2 Research Objective

We started preforming an experiment aimed to demonstrate electrostriction in BEC. We stopped the experiment in order to progress with achieving DFG of ^{40}K. I will though describe the theory behind the experiment, since we might continue with it in the future. The theoretical background relevant to this experiment in covered in many textbooks such as [33].

As for today we have several ideas for experiments. In this report I detail on the experiment we intend to preform first, aiming at characterizing dissipation in BEC and Fermi superfluid collisions, giving access to the Fermi superfluid dispersion relation.

2.1 Electrostriction in ^{87}Rb BEC

Coherent light with a frequency corresponding to an atomic transition has a high probability being absorbed by the atoms. Light far-detunned from resonance will hardly be absorbed, but will still be affected by the atoms, while experiencing an effective refractive index n_{eff} of:

$$n_{eff} = 1 + \frac{\Re e(\tilde{\chi})}{2} \tag{16}$$

$\tilde{\chi}$ is the Fourier transform of the atoms electric susceptibility. It is given in general, solving the optical Bloch equations while concentrating on only two atomic levels, by:

$$\tilde{\chi} = i\frac{3}{8\pi^2} n\lambda^3 (\rho_{11}^0 - \rho_{22}^0) \frac{\gamma}{\gamma + i\delta} \tag{17}$$

Here n is the atoms density, λ - the light wavelength, ρ_{11}^0 and ρ_{22}^0 - the populations[7] of the two atomic states denoted $|1\rangle$ and $|2\rangle$, γ - the width of the atomic transition, and δ - the detunning of the light.

For far-detunned light $\tilde{\chi}$ behaves as $\sim \frac{n}{\delta}$, while absorption - as $\sim \frac{n}{\delta^2}$. Since avoiding light absorption restricts δ to be high, one needs a high atom density n in order of being considerably affected by refraction. Atomic BEC provides us with the most dense atomic system, making a suitable candidate for exploring the regime of no absorption but considerable refraction of light by atoms. This will be referred to as the "electrostriction regime" in what follows.

One can observe two processes stemming from the light-atom interaction in the electrostriction regime:

[7] The diagonal elements of the density matrix.

- Lensing: light scatters-off from the atoms (the atoms act as a lens).
- Electrostriction: the atomic cloud deforms due to the momentum it gains from the scattered light.

Lensing is well-known and has applications in phase-contrast non-destructive imaging of atoms (see e.g. [31]). Electrostriction was not measured yet. We intend on measuring it using our ^{87}Rb BEC.

Several theoretical works [34],[35],[36],[37],[38] discuss the dipole interaction between atoms having an induced dipole moment by an external laser beam. The emergent force from this is also termed electrostriction, but should not be confused with the effect I discuss here.

Electrostriction - theory and experiment

Considering a $\hat{y}$ polarized plane wave of wave vector k incident at a BEC of density $n(\vec{r})$ along $\hat{z}$. The incident vector potential:

$$\vec{A} = A_0 \hat{y} e^{i(kz-\omega t)} \tag{18}$$

and a zero scalar potential $\varphi = 0$ (coulomb gauge) passes through the BEC acquiring a phase of:

$$\begin{aligned} \phi &= k \int dz n_{ref} = \phi_0 + \alpha n_{2D} \\ n_{2D} &= \int dz n \\ \alpha &= k \frac{3}{16\pi^2} \lambda^3 \frac{\delta\gamma}{\gamma^2 + \delta^2} \approx \frac{\sigma_0}{4} \frac{\gamma}{\delta} \\ \sigma_0 &= \frac{3\lambda^2}{2\pi} \end{aligned} \tag{19}$$

Here I used (16),(17) and assumed a low intensity and far detunned $|\delta| \gg \gamma$ laser so the population is mainly ground state $\rho_{11}^0 \approx 1$, $\rho_{22}^0 \approx 0$. The wave just after passing the BEC is thus described by a vector potential:

$$\vec{A} = A_0 \hat{y} e^{i(\phi + kz - \omega t)} \tag{20}$$

The corresponding electric and magnetic fields take the form[8]:

$$\begin{aligned} \vec{E} &= -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} = i\omega \vec{A} \\ \vec{B} &= \vec{\nabla} \times \vec{A} = iA_0 (k\hat{z} + \vec{\nabla}_{x,y}\phi) \times \hat{y} e^{i(\phi + kz - \omega t)} \end{aligned} \tag{21}$$

[8] I used the identity $\vec{\nabla} \times (\psi \vec{a}) = \vec{\nabla}\psi \times \vec{a} + \psi \vec{\nabla} \times \vec{a}$ in evaluating the second line in (21). I use $\vec{\nabla}_{x,y} = \hat{x}\partial_x + \hat{y}\partial_y$.

and the poynting vector is[9]:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}^* \approx \frac{1}{\mu_0}\omega|A_0|^2(k\hat{z} + \vec{\nabla}_{x,y}\phi) \tag{22}$$

