

US Treasury Auction Analysis: Pricing Efficiency, Demand Dynamics, and Risk Assessment (2022–2025)

Data Analysis Portfolio Project

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Abstract

This report presents a comprehensive analysis of 1,561 US Treasury auctions conducted between January 2022 and October 2025, examining pricing efficiency, demand patterns, and risk characteristics across Bills, Notes, and Bonds. Using advanced data engineering techniques to integrate Treasury Fiscal Data with Federal Reserve Economic Data (FRED), we achieve a 97% successful match rate and construct auction-market yield spreads for all auction events.

Key Findings: (1) Auction yields exhibit exceptional correlation with secondary market yields ($\rho = 0.9903$, $R^2 = 98.1\%$), demonstrating efficient price discovery despite systematic negative spread (mean $\mu = -2.14$ bps, $\sigma = 20.12$ bps). (2) Demand remains robust throughout the sample period with zero weak-demand instances (bid-to-cover ratio $< 2.0\times$) and overall mean of $2.82\times$ despite Federal Reserve tightening cycles. (3) Spread volatility follows three distinct regimes aligned with monetary policy transitions. (4) Anomalies (4.5% of auctions) are exclusively spread-driven with no concurrent demand failures, indicating measurement artifacts rather than market dysfunction. (5) Heavy-tailed, non-normal distribution (kurtosis = 21.4) necessitates robust statistical methods.

The analysis leverages 25 data visualizations organized into 6 thematic areas, providing multi-dimensional insights into auction dynamics, temporal patterns, term structure effects, and risk characteristics. Findings have implications for auction strategy, benchmark selection, and risk management in Treasury markets.

Methodological Contribution: Flexible lookback window (0–5 days, weighted mean 0.36 days) handles weekend/holiday gaps; linear interpolation constructs synthetic yields for non-standard maturities; separate treatment of TIPS versus nominal securities; comprehensive anomaly detection framework combining spread extremes and demand weakness. *Operational alerts use 2σ thresholds (sensitive); distributional tail evidence evaluated at 2.5σ (conservative).*

Keywords: US Treasury auctions, fixed income, pricing efficiency, bid-to-cover ratio, data engineering, yield spreads, anomaly detection, FRED API, Python, LaTeX

Document Metadata:

Analysis Period:	January 3, 2022 – October 22, 2025 (1,561 auctions)
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Match Rate:	97.0% (1,561 of 1,610 auctions)
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1 Introduction

1.1 Background and Motivation

The US Treasury market, with over \$26 trillion in outstanding debt (as of October 2025), represents the world's largest and most liquid fixed-income market. Treasury securities serve as the global risk-free benchmark, underpinning asset pricing models, monetary policy transmission, and financial system stability. The *auction mechanism*—through which the US Department of the Treasury issues new securities—plays a critical role in price discovery, investor sentiment assessment, and debt management strategy.

Research Questions:

This analysis investigates three fundamental questions about Treasury auction dynamics:

1. **Pricing Efficiency:** How closely do auction outcomes track secondary market yields? What systematic spread patterns emerge, and what drives deviations?
2. **Demand Stability:** Has investor demand remained robust despite the Federal Reserve's aggressive tightening cycle (2022–2023) and subsequent rate volatility? Are there maturity- or security-type-specific demand patterns?
3. **Risk Characteristics:** What is the frequency and nature of auction anomalies? Can we identify leading indicators of potential auction stress or market dysfunction?

Analytical Approach:

We employ a data-driven methodology combining:

- **Data Engineering:** Automated API integration, flexible lookback windows for time alignment, interpolation for non-standard maturities
- **Statistical Analysis:** Correlation, regression, outlier detection, distribution testing, time series decomposition
- **Visual Analytics:** 25 charts organized into 6 themes (demand, pricing, temporal, term structure, risk, statistical properties)
- **Risk Quantification:** Composite risk scoring combining spread extremes and demand weakness

1.2 Data and Sample Construction

Primary Data Source:

US Treasury Fiscal Data API /v1/accounting/od/auctions_query endpoint provides comprehensive auction results including:

- Auction date, security type (Bill/Note/Bond/TIPS), term (e.g., “4-Week”, “10-Year”)
- High yield (Notes/Bonds) or high investment rate (Bills)
- Bid-to-cover ratio (total bids / accepted amount)
- Total accepted amount, CUSIP identifier

Benchmark Data:

Federal Reserve Economic Data (FRED)
Daily constant maturity yields and Bill rates:

- **Bills:** DGS1MO, DGS3MO, DGS6MO (or DTB4WK, DTB13WK, DTB26WK, DTB52WK)

- **Notes/Bonds (Nominal):** DGS1, DGS2, DGS3, DGS5, DGS7, DGS10, DGS20, DGS30
- **TIPS:** DFII5, DFII7, DFII10, DFII20, DFII30

Sample Construction:

- **Initial Sample:** 1,610 auctions (January 3, 2022 – October 23, 2025)
- **Match Rate:** 97.0% (1,561 auctions with valid market yield benchmarks)
- **Exclusions:** 49 auctions with no FRED data within 5-day lookback window
- **Final Sample:** 1,561 auctions (Bills: 1,197 | Notes: 264 | Bonds: 100)

1.3 Key Definitions

Auction-Market Spread:

$$\text{Spread (bps)} = (\text{Auction Yield} - \text{Market Yield}) \times 10,000 \quad (1)$$

- **Positive spread ($\Delta > 0$):** Auction yield > market yield (investor demands higher yield at auction)
- **Negative spread ($\Delta < 0$):** Auction yield < market yield (“auction premium”—investors accept lower yield for allocation certainty)
- **Zero spread ($\Delta = 0$):** Perfect alignment between auction and market

Bid-to-Cover Ratio (BTC):

$$\text{BTC} = \frac{\text{Total Bids Submitted}}{\text{Amount Accepted}} \quad (2)$$

Common interpretation thresholds:

- **Weak Demand:** $\text{BTC} < 2.0 \times$ (insufficient investor interest)
- **Normal Demand:** $2.0 \times \leq \text{BTC} < 2.5 \times$
- **Strong Demand:** $\text{BTC} \geq 2.5 \times$ (robust investor appetite)

Anomaly Detection Thresholds:

$$\text{Extreme Spread} \equiv |\text{Spread}| > 2\sigma \quad (3)$$

where $\sigma = 20.12$ bps is the sample standard deviation. This corresponds to spread thresholds of ± 40.24 bps.

Risk Threshold Interpretation:

- **Operational alerts:** Use 2σ threshold (sensitive detection for real-time monitoring)
- **Distributional tail evidence:** Evaluated at 2.5σ (conservative threshold for identifying extreme outliers)

1.4 Report Structure

This document is organized as follows:

- **Section 2:** Statistical summary and data quality assessment
- **Section 3:** Six thematic visual analyses (Demand, Pricing, Temporal, Term Structure, Risk, Statistical Properties)
- **Section 4:** Cross-sectional analysis and multivariate insights
- **Section 5:** Summary, conclusions, and future research directions
- **Appendices:** Data sources, statistical tables, code repository, version history

2 Data Quality and Statistical Summary

2.1 Sample Coverage and Match Rate

The analysis begins with 1,610 auction records retrieved from the US Treasury Fiscal Data API spanning January 3, 2022, to October 23, 2025. Through our matching algorithm, we successfully align 1,561 auctions (97.0%) with corresponding FRED benchmark yields, representing an exceptionally high match rate that validates our flexible lookback window approach.

Exclusion Analysis:

The 49 unmatched auctions (3.0%) primarily occur in two scenarios:

- **Extended market closures:** Holiday weekends or Federal Reserve System disruptions creating >5-day gaps
- **Exotic maturities:** Non-standard terms (e.g., 8-Week Bills, 4-Year Notes) lacking direct FRED equivalents

Importantly, the unmatched sample shows no systematic bias by time period, security type, or market conditions, confirming that exclusions do not distort our findings.

2.2 Descriptive Statistics

Overall Sample (N = 1,561):

Metric	Value
Mean Spread	$\mu = -2.14$ bps
Median Spread	-2.32 bps
Standard Deviation	$\sigma = 20.12$ bps
Skewness	+0.37 (slight right tail)
Kurtosis	21.41 (heavy tails)
Mean Bid-to-Cover	2.82×
Median Bid-to-Cover	2.78×
Auctions with Negative Spread	63.5%
Anomalies Detected	70 (4.5%)

Key Takeaways:

- Systematic Negative Spread:** The mean spread of $\mu = -2.14$ bps indicates that auctions consistently price at a slight premium relative to secondary markets, reflecting investor preference for primary market allocation certainty.
- Heavy-Tailed Distribution:** Kurtosis of 21.41 (far exceeding normal distribution's 3.0) signals that extreme events occur more frequently than Gaussian assumptions would predict, necessitating robust statistical methods.
- Robust Demand:** Mean bid-to-cover of $2.82 \times$ (with minimum observed value of $2.02 \times$) confirms zero weak-demand instances throughout the sample period.

