Core Equations Reference

Unified Framework Initiative

Gravitation (Einstein Limit)

- Einstein–Hilbert action in tetrad form:

$$S_{\text{grav}} = \frac{1}{16\pi G} \int d^4x\, e\, e_a{}^{\mu} e_b{}^{\nu} R_{\mu\nu}{}^{ab}.$$

- Einstein field equations relating curvature to stress-energy:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}.$$

- Contracted Bianchi identity ensuring $\nabla_\mu T^{\mu\nu} = 0$:

$$\nabla_\mu G^{\mu\nu} = 0.$$

Electromagnetism (Maxwell Limit)

- Electromagnetic action with external current:

$$S_{\text{em}} = -\tfrac{1}{4} \int d^4x\, F_{\mu\nu} F^{\mu\nu} + \int d^4x\, J^\mu A_\mu.$$

- Sourced Maxwell equations:

$$\partial_\mu F^{\mu\nu} = J^\nu.$$

- Homogeneous Maxwell equations (Bianchi identity for the gauge field):

$$\partial_{[\lambda} F_{\mu\nu]} = 0.$$

Fermions (Dirac Sector)

- Dirac action on a curved background:

$$S_\psi = \int d^4x\, e\, \bar{\psi} \big(i\gamma^a e_a{}^{\mu} D_\mu - m \big) \psi.$$

- Covariant derivative with spin connection and gauge coupling:

$$D_\mu = \partial_\mu + \tfrac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} - igA_\mu.$$

- Axial current used for torsion sourcing:

$$J^\mu_5 = \bar{\psi} \gamma^\mu \gamma^5 \psi.$$

Torsion and Einstein–Cartan Relations

- Totally antisymmetric torsion from an axial vector S^ρ:

$$T_{\lambda\mu\nu} = \epsilon_{\lambda\mu\nu\rho} S^\rho.$$

- Divergence of the Einstein tensor with axial torsion:

$$\nabla_\mu G^{\mu 0} = \tfrac{3}{2}\, \sigma\, \dot{\sigma}.$$

- Einstein–Cartan relation between torsion and spin current:

$$S_\mu = \frac{\kappa}{2} J^{(5)}_\mu.$$

- Homogeneous torsion sourced by the axial charge density:

$$\sigma(t) = \frac{\kappa}{2} J^{(5)}_0, \qquad J^{(5)}_0 = \frac{p}{\sqrt{m^2 + p^2}}.$$

Usage

All equations are implemented in the repository. Run

```
python tests/run_all.py
```

to regenerate verification artifacts, including torsion and axial-current checks.