The difference in the Poynting vector after and before passing the BEC gives the electrostriction force per unit area felt by the atoms[10]:

$$\frac{1}{c}\Delta\vec{S} = \frac{k}{\mu_0}|A_0|^2\vec{\nabla}_{x,y}\phi = \frac{\sigma_0}{2}\frac{\gamma}{\delta}\frac{I}{\omega}\vec{\nabla}_{x,y}n_{2D} \tag{23}$$

where I used the speed of light $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ and the relation between beam intensity I and its electric field $\vec{E}$:

$$I = \frac{c\varepsilon_0}{2}\vec{E}^2 \tag{24}$$

Each atom feels a force of:

$$\begin{aligned} \vec{f}_{es} &= \frac{1}{c}\frac{\Delta\vec{S}}{n_{2D}} = \frac{\sigma_0}{2}\frac{\gamma}{\delta}\frac{I}{\omega}\frac{\vec{\nabla}_{x,y}n_{2D}}{n_{2D}} = -\vec{\nabla}U_{es} \\ U_{es} &= -\frac{\sigma_0}{2}\frac{\gamma}{\delta}\frac{I}{\omega}\ln\left(\frac{n_{2D}}{n_{2D}(\vec{0})}\right) = -\frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\ln\left(\frac{n_{2D}}{n_{2D}(\vec{0})}\right) \end{aligned} \tag{25}$$

where I introduced the electrostriction effective potential U_{es} and used the peak density $n_{2D}(\vec{0})$. Using a Gaussian beam, the atoms will experience also a dipole force:

$$\begin{aligned} \vec{f}_{dipole} &= \frac{3\pi}{2}\frac{\gamma}{\delta}\frac{\vec{\nabla}I}{ck^3} = -\vec{\nabla}U_{dipole} \\ U_{dipole} &= -\frac{\sigma_0}{4}\frac{\gamma}{\delta}\frac{I}{\omega} = -\frac{\hbar\gamma^2}{8\delta}\frac{I}{I_s} \end{aligned} \tag{26}$$

The effect of U_{es} might be observed on a BEC trapped in a FORT by two ways:

[9] I assumed $\vec{\nabla}_{x,y}\phi$ is smaller than $k\hat{z}$ and thus $\vec{B}$ approximately points towards $\hat{z} \times \hat{y}$. With this in hand, $\vec{E} = \alpha\hat{y}$ and $\vec{B} = \vec{\beta} \times \hat{y}$, with $\vec{\beta} = \beta\hat{z}$ makes $\vec{E} \times \vec{B}^* \approx \alpha\beta^*\hat{y} \times (\hat{z} \times \hat{y}) = \alpha\beta^*\hat{z} = \alpha\vec{\beta}^*$.

[10] The force is the time derivative of the momentum gained by the atoms $\vec{F} = \frac{d\vec{p}}{dt}$. The momentum gained by the atoms equals the number of scattered photons times the single photon momentum $p = \hbar k N_\gamma$. The number of scattered photons equals the light energy over the single photon energy $N_\gamma = \frac{E}{\hbar\omega}$. Plugging it all back and using $\omega = ck$, we get $\frac{|\vec{F}|}{\mathtt{area}} = \frac{1}{c}\frac{dE}{dt\cdot\mathtt{area}}$ where the last term is the Poynting vector.

- **Shape change:** Adiabatically ramping up a laser that causes U_{es} allowing the BEC shape to adjust to the joint potential of the FORT and U_{es}. Observing the BEC shape change by absorption imaging after no TOF[11].
- **Parametric excitation:** Shining a laser that causes U_{es} with a periodic intensity at a frequency matching the FORT frequency for some time. Observing heating of the BEC.

An interacting zero-temperature BEC consisting of N atoms of mass m trapped in a harmonic trap of angular frequencies ω_i will have a density of:

$$\begin{aligned} n &= \frac{15}{8\pi}\frac{N}{\prod_i x_{i,c,0}} \max\left(1-\sum_{i=1}^{3}\frac{x_i^2}{x_{i,c,0}^2}, 0\right) \\ x_{i,c,0} &= \sqrt{\frac{2\mu}{m\omega_i^2}} \\ \mu^{\frac{5}{2}} &= \frac{15\hbar^2\sqrt{m}}{2^{\frac{5}{2}}} N_0\omega_1\omega_2\omega_3 a \end{aligned} \tag{27}$$

and line density of:

$$n_{2D} = n_{2D}(\vec{0}) \max\left(1-\frac{x^2}{x_{c,0}^2}-\frac{y^2}{y_{c,0}^2}, 0\right)^{\frac{3}{2}} \tag{28}$$

The corresponding U_{es} is thus:

$$U_{es} = -\frac{3\hbar\gamma^2}{8\delta}\frac{I}{I_s}\ln\left(\max\left(1-\frac{x^2}{x_{c,0}^2}-\frac{y^2}{y_{c,0}^2}, 0\right)\right) \tag{29}$$

Expanding the joined potential around the origin one finds shifted trap angular frequencies:

$$\tilde{\omega}_i = \sqrt{\omega_i^2 + \frac{3\hbar\gamma^2}{8\delta}\frac{I}{I_s}\frac{2}{m\tilde{x}_{i,c,0}^2}} \tag{30}$$

where the BEC size and chemical potential will self-consistently change to:

$$\begin{aligned} \tilde{x}_{i,c,0} &= \sqrt{\frac{2\tilde{\mu}}{m\tilde{\omega}_i^2}} \\ \tilde{\mu}^{\frac{5}{2}} &= \frac{15\hbar^2\sqrt{m}}{2^{\frac{5}{2}}} N_0\tilde{\omega}_1\tilde{\omega}_2\omega_3 a \end{aligned} \tag{31}$$

[11] But after shutting-off the FORT, avoiding AC stark shift in images.

