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CALCULATION OF MAGNETIC FIELD DEPENDENT TRANSITION PROBABILITY FOR RUBIDIUM ATOMS

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Abstract

In this work, hyperfine structure and magnetic field interaction terms have been taken into account to estimate the transition probability. The mixed state Hamiltonian is considered as a matrix form Hamiltonian, which can be represented as the eigen value problem. The solutions of these eigen value problem are evaluated in terms of energy with respect to magnetic field and eigen function. These solutions are used to estimate the magnetic field dependent transition probability for rubidium atoms.

AMS Subject Classification: 81V80, 81V45

Key Words and Phrases: Magnetic field dependent transition probability, Eigen value problem,rubidium atoms.

1 Introduction

The variation of transition probability with respect to magnetic field has important implication in atomic systems [1],[2]. Alkali atoms are used in many experiments like atomic clock, laser cooling and Bose- Einstein condensation [3]. In our work we focus on the study of rubidium spectra and the individual atomic transition lines of rubidium atoms. In this work we have studied the magnetic field dependent intensity variation of rubidium hyperfine split $D_2(5^2S_{1/2} \rightarrow 5^2P_{3/2})$ lines on the basis of magnetic field dependent transition probability. In order to explain the relative intensity of the spectral lines with respect to magnetic field , we have calculated the transition probability for individual atomic transition and estimated the relative intensity with respect to magnetic field.

2 Theoretical Formalism

The hyperfine structure Hamiltonian with the magnetic field interaction term is [4] written as:

$$H = D_J(\mathbf{I}\cdot\mathbf{J}) + Q_J \frac{3(\mathbf{I}\cdot\mathbf{J})^2 + \frac{3}{2}(\mathbf{I}\cdot\mathbf{J}) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} + H_B \quad (1)$$

where D_J is the magnetic dipole constant and Q_J is the electric quadrupole term. H_B is the magnetic field interaction Hamiltonian. The magnetic field interaction term can be written as;

$$H_B = (\frac{-\mu_B}{\hbar})\mathbf{B}\cdot(\mathbf{L} + g_S\mathbf{S} + g_I\mathbf{I}) \quad (2)$$

Here B is the static external magnetic field, g_S and g_I are the electron and nuclear spin Landé factors, μ_B is the Bohr magneton. Theory of magnetic field dependent transition probability was discussed by Tremblay et al. [5]. In the presence of magnetic field the magnetic sublevels of hyperfine structure are mixed with other levels. The ground state modified wave function can be written as:

$$|\psi(F_g, m_g)\rangle = \sum_{F'_g} A_{F_gF'_g}|F'_g, m_g\rangle \quad (3)$$

Similarly the wave function can be written for the excited state:

$$|\psi(F_e, m_e)\rangle = \sum_{F'_e} A_{F_eF'_e}|F'_e, m_e\rangle \quad (4)$$

where $A_{F_gF'_g}$ and $A_{F_eF'_e}$ are mixing coefficients which depend on the magnetic field. $|F'_g, m_g\rangle$ and $|F'_e, m_e\rangle$ are state vectors corresponding to unperturbed states. F is no longer a good quantum number after the application of magnetic field. Magnetic sublevel quantum numbers (m) can be used to describe this system.The coefficients of $A_{F_gF_g'}$ and $A_{F_eF_e'}$ can be found from the eigen vectors of equation (1) and the eigen values correspond to energy. The spontaneous emission rate S_{eg} is related to the transition probability, which is equivalent to the modified transition probability due to magnetic field.

$$|\langle e|P_r|g\rangle|^2 \approx S_{eg} \approx k^2[\psi(F_e, m_e); \psi(F_g, m_g); r] \tag{5}$$

Modified transition probability can be represented as

$$k[\psi(F_e, m_e); \psi(F_g, m_g); r] = \sum_{F_g'F_e'} A_{F_eF_e'} k(F_e', m_e; F_g', m_g; r) A_{F_gF_g'} \tag{6}$$

where

$$k(F_e', m_e; F_g', m_g; r) = (-1)^{1+I+J_e+F_e+F_g-m_e}\sqrt{2J_e+1}\sqrt{2F_e+1}\sqrt{2F_g+1} \begin{pmatrix} F_e & 1 & F_g \\ -m_e & r & m_g \end{pmatrix} \begin{Bmatrix} F_e & 1 & F_g \\ J_g & I & J_e \end{Bmatrix} \tag{7}$$

The matrices mentioned in the above equation indicate the 3j and 6j symbols.

3 Results and Discussion

The eigen values corresponding to different magnetic fields are computed for rubidium atomic transitions. Figures 1 and 2 show the energy for different rubidium atomic transitions with respect to magnetic field. It is clearly seen from the figure that energy shows nonlinear behavior with the increase in magnetic field. The magnetic field dependent transition probability which is computed for various rubidium atomic transitions are shown in figure 3. From these results we can say that transition probability varies with respect to magnetic field. As the magnetic field increases the variation is very prominent, which is the reason to observe nonlinear effects with the application of magnetic field.

