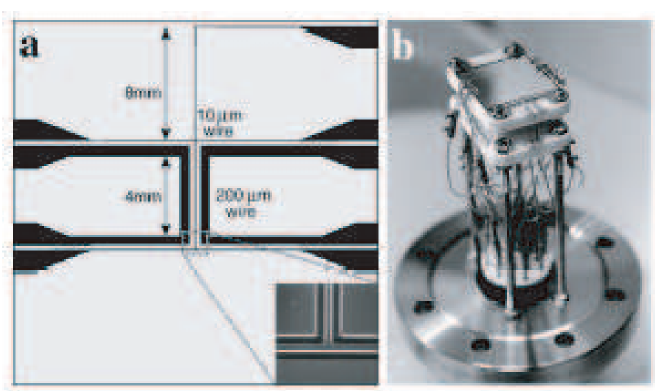
Fabrication technologies:
Substrate : Good heat conduction. Si, GaAS, Sapphire (glass also used)
Pattern design :
●UV lithography, electroplating for wires of size $\simeq 10\mu m$
Example of fabrication process

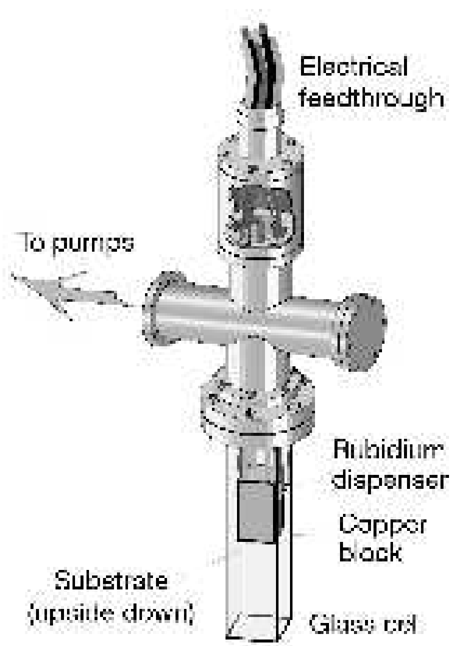


UV photolithography   Electrodeposition   Realisation of a miror



●Electron beam lithography, metal evaporation for size $< 1\mu m$

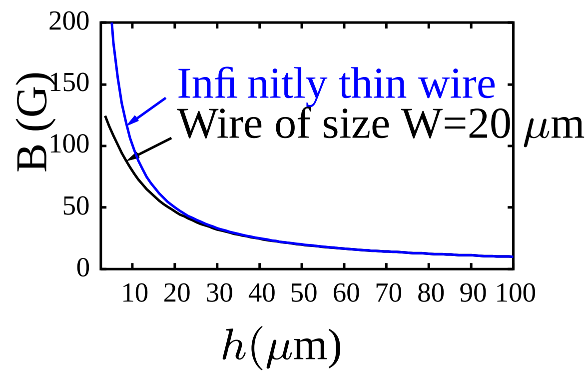




Maximum confi nement

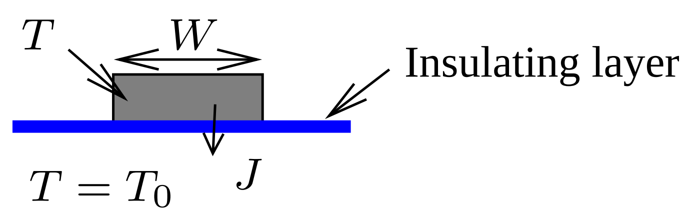
For infinilty small wire : $b' = \frac{\mu_0 I}{2\pi h^2}$
For a wire of size W : $b'_{max} \simeq \frac{\mu_0 I}{2\pi W^2}$

How large can be I ?
Limited by heat dissipation process



Heat dissipation
Assumptions : Substrate at temperature T_0. Insulating layer : thermal conductivity K.
Energy conservation for a unit length along the wire :

- Energy released by Joules effect in a second : $W = \frac{\rho}{We} I^2$
- Energy flux from the wire to the substrate : $J = KW(T - T_0)$
- Energy stored in the wire : $E = cWeT$



$$\Rightarrow \frac{dT}{dt} = \frac{\rho I^2}{c(We)^2} - \frac{K}{ce}(T - T_0).$$

Equilibrium temperature : $T_{eq} = T_0 + I^2 \frac{\rho}{KW^2 e}$ Growing time : $\tau = \frac{ce}{K}$

500 nm $SiO2$ on Si : $K \simeq 10^6 W/Km^2$.
$\tau \simeq 0.7$ μs for $e = 1\mu$m.
For $T_{max} = T_0 + 30K$, $W = 20\mu$m, $e = 5\mu$m, $I_{max} = 0.6A$
Scaling law for squarre wire ($e = W$)

$$T - T_0 = \Delta T_{max} \Rightarrow I_{max} \propto W^{3/2} \Rightarrow \boxed{b'_{max} \propto \frac{1}{\sqrt{W}}}$$

Heat transfer

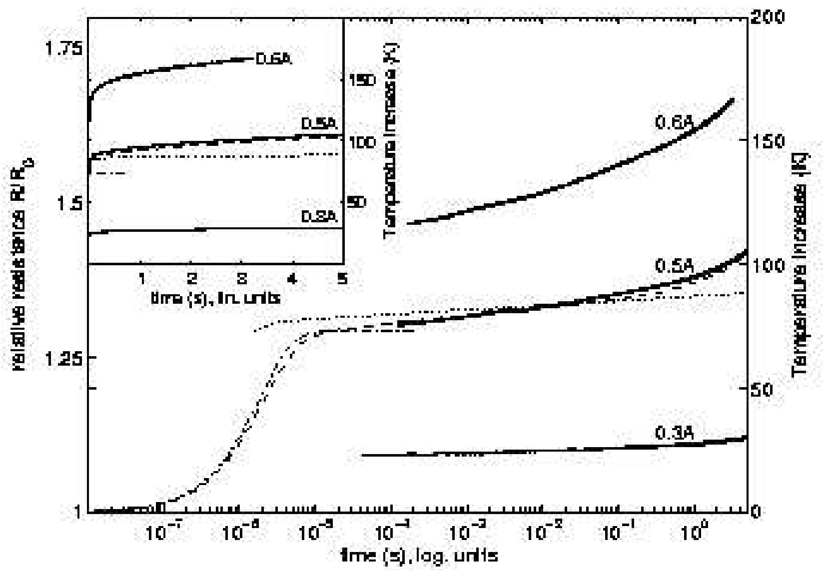
Limit of the model

- **Temperature dependence of ρ :** $\rho = \rho_0(1 + \alpha(T - T_0))$ $\alpha = 3.5 \times 10^{-3}$ K^{-1} for gold

  $\rho \uparrow$ when $T \uparrow$. $\rightarrow$ instability if $I > I_{crit} = \sqrt{\frac{KeW^2}{\alpha\rho_0}}$.

  For $W = 20\mu$m, $e = 5\mu$m, $I_c \simeq 2$A.

- **Heating of the substrate**
  Slow increase of temperature on time scales of seconds.

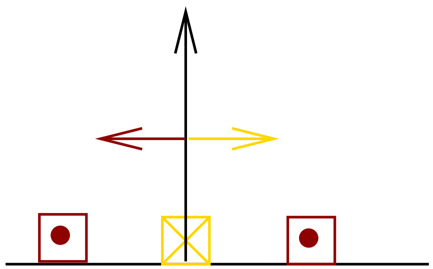


Importance of the substrate mount.

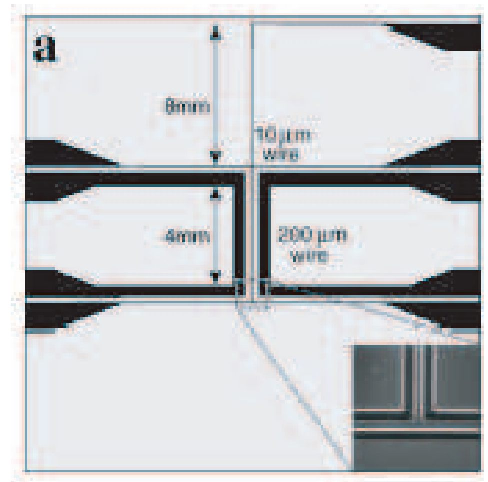
Other guide geometries

- A three wire guide

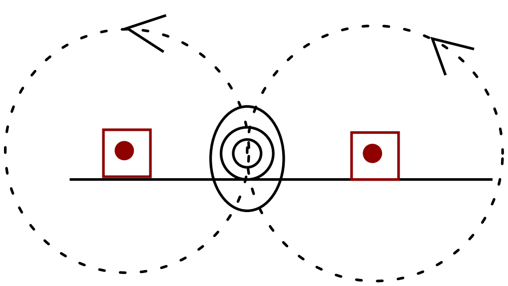
Field perpendicular to the wire produced by side wires.

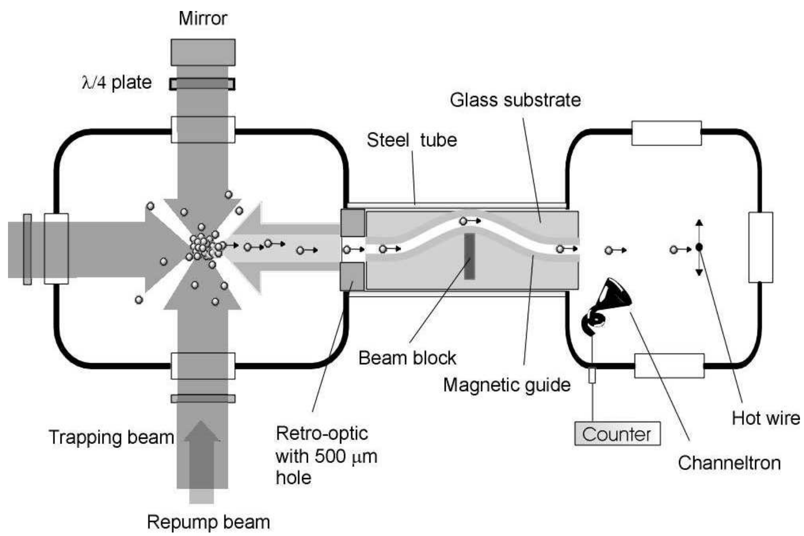


Possibility to realize curved guide.



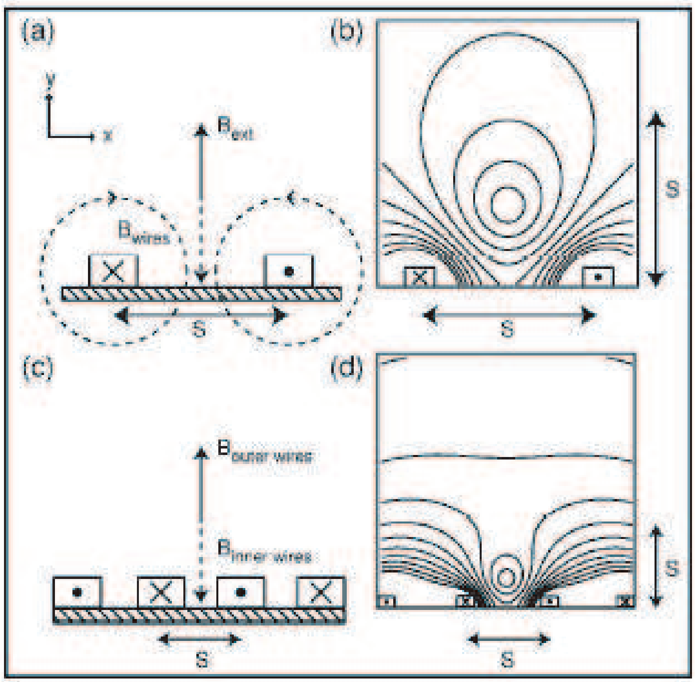
- A 2 wires guide



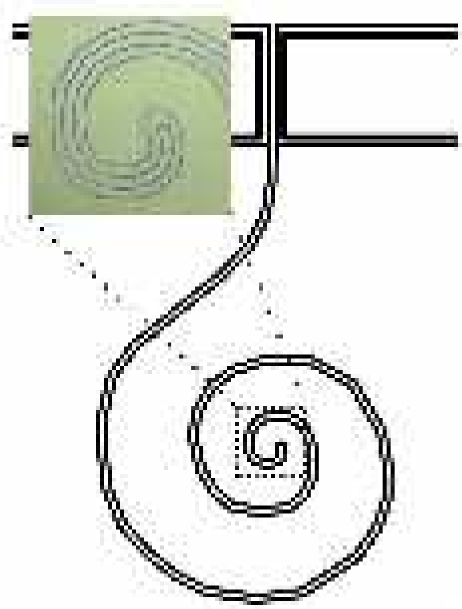


Other geometries

- A 2 wire guide with opposite current



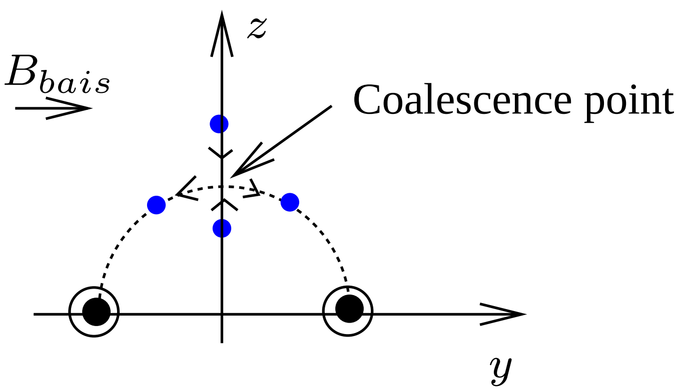
Atoms have been guided along a spiral
(group of J.Schmiedmayer)

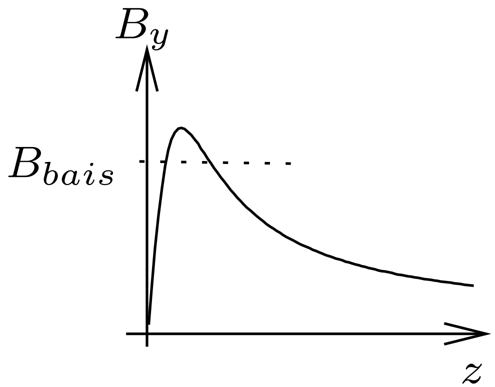


Experiment of M.Printiss : $S = 200\ \mu$m, $\nabla \mathbf{B} \simeq 800$ G/cm

Towards atom interferometry ?