Solving for $\tilde{\omega}_i$, $\tilde{x}_{i,c,0}$ and $\tilde{\mu}$ can be done numerically. Note the changes appear only along the directions perpendicular the the beam propagation.

In order of comparing the effects of electrostriction and dipole force, we consider the intensity of a gaussian beam focused on the atoms with a waist of w_0:

$$I = I_0 e^{-2\frac{x^2+y^2}{w_0^2}} \tag{32}$$

Expanding the joined potential (FORT and dipole) around the origin one finds shifted trap angular frequencies:

$$\breve{\omega}_i = \sqrt{\omega_i^2 + \frac{1}{m w_0^2}\frac{\hbar\gamma^2}{2\delta}\frac{I_0}{I_s}} \tag{33}$$

We want the change in trap frequency due to the dipole force be small compared to the bare trap frequencies. This restricts the beam waist making the beam homogenous enough suppressing the dipole force. I note the dipole and electrostriction forces have the same sign and scale the same with beam intensity and detunning, making the differentiation between them possible only by the gradients of beam intensity and atom density.

One also needs to make sure that the atoms do not scatter photons from the beam during illumination. Adiabatic illumination occurs for beam ramping during a time of $\tau = \frac{2\pi}{\min_i(\omega_i)}$. Durring this time the atoms scatter an average number of photons given by (see (4)):

$$\tau\Gamma_{Scat} = -\frac{\gamma}{\hbar\delta}U_{dipole}\tau = \frac{\gamma^3}{8\delta^2}\frac{I}{I_s}\tau \tag{34}$$

We thus need $\tau\Gamma_{Scat} \ll 1$ in order of avoiding considerable photon scattering that can heat and deform the BEC.

Considering a BEC of $N = 3 \cdot 10^5$ ^{87}Rb atoms at a trap frequency of $\omega = 2\pi \times 50Hz$, shined by a beam detunned from the D2 line, a numerical analysis shows it is preferable using the most powerful, detunned and large waist beam possible. The requirement for no light scattering considerably restricts the ability to observe electrostriction. This makes the choice of beam intensity a function of the detunning. Requiring each atom will scatter 0.1 photons during the measurement, the optimal intensity and the corresponding BEC size change follow the graphs in Figures 26 and 27.

Our technical challenge is to shine the atoms with a powerful, detunned and large waist beam. We intend to use a $3mW$ beam (maximal power we achieve today with our offset locked imaging laser) detunned by $\delta =$

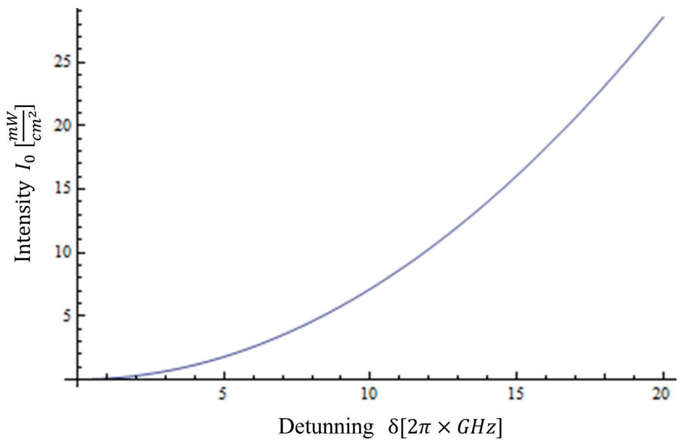

Figure 26: Optimal intensity for electrostriction beam as function of detunning.

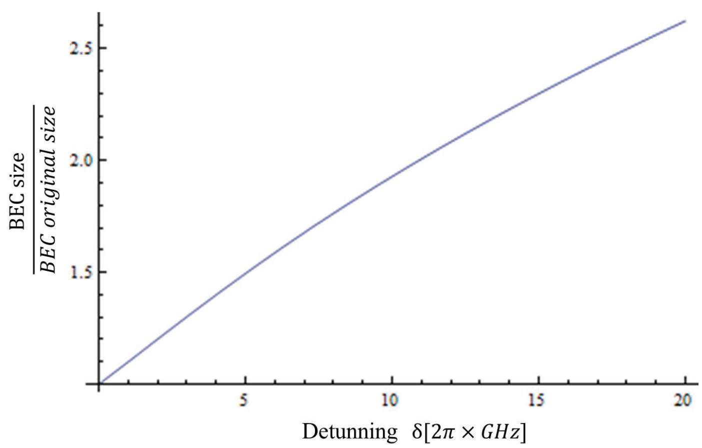

Figure 27: BEC size change due to electrostriction for optimal intensity as function of detunning.