The line intensities vary with respect to magnetic field. In figures 4 and 5, the variation of intensities due to magnetic field are plotted for ^{87}Rb and ^{85}Rb atoms. The individual magnetic sublevel transitions are computed and the resultant values of the intensity are calculated by adding the individual transitions together.

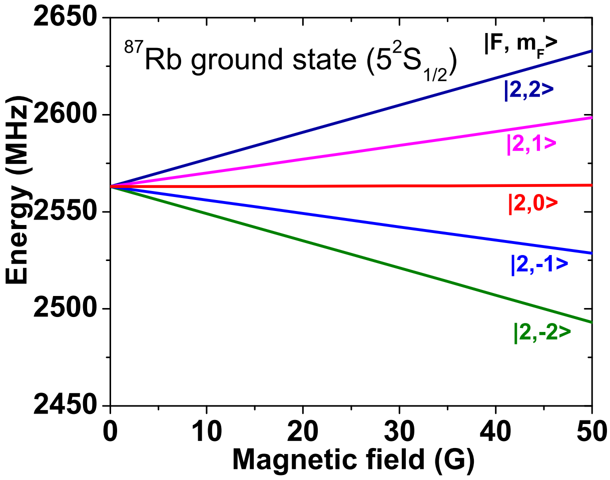

Figure 1: ^{87}Rb ground state ($5^2S_{1/2}$) magnetic sublevels.

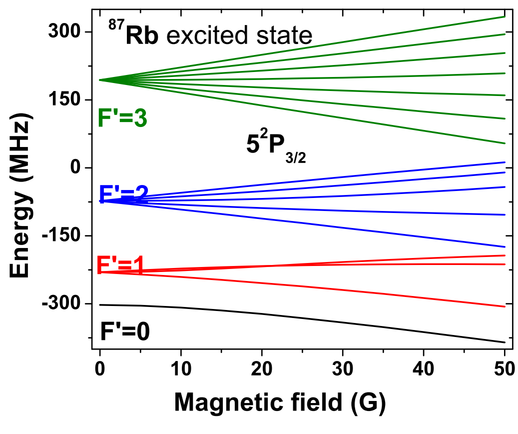

Figure 2: ^{87}Rb excited state ($5^2P_{3/2}$) magnetic sublevels.

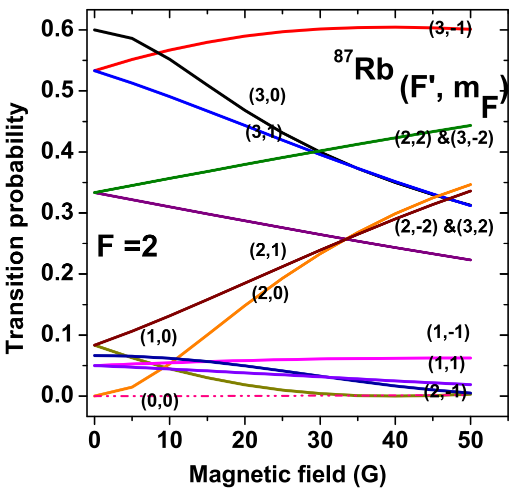

Figure 3: Magnetic field dependent transition probabilities for ^{87}Rb D_2 line $F = 2$ to $F^{'} = 0, 1, 2, 3$ transitions

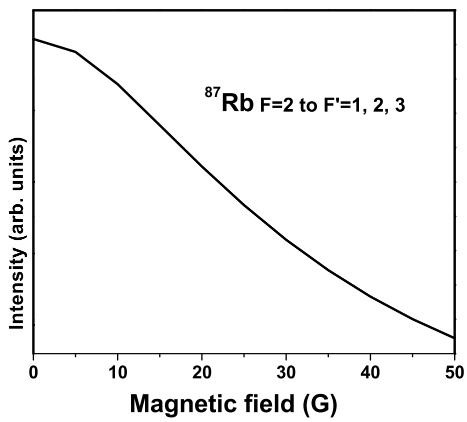

Figure 4: Variation of intensities as a function of magnetic field for ^{87}Rb

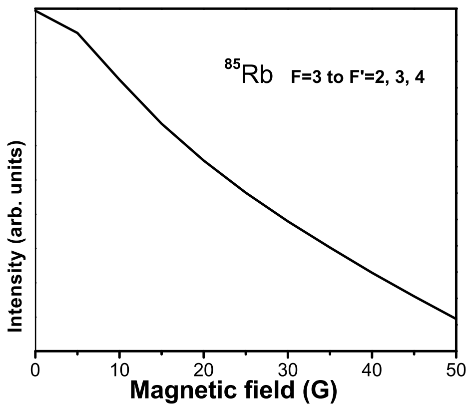

Figure 5: Variation of intensities as a function of magnetic field for ^{85}Rb

4 Summary and Conclusions

It is found that the magnetic field dependent transition probability can be computed by this method. There are nonlinear effects which will be present with the increase in magnetic field. The variation in intensity of the atomic spectral lines with respect to magnetic field can be estimated by this calculation.

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