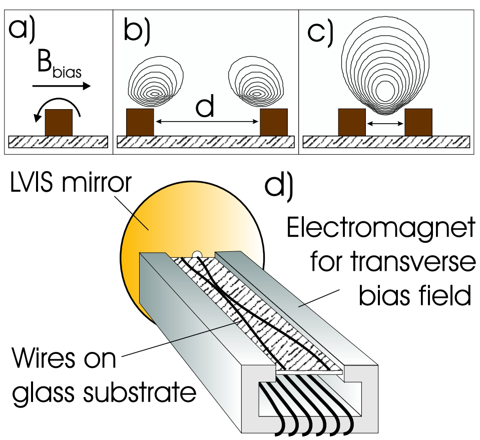
2 wires and a magnetic field

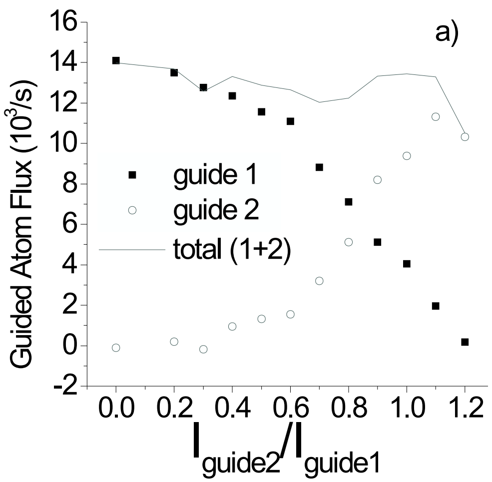




Possibility to split a wave packet in 2. $\longrightarrow$ interferometer
Tunnel coupling between the potential minima near the coalescence point ?
Difficulty : Very sensitive to magnetic field fluctuations

Demonstration of an atomic beam splitter



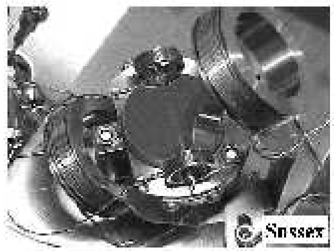


Thermal atoms. No demonstration of coherence

Realization of a longitudinal confinement

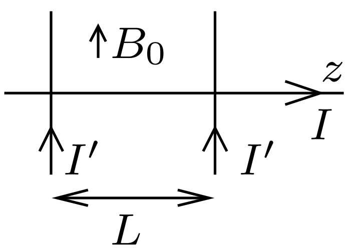
Longitudinal magnetic field presenting a minimum

- **Use of coils**

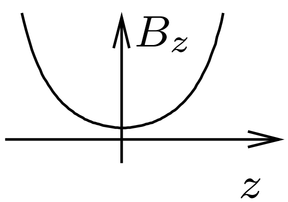


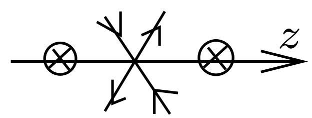
- **Use of wires : the Z shape trap**

Field produced by the bars for $h \ll L$



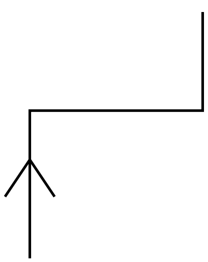
Z-shape wire





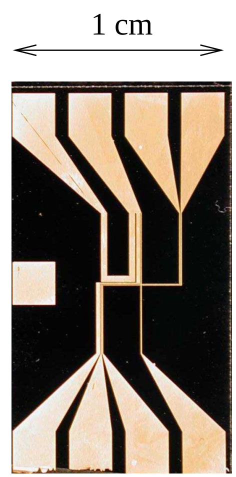
$$B_z \simeq \frac{h\mu_0 I}{L^2}$$

$$\partial^2 B_z/\partial^2 z \propto \frac{h\mu_0 I}{L^4}$$



For $h \ll L$

$$\partial^2 B_z/\partial z^2 = \frac{48\mu_0 I}{\pi}\frac{h}{L^4}$$



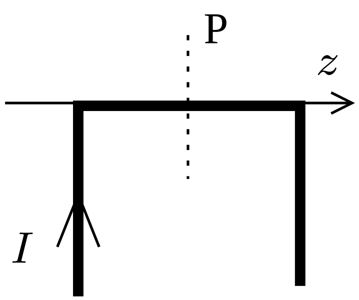
$L = 2.8$ cm, $I = 300$ mA
$B_0 = 14$ G, $B_{bias} = 1$ G
$h = 170$ μm
$\omega_{perp} = 3.5$ kHz
$\omega_z = 16$ Hz

Advantage : less electrical connections

Addition of an extra longitudinal field to prevent spin flip

Other geometries

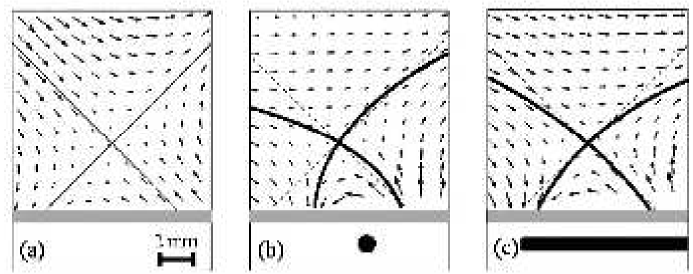
- **A micro quadrupole trap**



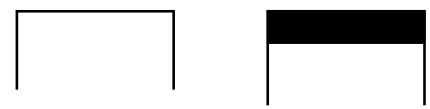
$B_z = 0$ in the plane P
B_z linear in z

For $L = 2.5\text{mm}$:
$b'_x = 85$ G/cm
$b'_y = -74$G/cm
$b'_z = -11$ G/cm

Larger domain of quadrupole fi eld for a ruban shape

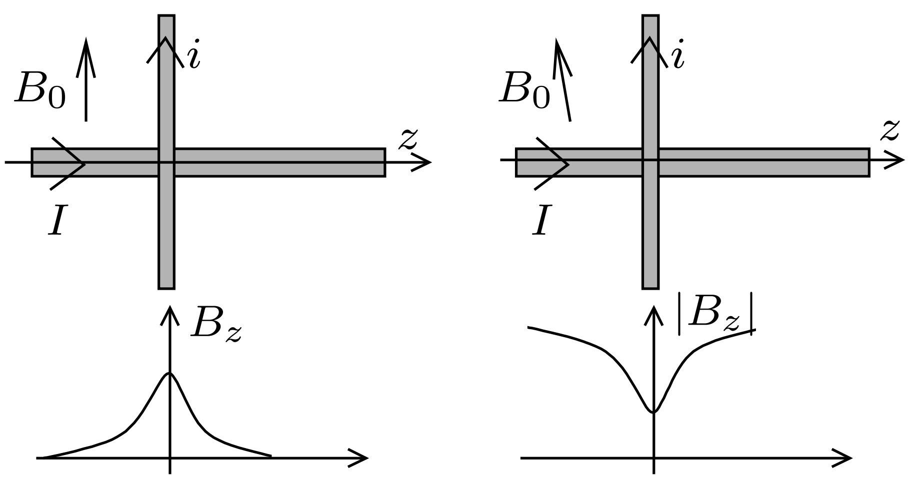


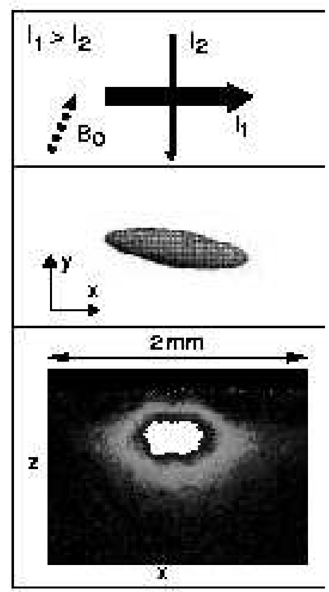
ideal quadrupole | wire | ruban



Used for MOT

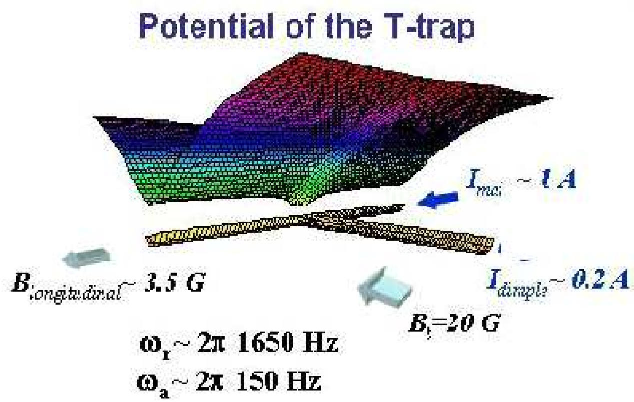
- **Ioffe trap with only two wires**

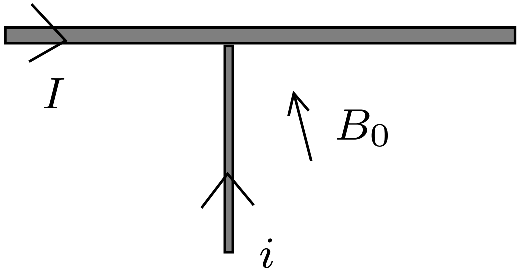




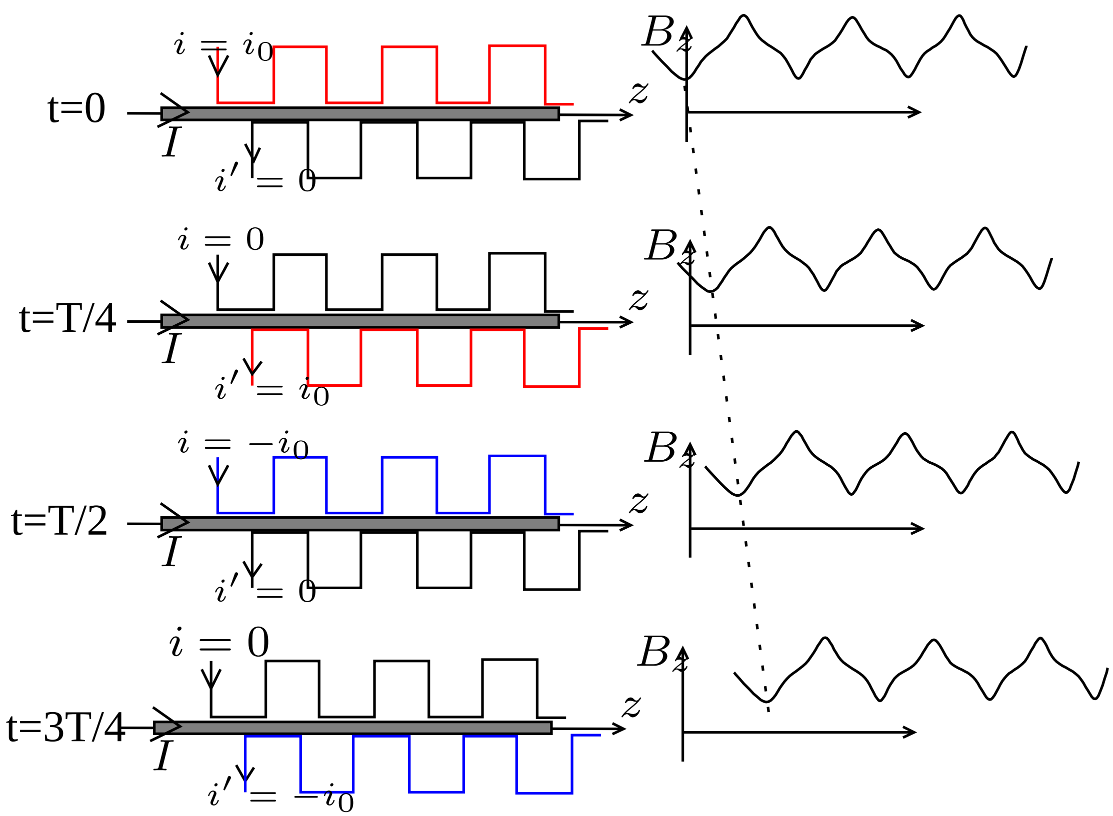
Realization of a longitudinal confinement

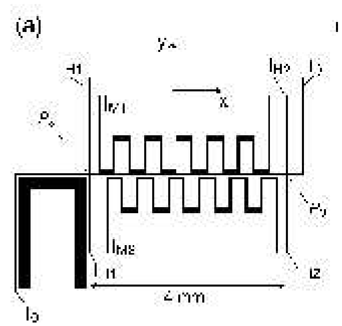
- **T trap**
  2 wires trap simplified in T trap.
  Advantage : less electrical connections

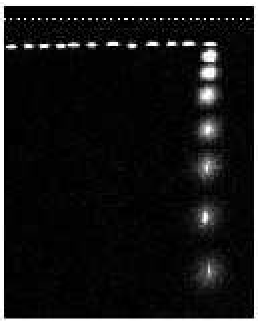




- **Conveyor belt for atoms**





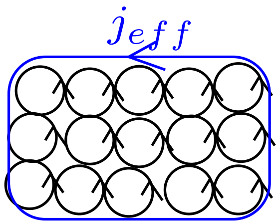


Use of magnetic material

Equivalence between magnetization and a fictitious current distribution

Case of a layer of magnetic material with **magnetization perpendicular to the surface.**

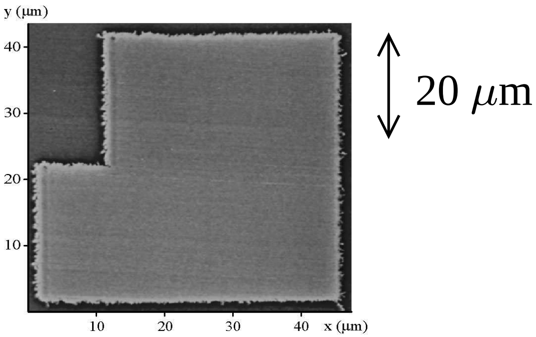
Geometry equivalent to a Z-shape trap



equivalent current distribution :

$$\mathbf{j}_{eff} = \nabla \times \mathbf{M}$$

Zone of unifrom magnetisation M :
equivalent current going around : $I = eM$



Possibility to write the magnetic pattern with laser (coercitive field decreases with temperature)
Size of pattern : limited to about 1 μm.
Large magnetic field gradients : 10^6 G/cm

Not tested on atoms yet.

Advantages of magnetic structures

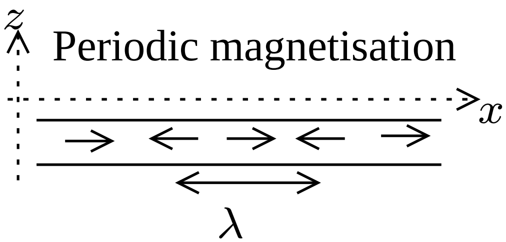
Absence of conductors nearby the atoms $\longrightarrow$ no thermally excited currents.
Stable in time

Disadvantages of magnetic structures

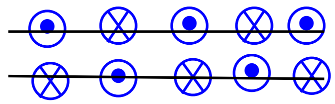
No possibility to turn on and off the magnetic field.

Micro-magnetic traps produced by **video tapes**.

