Rendering Participating Media

Data Visualization Seminar

Barnabás Börcsök

Chair of Computer Graphics and Visualization
Department of Informatics
Technical University of Munich

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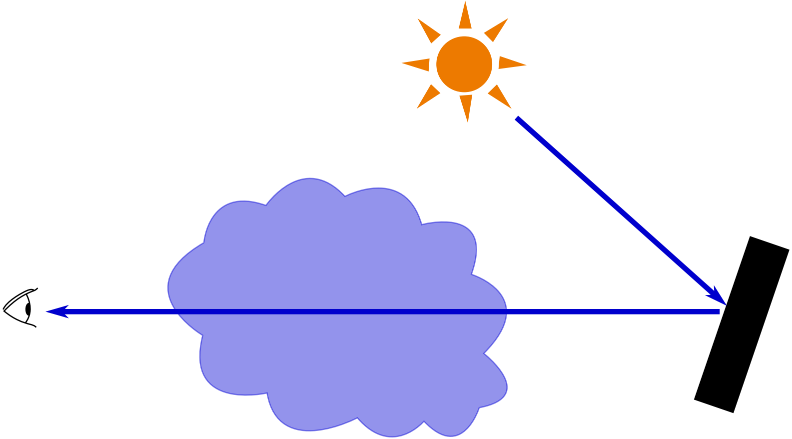
Motivation

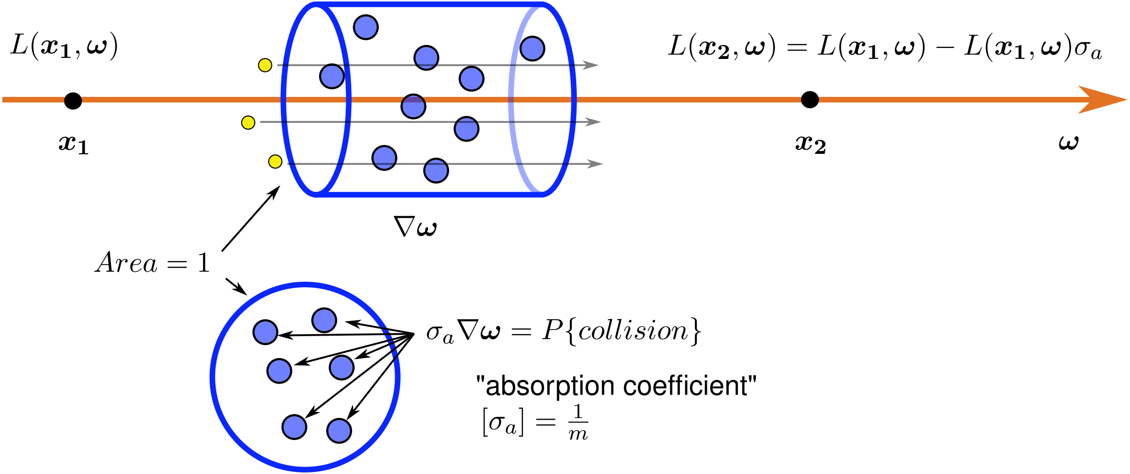



Motivation

Propagation of light in a medium



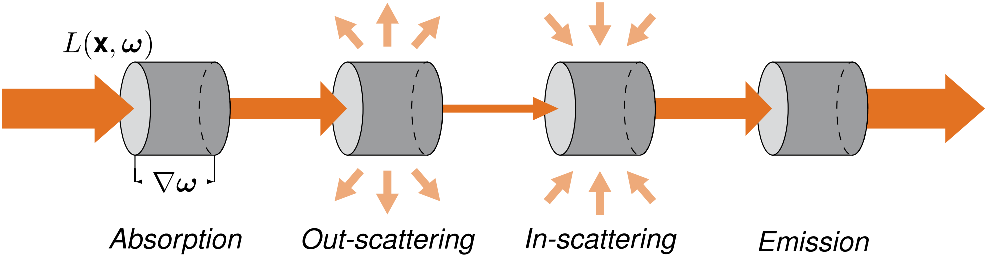
Change of radiance in a differential volume



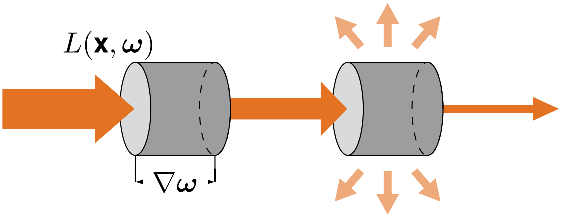
"absorption coefficient"

$[\sigma_a] = \frac{1}{m}$

Possible interactions
between the volume and the light traveling through the medium



Summing up the losses



Absorption *Out-scattering*

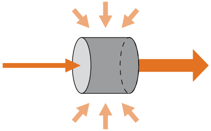
σ_a : Absorption coefficient
σ_s : Scattering coefficient
$\sigma_a + \sigma_s = \sigma_t$: Extinction coefficient

We lose $\sigma_t(\boldsymbol{x})L(\boldsymbol{x}, \boldsymbol{\omega})$ radiance due to *absorption* and *out-scattering*.

