2) (a) (i) $E[XY] = \sum xy P_r(X=x) P_r(Y=y) = P_r(X=1, Y=1)$.

$E[X] = \sum x P_r(X=x, Y=y) = P_r(X=1, Y=0) + P_r(X=1, Y=1) = P_r(X=1)$

$E[Y] = \sum y P_r(X=x, Y=y) = P_r(X=1, Y=1) + P_r(X=0, Y=1) = P_r(Y=1)$

$\Rightarrow P_r(X=1, Y=1) = E[XY] = E[X]E[Y] = \underline{P_r(X=1) \cdot P_r(Y=1)} \rightarrow$ independent

and for $X=X$, $Y=1-y$. $E[XY] = \sum x(1-y) P_r(X=x) P_r(Y=y) = P_r(X=1, Y=0)$

$E[X] = \sum x P_r(X=x, Y=y) = P_r(X=1)$, $E[Y] = \sum (1-y) P_r(X=x, Y=y) = P_r(Y=0)$

$\Rightarrow P_r(X=1, Y=0) = E[XY] = E[X]E[Y] = \underline{P_r(X=1) P_r(Y=0)} \rightarrow$ independent.

for $X=1-X$, $Y=1-y$ and $X=1-X$, $Y=y$, the same as above.

$\Rightarrow E[XY] = E[X]E[Y] \rightarrow$ $\underline{X, Y \text{ are independent.}}$ #

(b)

(i) $P(H=h) = \sum_i P(H=h, D=d_i) \Rightarrow P(H=h, D=d) \leq P(H=h)$ #

(ii) $P(H=h \mid D=d) = \dfrac{P(H=h, D=d)}{P(D=d)} \Rightarrow A$, $0 \leq a \leq 1$. $\Rightarrow$ $\underline{\text{depends}}$ #

(iii) $\dfrac{P(H=h, D=d)}{P(D=d)} \geq P(H=h, D=d) = P(D=d \mid H=h) \cdot P(H=h) \Rightarrow$ $\underline{\geq}$ #

3)

(a) $A = U \Lambda U^T$. $X^T \cdot A \cdot X = \sum_{i=1}^{d} X^T \cdot \lambda_i u_i u_i^T \cdot X = \sum_{i=1}^{d} \lambda_i (u_i^T X)^T u_i X$.

$\because (u_i^T X)^T \cdot u_i X > 0$, $\Rightarrow$ $\underline{\lambda_i \geq 0 \Rightarrow A = PSD.}$

If A is PSD, then $X^T A \cdot X = \sum_{i=1}^{d} \lambda_i (u_i^T X)^T \cdot u_i X \geq 0$ $\underline{\text{for } \lambda_i \geq 0}$ #

(b) Same as (a) $\underline{\text{with } \lambda_i > 0.}$ #

4)

(a) $f(x) = a^T x + b$. $\Rightarrow f(tx + (1-t)y) = a^T t x + a^T (1-t) y + b$.

$= t a^T x + (1-t) a^T y + tb + (1-t) b \Rightarrow f(x)$ is convex and concave.

$f(x)$ is not strickly convex.