pstat274_hw03_aoxu

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2022-10-18

Problem 1

$X_t = \frac{2}{3}X_{t-1} + \frac{1}{2}X_{t-2} + Z_t$

Since it's AR(2) model, then it's invertible since AR(2) is always invertible.

Then, $\phi(z)$ = 1 - $\frac{2}{3}z$ - $\frac{1}{2}z^2$

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polyroot(c(1, -2/3, -0.5))
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## [1]  0.8968053+0i -2.2301386+0i
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Since 0<0.8968053 < 1, then it's not stationary.

Therefore, it's not stationary but invertible.

Problem 2

Only the statement II is true.

I is false: For AR(3), the pacf at lag3, $\alpha(3)$ = $\phi_{33} \neq$ 0, so I is false.

II is true: The lag(4) for AR(3) must be 0 since there is no term after lag(3).

III is false: If partial Autocorrelation for lag 4 is greater than zero, then it's not AR(3).

Problem 3

$X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$

From lecture slide, we could get that $\rho_x(k) = \frac{(\phi_1+\theta_1)(1+\phi_1\theta_1)}{(1+2\phi_1\theta_1+\theta_1^2)}\phi_1^{k-1}$.

Then since $\rho_x(1)$ = 0.7 and $\rho_x(2)$ = 0.73, we could get that $\rho_x(1)$ = $\frac{(\phi_1+\theta_1)(1+\phi_1\theta_1)}{(1+2\phi_1\theta_1+\theta_1^2)}$ = 0.7 while $\rho_x(2)$ = $\frac{(\phi_1+\theta_1)(1+\phi_1\theta_1)}{(1+2\phi_1\theta_1+\theta_1^2)}\phi_1$ = 0.3.

Therefore, we get ϕ_1 = $\frac{\rho_x(2)}{\rho_x(1)}$ = $\frac{0.3}{0.7}$ = $\frac{3}{7}$

Problem 4

Since $\phi_{11} = -0.60$, $\phi_{22} = 0.36$, and $\phi_{kk} = 0$ for $k \geq 3$, then we could get that it's AR(2) model and $\phi_2 = \phi_{22} = 0.36 = \frac{\rho_x(2)-(\rho_x(1))^2}{1-(\rho_x(1))^2}$. $\rho_x(1) = \phi_{11} = -0.6$. So we get $\rho_x(2) = 0.5904$.

By using Yule-Walker equations, we get that

$$\begin{cases} \rho_x(1) = & \phi_1 + \phi_2\rho_x(1) = -0.6 \\ \rho_x(2) = & \phi_1\rho_x(1) + \phi_2 = 0.5904 \end{cases}$$

Then we get that $\phi_1 = -0.384$ and $\phi_2 = 0.36$.

Therefore, $X_t = -0.384X_{t-1} + 0.36X_{t-2} + Z_t$ where Z_t ~ $WN(0, \sigma_z^2)$

Problem 5

Only E is false.

A is true.

Since $X_t = 0.8X_{t-1} + 2 + Z_t - 0.5Z_{t-1}$, then we get $\phi_1 = 0.8$ and $\theta_1 = -0.5$

$\rho_x(1)$ = $\frac{(\phi_1+\theta_1)(1+\phi_1\theta_1)}{(1+2\phi_1\theta_1+\theta_1^2)}$ = $\frac{(0.8-0.5)(1-0.4)}{(1+2(-0.4)+0.25)}$ = 0.4

B is true.

$\rho_x(k) = \frac{(\phi_1+\theta_1)(1+\phi_1\theta_1)}{(1+2\phi_1\theta_1+\theta_1^2)}\phi_1^{k-1}$, since $\phi_1 = 0.8 < 1$, then we get that $\phi_1^{k-1} < 1$ for all $k \geq 2$, then $\rho_x(k) < \rho_x(1)$

C is true.