$\sigma_t \implies$ Homogeneous

$\sigma_t(\boldsymbol{x}) \implies$ Heterogeneous

In-scattered radiance



$$L_s(\boldsymbol{x}, \boldsymbol{\omega}) = \int_{S^2} f_p(\boldsymbol{x}, \boldsymbol{\omega}, \boldsymbol{\omega}') L_i(\boldsymbol{x}, \boldsymbol{\omega}') d\boldsymbol{\omega}'$$

Phase function

$f_p(\boldsymbol{x}, \boldsymbol{\omega}, \boldsymbol{\omega}')$

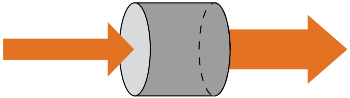
$\approx BSDF$

(in surface rendering)

- scattering at point $\boldsymbol{x}$, given incident ($\boldsymbol{\omega}$) and outgoing ($\boldsymbol{\omega}'$) directions
- $\int_{S^2} f_p = 1$
- $f_p(\theta)\big|_{\theta = \measuredangle(\boldsymbol{\omega}, \boldsymbol{\omega}')}$
- $f_p(\boldsymbol{x}, \boldsymbol{\omega}, \boldsymbol{\omega}') = 1/(4\pi)$, if the medium is *isotropic*

(otherwise, *anisotropic*)

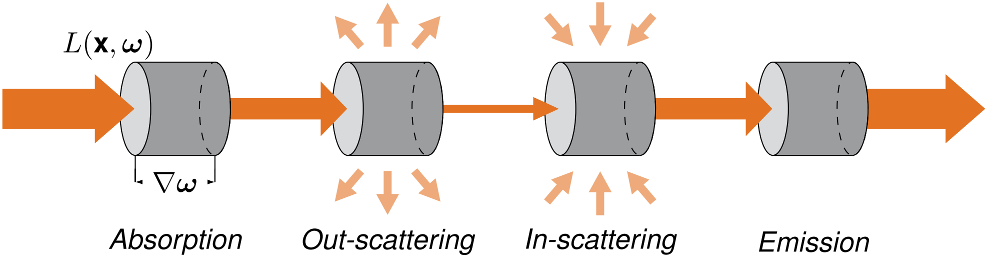
Emission



$$L_e(\boldsymbol{x}, \boldsymbol{\omega})$$

$$\sigma_a(\boldsymbol{x}) L_e(\boldsymbol{x}, \boldsymbol{\omega})$$

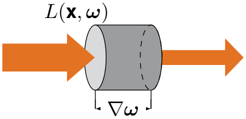
Assembling all the parts



- Loses $\sigma_a L(x, \omega)$ due to absorption
- Loses $\sigma_s L(x, \omega)$ due to out-scattering
- Gains $\sigma_s L_i(x, \omega)$ due to in-scattering
- Gains $\sigma_a L_e(x, \omega)$ due to emission

RTE – Radiative Transfer Equation

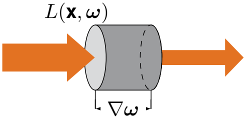
The change in radiance L traveling along direction $\boldsymbol{\omega}$ through a differential volume element at point $\boldsymbol{x}$.



$$(\boldsymbol{\omega}\nabla)L(\boldsymbol{x},\boldsymbol{\omega}) = -\sigma_t(\boldsymbol{x})L(\boldsymbol{x},\boldsymbol{\omega}) + \sigma_s(\boldsymbol{x})L_s(\boldsymbol{x},\boldsymbol{\omega}) + \sigma_a(\boldsymbol{x})L_e(\boldsymbol{x},\boldsymbol{\omega}) \tag{1}$$

RTE – Radiative Transfer Equation

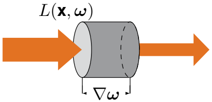
The change in radiance L traveling along direction $\boldsymbol{\omega}$ through a differential volume element at point $\boldsymbol{x}$.



$$(\boldsymbol{\omega}\nabla)L(\boldsymbol{x}, \boldsymbol{\omega}) = -\sigma_t(\boldsymbol{x})L(\boldsymbol{x}, \boldsymbol{\omega}) + \sigma_s(\boldsymbol{x})L_s(\boldsymbol{x}, \boldsymbol{\omega}) + \sigma_a(\boldsymbol{x})L_e(\boldsymbol{x}, \boldsymbol{\omega}) \tag{2}$$

Let's integrate it!

