pstat274_hw06_aoxu

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2022-11-07

Problem 1

Solution:

$\rho_w(1) = -0.04 \pm 1.96 * 0.08 = (-0.1968, 0.1168)$

$\rho_w(2) = -0.5 \pm 1.96 * 0.1 = (-0.696, -0.304)$

$\rho_w(3) = 0.03 \pm 1.96 * 0.11 = (-0.1856, 0.2456)$

$\rho_w(4) = -0.01 \pm 1.96 * 0.11 = (-0.2256, 0.2056)$

$\rho_w(5) = 0.01 \pm 1.96 * 0.11 = (-0.2056, 0.2256)$

$\rho_w(6) = 0.02 \pm 1.96 * 0.11 = (-0.1956, 0.2356)$

$\rho_w(7) = 0.03 \pm 1.96 * 0.11 = (-0.1856, 0.2456)$

$\rho_w(8) = -0.01 \pm 1.96 * 0.11 = (-0.2256, 0.2056)$

All intervals include 0 except $\rho_w(2)$, so it's MA(2) model.

$\phi_z(B)Z_t = \theta_z(B)W_t, \phi_z(B) = 1, \theta_z(B) = 1 + \theta_{z1}B\theta_{z2}B^2 = 1 + \theta_{z2}B^2$

$\phi_z(B)\phi(B)X_t = \theta(B)\theta_z(B)W_t \Longrightarrow (1 - B\phi_1)X_t = (1 + \theta_1 B)(1 + \theta_z B^2)W_t = (1 + \theta_1 B + \theta_z B^2 + \theta_1\theta_z B^3)W_t$

Therefore, it's ARMA(1,3) model.

Problem 2

$H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$

$\bar{X} \sim N(\mu, \frac{\sigma_X^2}{100}) \Longrightarrow \frac{\bar{X}-\mu}{\sqrt{\frac{var(\bar{X})}{100}}} \sim N(0, 1)$

$P(-1.96 \leq \frac{\bar{X}-\mu}{\sqrt{\frac{var(\bar{X})}{100}}} \leq 1.96) = 0.95 \Longrightarrow -1.96\sqrt{\frac{var(\bar{X})}{100}} \leq 0.271 - \mu \leq 1.96\sqrt{\frac{var(\bar{X})}{100}} \Longrightarrow$
$0.271 - 1.96\sqrt{\frac{var(\bar{X})}{100}} \leq \mu \leq 0.271 + 1.96\sqrt{\frac{var(\bar{X})}{100}}$

Since it's AR(1) model. we could get that $var(\bar{x}) \approx \gamma_x(0) + 2\sum_{n=1}^{\infty}\gamma_x(h) = \gamma_x(0) + 2\sum_{n=1}^{\infty}\phi^h\gamma_x(0)$=

$\gamma_x(0)(1 + 2\sum_{n=1}^{\infty}\phi * \phi^{n-1})$ = $\frac{\sigma^2}{1-\phi^2}$ $(1 + 2\frac{\phi}{1-\phi})$ = $\frac{2}{1-0.6^2}(1 + 2 * \frac{0.6}{1-0.6})$=12.5

Therefore, we could get that $0.271 - 1.96\sqrt{\frac{12.5}{100}} \leq \mu \leq 0.271 + 1.96\sqrt{\frac{12.5}{100}} \Longrightarrow -0.422 \leq \mu \leq 0.964.$

Since $\mu = 0$ is in the 95% confidence interval (-0.422,0.964), we could get that null hypothesis $\mu = 0$ cannot be rejected.

G1

Solution:

a. By using ACF: $\hat{\rho} \sim N(\rho(h), h^{-1})$, $\hat{\rho}(h) \pm \frac{1.96}{\sqrt{n}}, n = 200.$

$\rho(1) = 0.427 \pm \frac{1.96}{\sqrt{200}} = (0.2884, 0.5655)$

Since it doesn't contain 0 and $\rho(1) \neq 0$, so we get that $\{X_t - \mu\}$ is not a white noise.

b. AR(2)

$\hat{\mu} = \bar{x} = 3.82$

Since $\rho(1) = \phi(1) + \phi(2)\rho(1)$ and $\rho(2) = \phi(1)\rho(1) + \phi(2)$,

we could get that $\phi(1) = \frac{\rho(1)-\rho(1)\rho(2)}{1-\rho(1)^2}$ = 0.274 and $\phi(2) = \rho(2) - \frac{\rho(1)^2-\rho(1)^2\rho(2)}{1-\rho(1)^2}$ = 0.358

$\gamma(0) = \phi_1\gamma(1) + \phi_2\gamma(2) + \sigma_Z^2$

$\Longrightarrow \sigma_z^2 = \gamma(0)(1 - \phi_1\rho(1) - \phi_2\rho(2)) = 0.82$

c. AR(2) model

$\alpha(2) = \phi_2 = 0.358$

$\alpha(1) = \frac{\phi_1}{1-\phi_2} = 0.427$

$\alpha(k) = 0$ for $k \neq 1, 2$