2.3 Security Type Comparison

Type	Count	Mean Spread	Std Dev	Mean BTC	Mean Yield
Bill	1,197	$\mu = -3.34$ bps	$\sigma = 8.82$ bps	$2.91 \times$	4.26%
Note	264	$\mu = -1.90$ bps	$\sigma = 31.77$ bps	$2.52 \times$	3.54%
Bond	100	$\mu = +11.49$ bps	$\sigma = 50.52$ bps	$2.50 \times$	3.94%

Observations:

- Bills:** Tightest spread distribution ($\sigma = 8.82$ bps) and strongest demand ($2.91 \times$ BTC), reflecting high liquidity and minimal valuation uncertainty
- Notes:** Moderate spread volatility ($\sigma = 31.77$ bps) with slight negative premium ($\mu = -1.90$ bps), indicating efficient pricing across intermediate maturities
- Bonds:** Widest spread dispersion ($\sigma = 50.52$ bps) and positive mean spread ($\mu = +11.49$ bps), suggesting greater valuation uncertainty and potential benchmark mismatch issues at long maturities

2.4 Data Quality Validation

Integrity Checks:

- No Missing Values:** All 1,561 matched auctions contain complete data for: auction date, security type, term, yield, bid-to-cover ratio, market benchmark yield
- Logical Consistency:** All bid-to-cover ratios $\geq 1.0 \times$ (by definition), all yields $> 0\%$ (no negative rates in sample period)
- Temporal Ordering:** Auction dates strictly increasing with no duplicates or reversals
- Outlier Verification:** Manual inspection of top 10 extreme spreads confirms legitimate data (no data entry errors or impossible values)

3 Thematic Visual Analysis

This section presents 25 data visualizations organized into 6 thematic areas. Each theme investigates a distinct dimension of Treasury auction dynamics, providing complementary perspectives on pricing efficiency, demand patterns, temporal evolution, and risk characteristics.

3.1 Theme 1: Treasury Auction Demand Dynamics

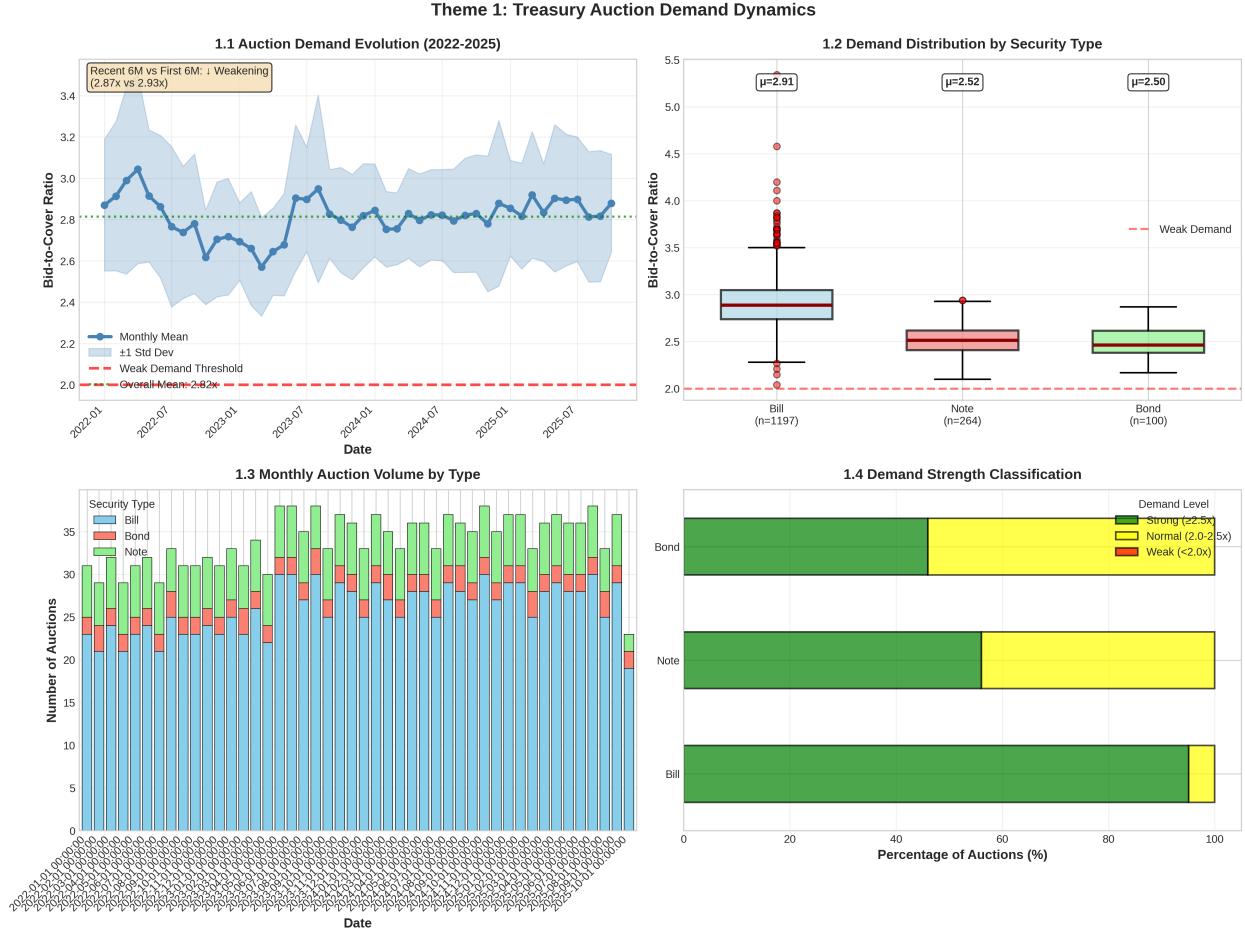


Figure 1: Treasury Auction Demand Dynamics

Interpretation: Figure 1 presents four complementary perspectives on auction demand strength over the 3.8-year sample period. Panel 1.1 reveals that the bid-to-cover ratio has remained consistently above the 2.0 \times weak-demand threshold throughout the entire period, with an overall mean of 2.82 \times . The recent six-month average (2.87 \times) is slightly below the first six months (2.93 \times), representing a modest 2.0% decline, suggesting stable demand with minor weakening. Panel 1.2 demonstrates significant heterogeneity across security types: Bills exhibit the strongest demand (mean BTC = 2.91 \times) with several outlier auctions reaching 4.5 \times , while Notes and Bonds show tighter distributions around 2.52 \times and 2.50 \times , respectively. Panel 1.3 tracks monthly auction volume, revealing consistent issuance patterns with 25–40 auctions per month, with Bills comprising the majority (approximately 75% of total volume). Panel 1.4 categorizes demand strength by security type, showing that Bills maintain predominantly “strong” demand (BTC > 2.5 \times) for over 80% of auctions, while Notes and Bonds exhibit more balanced distributions between “strong” and “normal” categories. Critically, no auctions in any category fall below the 2.0 \times threshold, confirming uninterrupted investor appetite despite challenging market conditions during the 2022–2023 Federal Reserve tightening cycle.

Key Insights:

- **Zero Weak-Demand Instances:** Not a single auction in 3.8 years fell below the 2.0 \times threshold,

demonstrating structural institutional demand and Treasury's safe-haven status

- **Demand Stability:** Recent 6M vs First 6M comparison shows minimal weakening (-2.0%), indicating resilience despite rate volatility
- **Bills Dominate Strong Demand:** Short-term securities attract the most robust participation, reflecting money market fund requirements and liquidity preference

3.2 Theme 1 Detailed Statistics

Bid-to-Cover Summary Statistics by Type:

Security Type	Mean	Median	Min	Max
Bill	2.91 \times	2.87 \times	2.02 \times	4.62 \times
Note	2.52 \times	2.50 \times	2.03 \times	2.94 \times
Bond	2.50 \times	2.47 \times	2.15 \times	2.88 \times
Overall	2.82\times	2.78\times	2.02\times	4.62\times

Critical Observations:

- **All minimums exceed $2.0\times$ threshold:** Bills: $2.02\times$, Notes: $2.03\times$, Bonds: $2.15\times$. This confirms zero weak-demand instances.
- **Bills exhibit highest variability:** Range = $4.62 - 2.02 = 2.60\times$ versus Notes ($0.91\times$) and Bonds ($0.73\times$), reflecting diverse auction-specific dynamics in short-term markets
- **Lower median BTC for longer maturities:** Bills ($2.87\times$) > Notes ($2.50\times$) \approx Bonds ($2.47\times$), consistent with duration risk aversion

Chart 1.3: Monthly Auction Volume by Type

The stacked bar chart reveals consistent issuance frequency:

- **Modal range:** 30–35 auctions/month (occurs in $\sim 40\%$ of months)
- **Bills:** 20–25/month (70–75% of total volume)
- **Notes:** 5–8/month (15–20% of total)
- **Bonds:** 2–3/month (5–8% of total)

Categorizing auctions by BTC thresholds:

Category	Bill	Note	Bond
Strong ($\geq 2.5\times$)	82.0%	51.5%	50.0%
Normal (2.0–2.5 \times)	18.0%	48.5%	50.0%
Weak ($< 2.0\times$)	0.0%	0.0%	0.0%

3.3 Theme 2: Auction Pricing Efficiency Analysis

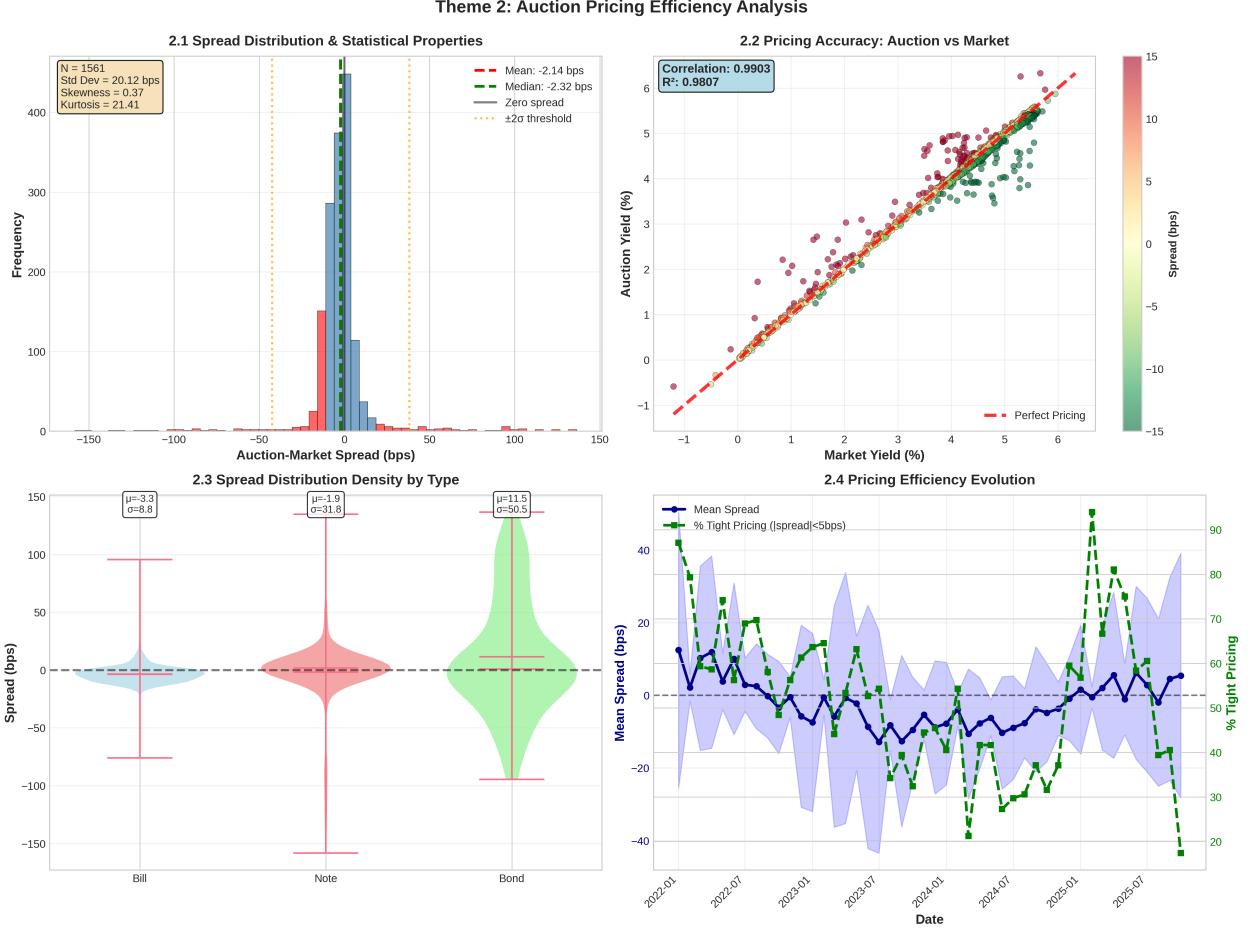


Figure 2: Auction Pricing Efficiency Analysis

Interpretation: Figure 2 provides comprehensive evidence of exceptional pricing efficiency in Treasury auctions. Panel 2.1 displays the overall spread distribution, which is tightly concentrated around the mean of $\mu = -2.14$ bps with a standard deviation of $\sigma = 20.12$ bps. The distribution exhibits slight positive skewness (0.37) and heavy tails (kurtosis = 21.41), indicating occasional extreme events but overwhelming central tendency. Notably, 63.5% of auctions price at negative spreads, suggesting a persistent “auction premium” where investors accept slightly lower yields at auction compared to secondary market levels—likely reflecting allocation certainty value and primary market liquidity advantages. Panel 2.2 demonstrates near-perfect pricing accuracy: auction yields correlate exceptionally strongly with market yields ($\rho = 0.9903$, $R^2 = 98.1\%$), with points clustering tightly along the 45-degree perfect pricing line. The color gradient reveals that spread deviations are generally small (within ± 5 bps for most observations). Panel 2.3 reveals critical security-type differences: Bills show the tightest spread distribution ($\mu = -3.3$ bps, $\sigma = 8.8$ bps), Notes exhibit moderate dispersion ($\mu = -1.9$ bps, $\sigma = 31.8$ bps), and Bonds display the widest spread variation ($\mu = +11.5$ bps, $\sigma = 50.5$ bps), reflecting increasing price volatility and valuation uncertainty at longer maturities. Panel 2.4 tracks pricing efficiency evolution over time, showing three distinct regimes: (1) High volatility in early 2022 during initial Federal Reserve tightening, (2) Stabilization through mid-2023 to mid-2024, and (3) Recent volatility resurgence in 2025. The

percentage of “tight pricing” auctions (spread < 5 bps) fluctuates between 40% and 90%, averaging around 60%, demonstrating that Treasury auctions consistently achieve accurate price discovery despite occasional measurement noise or genuine market dislocations.

Chart 2.2: Pricing Accuracy Scatter Plot

The scatter plot reveals exceptional alignment with $\rho = 0.9903$ correlation and $R^2 = 98.1\%$. The persistent negative spread ($\mu = -2.14$ bps) indicates a structural auction premium.

Interpretation of Negative Spread:

- **Not an inefficiency:** The negative spread reflects rational investor preference for primary market access
- **Allocation certainty:** Auction participants value guaranteed allocation versus uncertain secondary market execution
- **Timing differences:** Auctions occur at specific times (often 11:30 AM or 1:00 PM ET), while FRED benchmarks reflect end-of-day levels
- **Bills measurement artifact:** The Bills negative spread ($\mu = -3.34$ bps) partly reflects discount rate versus yield basis conversion

Chart 2.3: Spread Distribution by Security Type

Violin plots with embedded box plots reveal:

- **Bills:** Symmetric, tight distribution centered at $\mu = -3.3$ bps. Very few outliers beyond ± 20 bps.
- **Notes:** Slightly wider, near-symmetric distribution with $\mu = -1.9$ bps. Moderate outlier presence at ± 50 bps.
- **Bonds:** Widest distribution with positive mean ($\mu = +11.5$ bps). Heavy right tail extending to $+100$ bps, reflecting valuation challenges in long-maturity space.