Periodic magnetisation



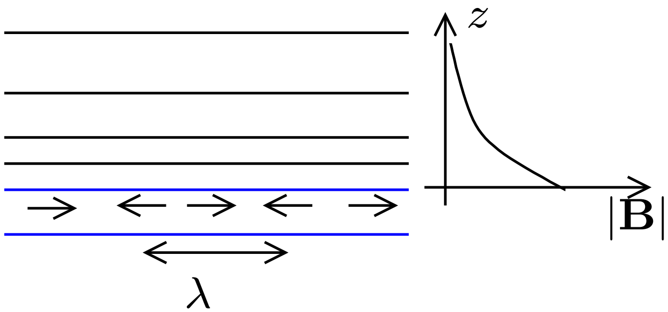
Equivalent currents



Magnetic field produced by a periodic current distribution :

$\nabla \times \mathbf{B} = \nabla.\mathbf{B} = 0 \rightarrow \vec{B} = \nabla\phi,\ \Delta\phi = 0$ Periodicity and $\Delta\phi = 0$: $\rightarrow \phi = \sum_n e^{i2n\pi x/\lambda} e^{-2n\pi z/\lambda}$

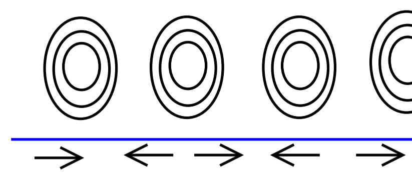
For $z > \lambda$, terms $n = \pm 1$ dominate : $\phi = \phi_1 e^{2i\pi x/\lambda} e^{-2\pi z/\lambda} + c.c. \Rightarrow \vec{B} = B_1 e^{-kz}(-\cos(kx)\mathbf{x^0} + \sin(\mathbf{kx})\mathbf{z^0})$.



Exponential decay of $|\mathbf{B}|$

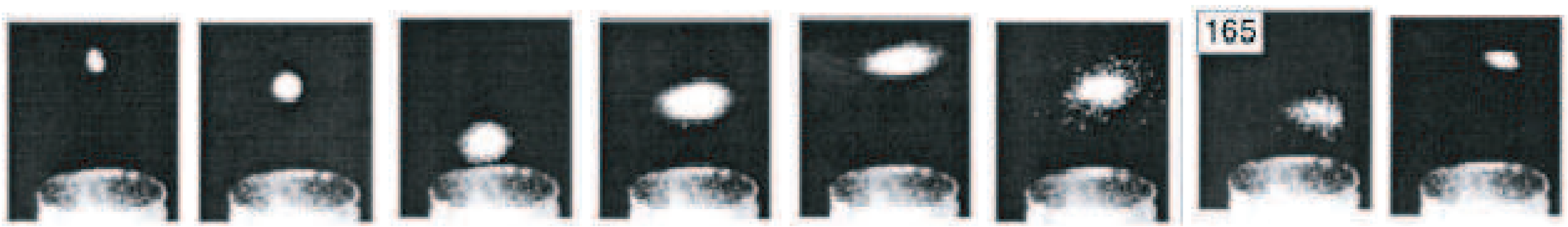
Used as a miror for atoms

Addition of a uniform fi eld $\mathbf{B_0}$ in the plane xz



Array of micro-traps

BEC achieved in such microtraps



Limitation of micro-traps

Advantages of microtraps realized by microstructures

- Very high confinement.
- Very compact apparatus.
- Large variety of trapping and guiding configurations

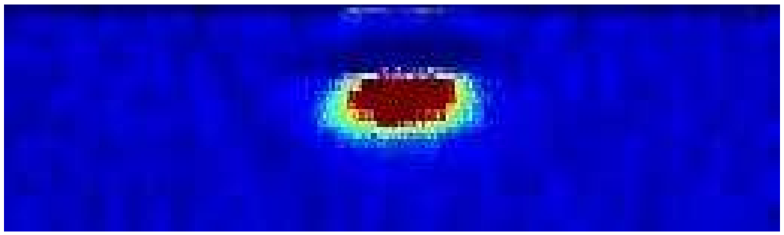
Limitations of micro-traps

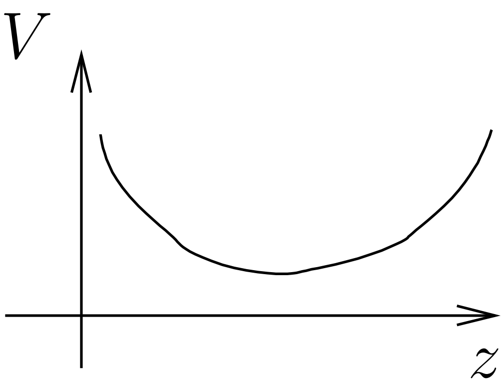
- Roughness of the magnetic potential
- Interaction with nearby materials at high temperature
  Coupling with thermally excited currents
- Van-der-Waals attraction force to the surface
  Effect of atoms adsorbed on the surface.

Roughness of the longitudinal potentiel in an atom guide

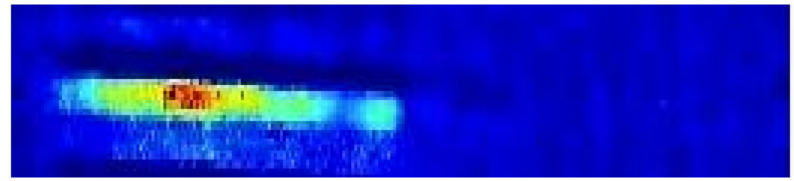
Cloud at small temperature get fragmented when brought close to wire

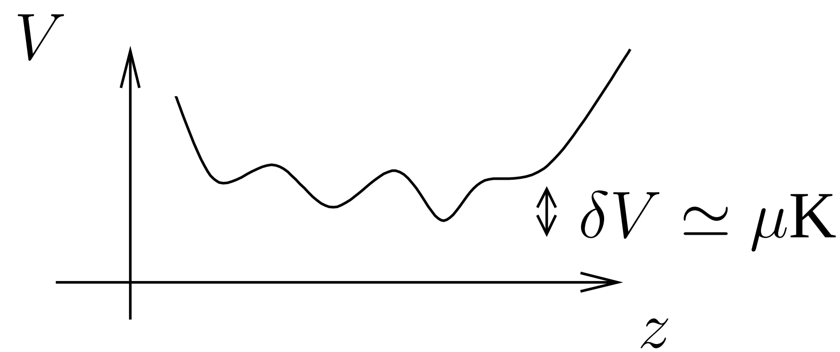
500μm from wire





50μm from wire



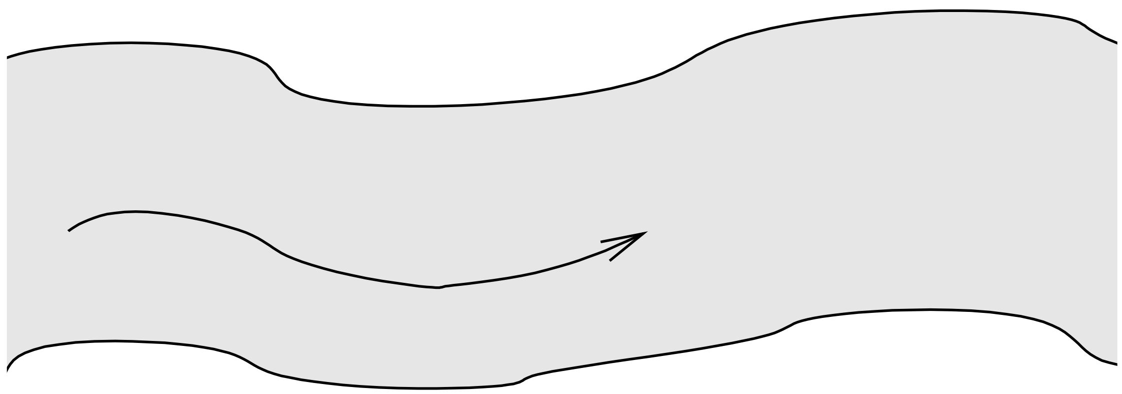


$$\delta V \propto I$$

Due to distortion of current flow in wire

Origin of the roughness

Roughness of the border edge



Induces deviation of the current flow inside the wire

$\longrightarrow$ rough longitudinal magnetic fi eld $\longrightarrow$ potential roughness

$\Delta B/B_0 \simeq 10^{-4}$: $\longrightarrow$ very sensitive to wire border fluctuations

Fluctuations of B_x produce a small displacement of the guide height.

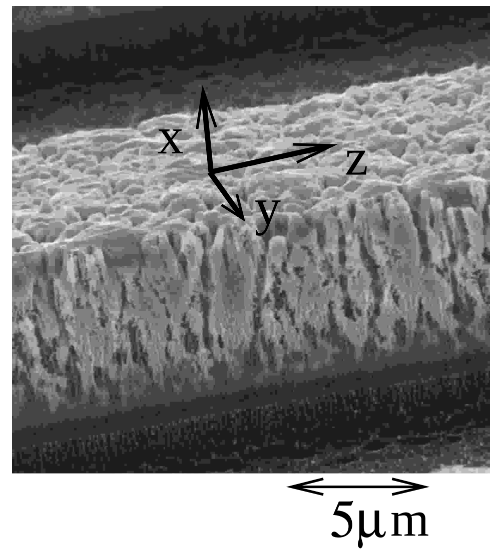
Origin of the roughness : experimental evidence

- Potential roughness measured with cold atomic cloud at thermal equilibrium . $n(z) \propto e^{-V(z)/k_B T}$

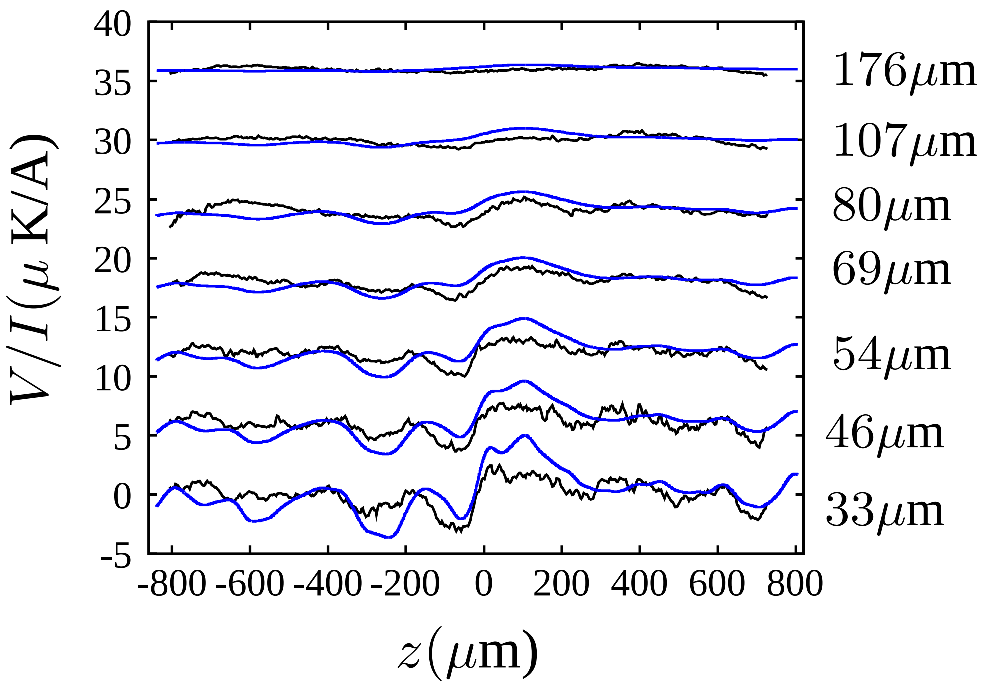
  Confining potential substracted.

- Wire border roughness measured with an SEM.

  Expected potential roughness computed.



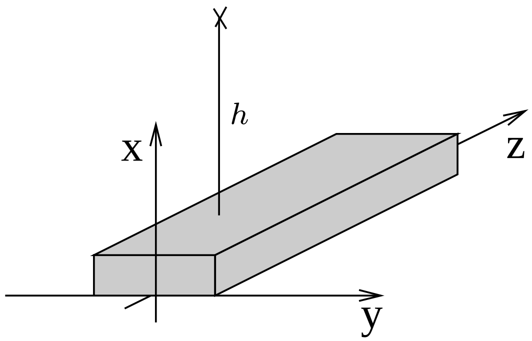
Potentials

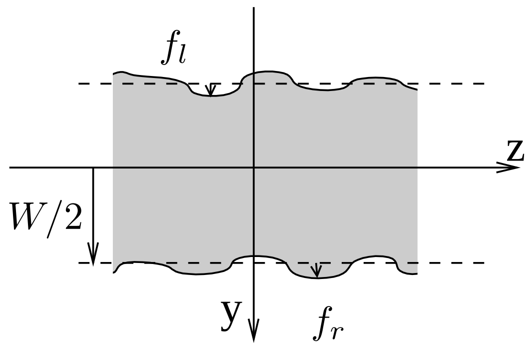


Potential measured with the atoms

Potential calculated from the measurement of the wire edge roughness

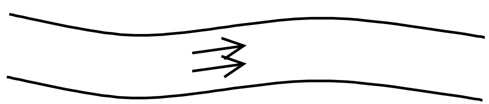
Scaling laws of the potential roughness





We consider only $h \gg W \rightarrow B_z = B_z[I_y(z)]$, $I_y = \int_x \int_y dx dy j_y$

• k component $f_{l/r} = f_k \cos(kz)$. $h \gg W \rightarrow k \ll 1/W$.

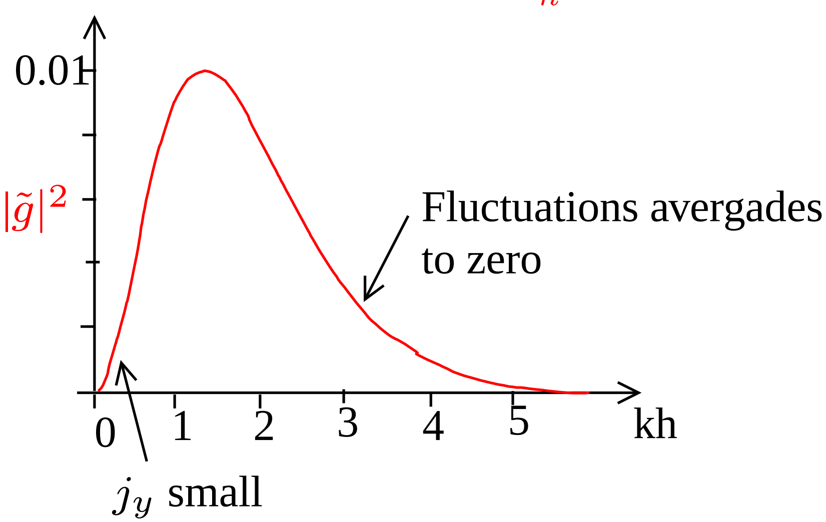


$I_{y_k} = -I f_k k \sin(kx)$

Only symmetric part $f_l + f_r$ contribute to I_y.

