MIT GCFP | GOLUB CENTER FOR FINANCE AND POLICY

Collective Defined Contribution Plan Contest

Model Overview

This crowd-sourced contest seeks an answer to the question, “What is the optimal investment strategy and risk-sharing policy that provides long-term public sector employees with the highest achievable scheduled benefit subject to limits on the probability and severity of benefit shortfalls?” The model provides a basic tool for contestants to quantify the statistical implications of different proposed investment strategies and risk-sharing policies on retirement income security.

The model relies on Monte Carlo simulation to generate probability distributions of benefit outcomes across workers and over time. The model tracks the money paid into the plan on behalf of each individual worker and the stochastic investment performance on those accumulated assets over his or her working life. The workforce includes employees of various ages and work histories, some of whom will work until they are fully vested and others that will separate earlier.

Some assumptions are fixed and other variables are chosen by contestants. We have fixed certain assumptions to ensure comparability across the entries and realistic predictions about achievable outcomes. In answering the qualitative questions, as well as changing certain parameters, contestants are encouraged to modify the code to reflect recommended changes in plan rules that improve outcomes. However, those changes cannot have the effect of altering the fixed assumptions. Entries must include the user’s version of the code.

1. Structure of model

This section briefly explains the logic of the model, which is written in R. Note that the number of Monte Carlo simulations will affect run time.

Step 1: **Initialization.** This step preloads model parameters and incorporates a basic investment strategy (which is not optimized). The fixed parameters are embedded in the R code.

The parameters include various economic, financial, and demographic assumptions. (Please see Tables 1 through 8 for details.) . The number of workers in each age bracket, from the minimum starting age to the retirement age, is initialized so as to match the steady-state worker distribution by age.[1] All workers are assumed to start at “time 0” with a zero balance in their notional accounts.[2] Contestants need to specify rules for asset allocations between stocks, bonds, and alternative assets. The code is set up with fixed asset allocations. Dynamic asset allocations that are contingent on model outcomes require additional programming.

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[1] The steady state distribution of workers (“steady_state_dist.csv”) is a table which granularly shows the number of workers, in the steady state, of different current ages (the first row represents the current age of 25 and the last row represents the current age of 62) and different ages at which the workers joined the plan (the first column represents the workers who joined the plan at age 25 and the last column represents the workers who joined the plan at age 62.) All cells are mutually exclusive and collectively exhaustive to represent all possible cases of any worker in service. They sum to the total size of the workforce: 100,000.

[2] Because of this, the model has to be iterated forward for many years before plan balances reach a steady state distribution. Proposals will first be checked in year 35, once the accounts reach a steady state.

Step 2: **Generate random returns.** A key driver of ultimate benefit payments is investment returns over time. Each year the model generates a return for each asset class, which is used to update the notional account for each worker and aggregate plan asset holdings. Those annual returns are randomly generated based on fixed assumptions about mean returns and the covariance matrix. Returns are assumed to be jointly normally distributed. A Cholesky Decomposition was used to implement the covariance structure numerically.[3]

Step 3: **Accumulation phase.** A key feature of Collective Defined Contributions Plans (CDCP) is that everyone's contributions are aggregated into one collective portfolio. While the portfolio is collectively managed based on the contestant's investment strategy, each cohort of workers has three notional accounts that the model tracks: CohortAssets, WorkerAssets, and FloorAssets. Note that all workers who belong to the same cohort share the same accounts, and each worker has a claim to his or her per capita share. Workers have no control over the accounts; they exist solely to keep track of the worker's savings contributions and investment credits.

Specifically, CohortAssets tracks the cumulative balance of a cohort based on contributions and investment performance. The investment performance is capped by the contestant-specified ceiling return, and returns in excess of that ceiling are transferred into the Reserve Fund. WorkerAssets tracks vested benefits of workers of the cohort who separate prior to full vesting. The assumed payout policy provides those separating workers with their own contributions plus returns from the investment portfolio as well as their employer's contributions with interest at the risk-free rate. FloorAssets tracks the scheduled benefit to fully vested retirees. Its balance accumulates contributions from the worker and the employer at a contestant-specified floor rate of return.

The Reserve Fund is assumed to be managed conservatively, earning the risk-free rate.

Step 4: **Distribution phase.** The distribution phase starts when a worker separates or retires. Fully vested workers receive full benefits, based on the higher amount of either CohortAssets or FloorAssets. If CohortAssets are less than FloorAssets, then the Reserve Fund makes up shortfalls if the Reserve Fund has sufficient assets at that time. If the Reserve Fund is insufficient, however, the worker only receives CohortAssets plus what is available in the Reserve Fund.

Separators without sufficient tenure to be vested only have a claim to WorkerAssets. When CohortAssets exceed WorkerAssets, the difference goes into the Reserve Fund.
Interpreting the adequacy of retirement savings requires translating lump sum accumulations into annual income, and comparing that income to pre-retirement earnings. The pre-specified fixed annuity factor multiplied by the asset distribution gives the lifetime annual income generated by those assets.