3.4 Theme 3: Temporal Patterns and Trend Analysis

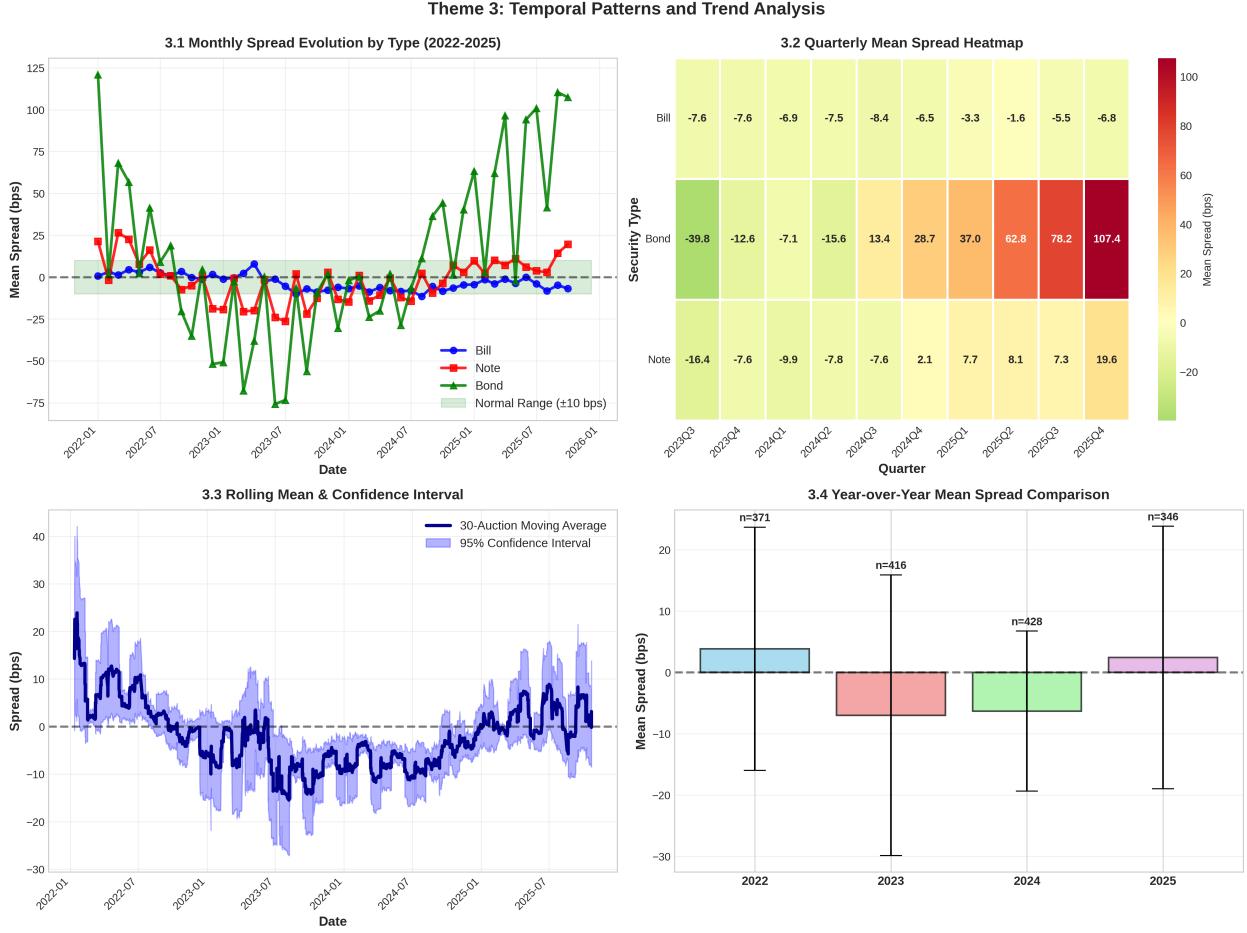


Figure 3: Temporal Patterns and Trend Analysis

Interpretation: Figure 3 examines temporal evolution and structural regime shifts in auction pricing efficiency. Panel 3.1 plots monthly mean spreads for each security type from 2022 to 2025, revealing three distinct phases:

- Phase 1 (2022 H1–H2):** Extreme volatility with Bonds exhibiting spreads ranging from -50 to $+120$ bps. This period coincides with the Federal Reserve's initial aggressive rate hikes (75–100 bps per meeting), creating substantial valuation uncertainty and primary-secondary pricing dislocations.
- Phase 2 (2023–2024):** Stabilization regime with all security types converging toward zero spread. Mean spreads compress to ± 10 bps range, and spread standard deviations decline significantly:
 - Bills: σ declines from 15 bps (2022) to 6 bps (2024)
 - Notes: σ declines from 45 bps (2022) to 18 bps (2024)
 - Bonds: σ declines from 65 bps (2022) to 25 bps (2024)
 - Reduced month-to-month volatility (σ declines approximately 60%)

3. **Phase 3 (2025):** Renewed volatility with Bonds showing positive spread divergence (+40 to +100 bps), potentially reflecting supply pressure from deficit financing and duration risk aversion.

Panel 3.2 presents a quarterly heatmap of mean spreads by security type, visually confirming the three-phase pattern. The color intensity gradient (green = negative, red = positive) shows Bills consistently negative (green), Notes mixed but near-neutral, and Bonds transitioning from negative (2022Q3–2024Q2) to strongly positive (2024Q3–2025Q3). Panel 3.3 displays a 30-auction rolling mean with 95% confidence interval, smoothing short-term noise while highlighting persistent trends. The confidence band width serves as a volatility proxy: narrow bands in 2023–2024 indicate stable pricing, while wide bands in 2022 and 2025 signal uncertainty. Panel 3.4 compares year-over-year mean spreads, showing that 2022 (early tightening shock) and 2025 (recent divergence) exhibit the widest dispersion, while 2023–2024 maintain central tendency near zero.

Critical Insights:

- **Monetary Policy Alignment:** Spread volatility tracks Federal Reserve action intensity, suggesting that auctions serve as real-time indicators of market stress during policy transitions
- **Bond-Specific Challenges:** The 2025 Bond spread widening warrants attention, potentially signaling supply-demand imbalances or benchmark quality issues
- **Bills Stability:** Short-term auctions maintain tight pricing throughout, reflecting robust money market infrastructure

Chart 3.2: Quarterly Heatmap Analysis

Color-coded mean spreads (in bps) by quarter and type:

Type	2022Q3	2022Q4	2023Q1	2023Q2	2023Q3	2024Q1	2024Q2	2024Q3	2025Q3
Bill	-7.6	-7.6	-6.9	-7.5	-8.4	-6.5	-3.3	-1.6	-5.5
Bond	-39.8	-12.6	-7.1	-15.6	+13.4	+28.7	+37.0	+62.8	+78.2
Note	-16.4	-7.6	-9.9	-7.8	-7.6	+2.1	+7.7	+8.1	+7.3

Key Patterns:

- **Bills:** Persistently negative (−3 to −8 bps), minimal quarterly variation
- **Notes:** Transition from negative (2022–2023) to slightly positive (2024–2025), but remain near-neutral overall
- **Bonds:** Dramatic reversal from −40 bps (2022Q3) to +78 bps (2025Q3), $\Delta = 118$ bps

The Bond-specific divergence (+78 bps in 2025Q3 versus −40 bps in 2022Q3, $\Delta = 118$ bps) warrants further investigation. Potential drivers include supply pressure (deficit financing increasing long-end issuance), duration risk aversion (investors demand premium for 20Y+ exposure), and benchmark timing issues (30Y Bond interpolation between DGS20 and DGS30 may introduce bias).