NB : If $k \gg W$, then current distortion localised near the borders

$\rightarrow B_k = \mu_0 I f_k g(k,h) = f_k \frac{\mu_0 I}{h^2} \tilde{g}(kh)$



Case of white noise rougness of f

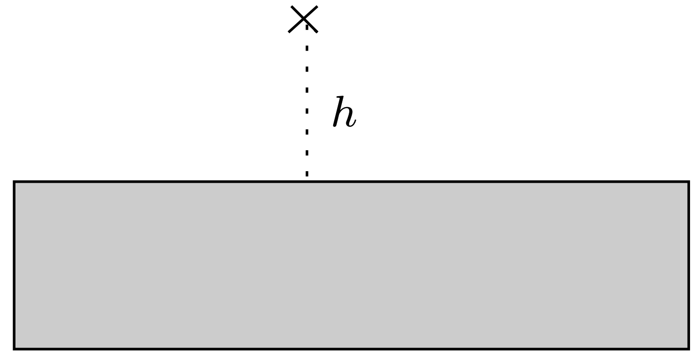
$$|f_k|^2 = J_f = J_0$$

$$\langle B_z^2 \rangle = 0.044 J_0 \frac{(\mu_0 I)^2}{h^5}$$

Fluctuations of B_z peacked at $k \simeq 1/h$

Effect of Casimir Polder

Interaction between an atom and a dielectric wall :



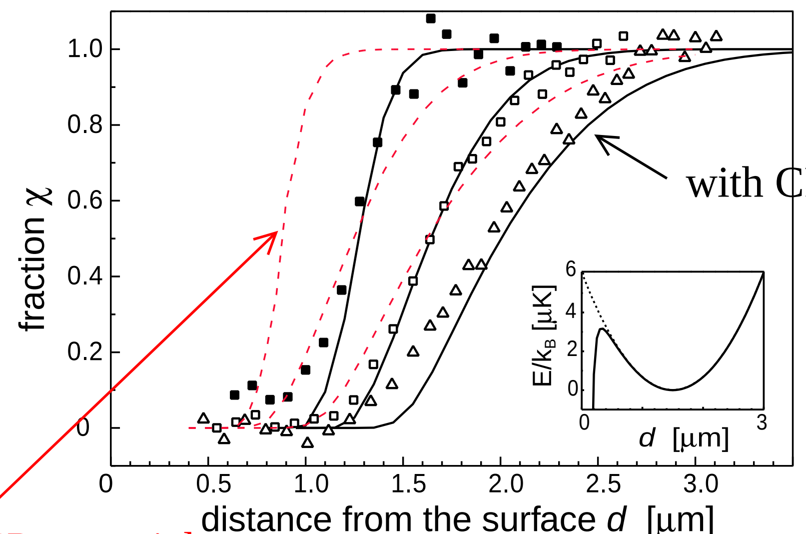
- For $h \ll \lambda_{at}$: Van der Waals force $V_{VdW} = -\frac{C_3}{R^3}$
- For $h \gg \lambda_{at}$: Casimir-Polder potential $V(h) = -\frac{C_4}{R^4}$

λ_{at} atomic transition wavelength $\quad C_4 = K(\epsilon)\alpha \quad \alpha$:polarisability $\quad C_4 \simeq 6 \times 10^{-4} \mu\mathrm{K}\mu\mathrm{m}^4$

ϵ : dielectric constant $\quad$ for Rb and $\epsilon = 4$

Experimental study

Effect of Casimir-Polder force :
smaller depth of the potential
$\longrightarrow$ losses of atoms by evaporation
Measured loss rate compared with calculation based on 1D evaporation model



with CP potential

Without CP potential

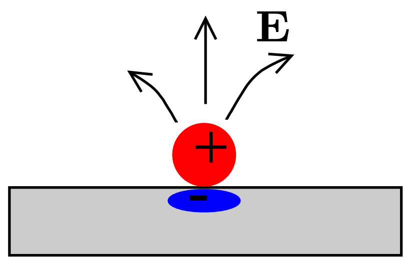
Limit the distance of approach to the surface

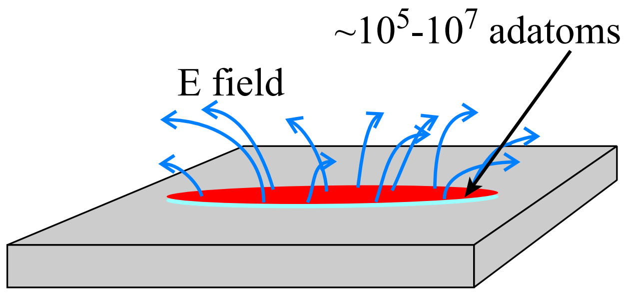
Effect of adsorption of atoms on the surface

Atoms adsorbed on some surfaces get polarized $\longrightarrow$ electric field $E \propto \frac{d}{r^3}$

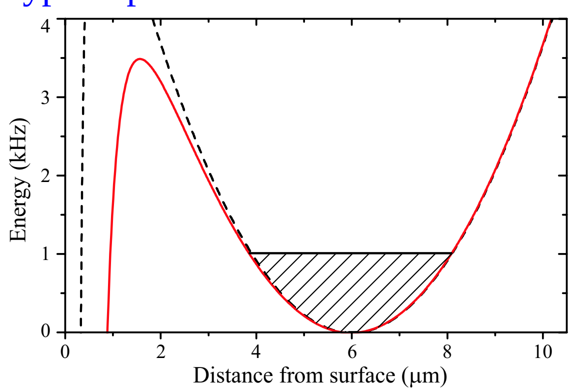
$\Rightarrow$ interaction energy with atoms : $U = -\frac{\alpha}{2} E^2$

$\Rightarrow$ Attraction towards the surface.





Typical potential for 10^7 Rb atoms adsorbed on 4μm$\times 150\mu$m area



Stability of a trapped cloud

- Background gas
- Fluctuations of the trap
- Spin flip transitions induced by magnetic field fluctuations
- interaction with environment at temperature $T = 320$ K

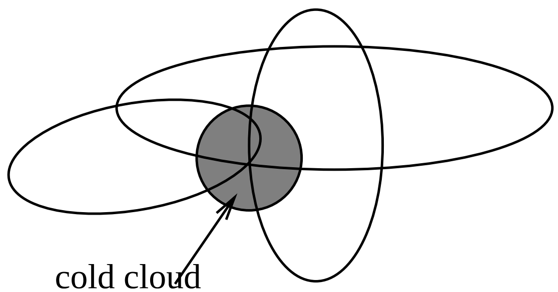
Effect of the background gas

- Losses produced by collisions
  Atoms of background gas at $T \simeq 320$ K$\gg$ depth of the trap.
  $\longrightarrow$ After a collision with a background atom, the trapped atom is lost almost certainly.

$$\Gamma = n_{bg} \bar{v}_T \sigma$$

  where $n_{bg} \propto P$ is the density of background gas at pressure P and $\bar{v}_T = \sqrt{\pi k_B T / 2m}$ is the thermal velocity.
  $\Gamma \propto P$.
  For Rb atoms, for $P = 10^{-9}$ mbar, $\Gamma \simeq 0.1$ s^{-1}.

- Heating produced by large scattering impact collisions
  Production of a hot atoms cloud $\longrightarrow$ heating rate of the cold cloud.



  Production of a cloud of hot atoms : **the Oort cloud**
  Very difficult to quantify. Role of multiple scattering : effect more important for dense cloud
  Solution : Trap depth small enough. Use of an RF shield.
  A typical value : $dT/dt \simeq 1$ μK/s

Heating induced by trap fluctuations

Technical fluctuation of current in coils $\Rightarrow$ fluctuation of trap parameter
Examples :

Transverse magnetic field $B \longrightarrow$ position fluctuation $\delta x = B/b'$
Fluctuation in current in the dipole coils $\longrightarrow$ fluctuation of ω_z.

Produces heating

- trap position fluctuations
  Fluctuating force : $F(t) = m\omega^2 \delta x$.
  Equation : $\ddot{x} = -\omega^2 x + F(t)/m$ Solution $x = x_0 \cos(\omega t) + \int_0^\infty \frac{F(\tau)}{m\omega} \sin(\omega(t-\tau))d\tau$
  Calculation of $\langle x^2 \rangle$ (statistical average over noise) :

$$\langle x^2 \rangle(t) = x_0^2 \cos^2(\omega t) + \int_0^t \int_0^t d\tau_1 d\tau_2 \frac{\langle F(\tau_1)F(\tau_2)\rangle}{m^2\omega^2} \frac{1}{2}(\cos(\omega(\tau_1 - \tau_2)) - \cos(2\omega t))$$

For $t \gg t_{cor}$ the correlation time of F, using $J_F(\omega) = \int_\infty^\infty dt e^{-i\omega t}\langle F(0)F(t)\rangle$ and averaging over a period, we have

$$\langle x^2 \rangle(t) = x_0^2/2 + t\frac{J_F(\omega)}{2m^2\omega^2}$$

Thus, the potential energy $E_p = m\omega^2 x^2/2$ average over a period fulfills $\frac{dE_p}{dt} = t\frac{J_F(\omega)}{4m}$ After average over an oscillating period, $E_p = E_c = E/2$ so that

$$\frac{dE}{dt} = \frac{J_F}{2m}$$

Heating linear in time. Driven oscillator. Excitation of the dipole mode
Heating rate for a fluctuating homogeneous magnetic field : $\frac{dE}{dt} = \frac{m^2\omega^4}{b'^2} J_B(\omega)$

oscillation frequencies fluctuations

$\omega^2 = \omega_0^2(1 + \epsilon(t))$
Calculation easier using quantum mechanics

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega_0^2 (1 + \epsilon) x^2$$

Using $x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^+)$,

$$H = \hbar\omega_0 (1 + \epsilon/2) a^+ a + \frac{1}{2} m\omega_0^2 \epsilon \frac{\hbar}{2m\omega} (a^2 + a^{+2})$$

$|n\rangle$ coupled to $|n - 2\rangle$, $|n + 2\rangle$

Transition rates induced by a fluctuating coupling

Sates $|i\rangle$ and $|f\rangle$ coupled by $v(t)A_{if}$, v fluctuates.
Initial state : $\psi = |i\rangle$.
Perturbation theory :
At time t, $|\psi\rangle = e^{-i\omega_i t}|i\rangle + c_f e^{-i\omega_f t}|f\rangle$,

$$c_i = \frac{1}{i\hbar}\int_0^t A_{if} v(\tau) e^{i(\omega_i-\omega_f)\tau} d\tau.$$

Population in f (averaged over random realization of v),

$$\langle |c_f|^2\rangle = \frac{1}{\hbar^2}\int_0^t\int_0^t d\tau_1 d\tau_2 |A_{if}|^2 \langle v(\tau_1)v(\tau_2)\rangle e^{-i\omega_{if}(\tau_1-\tau_2)}$$

For $t \gg t_{cor}$ the correlation time of v,

$$\langle |c_f|^2\rangle = \frac{|A_{if}|^2}{2\hbar^2}\int_0^{2t} J_v(\omega_{if}) d\tau$$

Thus the transition rate is

$$\Gamma_{if} = \frac{|A_{if}|^2}{\hbar^2} J_v(\omega_{if})$$

Link with Fermi Golden rule :
$v = \sum_i v_i e^{-i\omega_i t}$. In a dressed state approach, transition $|i\rangle \longrightarrow |f\rangle$ due to absorption of a quanta in a mode of v. Time independent coupling $|i, n_i\rangle \longrightarrow |f, n_i - 1\rangle$. Fermi Golden rule gives transition rate.

Heating rate induced by oscillation frequencies fluctuations

Here $V = \frac{\hbar\omega}{4}\epsilon(t)(a^2 + {a^+}^2)$.
Initial state $|n\rangle$:

$$\Gamma_{n\to n+2} = \frac{\omega^2}{16} J_\epsilon(2\omega)(n+1)(n+2)$$

$$\Gamma_{n\to n-2} = \frac{\omega^2}{16} J_\epsilon(2\omega)n(n-1)$$

Heating rate (thermal average over initil state)

$$\frac{dE}{dt} = \sum_n p_n 2\hbar\omega \frac{\omega^2}{16}((n+1)(n+2) - n(n-1))J_\epsilon(2\omega)$$

$$\frac{dE}{dt} = \frac{\omega^2}{2} J_\epsilon(2\omega)\langle E\rangle.$$

Exponential heating. Parametric driving

Higher order : heating for $\omega_{mod} = 2\omega/p$, p integer.

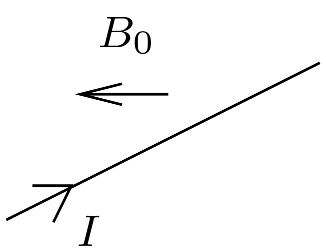
1 D calculation. For a three dimensionnal harmonic trap, $E_x = E/3$ and heating rate decreased by a factor 3.

Heating produced by current fluctuations in a wire guide

• Heating due to position fluctuation

Using $F = m\omega^2\delta x = \frac{m\omega^2\mu_0}{2\pi B_0} I$,

$$J_F = \left(\frac{m\omega^2\mu_0}{2\pi B_0}\right)^2 J_I .$$



$h = \frac{\mu_0 I}{2\pi B_0}$

Heating rate,

$$\frac{dE}{dt} = \frac{h^2 m\omega^4}{2}\frac{1}{I_0^2} J_I$$

Case $I_0 = 1$ A, $h = 10$ μm, $\omega = 2\pi \times 10$ kHz, $J_I = 1(\mu\mathrm{A}^2)/\mathrm{Hz}$: $dE/dt = 8$ μK/s.
Very good power supply : $J_I \simeq 10(n\mathrm{A}^2)/\mathrm{Hz}$

Do not happen if B_0 and I have correlated fluctuations : $\Delta I/I = \Delta B_0/B_0$ (circuit in series)

• Heating due to fluctuations of oscillation frequency

$\omega \propto 1/I$. Thus

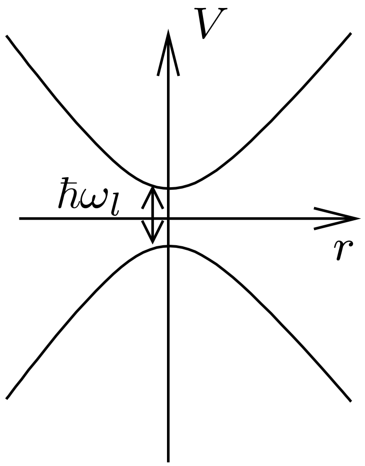
$$\frac{dE}{dt} = E\frac{\omega^2}{I^2} J_I .$$

Ratio of heating rates :

$$\frac{dE/dt)_{dip}}{dE/dt)_{param}} = \frac{m\omega^2 h^2}{2E} = \frac{h^2}{2l^2} \gg 1,$$

for cloud of size $l \ll h$.