$-2\pi \times 20GHz$ (maximal detunning we can measure with our spectrum analyzer) with a waist of $w_0 \approx 2mm$ (maximal waist allowed by maximal power and required intensity). At this working point we expect to observe a BEC growing to more than 2.5 times its original size along the directions perpendicular to the electrostriction beam (from $10\mu m$ to $27\mu m$). We expect each atom to scatter 0.1 photons durring the measurement[12]. This is expected to push 10% of the atoms in the beam direction at the recoil velocity $v_{recoil} = 6\frac{mm}{s}$, which will travel a considerable amount of $\tau v_{recoil} = 120\mu m$ till imaging. The trap angular frequencies are expected to change by a negligible factor of $2 \cdot 10^{-4}$ due to the dipole force.

The atoms that scatter photons might make heating measurements harder to interpret, while not affecting BEC reshaping (since these effects act on perpendicular axes), making us prefer BEC shape changes rather than parametric excitation.

Our strategy for suppressing the dipole force effect is based on using homogenous beams. This might be challenging tough, since speckles might be introduces to the beam by scattering (from optics scratches, dust, etc.), spoiling homogeneity. We thus need to use a beam coming out of a single mode fiber and passing a minimal number of optical elements. We should verify the beam is homogenous enough by beam profiling. Speckles spacially smaller than the condensate size will cause a random noise on the atomic density, which cannot mimic the effect of electrostriction. Speckles larger than the condensate will not cause much inhomogeneity. Our main concern is speckles similar in size to the condensate. Illuminating with an electrostriction beam with a speckle at the middle of the condensate will cause a trap frequency shift according to (see (30) and (33)):

$$\tilde{\omega}_i^2 = \omega_i^2 + \frac{3\hbar\gamma^2}{8\delta}\frac{I}{I_s}\frac{2}{m\tilde{x}_{i,c,0}^2} + \frac{1}{mw_0^2}\frac{\hbar\gamma^2}{2\delta}\frac{\Delta I}{I_s} \tag{35}$$

where the speckle is of intensity ΔI and size w_0. We thus expect our error of measuring electrostriction to scale as $\frac{\Delta I}{I_0}$, and should keep this ratio low.

Further research

Demonstrating electrostriction using cold atoms opens several interesting research routs:

- The electrostriction force can be tuned to be strongly attractive (using higher intensity and red detunned laser), and might cause the BEC to collapse.

[12] Heating will occur only if we wait between the electrostriction pulse and the BEC imaging. We do not intend to wait though. If we do wait, the cloud will be heated by a small amount of 10% of the recoil temperature: $0.1T_{recoil} = 36nK$.

- It might also cause self-trapping of the BEC (in directions transverse to the electrostriction beam), i.e trapping by the electrostriction force without a FORT.

- Electrostriction causes the BEC to interact with itself. Such self-interaction might introduce non-linear dynamics to the system. We might thus manage to probe non-linear interesting effects such as duffing, pulling or auto-resonance.

In order of looking for collapse and self-trapping I substitute $\tilde{x}_{i,c,0}$ and $\tilde{\mu}$ from the equilibrium equations (30),(31) getting an equation for $\tilde{\omega}_i$:

$$\tilde{\omega}_i^2 = \omega_i^2 + \frac{3\hbar\gamma^2}{8\delta}\frac{I}{I_s}\frac{\tilde{\omega}_i^2}{\left(\frac{15\hbar^2\sqrt{m}}{2^{\frac{5}{2}}}N_0 a\right)^{\frac{2}{5}}\tilde{\omega}_1^{\frac{2}{5}}\tilde{\omega}_2^{\frac{2}{5}}\omega_3^{\frac{2}{5}}} \tag{36}$$

which in the radially symmetric FORT case $\omega_1 = \omega_2 = \omega_t$ becomes:

$$\tilde{\omega}^2 = \omega_t^2 + \frac{3\hbar\gamma^2}{8\delta}\frac{I}{I_s}\frac{\left(\tilde{\omega}^2\right)^{\frac{3}{5}}}{\left(\frac{15\hbar^2\sqrt{m}}{2^{\frac{5}{2}}}N_0 a\right)^{\frac{2}{5}}\omega_3^{\frac{2}{5}}} \tag{37}$$

Collapse will occur if by increasing I the solution of (37) will tend to infinity till, for some value of I, it will not exist anymore. One sees though this is not the case: the right hand side scales as $\left(\tilde{\omega}^2\right)^{\frac{3}{5}}$ and will always reach the left hand side which scales as $\tilde{\omega}^2$ (for red detunned laser). I thus expect no BEC collapse due to electrostriction attraction. This stems from the inter-atomic repulsion and from the longitudinal direction being free of electrostriction force: the atoms will initially collapse in the transverse direction, but then will be repelled and spread along the longitudinal direction, reducing the local density and thus the electrostriction force. It might be that by illuminating the atoms with two perpendicular electrostriction beams, electrostriction might cause a collapse when it gets strong enough to overcome the inter-atomic repulsion. I did not analyze this option yet.