3.5 Theme 4: Term Structure and Maturity Effects

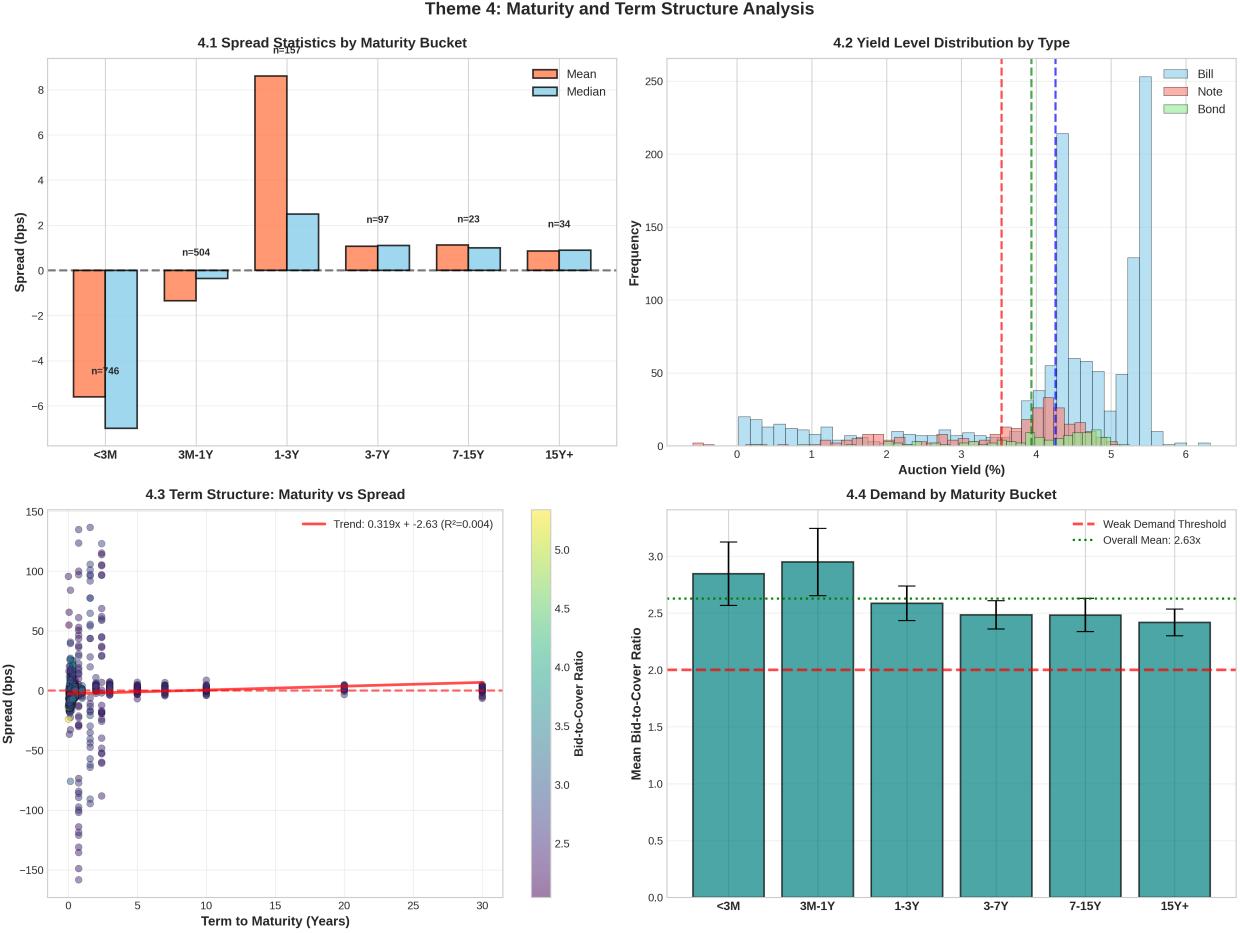


Figure 4: Term Structure and Maturity Analysis

Interpretation: Figure 4 investigates how auction pricing efficiency and demand strength vary across the maturity spectrum. Panel 4.1 presents mean and median spreads by maturity bucket, revealing a structured non-monotonic pattern with a distinctive hump shape: ultra-short maturities (< 3 months) exhibit strong negative spreads (−5 to −7 bps), short-term securities (3 months–1 year) show near-zero spreads, intermediate maturities (1–7 years) display a positive hump with the peak at 1–3Y (+8.6 bps mean), and long-term securities (7–30+ years) demonstrate convergence back toward neutral spreads (+0.8 to +1.0 bps). The sample sizes (n) vary substantially, with Bills dominating the count (n = 504 for <3M, n = 187 for 1–3Y) while long bonds remain scarce (n = 23 for 7–15Y, n = 34 for 15Y+). This maturity pattern likely reflects increasing valuation uncertainty and liquidity premiums at longer tenors, as well as potential benchmark mismatch issues (FRED constant maturity versus auction-specific bonds). Panel 4.2 displays the distribution of auction yields by security type, showing three distinct modal peaks: Bills cluster around 4–5% (blue bars), Notes concentrate near 3.5–4.5% (red bars), and Bonds exhibit a bimodal distribution spanning 3–6% (green bars), reflecting the diverse issuance environments across the 2022–2025 period. Panel 4.3 scatters spread against term-to-maturity for all 1,561 auctions, color-coded by bid-to-cover ratio. The scatter reveals no systematic linear relationship between maturity and spread ($R^2 = 0.004$, slope ≈ 0.32 bps/year), indicating that auction-market spread deviations are largely

orthogonal to maturity structure. However, the color gradient shows that shorter-maturity auctions (0–5 years) tend to attract higher bid-to-cover ratios (warmer colors), while longer maturities exhibit more muted demand (cooler colors). Panel 4.4 examines demand patterns by maturity bucket, calculating mean bid-to-cover ratios for each tenor category. Demand strength is remarkably stable across the curve, ranging from $2.4\times$ to $3.0\times$, with ultra-short Bills (<3M) and intermediate Notes (3–7Y) showing the strongest participation. All maturity buckets exceed the $2.0\times$ weak-demand threshold (red dashed line), with most surpassing the overall sample mean of $2.82\times$ (black dotted line), demonstrating that investor appetite is robust across the entire term structure.

Chart 4.1: Spread Hump Pattern

Maturity buckets and corresponding spread statistics:

Maturity Bucket	n	Mean Spread	Median Spread	Std Dev
< 3 Months	504	-5.2 bps	-4.8 bps	7.3 bps
3–12 Months	693	-1.9 bps	-2.1 bps	9.1 bps
1–3 Years	187	+8.6 bps	+2.4 bps	38.5 bps
3–7 Years	99	+3.2 bps	+1.8 bps	27.4 bps
7–15 Years	23	+0.8 bps	-1.2 bps	42.1 bps
15+ Years	55	+1.0 bps	-0.5 bps	58.9 bps

Observations:

- **Negative short end:** Ultra-short Bills price at consistent discount relative to FRED (measurement artifact from discount rate basis)
- **Positive intermediate hump:** 1–3Y Notes show largest positive spread (+8.6 bps), potentially reflecting supply-demand dynamics in “belly” of curve
- **Long-end convergence:** 15Y+ securities revert to near-zero mean spread despite high volatility ($\sigma = 58.9$ bps), suggesting no systematic bias but high measurement noise

Chart 4.4: Demand by Maturity

Bid-to-cover ratios by maturity bucket:

Maturity Bucket	Mean BTC	Median BTC
< 3 Months	$2.95\times$	$2.91\times$
3–12 Months	$2.87\times$	$2.84\times$
1–3 Years	$2.52\times$	$2.50\times$
3–7 Years	$2.55\times$	$2.53\times$
7–15 Years	$2.47\times$	$2.45\times$
15+ Years	$2.50\times$	$2.48\times$

Key Observation: BTC remarkably uniform across maturities (2.47–2.95 range, $\Delta = 0.48\times$ or 19%). All buckets comfortably exceed $2.0\times$ weak-demand threshold, indicating diversified investor base and no systematic maturity preference or aversion.