Spin flip transitions induced by fluctuating magnetic field



$\mathbf{B}$ along z in the trap

b_x and b_y couple trap state other Zeeman level

$\longrightarrow$ transition to untrapped states : losses

Transition rate : $\Gamma \simeq \frac{\mu_B^2}{\hbar^2} J_{B_x}(\omega_l)$

Case of current fluctuations in a micro-trap : $\Gamma \simeq \left(\frac{\mu_B \mu_0}{2\pi h}\right)^2 J_I.$ For $h = 1$ μm, $J_I = 0.3$ (nA)^2/Hz , $\Gamma \simeq 1.3$ s^{-1}

Interaction with environement at temperature $T \simeq 320$ K

• blackbody radiation

Field at $\omega \simeq \omega_l = \mu_B B_b$: induce transitions to an untrapped state $\Gamma \simeq \frac{\mu_B^2}{\hbar^2} J_B$
Black body field fluctuations :
Mode n of the electromagnetic field. $B_x = B_{x n_0}(a + a^+)/\sqrt{2}$.
Average over thermal state (no coherence between different modes)

$$\langle B_x(t) B_x(0)\rangle = \sum_n \frac{B_{x n_0}^2}{2} \langle (e^{-i\omega_n t} a_n + e^{i\omega t} a_n^+)(a_n + a_n^+)\rangle = \sum_n B_{x n_0}^2 (n p_n \cos(\omega_n t) + \underbrace{\frac{e^{-i\omega t}}{2}}_{\text{vaccum contribution}}).$$

Spetcral density : $J_{B_x} = \int \langle B_x(t) B_x(0)\rangle e^{-i\omega t} dt = \sum_n B_{x n_0}^2 n p_n \pi \delta(\omega - \omega_n)$
Repartition of energy between E and B and isotropie : $\frac{3}{\mu_0} J_{B_x} = \pi \sum_n \frac{\hbar\omega}{L^3} n p_n \delta(\omega - \omega_n) = \pi \frac{dE}{L^3 d\omega}$,
where $dE = \frac{\hbar\omega e^{-\hbar\omega/k_B T}}{1-e^{-\hbar\omega/k_B T}} \frac{\omega^2}{\pi^2 c^3} L^3 d\omega$. is the enrgy in the range $\omega \to \omega + d\omega$.

$$J_{B_x} = \frac{\mu_0 \hbar \omega^3}{3\pi c^3} \frac{e^{-\hbar\omega/k_B T}}{1 - e^{-\hbar\omega/k_B T}} \simeq \frac{k_B T \omega^2 \mu_0}{3\pi c^3}$$

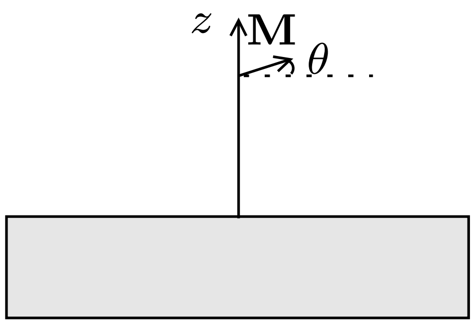
Induced loss rate : $\Gamma = 7 \times 10^{-18}$ s^{-1} for $\omega_l = 2\pi \times 1$ MHz.
Negligible

Interaction with nearby conductors

Atom chip : atoms close to a surface at temperature T : $h \simeq 1\mu$m..100μm

Effect of thermally excited current in the wire or substrate ? We need to know J_B

Fluctuating magnetic field close to a conductor



Small oscillating dipole. $\theta \ll 1$: $H = \frac{p_\theta^2}{2I} + \frac{K}{2}\theta^2$,
$\omega^2 = K/I$.

Interaction with the metal : $V = -\mathbf{B}.\mathbf{M} \simeq \theta M B_z$

- damping of oscillations
- surface at temperature $T \longrightarrow$ fluctuating currents $\longrightarrow$ fluctuating field $\longrightarrow$ heating of the dipole

- Heating due to the fluctuating field : Using previous results : $dE/dt)_{fluctu} = \frac{1}{2I} J_B(\omega)$
- Energy dissipation due to damping
  Linear response : $B = \theta_a M(\mathcal{R}_e(\alpha)\cos(\omega t) + \mathcal{I}_m(\alpha)\sin(\omega t))$ for $\theta = \theta_a \cos(\omega t)$.

  Energy decrease : $\left.\frac{dE}{dt}\right)_{diss} = \frac{\partial V}{\partial t} = \theta_a^2 M^2 \omega \cos(\omega t)(-\mathcal{R}_e(\alpha)\sin(\omega t) + \mathcal{I}_m(\alpha)\cos(\omega t))$

  Average over a period : $\left.\frac{dE}{dt}\right)_{diss} = \theta_a^2 M^2 \omega \mathcal{I}_m(\alpha)/2$

At thermal equilibrium $dE/dt)_{diss} = dE/dt)_{fluctu}$

$\Rightarrow$ $$\boxed{J_B = 2\frac{k_B T}{\omega}\mathcal{I}_m(\alpha)}$$ Theorem fluctuation/dissipation

we used $\theta_a^2 = 2\langle E\rangle/K \simeq 2k_B T/K$ for $k_B T \gg \hbar\omega$

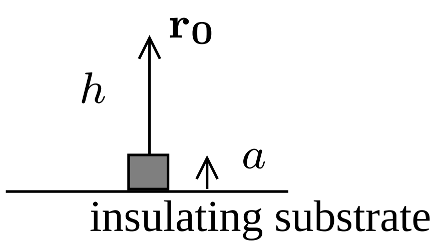
Scaling low for the fluctuation of magnetic field near conductors

$J_B = 2\frac{k_B T}{\omega}\mathcal{I}_m\alpha(\omega)$
$\mathcal{I}_m\alpha(\omega)$: component in quadrature of the magnetic field produced in $\mathbf{r_0}$ by the current induced by an oscillating dipole in $\mathbf{r_0}$.

Case of low frequencies :
Oscillating dipole $\longrightarrow$ oscillating magnetic field $B_d \propto 1/|\mathbf{r}-\mathbf{r_0}|^3$
$\longrightarrow$ induced electric field $E \propto \omega/|\mathbf{r}-\mathbf{r_0}|^2$ $(\nabla vect\mathbf{E} = -\partial\mathbf{B}\partial t)$
$\longrightarrow$ induced currents $j \propto \omega/(|\mathbf{r}-\mathbf{r_0}|^2\rho)$ $(\mathbf{j} = \mathbf{E}/\rho)$: Foucault currents
$\longrightarrow$ induced magnetic field in $\mathbf{r}$

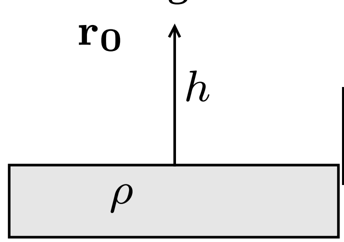
$$B_{ind} \propto \mathcal{I}_m\alpha \propto \frac{\omega}{\rho}\int d^3\mathbf{r}\frac{1}{|\mathbf{r}-\mathbf{r_0}|^4}$$

fluctuations produced by a wire



$$\boxed{J_B \propto \frac{k_B T a^2}{\rho h^3}}$$

fluctuations produced by a half space



$$\boxed{J_B \propto \frac{k_B T}{\rho h}}$$

Case of high frequencies

Electromagnetic field attenuated in the metal after a distance $\delta(\omega)$. $\delta = \sqrt{2\rho/\mu_0\omega}$:skin depth
If $h > \delta$, then previous scaling laws wrong.
For $\omega = 1.4$ MHz (1G$/\mu_B$), in gold, $\delta = 60\ \mu$m.

Correct calculation of α : Maxwell equations for evanescent waves.

Calculations for a half metallic space :

- $h \gg \delta$:

$$J_{B_i} = \frac{\mu_0^2 k_B T}{16\pi\rho} s_i \frac{3\delta(\omega)^3}{z^4}$$

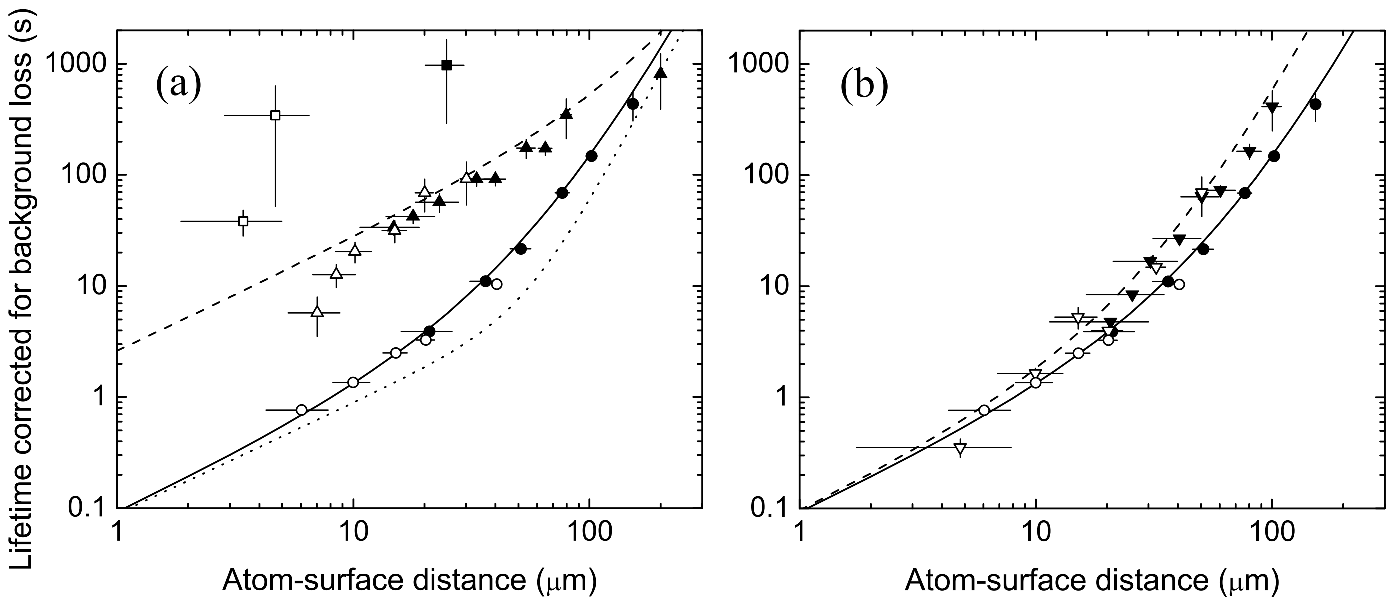
- $h \ll \delta$:

$$J_{B_i} = \frac{\mu_0^2 k_B T}{16\pi\rho} s_i \frac{1}{\delta}$$

$s_i = 1/2$ for i=x,y and $s_z = 1$, (z normal to surface)

Losses induced by thermally excited currents

Magnetic field fluctuation at $\omega = \omega_l$ produces losses.



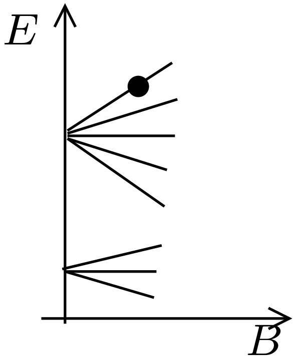
(a) : $\omega_l = 1.8$ MHz . Copper, titanium and silicon

(b) : $\omega_l = 1.8$ MHz and $\omega_l = 6.24$ MHz. Copper

Stability of a magnetically trapped cloud with respect to collisions

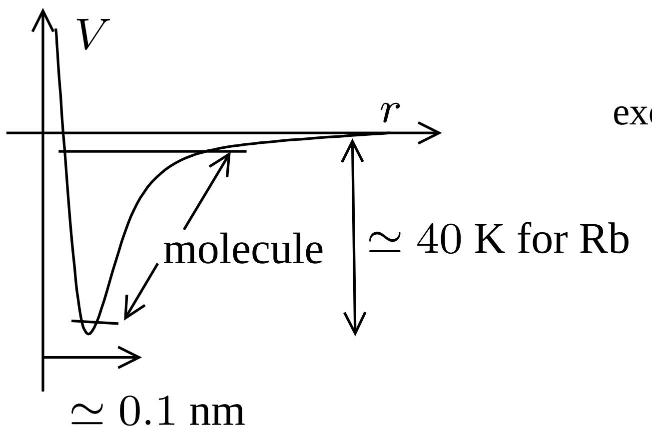
Magnetically trapped cloud : metastable state

- Atoms are not in the lower energy state



Exothermic 2 body collisions to an untrapped state.

- Molecules (and ultimately a solid) should form



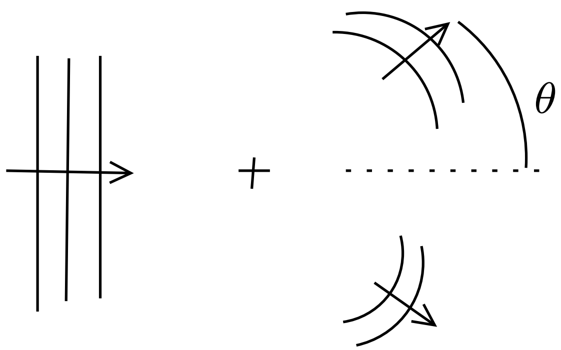
exothermic 3 body collisions

A little of collision theory

2 body elastic collision by a central potential.

$\psi(\mathbf{r_1},\mathbf{r_2}) = \phi(\frac{\mathbf{r_1}+\mathbf{r_2}}{2},\mathbf{r_1}-\mathbf{r_2}) = \phi(R,r)$

Stationnary state : $\phi = e^{iKR}\varphi_{k_{diff}}(r)$, $\frac{\hbar k^2}{m}\varphi_{k_{diff}} = -\frac{\hbar^2}{m}\Delta\varphi_{k_{diff}} + V(\mathbf{r})\varphi_{k_{diff}}$



Asymptotic behavior : $\varphi_{k_{diff}} \longrightarrow e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r}$

$f(\theta)$: scattering amplitude

Flux in the solid angle $d\Omega$:

$\frac{\hbar}{im}\left(\varphi^*\nabla\varphi - \varphi\nabla\varphi^*\right).d\mathbf{S} \simeq \frac{2k}{m}|f(\theta,\phi)|^2 d\Omega$

Collisionnal cross section : $\frac{\Gamma_{out}}{\phi_{in}} = \int\int |f(\theta,\phi)|^2 d\Omega = \sigma$

Low energy behavior

$k \longrightarrow 0 : \begin{array}{l} f(\theta,\phi) = f(k) \\ f(k) \longrightarrow f_0 \end{array} \quad \Rightarrow \sigma \longrightarrow \sigma_0$ independant of k

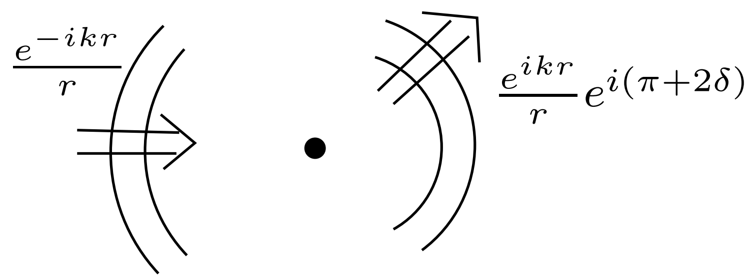
Identical atoms

Symmetrisation of the wave function.

