EECS 545 F17 HW2.   Name: Po-Jen Lai   Members: Ming-Ko Cho, Yi-Lun Wu, Chun-Yu Hsiung

1) (a)

(i) $f(x) = e^x - 1$. $\nabla^2 f(x) = e^x > 0$ for all x.

Also, $f(x)$ is twice continuously differentiable $\Rightarrow$ $f(x)$ is strictly convex #

(ii)

$\nabla^2 f(x) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\Rightarrow$ neither PSD or NSD. $\Rightarrow$ neither concex nor concave #

and also strictly concave #

It's strictly concave # because $\{(x_1, x_2) \in \mathbb{R}^2_{++} \mid x_1 x_2 \geq \alpha\}$ are convex. It's not strictly convex #

(iii)

$$\nabla^2 f(x) = \begin{bmatrix} \alpha(\alpha-1)x_1^{\alpha-2}x_2^{(1-\alpha)} & \alpha(\alpha-1)x_1^{\alpha-1}x_2^{-\alpha} \\ \alpha(\alpha-1)x_1^{\alpha-1}x_2^{-\alpha} & (1-\alpha)(-\alpha)x_1^{\alpha}x_2^{-\alpha-1} \end{bmatrix} = -\alpha(1-\alpha)x_1^{\alpha}x_2^{1-\alpha}\begin{bmatrix} x_1^{-1} \\ -x_2^{-1} \end{bmatrix}\begin{bmatrix} x_1^{-1} \\ -x_2^{-1} \end{bmatrix}^T$$

$\Rightarrow -\nabla^2 f(x) \succeq 0 \Rightarrow f$ is concave.

(b)

(i)

$$J(\theta) = \frac{1}{2}(A^T\theta - y)^T(A^T\theta - y) = \frac{1}{2}(\theta^T A A^T \theta - \theta^T A y - y^T A^T \theta - y^T y)$$

$\nabla_\theta J(\theta) = AA^T\theta - Ay$ #, $\theta^{(t+1)} = \theta^{(t)} - \alpha(AA^T\theta - Ay)$ #

(ii)

To minimize $J \Rightarrow \nabla_\theta J(\theta) = 0 \Rightarrow AA^T\theta = Ay \Rightarrow \theta = (A^T)^{-1}y$ #

2)

N: numbers of skip.

W: Win.

We know $P(W|N=0) = \frac{1}{2}$. now we assume k Rose Gold have been skipped.

$$\Rightarrow P(W|N=n) = \frac{50-k}{100-n}.$$

$$P(W|N=n+1) = \frac{50-k}{100-(n+1)} \cdot \frac{100-n-(50-k)}{100-n} + \frac{49-k}{100-(n+1)} \cdot \frac{50-k}{100-n} = \frac{50-k}{100-n}.$$

$$\Rightarrow P(W|N=n) = P(W|N=n+1) = P(W|N=0) = \frac{1}{2} \#$$

$\therefore$ The probability of correct prediction is always $\frac{1}{2}$ regardless of N.