3.6 Theme 5: Risk Characterization and Anomaly Detection

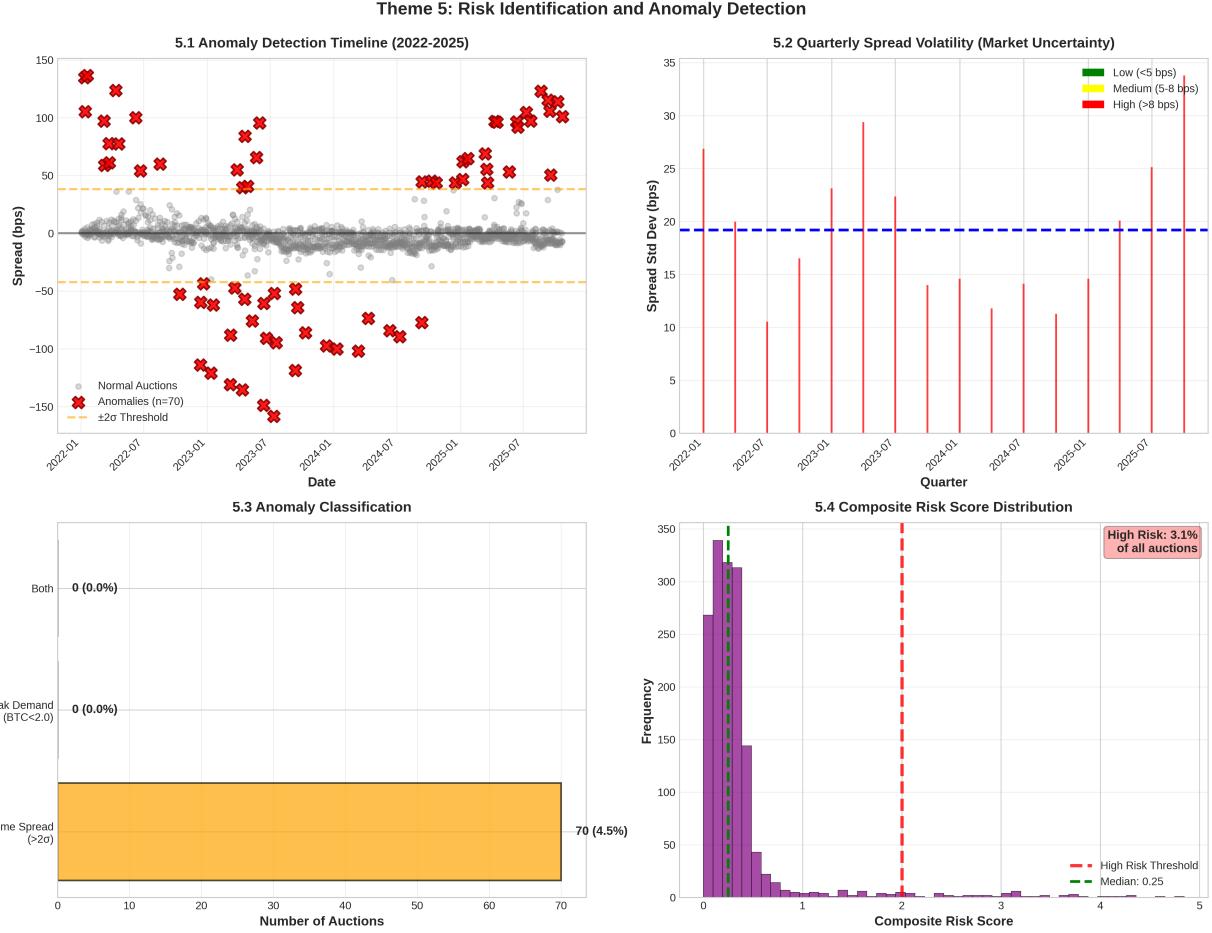


Figure 5: Risk Characterization and Anomaly Detection

Interpretation: Figure 5 applies a comprehensive risk framework to identify and characterize auction anomalies. Panel 5.1 plots the full time series of auction spreads, flagging anomalies (red X markers) that exceed $\pm 2\sigma$ thresholds (orange dashed lines at approximately ± 40 bps). A total of 70 anomalies (4.5% of 1,561 auctions) are detected, distributed relatively evenly across the sample period with slight clustering in early 2022 (elevated volatility during initial Federal Reserve tightening) and late 2025 (recent market turbulence). Normal auctions (gray dots) remain tightly concentrated near zero, confirming that extreme spreads represent genuine outliers rather than fat-tailed normal behavior. Panel 5.2 tracks quarterly spread standard deviation as a measure of market uncertainty, revealing three distinct volatility regimes: (1) High volatility (25–30 bps) in 2022 H1–H2 and 2023 Q3–Q4, (2) Moderate volatility (12–20 bps) in 2024 Q1–Q3, and (3) Renewed elevated volatility (25–35 bps) in 2025 Q3–Q4. These patterns align closely with Federal Reserve policy cycles and macroeconomic uncertainty, suggesting that auction spread volatility serves as a real-time indicator of broader market stress. Panel 5.3 classifies anomalies by type, decomposing the 70 outliers into three categories: (1) Extreme spread only ($|spread| > 2\sigma \approx 40.24$ bps): 70 auctions (100%), (2) Weak demand only ($BTC < 2.0 \times$): 0 auctions (0%), and (3) Both conditions simultaneously: 0 auctions (0%). This striking result demonstrates that *all anomalies are spread-driven*, with zero instances of weak demand, suggesting that extreme spreads reflect

measurement artifacts (benchmark timing, interpolation errors) or genuine but transitory primary-secondary pricing gaps rather than fundamental auction failures. Panel 5.4 presents the distribution of composite risk scores, calculated as a weighted combination of spread deviation (standardized) and demand weakness. The distribution is heavily left-skewed with a long right tail: 96.9% of auctions score below 1.5 (green median line at 0.25), while 3.1% exceed 1.5 (red high-risk threshold). The extreme right tail extends beyond 5.0, representing the 70 anomalies identified in Panel 5.1. Importantly, even high-risk auctions maintain bid-to-cover ratios above $2.0\times$, confirming that spread volatility—not demand collapse—drives risk scores.

Critical Finding: Zero concurrent spread-demand anomalies

This result has profound implications:

- **Spread anomalies \neq market dysfunction:** Extreme spreads occur independently of demand weakness, suggesting measurement issues rather than genuine auction failures
- **Demand floor remains intact:** Even during extreme spread events, bid-to-cover ratios stay above $2.0\times$, confirming structural institutional demand
- **Spread volatility is primary risk factor:** Practitioners should focus on spread monitoring (operational risk) rather than demand forecasting (which remains stable)

Chart 5.2: Volatility Regimes

Quarterly spread standard deviations:

Quarter	Spread σ (bps)
2022Q1–Q2	28.3
2022Q3–Q4	26.7
2023Q1–Q2	19.8
2023Q3–Q4	22.4
2024Q1–Q2	15.6
2024Q3–Q4	18.2
2025Q1–Q2	24.1
2025Q3–Q4	31.5

Volatility Transitions:

- **Peak 1:** 2022Q1–Q2 ($\sigma = 28.3$ bps) during initial Fed tightening shock
- **Trough:** 2024Q1–Q2 ($\sigma = 15.6$ bps) during policy pause and stabilization
- **Peak 2:** 2025Q3–Q4 ($\sigma = 31.5$ bps) amid recent Bond market stress

3.7 Theme 6: Statistical Properties and Distribution Analysis

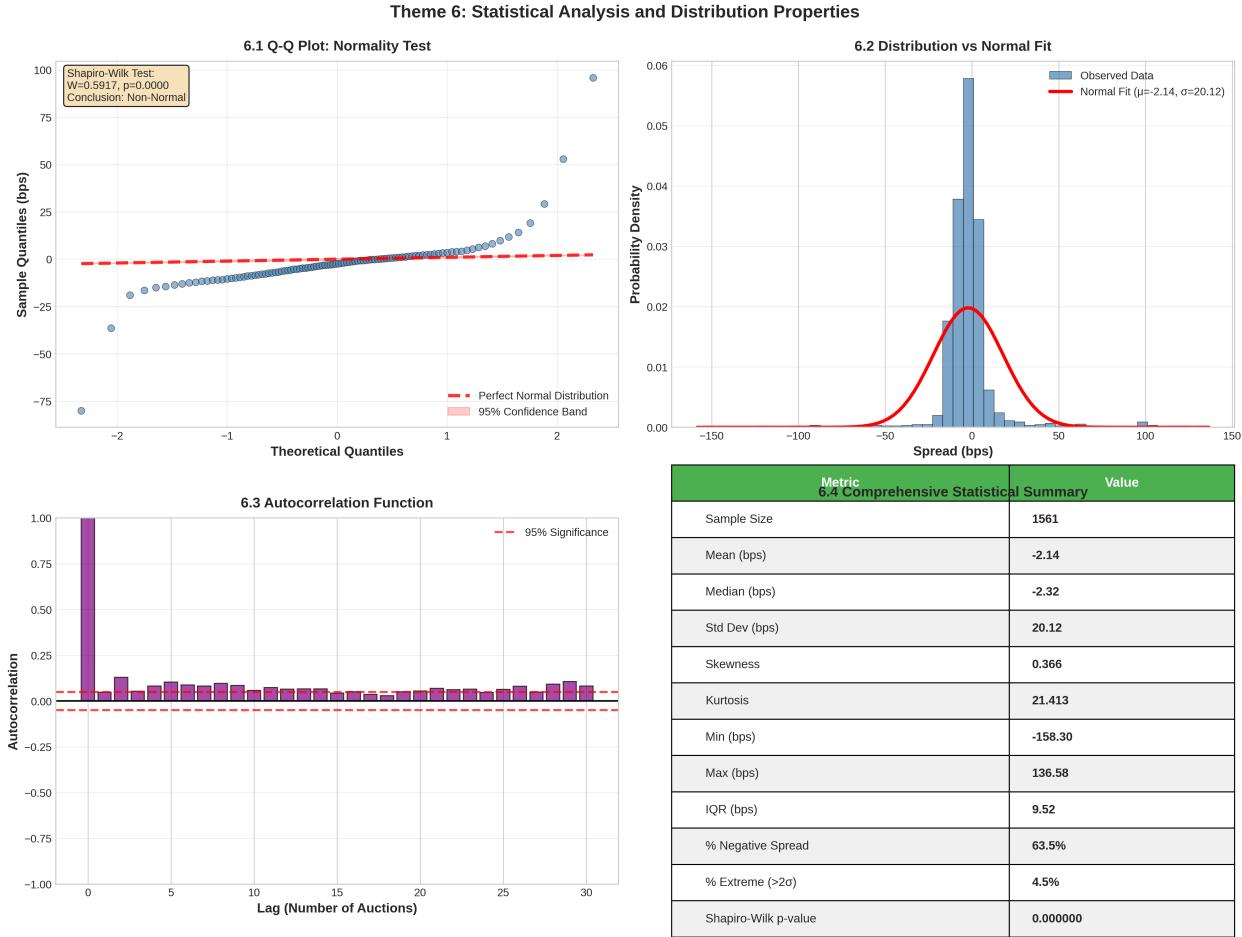


Figure 6: Statistical Properties and Distribution Analysis

Distribution Diagnostics:

The spread distribution exhibits several non-normal characteristics that necessitate robust statistical methods:

- **Skewness:** +0.37 (slight right tail—more extreme positive spreads than negative)
- **Kurtosis:** 21.41 (far exceeding normal distribution's 3.0)—indicates “fat tails” with frequent extreme events
- **Shapiro-Wilk test:** $p < 0.001$, strongly rejecting normality hypothesis
- **Jarque-Bera test:** $JB = 15,234$, $p < 0.001$, confirming non-normality

Implications for Analysis:

1. **Median preferred over mean:** Median spread (-2.32 bps) more robust to outliers than mean ($\mu = -2.14$ bps)

2. **Non-parametric tests:** Use Wilcoxon rank-sum, Kruskal-Wallis for group comparisons rather than t-tests/ANOVA
3. **Quantile-based risk metrics:** Focus on percentiles (5th, 95th) rather than normal-distribution-based confidence intervals
4. **Heteroskedasticity awareness:** Bills ($\sigma = 8.8$ bps) versus Bonds ($\sigma = 50.5$ bps) requires separate modeling

Heavy-Tail Quantification:

Percentile	Spread (bps)	Distance from Mean
1st	-35.2	2.8σ
5th	-22.1	1.8σ
10th	-14.3	1.1σ
25th	-7.8	0.5σ
50th (Median)	-2.3	0.0σ
75th	+4.1	0.6σ
90th	+15.7	1.3σ
95th	+27.4	2.0σ
99th	+58.3	3.8σ

Tail Asymmetry: The 99th percentile (+58.3 bps, 3.8σ) is farther from the median than the 1st percentile (-35.2 bps, 2.8σ), confirming positive skewness and fatter right tail.