Integrating the loss of radiance



$$L(\boldsymbol{x} + \nabla\boldsymbol{\omega}, \boldsymbol{\omega}) = L(\boldsymbol{x}, \boldsymbol{\omega}) - \sigma_t(\boldsymbol{x}) L(\boldsymbol{x}, \boldsymbol{\omega}) \nabla\boldsymbol{\omega}$$

$$L(\boldsymbol{x} + dx) = L(\boldsymbol{x}) - L(\boldsymbol{x})\sigma_t(\boldsymbol{x}) dx \Big|_{dx = \nabla\boldsymbol{\omega}, L(\boldsymbol{x}) = L(\boldsymbol{x}, \boldsymbol{\omega})}$$

$$\frac{L(\boldsymbol{x} + dx) - L(\boldsymbol{x})}{dx} = \boxed{\frac{dL(\boldsymbol{x})}{dx} = -L(\boldsymbol{x})\sigma_t(\boldsymbol{x})} \text{ ("exponential extinction")} \tag{3}$$

$$\int_{L(x)}^{L(x+S)} \frac{1}{L} dL = -\int_0^S \sigma_t(\boldsymbol{x}) dx$$

$$ln(L(\boldsymbol{x} + S)) - ln(L(\boldsymbol{x})) = -\int_0^S \sigma_t(\boldsymbol{x}) dx$$

Transmittance

The Beer-Lambert Law

$$\implies L(\boldsymbol{x}+S) = L(\boldsymbol{x})e^{-\int_0^S \sigma_t(\boldsymbol{x}+s)ds}$$

Usually written as:

$e^{-\int_0^y \sigma_t(\boldsymbol{x}-s\boldsymbol{\omega})ds} = T(\boldsymbol{x},\boldsymbol{y})$

"transmittance coefficient" $T(\boldsymbol{x},\boldsymbol{y})$

net reduction factor between $\boldsymbol{x}$ and $\boldsymbol{y}$

due to absorption and out-scattering

$\int_0^y \sigma_t(\boldsymbol{x}-s\boldsymbol{\omega})ds = \tau(\boldsymbol{x},\boldsymbol{y})$

"optical thickness" τ

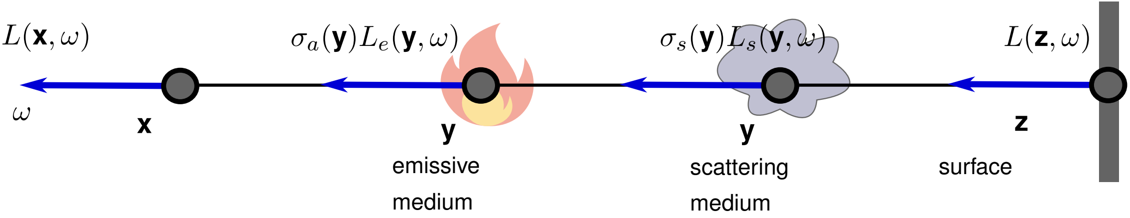
$$T(t) = e^{-\tau(t)} = e^{-\int_0^t \sigma_t(\boldsymbol{x}-s\boldsymbol{\omega})ds}$$

over distance t

RTE – Radiative Transfer Equation
The integral version

$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \int_0^\infty \underbrace{e^{-\int_0^y \sigma_t(\boldsymbol{x}-s\boldsymbol{\omega})ds}}_{\text{Transmittance } T(\boldsymbol{x},\boldsymbol{y})} \Big[\underbrace{\sigma_s(\boldsymbol{y}) L_s(\boldsymbol{y}, \boldsymbol{\omega})}_{\text{in-scatter}} + \underbrace{\sigma_a(\boldsymbol{y}) L_e(\boldsymbol{y}, \boldsymbol{\omega})}_{\text{emission}} \Big] d\boldsymbol{y} \tag{4}$$

VRE – Volume Rendering Equation



$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \int_0^z T(\boldsymbol{x}, \boldsymbol{y})\big[\sigma_a(\boldsymbol{y})L_e(\boldsymbol{y}, \boldsymbol{\omega}) + \sigma_s(\boldsymbol{y})L_s(\boldsymbol{y}, \boldsymbol{\omega})\big]dy + T(\boldsymbol{x}, \mathbf{z})L(\mathbf{z}, \boldsymbol{\omega}) \qquad (5)$$

