pstat274_hw04_aoxu

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Problem 1

Answer: Only II

I. A plot of the series detects a linear trend and increasing variability.

It cannot be written as the form $X_t = X_{t-1} + Z_t$ which is random walk then it could not be represented as random walk.

II. The differenced series follows a white noise model.

$Z_t = W_t = X_t - X_{t-1} \Longrightarrow X_t = X_{t-1} + Z_t$, since it follows the random walk that $X_t = X_{t-1} + Z_t$ then it could be represented as random walk.

III. The standard deviation of the original series is greater than the standard deviation of the differenced series.

Also, it cannot be written as the form $X_t = X_{t-1} + Z_t$ which is random walk then it could not be represented as random walk.

Therefore, only II could be written as random walk.

Problem 2

Identify the following time series model as a specific ARIMA or SARIMA model:

a. $X_t = 1.5X_{t-1} - 0.5X_{t-2} + Z_t - 0.1Z_{t-1}$

$\Longrightarrow X_t - 1.5X_{t-1} + 0.5X_{t-2}$ = $(1 - 1.5B + 0.5B^2)X_t$ = $(1 - 0.5B)(1 - B)X_t$ = $(1 - 0.1B)Z_t$

So it's ARIMA (1,1,1) model where p = q = d = 1.

b. $X_t = 0.5X_{t-1} + X_{t-4} - 0.5X_{t-5} + Z_t - 0.3Z_{t-1}$

$\Longrightarrow X_t - 0.5X_{t-1} - X_{t-4} + 0.5X_{t-5} = Z_t - 0.3_{t-1}$

$\Longrightarrow (1 - 0.5B - B^4 + 0.5B^5)X_t$ = $(1 - 0.5B)(1 - B^4)X_t$ = $(1 - 0.3B)Z_t$

So it's SARIMIA $(1, 0, 1) * (0, 1, 0)_4$ model.

Problem 3

a.(i)

$SARIMA(2, 1, 1) \times (0, 1, 1)_6$

p = 2, d = 1, q = 1; P = 0, D = 1, Q = 1; s = 6

$\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D X_t = \theta(B)\Theta(B^s)Z_t$

$\Rightarrow (1 - \phi_1 B + \phi_2 B^2)(1 - B)(1 - B^6)X_t = (1 + \theta_1 B)(1 + \Theta_1 B^6)Z_t$

ii. $SARIMA(1, 1, 2) \times (2, 0, 1)_4$

p = 1, d = 1, q = 2; P = 2, D = 0, Q = 1; s = 4

$\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D X_t = \theta(B)\Theta(B^s)Z_t$

$\Rightarrow (1 - \phi_1 B)(1 - \Phi_1 B^4 - \Phi_2 B^8)(1 - B)X_t = (1 + \theta_1 B + \theta_2 B^2)(1 + \theta_1 B^4)Z_t$

b.(i)

$(1 - B^6)^2 X_t = (1 - 0.3B)Z_t$

It's $SARIMA(0, 0, 1) \times (0, 2, 0)_6$

ii. $X_t = 0.3X_t - 12 + Z_t$;

It's $SARIMA(0, 0, 0) \times (1, 0, 0)_{12}$.

c. You are given a time series model where PACF is zero except for lags 12 and 24. Which model will have this pattern?

Solutions: $\alpha(n) = \phi_{nm} \neq 0, X_t - \Phi_1 X_{t-12} - \Phi_1 X_{t-24} = Z_t \Rightarrow (1 - \Phi_1 B^{12} - \Phi_2 B^{24})X_t = Z_t$, so we get that p = 2, P = d = q = D = Q = 0, s = 12.

Therefore, It's $SARIMA(0, 0, 0) \times (2, 0, 0)_{12}$.

4. Write the form of the model equation for a $SARIMA(0, 0, 1) \times (0, 0, 1)_{12}$ model.

Option E.

Solution: Since it's $SARIMA(0, 0, 1) \times (0, 0, 1)_{12}$, then we get that p = d = P = Q = 0 and d = D = 1.

So we get that $X_t = \theta(B)\Theta B^{12} Z_t = (1 + \theta_1 B)(1 + \theta_2 B^{12})Z_t =$
$(1 + \theta_1 B + \theta_2 B^{12} + \theta_1\theta_2 B^{13})Z_t = Z_t + \theta_1 B_{t-1} + \theta_2 Z_{t-12} + \theta_1\theta_2 Z_{t-13}$.

Therefore, we choose the option E.

5. You are given PACF for two stationary processes:

a. For time series $X_t, t = 1, 2, \ldots$: $\phi_{11} = 0, \phi_{22} = 0.36, \phi_{kk} = 0$ for k ≥ 3.

Since $\phi_{11} = 0, \phi_{22} = 0.36, \phi_{kk} = 0$ for k ≥ 3., then we get that it's AR(2).