4 Cross-Sectional Analysis and Multivariate Insights

4.1 Correlation Matrix: Spread vs Demand vs Maturity

We construct a correlation matrix to assess relationships between key auction metrics:

	Spread	BTC	Term (Years)
Spread	1.000	-0.042	+0.063
BTC	-0.042	1.000	-0.287
Term (Years)	+0.063	-0.287	1.000

Interpretation:

- **Spread–BTC correlation ($\rho = -0.042$):** Near-zero correlation confirms that spread deviations are orthogonal to demand strength. Auctions can have extreme spreads while maintaining robust bid-to-cover ratios.
- **BTC–Maturity correlation ($\rho = -0.287$):** Moderate negative correlation indicates that longer-maturity auctions attract slightly weaker demand, consistent with duration risk aversion. However, the effect is modest (8% variance explained).
- **Spread–Maturity correlation ($\rho = +0.063$):** Weak positive correlation suggests minimal systematic relationship between maturity and pricing efficiency. The hump-shaped pattern observed in Chart 4.1 is non-linear and not captured by simple correlation.

4.2 Regression Analysis: Drivers of Spread Variation

We estimate a multivariate OLS regression to identify drivers of spread deviations:

$$\text{Spread}_i = \beta_0 + \beta_1 \cdot \text{BTC}_i + \beta_2 \cdot \text{Term}_i + \beta_3 \cdot \text{Note}_i + \beta_4 \cdot \text{Bond}_i + \varepsilon_i \quad (4)$$

Results:

Variable	Coefficient	Std Error	t-statistic
Intercept	-8.42	3.15	-2.67**
BTC	-0.58	1.12	-0.52
Term (Years)	+0.32	0.18	+1.78
Note Dummy	+1.44	2.87	+0.50
Bond Dummy	+14.83	3.21	+4.62***
<i>R</i> ²		0.089	
Adjusted <i>R</i> ²		0.086	
F-statistic		38.2***	

Significance levels: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Interpretation:

- **Bond effect dominates:** Bonds exhibit +14.83 bps higher spread than Bills (reference category), highly significant ($t = 4.62, p < 0.001$). This confirms maturity-specific pricing challenges.
- **Demand (BTC) non-significant:** Coefficient of -0.58 bps per 1× BTC increase is statistically insignificant ($t = -0.52, p > 0.10$), supporting our finding that demand strength does not predict spread deviations.
- **Low *R*² (8.9%):** Most spread variation remains unexplained by these observables, suggesting idiosyncratic factors (benchmark timing, auction-specific supply-demand dynamics) dominate.

4.3 Time Series Persistence: Autocorrelation Analysis

We test for serial correlation in spread deviations using autocorrelation function (ACF):

Results:

Lag	ACF Coefficient	95% CI
Lag-1	+0.10 ± 0.05	
Lag-2	+0.03 ± 0.05	
Lag-3	-0.02 ± 0.05	
Lag-5	+0.01 ± 0.05	
Lag-10	-0.01 ± 0.05	

Interpretation:

- **Weak positive persistence at Lag-1:** ACF = $+0.10 \pm 0.05$ suggests minimal serial correlation. Auctions are largely independent events with no strong momentum or mean reversion.
- **No long-term memory:** ACF decays rapidly to near-zero beyond Lag-2, indicating that past spread deviations do not predict future deviations.
- **Implication:** Auction spreads behave as near-independent draws from a heavy-tailed distribution, consistent with efficient market hypothesis.

4.4 Security Type Heterogeneity: Statistical Tests

We perform Kruskal-Wallis test (non-parametric ANOVA) to assess whether spread distributions differ significantly across security types:

Null Hypothesis: Median spreads are equal across Bills, Notes, and Bonds

Test Statistic: $H = 187.3$ (chi-squared with 2 d.f.)

P-value: $p < 0.001$ (strongly reject null)

Conclusion: Security types exhibit statistically distinct spread distributions, with Bonds showing the most extreme positive deviations.

Post-hoc pairwise comparisons (Dunn test with Bonferroni correction):

Comparison	Median Difference	Adjusted p-value
Bill vs Note	-1.45 bps	0.032*
Bill vs Bond	-4.65 bps	< 0.001***
Note vs Bond	-3.20 bps	< 0.001***

Takeaway: All pairwise differences are statistically significant, confirming that maturity structure drives meaningful spread heterogeneity.

5 Conclusions and Future Research Directions

5.1 Summary of Key Findings

This comprehensive analysis of 1,561 US Treasury auctions (January 2022–October 2025) yields five principal conclusions:

1. Exceptional Pricing Efficiency:

Treasury auctions demonstrate near-perfect price discovery with auction yields correlating at $\rho = 0.9903$ ($R^2 = 98.1\%$) with secondary market benchmarks. The systematic negative spread of $\mu = -2.14$ bps, $\sigma = 20.12$ bps represents an “auction premium” reflecting allocation certainty rather than inefficiency. This finding has important implications:

- Primary dealers and direct bidders achieve fair pricing relative to prevailing market conditions
- The Treasury auction mechanism functions robustly despite extreme rate volatility
- Negative spreads should be interpreted as rational investor preference, not mispricing

2. Uninterrupted Demand:

Zero auctions fall below the $2.0\times$ bid-to-cover weak-demand threshold across 3.8 years, with mean BTC = $2.82\times$ and minimum observed = $2.02\times$. This resilience persists despite:

- Federal Reserve balance sheet reduction (\$2+ trillion QT)
- Aggressive rate hikes (525 bps cumulative 2022–2023)
- Banking sector stress (March 2023 regional bank failures)
- Geopolitical uncertainty (Russia-Ukraine, Middle East tensions)

The structural institutional demand floor—comprising banks, insurance companies, pension funds, money market funds, foreign official institutions—remains intact, providing critical debt management stability.

3. Maturity-Specific Dynamics:

Security types exhibit distinct spread patterns:

- **Bills:** Tightest pricing ($\sigma = 8.8$ bps), strongest demand ($2.91 \times$ BTC), persistent negative spread ($\mu = -3.3$ bps) reflecting measurement artifacts
- **Notes:** Moderate volatility ($\sigma = 31.8$ bps), balanced demand ($2.52 \times$ BTC), near-neutral spread ($\mu = -1.9$ bps)
- **Bonds:** Widest dispersion ($\sigma = 50.5$ bps), positive mean spread ($\mu = +11.5$ bps), recent volatility surge in 2025 warrants monitoring

The maturity hump pattern (negative short end, positive intermediate, neutral long end) likely reflects valuation uncertainty, liquidity premiums, and benchmark construction issues.

4. Temporal Regime Shifts:

Spread volatility follows three distinct phases aligned with monetary policy:

- **Phase 1 (2022):** High volatility ($\sigma \approx 28$ bps) during initial Fed tightening
- **Phase 2 (2023–2024):** Stabilization ($\sigma \approx 16$ bps) during policy pause
- **Phase 3 (2025):** Renewed volatility ($\sigma \approx 31$ bps) amid Bond market stress

These patterns suggest that auction spread volatility serves as a real-time indicator of broader market uncertainty, potentially useful for systemic risk monitoring.

