Given H, occurs, Y is Binomial
$$(3, \frac{3}{4})$$
.

PMF for
$$X \sim Binomial(n,p)$$
: $P_{\times}(x) = \left(\binom{n}{x}p^{\times}(1-p)^{n-x} \times = 0,1,...,n\right)$ otherwise

$$P_{Y|H_0}(y) = \begin{cases} \binom{3}{y} \left(\frac{1}{2}\right)^{y} \left(\frac{1}{2}\right)^{3-y} & y = 0,1,2,3 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \binom{3}{0} \cdot \frac{1}{8} & y = 0 \\ \binom{3}{1} \cdot \frac{1}{8} & y = 1 \\ \binom{3}{2} \cdot \frac{1}{8} & y = 2 \\ \binom{3}{2} \cdot \frac{1}{8} & y = 3 \end{cases} = \begin{cases} \frac{1}{8} & y = 0 \\ \frac{3}{8} & y = 1 \\ \frac{3}{8} & y = 2 \\ \frac{1}{8} & y = 3 \end{cases}$$

$$P_{Y1H_1}(y) = \begin{cases} \left(\frac{3}{7}\right)\left(\frac{3}{4}\right)^{y}\left(\frac{1}{4}\right)^{3-y} & y = 0,1,2,3 = 0 \end{cases}$$
otherwise

$P_{Y1H_1}(y) = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}^{Y} \begin{pmatrix} \frac{1}{4} \end{pmatrix}^{3-y} \qquad y = 0,1,2,3 = 0$ $ML \ Rule$ $If \ P_{Y1H_1}(y) \ge P_{Y1H_0}(y), \ pick \ H_1. \ If \ not, \ pick \ H_0.$				$ \begin{pmatrix} 3 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 &$	
	Y				$\begin{pmatrix} 3 \\ 3 \end{pmatrix} \cdot \frac{27}{64} y = 3 \qquad \begin{pmatrix} \frac{27}{64} \\ \frac{27}{64} \end{pmatrix} y = 3$
	0	1	2	3	$D^{ML}(y) = (1 y = 2, 3)$
P41H0 (4)	18	3 8	3/8	18	$D^{ML}(y) = \begin{cases} 1 & y = 2,3 \\ 0 & y = 0,1 \end{cases}$
P41H1 (4)	<u>1</u> 64	9 64	27 64	27 64	Ao={0,1} A,= {2,3}

$$\begin{pmatrix}
3 & \frac{1}{8} & 4 & 3 \\
3 & \frac{1}{8} & 4 & 3
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{8} & 4 & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
3 & \frac{1}{64} & 4$$

$$D^{ML}(y) = \begin{cases} 1 & y = 2,3 \\ 0 & y = 0,1 \end{cases}$$

$$A_0 = \{0,1\} \quad A_1 = \{2,3\}$$

• Example: Given Ho occurs, Y is Binomial (3, \frac{1}{2}).

Given Ho occurs, Y is Binomial (3, \frac{3}{4}).

	Y					
	0	1	a	3		
P41H0 (4)	-1 80	3/8	3/8	18		
P41H1 (4)	164	913	27 64	27 64		

$$D^{\text{ML}}(y) = \begin{cases} 1 & y = 2,3 \\ 0 & y = 0,1 \end{cases}$$

$$A_0 = \{0,1\} \quad A_1 = \{2,3\}$$

→ What is the probability of error for the ML rule Peme?

Peme = PFA [P[Ho]] + Pmo [P[H,]] Not specified in problem statement.

- add to problem statement: IP[Ho] = \frac{1}{5} IP[HI] = \frac{4}{5}

$$P_{FA} = \sum_{y \in A_1} P_{Y1H_0}(y) = P_{Y1H_0}(2) + P_{Y1H_0}(3) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$$

$$P_{MD} = \sum_{y \in A_o} P_{Y|H_1}(y) = \frac{1}{64} + \frac{9}{64} = \frac{5}{32}$$

$$P_e^{ML} = \frac{1}{2} \cdot \frac{1}{5} + \frac{5}{32} \cdot \frac{4}{5} = \frac{9}{40}$$
 Can we so better?

· Example: Given Ho occurs, Y is Binomial (3, 1). Given H, occurs, Y is Binomial (3, 3).

	Y				
	0	-	ત	3	
Pylho (y)	-1 80	3/8	3/8	18	
P41H1 (4)	14	9 64	27 64	27 64	

$$D^{ML}(y) = \begin{cases} 1 & y = 2,3 \\ 0 & y = 0,1 \end{cases}$$

$$\Rightarrow \text{ Assume } P[H_0] = \frac{1}{5}$$

$$P[H_1] = \frac{4}{5}$$

$$P_e^{ML} = \frac{9}{40}$$

→ Determine MAP rule and probability of error. $D^{MAP}(y) = \begin{cases} 1 & y = 1, 2, 3 \\ 0 & y = 0 \end{cases}$ MAP Rule

If PyIH, (y) IP(H,] 2 PYIH. (y) IP(H.), pick H. If not, pick Ho.

	Y				
	0	l	2	3	
PyIH. (4) ID[H.]	$\frac{1}{8} \cdot \frac{1}{5} = \frac{1}{40}$	3 - 3 40	3 - 1 = 3 40	$\frac{1}{8} \cdot \frac{1}{5} = \frac{1}{40}$	
P,,,(y) P[H,]	1-4-5= -80	9 4 5 90	$\frac{27}{64} \cdot \frac{4}{5} = \frac{27}{80}$	$\frac{27}{64} \cdot \frac{4}{5} = \frac{27}{80}$	

$$\begin{cases} y = 1, 2, 3 \\ 0 \quad y = 0 \end{cases}$$

 $P_{e}^{MAP} = P_{FA} P[H_{o}] + P_{mo} P[H_{i}]$ $= \frac{7}{8} \cdot \frac{1}{5} + \frac{1}{64} \cdot \frac{4}{5} = \frac{3}{16}$ $MAP \ outperforms \ ML.$ $P_{mD} = \sum_{Y \in A_{o}} P_{Y \mid H_{i}}(y)$ $= P_{Y \mid H_{i}}(0) = \frac{1}{64}$

$$P_{MD} = \sum_{y \in A_o} P_{Y|H_1}(y)$$
$$= P_{Y|H_1}(0) = \frac{1}{64}$$

$$P_{FA} = \sum_{y \in A_1} P_{Y|H_0}(y) = P_{Y|H_0}(1) + P_{Y|H_0}(2) + P_{Y|H_0}(3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$$