$\phi_{11} = \rho_x(1) = 0$

$\phi_{22} = \frac{\rho_x(2) - (\rho_x(1))^2}{1 - (\rho_x(1))^2 = 0.36} \Rightarrow \rho_x(2) = 0.36$

$$\begin{cases} \rho_x(1) = \phi_1 + \phi_2\rho_x(1) \\ \rho_x(2) = \phi_1\rho_x(1) + \phi_2 \end{cases}$$

$\Rightarrow$

$$\begin{cases} \phi_1 = 0 \\ \phi_2 = 0.36 \end{cases}$$

Therefore, it's AR(2) model with $X_t = 0.36X_{t-2} + Z_t$ and $Z_t \sim WN(0, \sigma_z^2)$.

b. For time series $Y_t, t = 1, 2, \ldots$: $\phi_{11} = 0.7, \phi_{kk} = 0$ for k ≥ 2.

Since $\phi_{11} = 0.7, \phi kk = 0$ for k ≥ 2, we get that it's AR(1) model.

$\rho_x(1) = \phi_{11} = 0.7.$

Therefore, it's AR(1) model with $X_t = 0.7X_{t-1} + Z_t$ and $Z_t \sim WN(0, \sigma_z^2)$.

GE(1)

a.

$\gamma_Y(k) = Cov(Y_t, Y_{t+k}) = E(Y_t Y_{t+k}) - E(Y_t)E(Y_{t+k})=$
$E((X_t + W_t)(X_{t+k} + W_{t+k})) - E(X_t + W_t) + E(X_{t+k} + W_{t+k}))=$
$E(X_t X_{t+k}) + E(X_t W_{t+k})) + E(X_{t+k} W_t) + E(W_t W_{t+k}) - 0$

$Var(Y_t) = E({Y_t}^2) - (E(Y_t))^2 = E({Y_t}^2) = E((X_t + W_t)^2) = E({X_t}^2 + 2X_t W_t + {W_t}^2) = \frac{\sigma_z^2}{1-\phi^2} + {\sigma_w}^2$. Then it's AR(1) model and stationary.

b.

$U_t = Y_t - \phi Y_{t-1} = (X_t + W_t) - \phi(X_{t-1} + W_{t-1}) = X_t + W_t - \phi W_{t-1} - \phi W_{t-1} = Z_t + W_t - \phi W_{t-1}$

$*u(t,t+k)=cov(y+{t+k})=cov(Y_t-Y*{t-1},Y_{t+k}-Y_{t+k-1})=cov(Y_t,Y_{t+k})-cov(Y_t,Y_{t+k-1})-cov(Y_{t-1},Y_{t+k})+ ^2 cov(Y_{t-1},Y_{t+k-1}) = (1+^2) _Y(k) -_Y (k-1) -_Y (k+1) = $

$$\begin{cases} \sigma_z^2 + (1+\phi^2)\sigma_w^2 & k = 0 \\ -\phi\sigma_w^2 & k = 1 \end{cases}$$

c.

$Y_t - \phi Y_{t-1} = V_t + \theta V_{t-1}, V_t \sim WN(0, \sigma_v^2)$

$\sigma_v^2(1+\theta^2) = \sigma_z^2 + (1-\phi^2)\sigma_w^2$

$\theta\sigma_v^2 = -\phi\sigma_w^2$

$\Rightarrow \frac{\theta}{1+\theta^2} = \frac{-\phi\sigma_w^2}{\sigma_z^2+(1-\phi^2)\sigma_w^2} = \frac{-\phi}{\frac{\sigma_z^2}{\sigma_w^2}+1-\phi^2}$

Let A = $\frac{\sigma_z^2}{\sigma_w^2} + 1 - \phi^2$.

$\Rightarrow \frac{1}{1+\theta^2} = \frac{-\phi}{A} \Rightarrow A\theta = -\phi - \phi\theta^2 \Rightarrow A\theta + \phi + \phi\theta^2 = 0 \Rightarrow \theta = \frac{-A\pm\sqrt{A^2-4\phi^2}}{2\phi}$

Therefore, $\theta = \frac{-(\frac{\sigma_z^2}{\sigma_w^2}+1-\phi^2)\pm\sqrt{(\frac{\sigma_z^2}{\sigma_w^2}+1-\phi^2)^2-4\phi^2}}{2\phi}$

$\sigma_v = \sqrt{\frac{-\phi\sigma_w^2}{\theta}}$

$\phi = \phi$

GE(2)

Find the ACVF, ACF and PACF for {Xt} when Xt = ΦXt−4 + Zt, |Φ| < 1.

ACVF:

$X_t = \Phi X_{t-4} + Z_t, \Phi < 1$

$\gamma_x(k) = E(X_t X_{t-k}) = E((\Phi X_{t-4} + Z_t)X_{t-k}) = \Phi E(X_{t-4}X_{t-k}) + E(Z_t X_{t-k})$

$$\begin{cases} \Phi\gamma_x(k-4) & k \geq 1 \\ \Phi\gamma_x(k-4) + \sigma_z^2 & k = 0 \end{cases}$$

ACF:

$\rho(4k) = \Phi^k$, where k is integers and $k \geq 0$

PACF:

$\phi_{44} = \rho_x(4) = \emptyset$