For Bosons : $\sigma = 2\int\int |f(\theta,\phi)|^2 d\Omega$

For fermions : $\sigma \longrightarrow 0$ as $k \longrightarrow 0$

Phase shift and scattering length



Central potential : angular momentum is concerved.

Low energy : f isotrop $\Rightarrow$ only the wave function $L=0$ is scattered.

Only one parameter : phase shift of the scattered wave.

$\left\{ \begin{array}{l} f \simeq \frac{1}{2ik}(e^{2ik}-1) \\ \delta \simeq ka \end{array} \right.$

a : scattering length.

Collisions rates

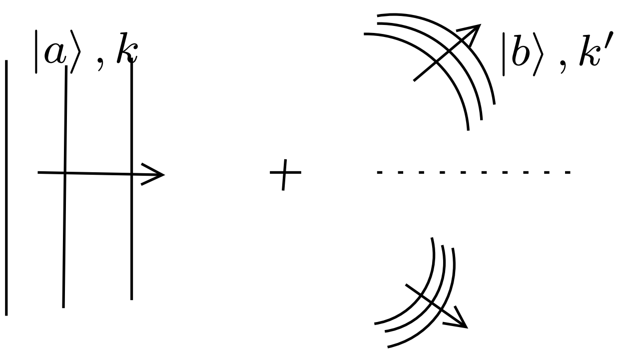
Elastic collision rate

Element of volume d^3r. Number of atoms of velocity between $\mathbf{v}$ and $\mathbf{v} + d^3\mathbf{v}$: $f(\mathbf{v})d^3\mathbf{v}$.

Collision rate for an atom of velocity v_1 $\gamma = \int d^3\mathbf{v}'\sigma|\mathbf{v_1} - \mathbf{v}'|f(\mathbf{v}')$.

Collision rate per atom : $\Gamma_{coll} \simeq n\sigma\sqrt{4k_BT}m$.

2 body inelastic collision.



Output flux : $\Gamma_{out} = \frac{2k'}{m}|f|^2 d\Omega$

Low energy behavior : f independant on incident energy.

Rate of inelsatic collision per atom :

$\Rightarrow \Gamma_{inel} = Gn$ independant on incident enrgy.

Ratio between elastic to inelastic collisions $\frac{\Gamma_{el}}{\Gamma_{inel}} \propto \sqrt{T}$.

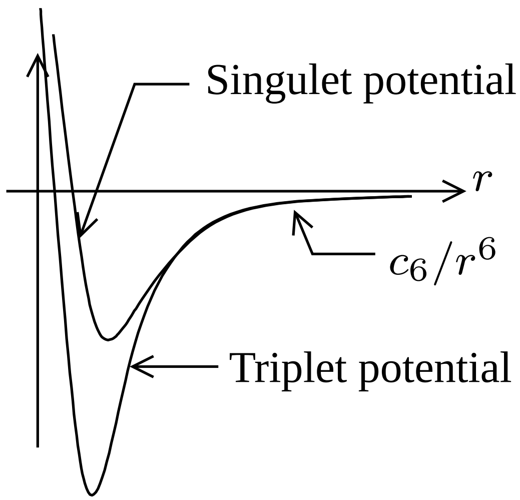
Interaction between two alkali

First approximation : $V = V_{el}$ electrostatic interaction

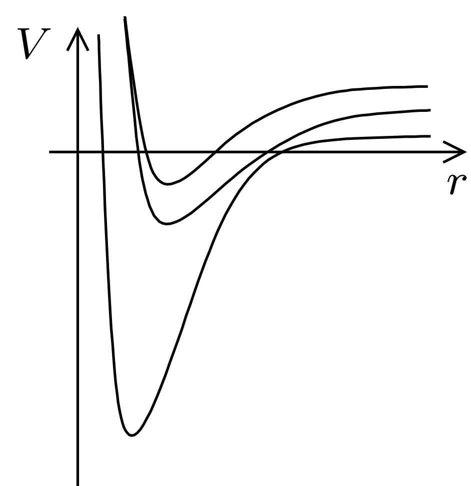
- Central potenial
- Depends on the spin via the symmetrisation of wave function

2 alkaline atoms : 2 spin 1/2.
Totla spin :

$S = 1$: Symmetric by exchange of spins ($|S = 1, m = 1\rangle = |1/2, 1/2\rangle$)
$S = 0$: Antisymmetric by exchange of spins



Singlet state : Wave function symmetric by exchange of electrons
Two electrons in the same lowest energy wave function
Triplet state : Wave function antisymmetric by exchange of electrons
An electron in the first excited state.



Hyperfine interaction.

At large distance, the hyperfine interaction in each atom dominates.
State discribed by the hyperfine level f_1, f_2 of each atom.

Selection rule and stable states

Total spin : $\vec{F} = \mathbf{I_1} + \mathbf{I_2} + \mathbf{S_1} + \mathbf{S_2}$
Potential invariant by a rotation of all the spins.
$\Rightarrow F, m_F$ concerved

$\Rightarrow m_{f_1} + m_{f_2}$ is conserved.

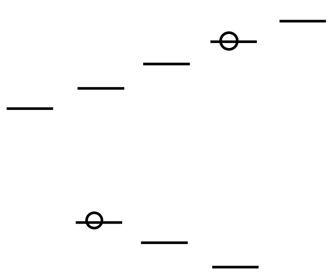
Stable states

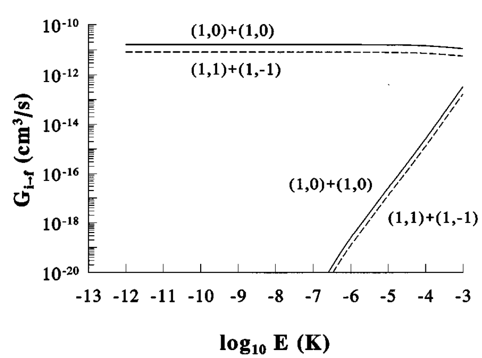
- $|F_{max}, m_{max}\rangle$: Maximum value of m_F. Pure triplet state

- $|F_{min}, m = F_{min}\rangle$: exchange collisions energetically forbidden

Unstable states
Any collision that preserved $m_{f_1} + m_{f_2}$ is possible.

In Na : $|2,1\rangle + |1,-1\rangle \rightarrow |1,0\rangle + |1,0\rangle$





Typical rates : $G \simeq 10^{-11}\text{cm}^3/\text{s}$
$\Rightarrow$ Mixture of states unstable
$\Rightarrow$ Any state different from $|F_{max}, m_{max}\rangle$ and $|F_{min}, m = F_{min}\rangle$ unstable

A special case : Rubidum 87

For Rb : exchange collisions rate very small : $G_{(2,2)+(1,-1)} = 2.3 \times 10^{-14}\mathrm{cm}^3\mathrm{s}^{-1}$.

Very naive explanation

Neglect hyperfine energy during scattering
Initial state :

$$
\begin{aligned}
|f_1, f_2, m_1, m_2\rangle &= \sum_{m_{i_1}, m_{i_2}} c_{m_{i_1}, m_{i_2}} \left| m_{i_1}, m_{i_2}, m_{s_1}, m_{s_2} \right\rangle \\
&= \sum_{m_{i_1}, m_{i_2}} \left| m_{i_1}, m_{i_2} \right\rangle \left(c_{0, m_{i_1}, m_{i_2}} |S=0\rangle + c_{0, m_{i_1}, m_{i_2}} \left| S=1, m_1+m_2-m_{i_1}-m_{i_2} \right\rangle \right)
\end{aligned}
$$

Scattering of $(c_0 |S=0\rangle + c_1 |S=1, m\rangle)$

$$\psi_{scatt} = c_0(e^{ikr} + f_s \frac{e^{ikr}}{r}) |S=0\rangle + c_1(e^{ikr} + f_T \frac{e^{ikr}}{r}) |S=1, m\rangle$$

For Rubidium, $a_T \simeq a_S \Rightarrow f_T \simeq f_T = f$

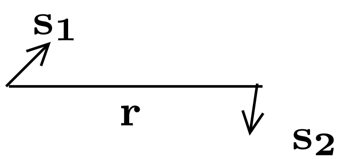
$\longrightarrow$ scattered component : $f \frac{e^{ikr}}{r} (c_0 |S=0\rangle + c_1 |S=1, m\rangle)$

No change of state during scattering

Scattering amplitude is f : independant of internal state.

Spin changing collisions

Interactions between magnetic moment of the electrons

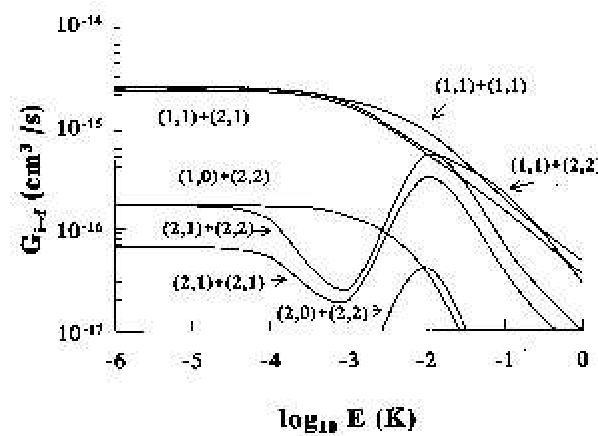


$$V = \frac{\mu_0 \mu_B}{4\pi r^3}(\mathbf{s_1}.\mathbf{s_2} - 3(\mathbf{s_1}.\mathbf{r})(\mathbf{s_1}.\mathbf{r}))$$

V is not conserved by a rotation of the spins only.
$\Rightarrow$ **F** is not conserved.

Transfer of angular momentum from spin degree of freedom to orbital momentum of the relative motion.

Usually very small. For example, in Na, $G \simeq 10^{-15}\mathrm{cm}^3\mathrm{s}^{-1}$.



Expected to decrease for low magnetic field in the lower hyperfine state. The energy realized becomes small

Case of Cesium

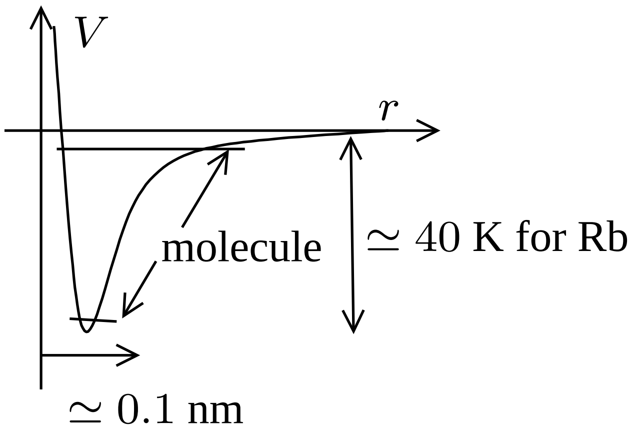
Important inelastic rate. Both for higher hyperfine state and lower hyperfine state. $G \simeq 10^{-12}\mathrm{cm}^3\mathrm{s}^-1.$

Prevent the realization of BEC in magnetic trap for Cs.

The spin-orbit coupling interaction has for Cs a similar role.

3 body losses

Formation of a molecule : 3 atoms required



Exothermic collision :
At + At + At $\longrightarrow$ At_2 + At

Collision rate per atom: $\Gamma_{3b} = Ln^2$.
Measured rate for Rb : $L = 1.8 \times 10^{-29}$ $\mathrm{cm}^6\mathrm{s}^{-1}$
Limit the lifetime of BEC.
Ratio with elastic collision rate :

$$\frac{\Gamma_{3b}}{\Gamma_{el}} \propto \frac{n}{\sqrt{T}}$$

In a harmonic trap,

$$\frac{\Gamma_{3b}}{\Gamma_{el}} \propto \frac{N\omega^3}{T^2}$$

Adiabatic decompression : $T \propto \omega$

$$\Rightarrow \frac{\Gamma_{3b}}{\Gamma_{el}} \propto N\omega$$

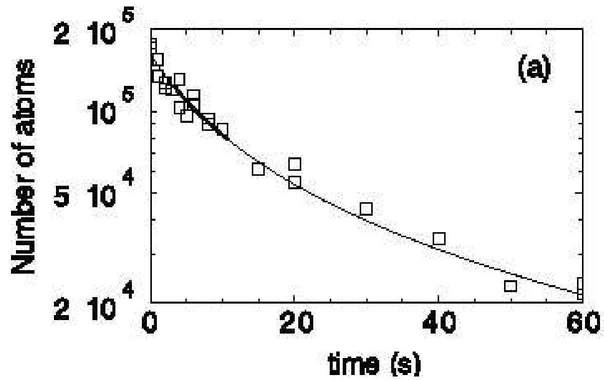
Experimental observations

Evolution of atom numbre :

$$\frac{dN}{dt} = -\Gamma N - G\bar{n}N - L\bar{n^2}N$$

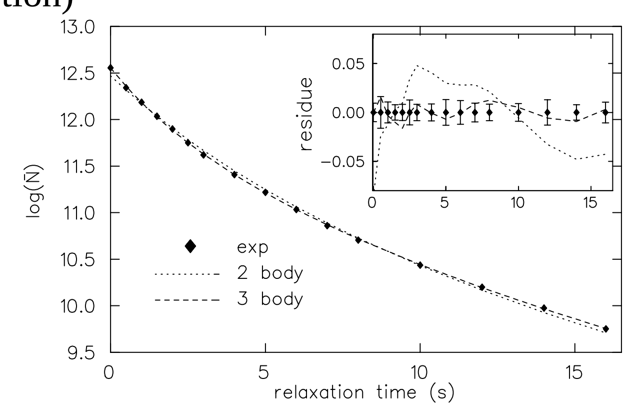
Inelastic process $\longrightarrow$ non exponential decay.

- 2 body spin relaxation losses in magntically trapped Cs



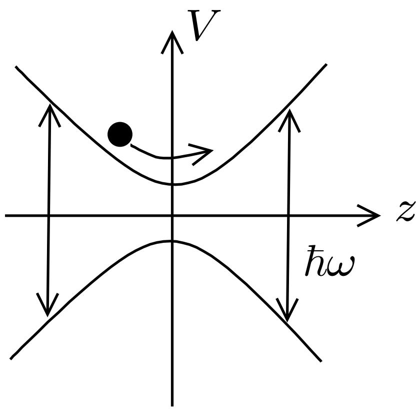
fit : Assume thermal distribution.

- 3 body decay in a Rubidium BEC
Very low rate $\longrightarrow$ high density requiered $\longrightarrow$ BEC used
$\bar{n}$ depends on N in a BEC (mean field interaction)



Use of radio-frequency magnetic fields to decrease potential depth

Spin 1/2



Magnetic field : $B_x \cos(\omega t)$

Can make a transition between $|\uparrow\rangle$ and $|\downarrow\rangle$

Transition resonnant at $\omega_l = \omega$

$\Longrightarrow$ atoms lost when they reach the equipotential $V = \hbar\omega/2$

RF magnetic field : current loop

Inside the vacuum chamber if metallic !