Self-trapping will occur if solutions for (37) exist even for $\omega_t = 0$, which is indeed the case by an argument similar to that given for the absence of BEC collapse. In order of observing self trapping, the electrostriction potential should have a large enough trapping frequency to support a reasonable condensate size. For the possible working point discussed above, a BEC in a longitudinal one dimensional trap with $\omega_3 = 2\pi 50Hz$ will be trapped by electrostriction also along the transverse directions with $\tilde{\omega} = 530Hz$. I note that without the longitudinal trapping, the Thomas-Fermi approximation fails and one cannot use (31). In addition, our treatment of the BEC as a thin lens also becomes invalid. This prevents me from predicting if one can trap atoms in two dimensions by electrostriction without one dimensional confinement. This can be examined in experiment though.

Non-linear dynamics seems to stem from electrostriction, but is still under our theoretical investigation.

Stuff to put in order: The electrostriction force acts perpendicular to the beam inducing it. A small periodic modulation of the beam intensity will thus cause a BEC density oscillation along the two directions perpendicular to the electrostriction beam. The steady state of such oscillation in an isotropic harmonic trap will take the form:

$$\begin{aligned} n(\vec{r},t) &= n(x(1+\epsilon\sin(\omega t+\phi_0)), y(1+\epsilon\sin(\omega t+\phi_0)), z, t=0) = \\ &= f(x^2(1+\epsilon\sin(\omega t+\phi_0))^2 + y^2(1+\epsilon\sin(\omega t+\phi_0))^2 + z^2) \approx \\ &= f(r^2) + 2\epsilon(x^2+y^2)\sin(\omega t+\phi_0) f'(r^2) + \mathcal{O}(\epsilon^2) = \\ &= f(r^2) + 2\epsilon r^2\sin^2\theta\sin(\omega t+\phi_0) f'(r^2) + \mathcal{O}(\epsilon^2) = \\ r^2\sin^2\theta &= \sqrt{\pi}\frac{4}{3} r^2 Y_0^0(\theta,\varphi) - \frac{4}{3}\sqrt{\frac{\pi}{5}} r^2 Y_0^2(\theta,\varphi) \end{aligned} \tag{38}$$

n is the atomic density, ϵ parameterizes the small oscillation amplitude along the radial direction, and ω - the oscillation angular frequency. Writing the density in this way shows the oscillation is decomposed of two BEC modes of oscillation corresponding to $n=1, l=0, m=0$ and $n=0, l=2, m=0$. These modes have angular frequencies $\sqrt{5}\omega_0$ and $\sqrt{2}\omega_0$ respectively [39], where ω_0 is the trap angular frequency. For a driving angular frequency equal to one of these modes we expect a resonantly increased oscillation amplitude, which will translate to increased atom heating after letting the oscillations decay to heat. We thus expect parametric excitation to show two peaks of heating at the angular frequencies mentioned. I note this result is modified for a BEC non-isotropic trap or for large modulations of the electrostriction beam intensity.

Another **wrong** derivation of the electrostriction force: Consider first a single atom with electric polarizability α in the presence of an applied electric field $\vec{E}$. The atom is induced an electric dipole moment $\vec{P} = \alpha\vec{E}$ which has an interaction energy of $U = -\vec{P}\cdot\vec{E} = -\alpha\vec{E}^2(\vec{r})$, when the atom is positioned at $\vec{r}$. If we consider $\vec{E}$ as given externally but let the dipole move in space, we have only the position $\vec{r}$ and momentum $\vec{q}$ as a dynamical coordinate in the Hamiltonian: $H[\vec{r}(t),\vec{q}(t)] = \frac{q^2}{2m} - \alpha\vec{E}^2(\vec{r})$, where m is the atom's mass. Hamilton equation thus reads:

$$
\begin{aligned}
\frac{\partial \vec{r}}{\partial t} &= \frac{\partial H}{\partial \vec{q}} = \frac{\vec{q}}{m} \\
\frac{\partial \vec{q}}{\partial t} &= -\frac{\partial H}{\partial \vec{r}} = \alpha \vec{\nabla} \vec{E}^2
\end{aligned}
\tag{39}
$$

which gives the dipole force equation: $\alpha \vec{\nabla} \vec{E}^2(\vec{r}) = m \frac{d^2 \vec{r}}{dt^2}$

Now lets consider a cloud of atoms rather than a single one.In the presence of an applied electric field $\vec{E}$, an atom acquires an electric dipole moment $\vec{P} = \alpha \vec{E}$, where the electric polarizability α is related to electric susceptibility χ through the free space electric permittivity ε_0: $\alpha = \varepsilon_0 \chi$. The energy density of the atomic dipoles in the applied electric field is $u = \frac{1}{2} \vec{E} \cdot \vec{D}$, where $\vec{D} = \varepsilon_0 \vec{E} + \vec{p}$ is the electric displacement field, and $\vec{p} = n\vec{P}$ - the dipole density of atoms with density n. Using the above relations we write the energy density as:

$$
u = \frac{1}{2} \varepsilon_0 \vec{E}^2 + \frac{1}{2} \varepsilon_0 n \chi \vec{E}^2 \tag{40}
$$