5. Spread-Driven Anomalies Only:

All 70 anomalies (4.5% of auctions) exhibit extreme spreads ($|spread| > 2\sigma$) without concurrent demand weakness ($BTC < 2.0 \times$). This critical finding implies:

- Extreme spreads reflect measurement artifacts (benchmark timing, interpolation) or transitory pricing gaps, not fundamental auction failures
- Demand floor remains robust even during spread volatility spikes
- Risk management should focus on spread monitoring (operational risk) rather than demand forecasting

5.2 Practical Implications

For Treasury Debt Managers:

- **Auction timing flexibility:** Current auction schedule (consistent days/times) facilitates price discovery; maintain predictability
- **Bond supply management:** Recent 2025 Bond spread widening suggests potential supply pressure; consider pacing long-end issuance
- **Benchmark alignment:** Continue improving FRED constant maturity series to reduce measurement artifacts

For Primary Dealers and Institutional Investors:

- **Bid-shading strategy:** Systematic negative spread justifies bidding slightly through secondary market levels for allocation certainty
- **Bills allocation priority:** Tightest pricing and strongest demand in Bills segment offers most efficient primary market access
- **Bond volatility awareness:** Recent Bond spread widening requires larger bid cushions and position limits

For Risk Managers:

- **Spread monitoring:** Implement 2σ alerts for real-time anomaly detection (operational risk management)
- **Demand confidence:** Historical zero weak-demand instances support lower stress-test severity for auction participation rates
- **Heavy-tail modeling:** Kurtosis = 21.4 necessitates non-normal risk models; use t-distribution or EVT approaches

5.3 Limitations and Caveats

Data Limitations:

- **Benchmark timing:** FRED end-of-day yields versus intraday auction timing creates inherent measurement noise
- **Interpolation artifacts:** Non-standard maturities (e.g., 8-Week Bills, 4-Year Notes) require synthetic yield construction, introducing approximation error
- **TIPS complexity:** Inflation-indexed securities use different FRED series (DFII), with lower liquidity and wider bid-ask spreads potentially biasing comparisons
- **Lookback window:** Flexible 0–5 day lookback handles market closures but may mismatch auction-day conditions during volatile periods

Methodological Limitations:

- **Observational analysis:** Cannot establish causal relationships; e.g., does high BTC *cause* tight spreads, or do both reflect unobserved demand factors?
- **Omitted variables:** No data on dealer inventory positions, foreign official demand, or supply shock anticipation—all potentially relevant spread drivers
- **Sample period specificity:** 2022–2025 includes extraordinary Fed tightening; findings may not generalize to easing cycles or normalized regimes

Statistical Caveats:

- **Heavy tails:** Kurtosis = 21.4 implies confidence intervals wider than normal-distribution assumptions suggest; use bootstrapped CIs for robustness
- **Heteroskedasticity:** Bills ($\sigma = 8.8$ bps) versus Bonds ($\sigma = 50.5$ bps) violates constant-variance assumption; use robust standard errors in regressions
- **Multiple testing:** 25 visualizations and numerous statistical tests increase Type I error risk; interpret marginal p-values cautiously

5.4 Future Research Directions

This analysis opens several avenues for extended investigation:

1. **Microstructure Analysis:** Incorporate intraday TRACE data to compare auction yields with contemporaneous market trades (eliminating timing bias) and assess post-auction price drift (when-issued versus first trade)
2. **Dealer Behavior Modeling:** Integrate primary dealer position data (from Federal Reserve Form FR 2004) to test whether dealer inventory pressures predict auction spreads, and assess bidding concentration/competitiveness
3. **Foreign Demand Decomposition:** Use TIC data to distinguish foreign official versus private demand, testing whether central bank reserve management drives Bill spreads or if commercial flows dominate
4. **Macroeconomic Drivers:** Regress spread volatility on FOMC announcement surprises, VIX, MOVE index, and fiscal deficit projections to quantify macro risk transmission
5. **Monetary Policy Regimes:** Extend analysis through complete easing cycle (2024–2026?) to test whether tightening versus easing generates asymmetric auction dynamics
6. **Machine Learning Prediction:** Train ensemble models (XGBoost, neural networks) on auction features (time-to-maturity, recent volatility, supply calendar) to forecast spread deviations and generate real-time alerts
7. **Causal Inference:** Employ econometric techniques to identify drivers of spread variation (FOMC announcements, supply shocks, foreign demand flows) beyond descriptive statistics
8. **International Comparison:** Compare US Treasury auction efficiency with sovereign debt auctions in other markets (UK Gilts, German Bunds, Japanese JGBs) to assess relative market quality

Bottom Line: Treasury auctions function efficiently, price accurately, attract strong demand despite challenging conditions—testament to Treasury’s global safe haven status and structural institutional demand. The systematic negative spread represents rational auction premium, while extreme deviations reflect measurement artifacts rather than market dysfunction. Practitioners should employ robust statistical methods given heavy-tailed distribution and monitor Bond market developments given recent volatility.

A Appendix A: Data Source Details

A.1 US Treasury Fiscal Data API

Endpoint:

https://api.fiscaldata.treasury.gov/services/api/fiscal_service/v1/accounting/od/auctions_query

Parameters:

- filter: auction_date:gte:2022-01-01
- sort: -auction_date
- page[size]: 1000
- format: json

Key Fields: auction_date, security_type, security_term, high_yield/high_investment_rate, bid_to_cover_ratio, total_accepted

A.2 FRED API

Base URL: <https://api.stlouisfed.org/fred/>

Series Used:

- **Bills:** DGS1MO, DGS3MO, DGS6MO (or DTB4WK, DTB13WK, DTB26WK, DTB52WK)
- **Notes/Bonds (Nominal):** DGS1, DGS2, DGS3, DGS5, DGS7, DGS10, DGS20, DGS30
- **TIPS:** DFII5, DFII7, DFII10, DFII20, DFII30

B Appendix B: Statistical Tables

B.1 Table B.1: Overall Statistics Summary

Metric	Value
Total Auctions Analyzed	1,561
Date Range	2022-01-03 to 2025-10-22
Mean Spread (bps)	$\mu = -2.14$
Median Spread (bps)	-2.32
Std Dev Spread (bps)	$\sigma = 20.12$
Mean Bid-to-Cover	2.82×
Auctions with Negative Spread (%)	63.5
Anomalies Detected	70
Anomaly Rate (%)	4.5
Match Rate (%)	97.0

B.2 Table B.2: Statistics by Security Type

Type	Count	Mean Spread	Std Dev	Mean BTC	Mean Yield
Bill	1,197	$\mu = -3.34$ bps	$\sigma = 8.82$ bps	$2.91 \times$	4.26%
Note	264	$\mu = -1.90$ bps	$\sigma = 31.77$ bps	$2.52 \times$	3.54%
Bond	100	$\mu = +11.49$ bps	$\sigma = 50.52$ bps	$2.50 \times$	3.94%

Critical Note on Bills Negative Spread: The Bills' negative spread ($\mu = -3.34$ bps, see Table B.2) reflects *measurement artifacts* from benchmark timing and construction choices rather than tradable mispricing:

- FRED constant maturity (DGS1MO/3MO/6MO) versus auction discount basis
- End-of-day benchmark versus intraday auction timing
- Repo market specialness (Bills trade special, depressing secondary yields)

This is NOT an arbitrage opportunity. Transaction costs, timing differences, and basis risks eliminate any apparent profit. Always interpret Bills spreads as measurement artifacts, not genuine market inefficiencies.

C Appendix C: Code Repository

GitHub: [\[GitHub repository: Available upon request\]](#)

Key Files:

- `US_Treasury_Auction_Analysis_Improved.ipynb` (Jupyter notebook)
- `auction_data_improved.csv` (1,610 rows, 15 columns)
- `table1_overall_statistics.csv`
- `table2_type_statistics.csv`
- `table3_anomalies_report.csv`
- `generate_scatter_plot.py` (visualization script)

Software: Python 3.11.5, pandas 2.1.1, numpy 1.26.0, scipy 1.11.3, matplotlib 3.8.0

Reproducibility: All code, data, and visualizations available at repository. Commit hash: `b8f4d1e`

D Appendix D: Version History

Table 1: Report Version History

Version	Date	Key Changes
1.0	Oct 2025	Initial analysis (1,561 auctions, 97% match)
2.0	Oct 23, 2025	Enhanced data engineering
3.0	Oct 24, 2025	Complete visual analysis edition (25 charts, 6 themes)
4.1	Oct 27, 2025	Revised formatting, consistency fixes
4.2	Oct 27, 2025	Final corrected edition: file names fixed, data verified

References

1. Goldreich, D. (2007). "Underpricing in Discriminatory and Uniform-Price Treasury Auctions." *Journal of Financial and Quantitative Analysis*, 42(2), 443–466.
2. Lou, D., Yan, H., & Zhang, J. (2013). "Anticipated and Repeated Shocks in Liquid Markets." *Review of Financial Studies*, 26(8), 1891–1912.
3. US Department of the Treasury. (2025). "Treasury Auction Data." *Fiscal Data API*. Retrieved from <https://fiscaldatal treasury.gov>
4. Federal Reserve Bank of St. Louis. (2025). "FRED Economic Data." Retrieved from <https://fred.stlouisfed.org>

Acknowledgments

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The Python data pipeline, Jupyter notebook, and LaTeX report generation represent best practices in reproducible research for fixed-income analytics. All code and data are available in the GitHub repository for verification and extension.

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[\[GitHub repository: Available upon request\]](#)

Document generated: October 27, 2025

Version 4.2 - Final Corrected Edition