How strong should be the RF field ?

What happens for an atom of spin F>1/2 ?

Stationary states at a given position

Stationary magnetic field : $B_z\mathbf{z^0}$. $\omega_l = 2\mu_B B_z$
RF field : $\mathbf{B}(t) = B_{RF}\cos(\omega t)\mathbf{x^0}$

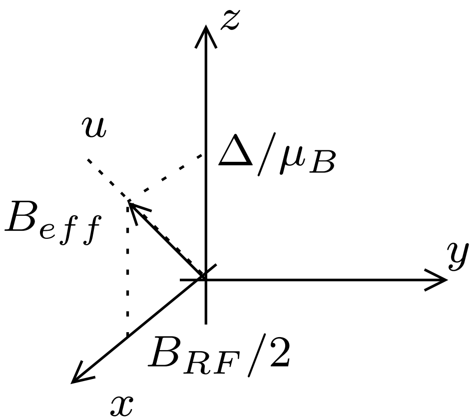
$$H = \frac{\omega_l}{2}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) + \mu_B B_{RF}\cos(\omega t)(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|)$$

Rotating wave approximation
$|\psi\rangle = c_\uparrow e^{-i\omega t/2}|\uparrow\rangle + c_\downarrow e^{i\omega t/2}|\downarrow\rangle$
Evolution :

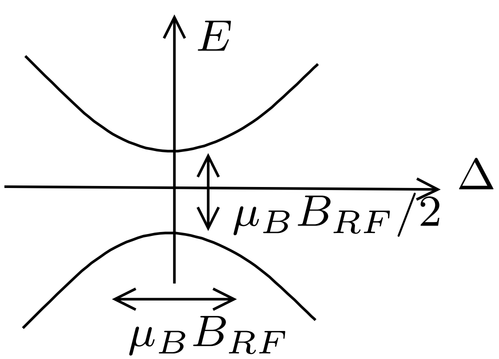
$$\left\{\begin{array}{l} i\frac{d}{dt}c_\uparrow = (\frac{\omega_l}{2} - \frac{\omega}{2})c_\uparrow + \frac{\mu_B B_{RF}}{2}c_\downarrow \\ i\frac{d}{dt}c_\downarrow = (-\frac{\omega_l}{2} + \frac{\omega}{2})c_\downarrow + \frac{\mu_B B_{RF}}{2}c_\uparrow \end{array}\right.$$

$\Delta = \omega_l - \omega, \quad \left|\tilde{\psi}\right\rangle = e^{i\omega S_z t}|\psi\rangle$ evolves with

$$\tilde{H} = \begin{pmatrix} \frac{\Delta}{2} & \frac{\mu_B B_{RF}}{2} \\ \frac{\mu_B B_{RF}}{2} & -\frac{\Delta}{2} \end{pmatrix}$$



Eigenstates : $|\uparrow\rangle_u, |\downarrow\rangle_u$
Energies : $\pm\frac{1}{2}\sqrt{\Delta^2 + (\mu_B B_{RF})^2}$
Case $\Delta = 0$: $|\uparrow\rangle_u = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$



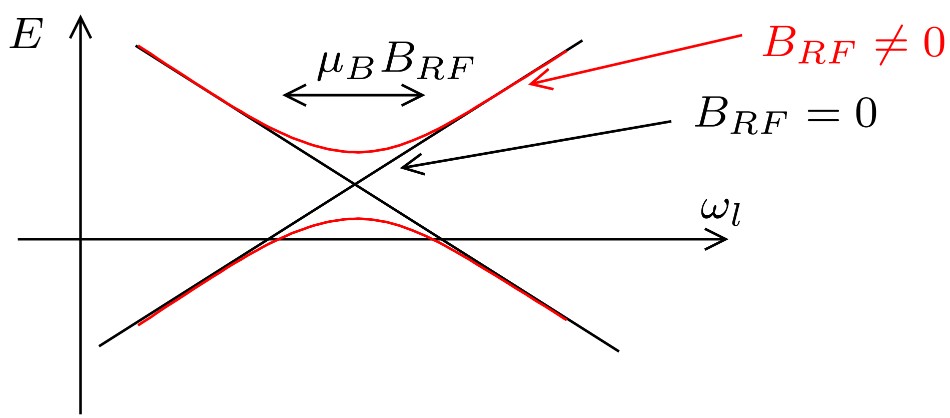
Dressed states picture

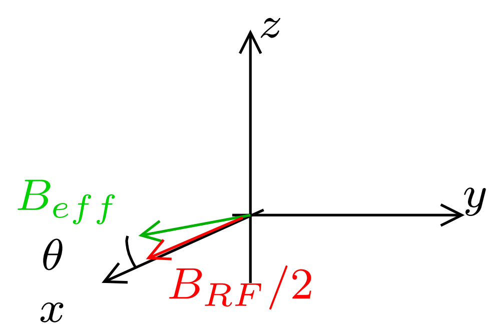
N : number of photons in the RF field

neglecting the state $|N-1,\downarrow\rangle$ equivalent to rotating wave approximation

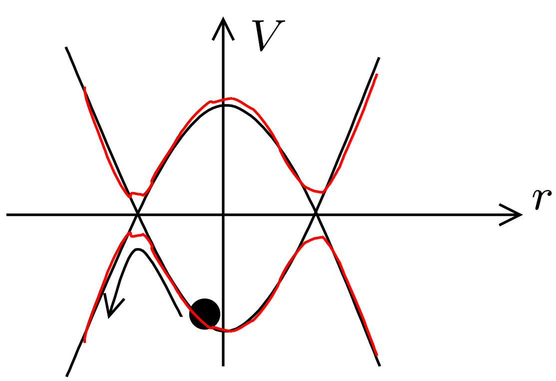
$$|N,\uparrow\rangle \xleftrightarrow{\mu_B B_{RF}/2} |N+1,\downarrow\rangle$$

Energies : $N\hbar\omega + \omega_l/2$ $\qquad$ $(N+1)\hbar\omega - \omega_l/2$





Dressed state potential



Adiabatic following :

$$\dot{\theta} \ll \omega_l = \mu_B B_{RF}/2\hbar$$

$$b'v \ll \frac{\mu_B B_{RF}^2}{\hbar}$$

$$\left\{ \begin{array}{l} b' = 100\mathrm{G/cm} \\ v = 15\mathrm{cm/s} \\ B_{RF} > 20\mathrm{mG} \end{array} \right.$$

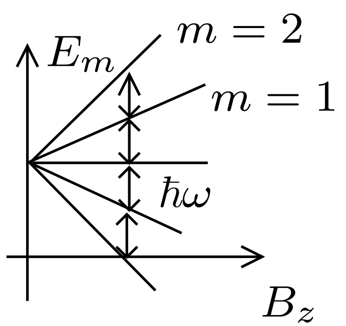
Another point of view : $\Delta T \gg 1/\omega_l$

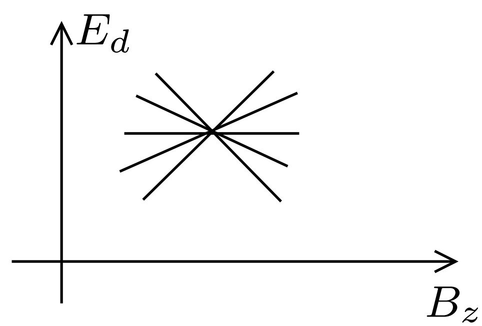
Case of an atom of spin F>1/2

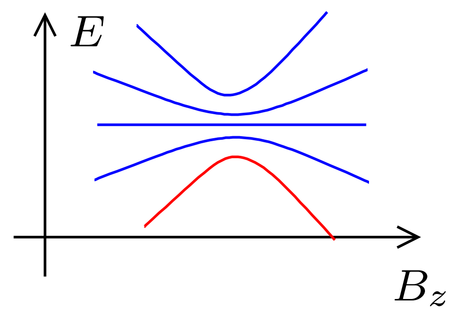
Case F=2

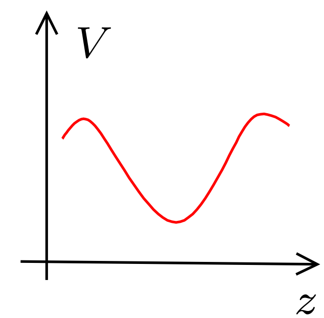
$$B_x \propto F_x = (F_- + F_+)/\sqrt{2}$$

$$|N, m=2\rangle \longleftrightarrow |N+1, m=1\rangle \longleftrightarrow |N+2, m=0\rangle \longleftrightarrow |N+3, m=-1\rangle \longleftrightarrow |N+4, m=-2\rangle$$

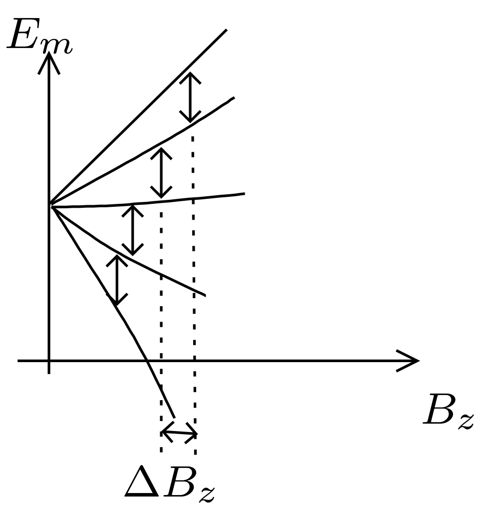


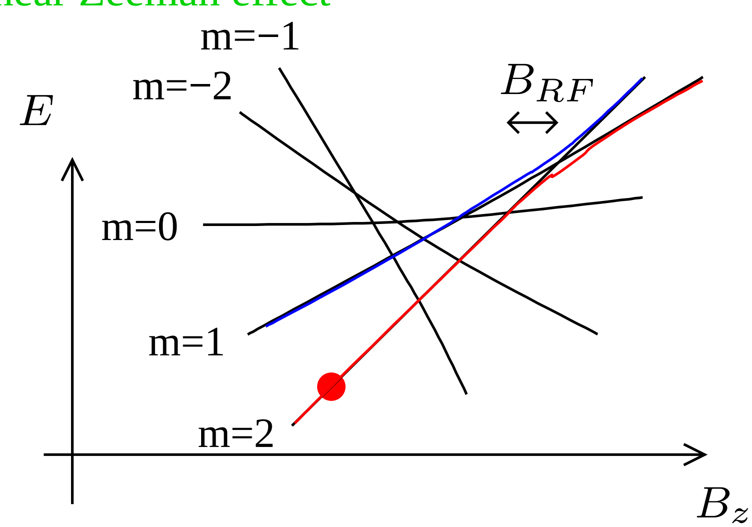






Case of high magnetic fields : non linear Zeeman effect





Atom stays tarpped

Happens when

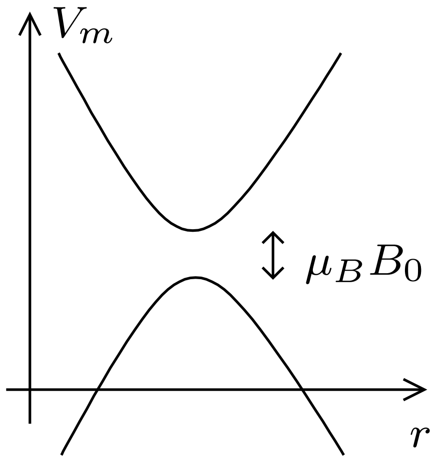
$$\Delta B_z > B_{RF}$$

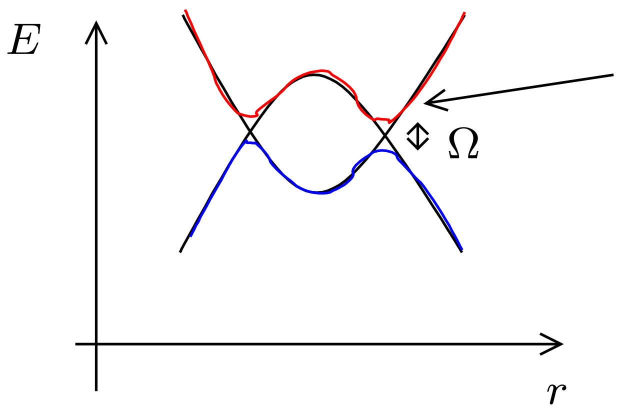
No problem if

$$B_{RF} \gg B_z \frac{\mu_B B_z}{E_{HF}}$$

$\longrightarrow$ Does not work at high fields

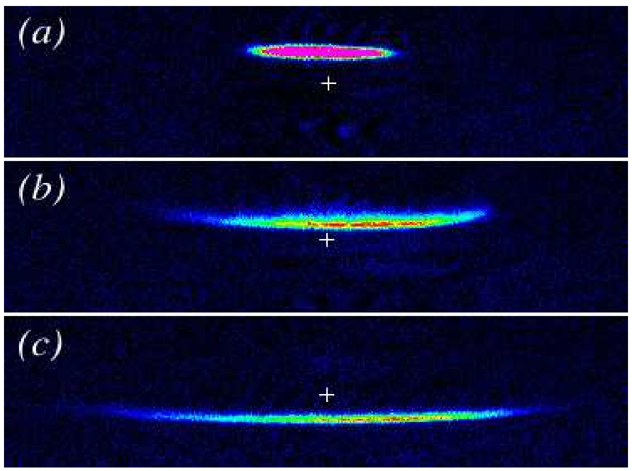
A trap for dressed states





Confi nement $\omega_{osc} = b' \sqrt{2\mu_B / m\hbar B_{RF}}$

losses : same calculation as for normal trap $\rightarrow \Gamma \propto e^{-\frac{\mu_B B_{RF}}{\omega_{osc}}}$



Loading : start from $\omega_{RF} < \mu_B B_0$

Increase ω_{RF}

Effect of gravity : atoms accumlate at the bottom

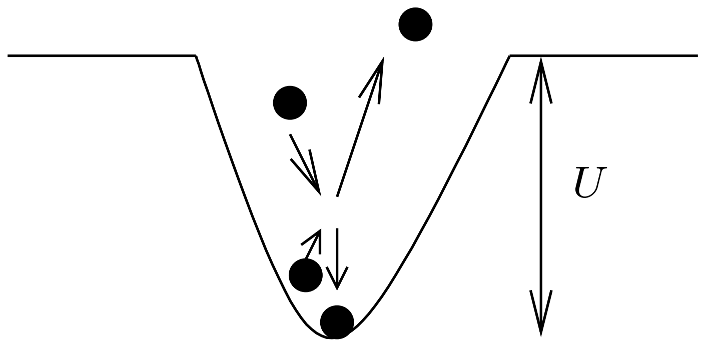
Evaporative cooling

Principle Thanks to elastic collisions, an atom can gain an energy larger than the depth of the trap U. He takes away an energy

$$\Delta E > U > \langle E/N \rangle.$$

Thus

$$\langle E/N \rangle \downarrow \rightarrow T \downarrow .$$



2 conflicting conditions :

- Strong decrease of T per lost atom : U large required
- Time of evaporation smaller than the life time of the cloud : U not too large

Forced evaporative cooling

While T decreases, U is decreased to maintain a high cooling time.
In the following we give a very simple model describing evaporative cooling.