The first term is the bare electric field energy, while the second is the interaction energy between the electric field and the induced atomic dipoles. We consider $\vec{E}$ as given externally but let the dipoles move in space, thus we have only the mass density $\rho(\vec{r}) = mn(\vec{r})$ and the velocity potential[13] $\varphi(\vec{r})$ as dynamical fields in the Hamiltonian: $H[\rho(\vec{r},t), \varphi(\vec{r},t)] = \int d^3\vec{r} \left(\frac{\rho(\vec{\nabla}\varphi)^2}{2} - \frac{1}{2} \varepsilon_0 \frac{\rho}{m} \chi \vec{E}^2 \right)$. Hamilton equations thus read:

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= \frac{\delta H}{\delta \varphi} = -\vec{\nabla}(\rho \vec{\nabla} \varphi) = -\vec{\nabla}(\rho \vec{v}) \\
\frac{\partial \varphi}{\partial t} &= -\frac{\delta H}{\delta \rho} = -\frac{1}{2}(\partial_i \varphi)^2 + \frac{1}{2m} \varepsilon_0 \chi \vec{E}^2 = -\frac{1}{2} v^2 + \frac{1}{2m} \varepsilon_0 \chi \vec{E}^2
\end{aligned}
\tag{41}
$$

The first equation is the continuity equation, while the second one can be written as the Euler equation of a fluid flow in the dipole force:

$$
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = \frac{1}{2m} \varepsilon_0 \chi \vec{\nabla} \vec{E}^2 \tag{42}
$$

A wrong thing to do is to use the density **field** $n(\vec{r})$ but forget about **field** theory and just claim the force is:

$$
\vec{F} = -\vec{\nabla} U = \frac{1}{2} \varepsilon_0 (\vec{\nabla} n) \chi \vec{E}^2 + \frac{1}{2} \varepsilon_0 n \chi \vec{\nabla} E^2 \tag{43}
$$

[13] The velocity field $\vec{v}$ is given by $\vec{v} = \vec{\nabla} \varphi$.

One can e.g. do this to gravity $u = ngz$ and claim there's a force component proportional to $\vec{\nabla}n$... Very surprisingly though, sticking with the above wrong derivation gives a result almost identical to the electrostriction force expression up to the fact it includes $\vec{\nabla}_{x,y,z}$ rather than $\vec{\nabla}_{x,y}$ (the force here has a $\hat{z}$ component, while electrostriction doesn't). There's no reason for this treatment, which assumes a given $\vec{E}$, to know about the change of $\vec{E}$ and its back-action on the particles, but somehow it seems to do.

Conspiracy: maybe there's a deep theoretical reason this treatment gives the back-action, and the force along $\hat{z}$ is the back-action of the light reflected back from the BEC?

Equivalence between electrostriction and interaction tunning in 2D: In 3D electrostriction cannot be described by a potential, since it acts only in directions transverse to the electrostriction beam, while the density changes also longitudinally. Tightly confining an atomic cloud in $\hat{z}$ will allow describing electrostriction by a potential:

$$U_{es} = -\frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\ln\left(\frac{n}{n(\vec{0})}\right) \tag{44}$$

in the 2D transverse plane. The atoms dynamics is thus described by the Gross-Pitaevski equation (GPE) including electrostriction and trapping:

$$i\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar}{2m}\nabla^2 + g|\psi|^2 + V + U_{es}\right)\psi \tag{45}$$

The dipole force is absent, since the electrostriction beam is a plane wave having no intensity gradient. Dealing for simplicity with BEC in a box potential $V = 0$, the atomic density $n = |\psi|^2$ is expected to be homogenous away from the trap boundaries. It is expected to stay homogenous also after applying an electrostriction beam, since U_{es} is constant in space if n is so. Any deformation of the density n will cause though an electrostriction force. As long as the density deformations $\delta n = n - n(\vec{0})$ are small compared to the density itself $\delta n \ll n(\vec{0})$, one can approximate the electrostriction potential:

$$U_{es} = -\frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\frac{\delta n}{n(\vec{0})} + \mathcal{O}\left(\frac{\delta n}{n(\vec{0})}\right)^2 \tag{46}$$

Plugging this in the GPE we get:

$$\begin{aligned} i\frac{\partial\psi}{\partial t} &= \left(-\frac{\hbar}{2m}\nabla^2 + gn(\vec{0}) + \tilde{g}\delta n\right)\psi \\ \tilde{g} &= g - \frac{\sigma_0}{2}\frac{\gamma}{\delta}\frac{I}{\omega}\frac{1}{n(\vec{0})} = g - \frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\frac{1}{n(\vec{0})} \end{aligned} \tag{47}$$

This has the form of a BEC in a box with a modified interaction strength parameter $\tilde{g}$ rather than g. As long as the small density change approximation holds, electrostriction just modifies the BEC interaction strength, mimicking the effect of Feshbach resonance in 2D. The background scattering length of ^{87}Rb atoms is $a_{bg} = 100a_0$. A $3mW$ electrostriction beam $2mm$ in radius, $\delta = 2\pi 20GHz$ detunned from the ^{87}Rb D2 line, will modify the scattering length by $\triangle a = -\frac{m}{4\pi\hbar^2}\frac{\sigma_0}{2}\frac{\gamma}{\delta}\frac{I}{\omega}\frac{1}{n(\vec{0})} = \pm 570a_0$, where the sign depends on the detunning sign. Note that the electrostriction can cancel the interatomic interaction and also change its sign making it attractive.

