EECS 545 F17 HW1

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1) (a)

(i) True. We know $I = I^T$, since $AA^{-1} = I \Rightarrow AA^{-1} = (AA^{-1})^T$, since $(AB)^T = B^T A^T \Rightarrow AA^{-1} = (A^{-1})^T A^T$

$\Rightarrow AA^{-1} = A^{-1}A = (A^{-1})^T A^T$. since $A = A^T \Rightarrow A^{-1}A = (A^{-1})^T A$. $\Rightarrow A^{-1}AA^{-1} = (A^{-1})^T A \cdot A^{-1} \Rightarrow \underline{A^{-1} = (A^{-1})^T}$ #

(ii) True. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. if. $\det(A) = 1$. $\begin{bmatrix} a & c \\ b & d \end{bmatrix} = A^T = A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$\Rightarrow d = a,\ b = -c \Rightarrow A$ has the form $\begin{bmatrix} a & -c \\ c & a \end{bmatrix}$, and $a^2 + c^2 = 1$

$\Rightarrow$ We can write A as $\underline{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$ # if $\det(A) = -1$. $\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$

$\Rightarrow a = -d,\ b = c \Rightarrow A$ has the form $\begin{bmatrix} a & c \\ c & -a \end{bmatrix}$, and $-(a^2 + c^2) = -1 \Rightarrow \underline{A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}}$ #

(iii) eigenvalues of A are $-15, -4.899, 4.899 \Rightarrow A$ is not positive-definite

$\Rightarrow A$ cannot be decomposed as CC^T. <u>the answer is false.</u> #

2)

(a)

(i) $E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$, since $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy \Rightarrow E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X,Y}(x,y)\,dx\,dy$

since. $f_{X|Y}(x|y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)} \Rightarrow \underline{E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot f_{X|Y}(x|y) \cdot f_Y(y)\,dx\,dy = E_Y[E_X[X|Y]]}$ #

(ii) $E[I[X\in C]] = \int_X I[X \in C]\,dP = \int_X 1 \cdot dP = \underline{P(X \in C)}$ #

(iii) $Var[X] = E[X^2] - (E[X])^2 = E_Y[E_X[X^2|Y]] - (E_Y[E_X[X|Y]])^2$

$= E_Y[E_X[X^2|Y]] - E_Y[(E_X[X|Y])^2] + E_Y[(E_X[X|Y])^2] - (E_Y[E_X[X|Y]])^2$

$= \underline{E_Y[Var_X[X|Y]] + Var_Y[E_X[X|Y]]}$ #

(iv) $E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_X(x) f_Y(y)\,dx\,dy = \int_{-\infty}^{\infty} f_X(x)\,dx \int_{-\infty}^{\infty} f_Y(y)\,dy = \underline{E[X]E[Y]}$ #