Evaporative cooling : a simple model

We assume :

- **Any atom of energy larger than U leaves the trap immediately.** This will be a good assumption for 3 dimensional evaporation and if $\gamma_{col} \ll \omega_{osc}$.
- **There is a quasi-thermal equilibrium of the cloud in the trap.** Energy distribution is given by the Boltzmann law.
- **The trap is harmonic**

We note $\eta = \frac{U}{k_B}T$. We assume $\eta \gg 1$.
Effect of evaporation only Number of atom decrease :

$$\dot{N} = -\Gamma_{ev} N.$$

Each lost atom carry an energy $E > U$. In average, the energy is $\langle \epsilon \rangle = k_B T(\eta + \kappa)$. κ is of the order of 1. Thus,

$$\dot{E} = -N\Gamma_{ev}(\eta + \kappa) k_B T.$$

$k_B T = E/3N$. Thus, if no other losses,

$$\frac{\dot{T}}{T} = \frac{\dot{N}}{N}\left(\frac{\eta + \kappa}{3} - 1\right) = \alpha \frac{\dot{N}}{N} \rightarrow T \propto N^{\alpha}$$

Scaling lows : $\Gamma_{el} \propto nv \propto N^{1-\alpha}$ $\quad D \propto \frac{n}{T^{3/2}} \propto N^{1-3\alpha}$

If $\alpha > 1$, Γ_{el} increases in time. Runaway evaporation

Role of atom losses due to background collisions ?

Equations in presence of a loss rate

Losses due to background gas :

$$\dot{N}_{bg} = -\Gamma N.$$

Not an energy dependent loss rate. Thus

$$\dot{E}_{bg} = -\Gamma E.$$

We thus have

$$\begin{cases} \dot{N} = -N\Gamma_{ev} - \Gamma N \\ \dot{E} = -\Gamma_{ev} E \frac{\eta+\kappa}{3} - \Gamma E \end{cases}$$

We will consider the quantities n and v which are the peak density and the thermal velocity $v = 2\sqrt{2k_B T/\pi m}$.

$$n \propto NT^{-3/2} \text{ and v} \propto \sqrt{\mathrm{T}}.$$

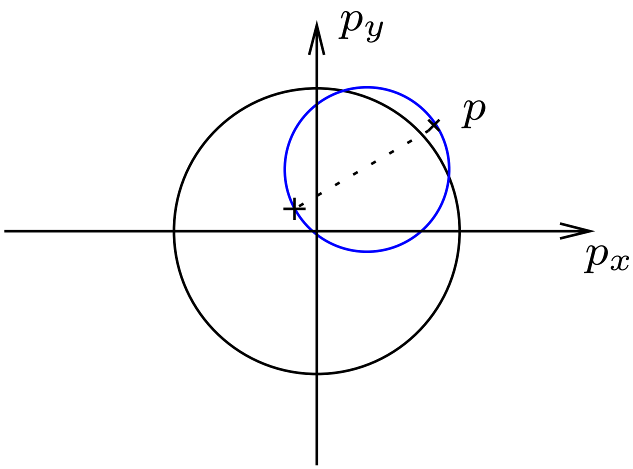
We obtain (using $E = 3Nk_B T$),

$$\begin{cases} \frac{\dot{n}}{n} = \frac{\eta+\kappa-5}{2}\Gamma_{ev} - \Gamma \\ \frac{\dot{v}}{v} = -\Gamma_{ev}\frac{\eta+\kappa-3}{6} \end{cases}$$

Calculation of Γ_{ev}

local loss rate.
Small region of volume δV.
Let us suppose the trap has an infinite depth.
Number of atoms of energy larger than $\eta k_B T$: $\delta N_{exc} \simeq n\delta V \frac{2}{\sqrt{\pi}}\sqrt{\eta}e^{-\eta}$.
Velocity of atoms of energy $\eta k_B T$: $\sqrt{\eta}\sqrt{2k_BT/m} = \sqrt{\pi}\sqrt{\eta}v/2$.
Collision rate for an atom of energy $\eta k_B T$: $\Gamma = n\sigma v\sqrt{\pi}\sqrt{\eta}/2$.

$$\Gamma\delta N_{exc} = n^2\delta V\sigma v\eta e^{-\eta} \simeq \text{loss rate from the high energy tail.}$$



After a collision, the probability that an atom stays in the high energy region is very small

Thermal equilibrium :

$$n^2\delta V\sigma v\eta e^{-\eta} = \delta\Gamma_{evap}.$$

Sum on the volume of the trap
At a position $\mathbf{r}$, $U = U(\mathbf{r})$.
Evaporation parameter : $\eta(\mathbf{r}) = \eta - U(\mathbf{r})$.
Density of atoms : $n(\mathbf{r}) = n_0 e^{-U(\mathbf{r})}$.

$$\delta\Gamma_{evap} \simeq nn_0 e^{-U(\mathbf{r})/k_BT}\delta V\eta e^{-(\eta-U(\mathbf{r}))}\sigma\bar{v}.$$

$\int \delta\Gamma_{evap} d^3\mathbf{r} = N\Gamma_{ev} \Rightarrow$

$$\boxed{\Gamma_{ev} \simeq n_0\sigma v\eta e^{-\eta}.}$$

Runnaway evaporation

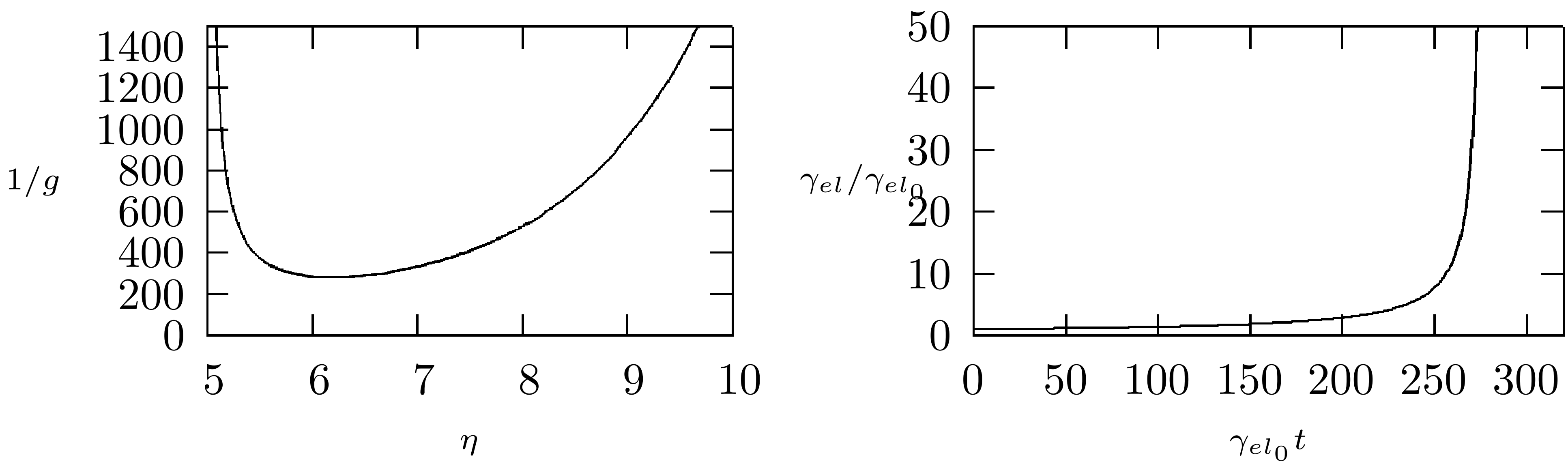
$$\begin{cases} \frac{\dot{n}}{n} = -\Gamma + \frac{\eta-4}{2} n\sigma v \eta e^{-\eta} \\ \frac{\dot{v}}{v} = -\frac{\eta-2}{6} n\sigma v \eta e^{\eta} \end{cases}$$

Solution :

$$nv = n(t=0)v(t=0)\frac{e^{-\Gamma t}}{1 + rg(e^{-\Gamma t} - 1)}$$

$r = \sqrt{2}n(t=0)v(t=0)\sigma/\Gamma = \Gamma_{el}(t=0)/\Gamma$ and $g = \eta e^{-\eta}\frac{\eta-5}{3\sqrt{2}}$.

If $r > 1/g$, divergence : Runnaway evaporation



$\boxed{\Gamma_{el}(t=0)/\Gamma > 300}$

Limit of the model

Limit of the model

- **Beyond the approximation $\eta \gg 1$ for the calculation of Γ_{evap}. Use kinetic theory. (Walravan 1996)**
- **Spilling. Losses of atoms due to decrease of U, even in the absence of collisions.**
- **Computation of κ. Beyond the approximation $\kappa = 1$.**

Presence of 2 body losses

Dipolar spin changing collisions. **Case for Hydrogen and Cesium.**
$\Gamma_{dip} \propto n \longrightarrow \Gamma_{dip}/\Gamma_{el} \propto \frac{1}{\sqrt{T}}$ **increases in time.** **no runaway regime**
Losses more important at the center of the trap where U minimum $\longrightarrow$ evaporative heating

For Rubidium : $\Gamma_{dip} = nG^2 = \Gamma_{el}$ for $T_c = \frac{\pi m G^2}{16\sigma^2} \simeq 5$ pK

Three body losses

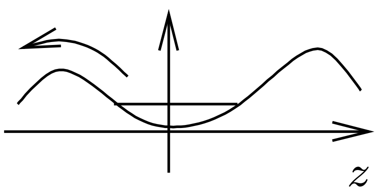
Actual limit for evaporation with alkali.
$\Gamma_{3b} \propto n^2$**. Are more and more important**
$\longrightarrow$ **decompression of the trap at the end of evaporation.**

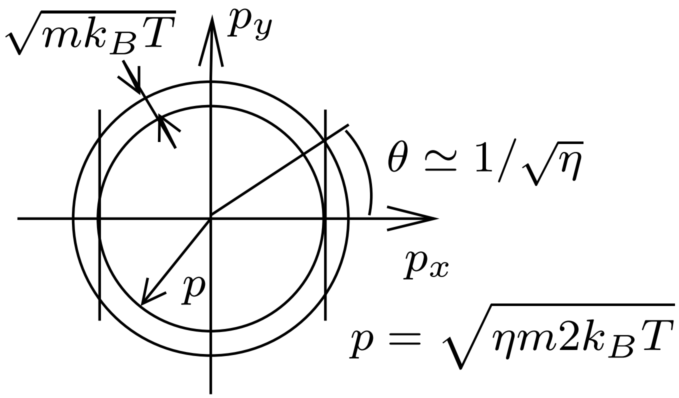
Hydrodynamic regime

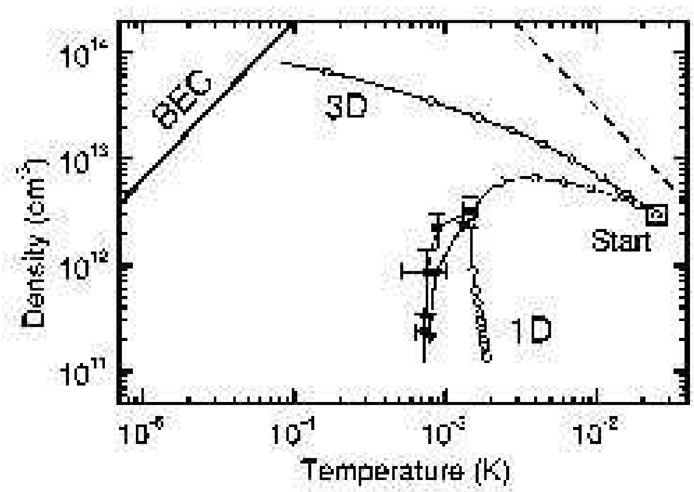
Model : $\Gamma_{el} \ll \omega$, ie $\frac{\sigma}{n} = l \gg L = \sqrt{k_B T}\omega\frac{1}{m}$.
$\longrightarrow$ **Spatial evaporation equivalent to a,n energy cut off.**
Case $\Gamma_{el} \ll \omega$?
Inhomogeneity of temperature. Diffusion equations for heat transport and hydrodynamic equations.

Role of the dimensionnality

Evaporation only in 1D : Case for the first experiment with H
Evaporation efficiency decreased :
$\Gamma_{evap_{1D}} = \Gamma_{evap_{3D}}/(4\eta)$

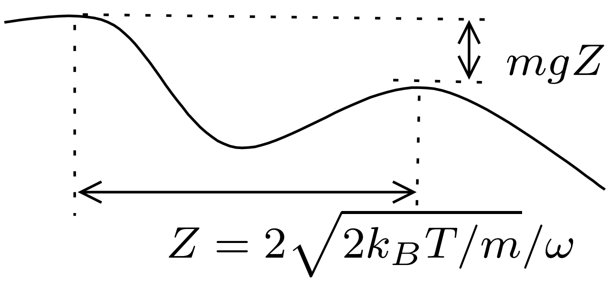


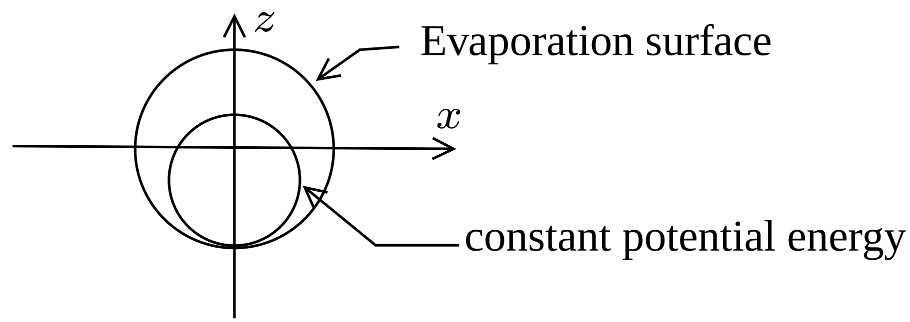




At high temperature, ergodicity $\rightarrow$ evaporation 3D

Case of RF evaporation





Evaporation becomes 1D if $mgZ > k_BT \Rightarrow k_BT < \frac{2\eta m g^2}{\omega^2}$
Case of Rubidium, with $\omega = 50$ **Hz,** $T_{1D} \simeq 20\ \mu$**K.**
Evaporation 1D at the end of cooling

Experimental strategy and results

Strategy :
Optimize the phase space density as a function of time.
Results :
Typically, $D \propto \frac{1}{N^{\gamma}}$**,** $\gamma \simeq 2, .., 3$**.**