Step 5: **Replace lost workers.** The model replaces separating and retiring workers (collectively referred to as "lost workers") at the end of each simulation year with new workers in according to a fixed age distribution for new hires. (See Table 3 for details.) This maintains the workforce at 100,000, and a steady state age distribution.

Step 6: **Record data.** In the R code, the user can choose a specific analysis year in which the model records various data. (See Section 4 for details.)

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[3] See Glasserman, Paul. (2013). *Monte Carlo methods in financial engineering* (Vol. 53), Chapter 3. Springer Science & Business Media.

2. Fixed parameters

Many assumptions about economic, financial, and plan variables are pre-specified and fixed to ensure realism and consistency across contest entries. For simplicity, some values that in fact vary over time or across employees are held fixed. All variables are on an annual basis unless otherwise noted. Tables 1 through 6 summarize the fixed parameters.

Table 1: Fixed economic assumptions

Inflation rate	2%
Risk-free nominal rate	2.5%
Base wage in 2018	$50,000
Real wage growth	0.5%
Nominal wage growth	2.5%
Annuitization factor	0.04

Table 2: Fixed plan assumptions

Workforce size (stable)	100,000
Minimum starting age	25
Retirement age	62
Years to max benefit	32
Employer contribution rate	10%
Employee contribution rate	10%
Initial buffer	$0

Table 3: Age distribution of new hires

25 to 34	0.5
35 to 44	0.25
45 to 54	0.2
55 to 61	0.05

Table 4: Separation probabilities by current age

26 to 44	0.02
45 to 54	0.02835
55 to 61	0.0942

Table 5: Statistics on annual returns and trading costs

	Stocks	Bonds	Alternatives
Mean nominal return (%)	6.5	3	5
Standard deviation (%)	20	9.8	18.2
Trading cost (%)	0.0005	0.0005	0.0015

Table 6: Cross-correlations of annual returns

	Stocks	Bonds	Alternatives
Correlation with stocks	1	-0.01	0.54
Correlation with bonds	-0.01	1	0.06
Correlation with alternatives	0.54	0.06	1

3. Contestant-specified parameters

For the purpose of making proposals comparable, contestants can only change certain program parameters and the investment strategy. Specifically, you will need to optimize plan performance by choosing the floor and ceiling rates of return that govern the evolution of CohortAssets and the Reserve Fund, along with the investment strategy. There are two levels of plan risk tolerances that you are asked to consider, and the optimal parameters and strategies will be different in those two cases. Table 7 and 8 summarize those variables.

Table 7: Return variables

R Object	Explanation
`user_rmin`	Minimum return (floor) when reserve assets are available
`user_rmax`	Maximum return (ceiling); surplus goes to reserve

Table 8: Asset allocation in the optimal portfolio

R Object	Explanation
`user_AssetShareStocks`	Proportion of assets to be stocks*
`user_AssetShareBonds`	Proportion of assets to be bonds*
`user_AssetShareAlternatives`	Proportion of assets to be alternatives*

*****A dynamic investment strategy requires additional programming by the user.

In addition to setting floor and ceiling rates and crafting an investment strategy, the code requires users to choose a number of other parameters. These are denoted in the code by double-hash (##) comment lines and by object names beginning with `user_`. All parameters requiring user input are located at the beginning of the R code.

Table 9: Other User-selected Parameters

R Object	Explanation
`user_Nyear`	Number of years to simulate
`user_NMonte`	Number of Monte Carlo simulations to run
`user_multiple`	Multiple of floor payment at which to cap Reserve Fund
`user_starting_age`	Starting age for the worker whose outcomes you want to analyze
`user_separating_age`	Separating age for the worker whose outcomes you want to analyze
`user_analysis_year`	The year for which you want to analyze outcomes

4. Outputs

The basic code produces several outputs that may be of interest to contestants:

1. Probabilities that realized benefits for the specified worker will a) fall short of scheduled benefits, b) fall short of 80% of the scheduled benefits, and c) fall short of 50% of the scheduled benefits.
2. Summary statistics on the distribution of likely lump-sum values of realized benefits for the specified worker
3. Summary statistics on the distribution of likely annuitized values of realized benefits for the specified worker
4. Summary statistics on the distribution of likely income replacement rate[4] for the specified worker
5. Summary statistics on the distribution of likely benefit shortfalls for the specified worker (a positive number indicates a shortfall; a negative one indicates a surplus)
6. Summary statistics on the distribution of likely values of the Reserve Fund in the specified year

Users may also add their own code to produce additional outputs of interest.

[4] The replacement rate is defined as the annuity amount that can be supported by assets at retirement or separation divided by the average wage over the last 5 years of employment. It is often used as an indicator of pension adequacy. The model assumes that the annual annuity payment that can be supported is 4% of accumulated assets.