Accepting the above result one can reconsider all results obtained from the GPE with $\tilde{g}$ instead of g, and get the BEC modified behavior due to electrostriction. This includes modification of:

- The BEC excitation spectrum in the bulk (Bogoliubuv spectrum) and specifically the superfluid critical velocity. The effect can be enhanced $\triangle a > 0$, decreased $0 > \triangle a > -a_{bg}$ or canceled $\triangle a = -a_{bg}$.

- Sound waves propagation. Sound can be made faster $\triangle a > 0$, slower $0 > \triangle a > -a_{bg}$ or absent $\triangle a = -a_{bg}$.

- BEC stability. For $\triangle a < -a_{bg}$ a BEC in a box becomes attractive and thus unstable. A collapse will occur, but will end up with some pattern formation rather than a total collapse - see details below.

In cases where the local density approximation (LDA) can be applied to the above theoretical result, electrostriction can also modify:

- Time of flight anisotropic expansion.

- The BEC excitation spectrum on the surface (harmonically trapped BEC eigenmodes).

- Solitons shape, size and propagation velocity.

- Vortices shape and size.

These modifications should also occur when the LDA is invalid, but will deserve a different theoretical analysis - some of which I analyze in what follows. I note:

- Electrostriction is accompanied with light scattering. This might make some of the above effects impractical to measure.

- Electrostriction changes the interaction on the $n^{-1/3}$ length scale and doesn't affect collisional cross-sections, which occur on the atomic interaction scale. Specifically, thermalization will not be effected by electrostriction.

- The 2D treatment above is valid for 1D as a special case.

Grey and dark solitons: Considering a BEC tightly confined in two transverse directions, which moves at constant velocity v along the longitudinal direction z without change in shape, one can rewrite the GPE in dimensionless parameters as in [40] (section 5.5 about solitons):

$$\begin{aligned}
&2iU\frac{df}{d\zeta} = \frac{d^2f}{d\zeta^2} + f\left(1 - |f|^2 - A\ln\left(\frac{|f|^2}{|f(\vec{0})|^2}\right)\right) \\
&\psi = \sqrt{n}fe^{-i\mu t/\hbar} \\
&\zeta = \frac{z - vt}{\xi} \\
&U = \frac{mv\xi}{\hbar} \\
&\xi = \frac{\hbar}{\sqrt{2mgn}} \\
&A = \frac{1}{gn}\frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}
\end{aligned} \tag{48}$$

Multiplying (48) by f^* and subtracting the obtained equation from its complex conjugate gives:

$$2iU\frac{d}{d\zeta}|f|^2 = 2iU\left(\frac{df}{d\zeta}f^* + f\frac{df^*}{d\zeta}\right) = \frac{d^2f}{d\zeta^2}f^* - \frac{d^2f^*}{d\zeta^2}f = \frac{d}{d\zeta}\left(f^*\frac{df}{d\zeta} - f\frac{df^*}{d\zeta}\right) \tag{49}$$

Integrating this equation over ζ using the boundary conditions:

$$\begin{aligned}
&\lim_{\zeta\to\pm\infty}|f| = 1 \\
&\lim_{\zeta\to\pm\infty}\frac{df}{d\zeta} = 0
\end{aligned} \tag{50}$$

one gets:

$$2iU(1 - |f|^2) + f^*\frac{df}{d\zeta} - f\frac{df^*}{d\zeta} = 0 \tag{51}$$

The imaginary part of (48) is:

$$2U\frac{df_1}{d\zeta} = \frac{d^2 f_2}{d\zeta^2} + f_2\left(1 - f_1^2 - f_2^2 - A\ln\left(\frac{f_1^2+f_2^2}{f_1(\vec{0})^2+f_2(\vec{0})^2}\right)\right)$$
$$f = f_1 + if_2 \tag{52}$$

Now I an stuck. Without the electrostriction term, equations (51) and (52) take the same form when looking for $f_2 = const$ solutions.The electrostriction term makes them different and one cannot find a $f_2 = const$ solution. I do not know how to proceed.

Instability in 1D BEC due to electrostriction-induced effective attraction: Following [41], the 1D GPE with attractive interactions has a stationary solution, which is unstable. Considering perturbations to the stationary solution with definite wave length and frequency, one can calculate the dispersion relation of such perturbations. Applying this formalism to the 1D version of (47) one gets a dispersion relation:

$$\omega^2 = \left(\frac{\hbar k^2}{2m}\right)^2\left(1 + \frac{2n(0)\tilde{g}}{\frac{\hbar k^2}{2m}}\right) \tag{53}$$

where $\tilde{g}$ was defined in (47). One can see that when the electrostriction-induced effective attraction overcomes the repulsive background interaction $\tilde{g} < 0$ $\left(g < \frac{\sigma_0}{2}\frac{\gamma}{\delta}\frac{I}{\omega}\right)$, the angular frequency ω becomes imaginary for $0 < k < k_c$ and gets a maximal (imaginary) value at $k = k_p$, where:

$$\frac{\hbar k_c^2}{2m} = -2n(0)\tilde{g}$$
$$k_p^2 = \frac{k_c^2}{2} \tag{54}$$