It follows the model that $X_t - \phi_1X_{t-1} = Z_t - \theta_1Z_{t-1}$ where $\phi_1 = -0.8$ and $\theta_1 = 0.5$, then it's ARMA(1,1).

D is true.

ϕ_z = 1-0.8Z = 0, then $\phi_1 = 0.8 < 1$, so it's stationary.

E is false.

μ = $\frac{C}{1-\phi_1}$ = $\frac{2}{0.2}$ = 10 $\neq$ 2

Problem 6

I and III are parameter redundant.

I

X_t = $\frac{1}{2}X_{t-1}$ + Z_t - $\frac{1}{2}Z_{t-1}$

It could be converted to X_t-$\frac{1}{2}X_{t-1}$ = Z_t - $\frac{1}{2}Z_{t-1}$, since $\phi(z) = 1 - \frac{z}{2} = \theta(z)$. So they share one common factor, then it is parameter redundant.

II

X_t = $\frac{1}{2}X_{t-1}$ + Z_t - $\frac{1}{9}Z_{t-2}$

It could be converted to X_t-$\frac{1}{2}X_{t-1}$ = Z_t - $\frac{1}{9}Z_{t-2}$, since $\phi(z) = 1 - \frac{z}{2}$ $\theta(z) = 1 - \frac{z^2}{9}$. They don't share the common factor, then it's not parameter redundant.

III

X_t = $\frac{-5}{6}X_{t-1}$ - $\frac{1}{6}X_{t-2}$ + Z_t +$\frac{8}{12}Z_{t-1}$+ $\frac{1}{12}Z_{t-2}$

Since $\phi(z)$ = 1+$\frac{5z}{6}$+$\frac{z^2}{6}$ = (1+$\frac{z}{2}$)(1+$\frac{z}{3}$),

$\theta(z)$ = 1+$\frac{8z}{12}$+$\frac{z^2}{12}$ = (1+$\frac{z}{2}$)(1+$\frac{z}{6}$),

they share one common factor 1+$\frac{z}{6}$ so it's parameter redundant.

GE 1

X_t = Z_t + θZ_{t-1}, Z_t ~ WN(0,σ^2) $\gamma_x(k)$ = $Cov(X_t, X_{t+k})$ = $E((Z_t + \theta Z_{t-1})(Z_{t+k} + \theta Z_{t+k-1}))$ = $E(Z_t Z_{t+k}) + \theta(E(Z_t Z_{t+k-1} + E(Z_{t-1}Z_{t+k})) + \theta^2 E(Z_{t-1}Z_{t+k-1})$

$$\begin{cases} =\sigma^2(1+\theta)^2 & k=0 \\ =\sigma^2\theta & |k|=1 \\ =0 & otherwise \end{cases}$$

$\gamma_y(k)$ = $Cov(Y_t, Y_{t+k})$ = $E((Z_t + \theta^{-1}Z_{t-1})(Z_{t+k} + \theta^{-1}Z_{t+k-1}))$ = $\gamma_z(k) + \theta_1\gamma_z(k+1) + \theta^{-1}\gamma_z(k-1) + \theta^{-2}\gamma_z(k)$

$$\begin{cases} =\sigma^2(1+\theta)^2 & k=0 \\ =\sigma^2\theta & |k|=1 \\ =0 & otherwise \end{cases}$$

Then we get that $\gamma_x(k) = \gamma_y(k)$.

GE 2

$X_t = \phi X_{t-1} + Z_t$ $Y_t = X_{2t}$ = $\phi X_{2t-1-1} + Z_{2t-1} + Z_{2t}$ = $\phi^2 X_{2t-2} + \phi Z_{2t-1} + Z_{2t}$ = $\phi^2 Y_{t-1} + \phi Z_{2t-1} + Z_{2t}$

Then, we get that $\phi^* = \phi^2$ and $Z_t^* = \phi Z_{2t-1} + Z_{2t}$