A modulation having a wave number k_p stemming from a BEC density fluctuation or an electrostriction beam intensity fluctuation, will grow exponentially faster than in any other wave number. The BEC density profile will thus get increasingly modulated at wave number k_p and will reach a point where the small δn approximation used in deriving (47) breaks down. For large enough δn, the linear scaling of the repulsive background interaction dominates the logarithmic scaling of the electrostriction force, and we thus expect the modulation to stop growing and stabilize. I did not solve for the resulting BEC density pattern, but can get a hint to its qualitative behavior. Assuming the Thomas-Fermi limit applies, the density should obey:

$$n = \frac{\mu - V}{g} \tag{55}$$

at any point in space. V has linear and logarithmic components in n, and thus the equation can have two, one or no finite solutions at each point. The only inhomogeneous solution is thus a density that has only two values, and jumps between them at different points in space. Considering the above argument we expect the resulting modulation to be periodic with $k \approx k_p$. This solution clearly violates[14] the Thomas-Fermi limit validity, and thus this should be treated as a qualitative description of the steady modulated state.

The GPE (45) in D dimensions can be derived as the equation of motion resulting from the action:

$$\begin{aligned} & -i\hbar \int (\psi^* \frac{\partial \psi}{\partial t} + \varepsilon) d^D r dt \\ & \varepsilon = \frac{\hbar^2}{2m} |\vec{\nabla}\psi|^2 + \frac{g}{2}|\psi|^4 + V|\psi|^2 + U_{es}|\psi|^2 = \\ & -\frac{\hbar}{2m}|\vec{\nabla}\sqrt{n}|^2 + \frac{g}{2}n^2 + Vn + U_{es}n \end{aligned} \tag{56}$$

Searching for a stationary solution of the GPE thus amounts in minimizing the energy $\int \varepsilon d^D r$, under the constraint of normalized ψ. I will approximately minimize the energy by a variational method, using the ansatz:

$$\begin{aligned} & \psi(\vec{r}) = \psi_0 e^{-\frac{r^2}{2w^2 a_{ho}^2}} \\ & \int |\psi|^2 d^D r = 1 \end{aligned} \tag{57}$$

where w is the variational parameter for the condensate width, and $a_{ho} = \sqrt{\frac{\hbar}{m\omega}}$ is the harmonic oscillator length. Preforming this ansatz to the PGE energy in $D = 3$ for instance gives:

$$\frac{\varepsilon_{3D}}{N\hbar\omega} = \frac{3}{4}(w^{-2} + w^2) + (2\pi)^{-1/2}\frac{Na}{a_{ho}}w^{-3} + \frac{3}{8\omega}\frac{\gamma^2}{\delta}\frac{I}{I_s} \tag{58}$$

where a is the scattering length. From this expression one can get the critical atom number for collapse, which occurs when the energy has no local minimum. In $3D$ this happens for attractive interactions $a < 0$ when $N \gtrsim 0.67\frac{a_{ho}}{a}$. Notice electrostriction has no effect on the collapse, since the energy contribution of electrostriction is constant in w. This is true in any other dimension as well. If we would have taken the small δn approximation

[14] Rapid density changes cost lost of kinetic energy, which is neglected in the Thomas-Fermi limit.

we would have **falsely** deduced electrostriction can in principle control the condition for collapse. Notice that our $3D$ treatment was inappropriate, since electrostriction is no really a potential force...

Force on BEC under sudden electrostriction pulse: A BEC of density (27) in an isotropic harmonic trap force of $\vec{f}_{ho} = -m\omega^2\vec{r}$ will feel an electrostriction force of:

$$\vec{f}_{es} = -\frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\frac{\vec{\nabla}_{x,y}n}{n} = -\frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\frac{2\vec{r}_{x,y}}{r^2 - x_{c,0}^2} \tag{59}$$

where $\vec{r}_{x,y} = (x, y, 0)$. The total force will thus be:

$$\vec{f}_{total} = -(m\omega^2 + \frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\frac{2}{r^2 - x_{c,0}^2})(\hat{x} + \hat{y}) - m\omega^2\hat{z} \tag{60}$$

Expanding this force around $\vec{r} = \vec{0}$ one gets the modified trap frequencies:

$$\tilde{\omega}_x = \tilde{\omega}_y = \sqrt{\omega^2 - \frac{\hbar\gamma^2}{4\delta m}\frac{I}{I_s}\frac{2}{x_{c,0}^2}} = \omega\sqrt{1 - \frac{\hbar\gamma^2}{4\delta}\frac{I}{I_s}\frac{2}{(15\hbar^2\sqrt{m}N_0\omega^3 a)^{\frac{2}{5}}}} \tag{61}$$

$$\tilde{\omega}_z = \omega_z$$

Polarization of e.s. beam?

At the end of any calculation of physical observables the cut-off will cancel.

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