Monte Carlo Integration

- $\int f(x)dx = \int \frac{f(x)}{p(x)}p(x)dx = E_N\left[\frac{f(x)}{p(x)}\right] \approx \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{p(x_i)}$
- Applied to the Volume Rendering Equation:

$$\langle L(\boldsymbol{x},\boldsymbol{\omega})\rangle = \frac{T(\boldsymbol{x},\boldsymbol{y})}{p(y)}\left[\sigma_a(\boldsymbol{y})L_e(\boldsymbol{y},\boldsymbol{\omega}) + \sigma_s(\boldsymbol{y})L_s(\boldsymbol{y},\boldsymbol{\omega})\right] + T(\boldsymbol{x},\mathbf{z})L(\mathbf{z},\boldsymbol{\omega})$$

- $p(y)$ is the PDF of sampling point y

$$\Longrightarrow \sum_{i=1}^{N}\left(\frac{T(\boldsymbol{x},\boldsymbol{y}_i)}{p(y_i)}\left[\sigma_a(\boldsymbol{y}_i)L_e(\boldsymbol{y}_i,\boldsymbol{\omega}) + \sigma_s(\boldsymbol{y}_i)L_s(\boldsymbol{y}_i,\boldsymbol{\omega})\right]\right) + T(\boldsymbol{x},\mathbf{z})L(\mathbf{z},\boldsymbol{\omega})$$

- We need:
  - Sampling distances
  - Estimating the transmittance T along a ray

Tracking
In homogeneous volumes

- Simulate how a photon bounces around inside a volume
- Explicitly modeling absorption and scattering effects

$$T(t) = e^{-\int_0^t \sigma_t(\boldsymbol{x} - s\boldsymbol{\omega})ds} = e^{-\int_0^t \sigma_t ds} \boxed{= e^{-\sigma_t t} = T(t)} \tag{6}$$

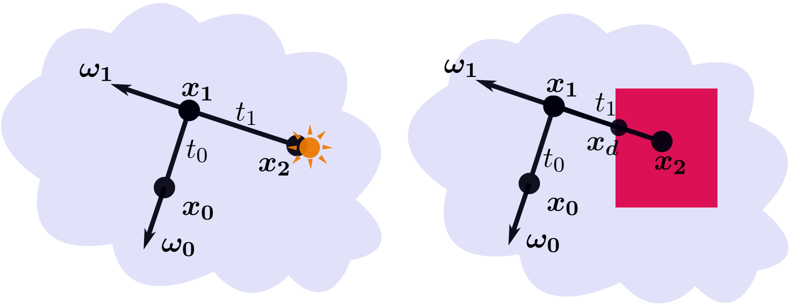
PDF $p(t) = \sigma_t e^{-\sigma_t t}$ (by normalizing)
Perfectly importance sample with $t' = -ln(1 - \zeta)/\sigma_t$ $\qquad \zeta \in [0, 1)$

$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \int_{t=0}^{d} p(t) \left[\frac{\sigma_a}{\sigma_t} L_e(\boldsymbol{x_t}, \omega) + \frac{\sigma_s}{\sigma_t} L_s(\boldsymbol{x_t}, \boldsymbol{\omega}) \right] dt + L_d(\boldsymbol{x_d}, \boldsymbol{\omega}) \tag{7}$$

$$\sigma_a + \sigma_s = 1; P_a = \frac{\sigma_a}{\sigma_t}; P_s = \frac{\sigma_a}{\sigma_t} \tag{8}$$

Closed-Form tracking

In homogeneous volumes



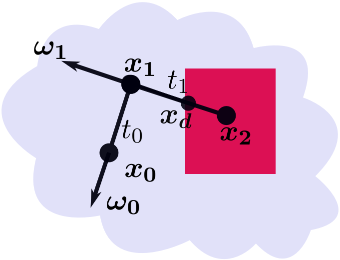
$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \int_{t=0}^{d} p(t) \Big[P_a L_e(\boldsymbol{x_t}, \omega) + P_s L_s(\boldsymbol{x_t}, \boldsymbol{\omega}) \Big] dt + L_d(\boldsymbol{x_d}, \boldsymbol{\omega}) \tag{9}$$

Regular tracking

In heterogeneous volumes

What happens if the volume is **not homogeneous**? $\implies \sigma_t(\boldsymbol{x})$

$\implies$ apply closed-form tracking to homogeneous sub-parts $\implies \sigma_t$



$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \int_{t=0}^{d} p(t) \left[\frac{\sigma_a}{\sigma_t} L_e(\boldsymbol{x_t}, \omega) + \frac{\sigma_s}{\sigma_t} L_s(\boldsymbol{x_t}, \boldsymbol{\omega}) \right] dt + L_d(\boldsymbol{x_d}, \boldsymbol{\omega}) \tag{10}$$

Delta tracking
Introducing null-collisions

1. Problem: the volume is heterogeneous
2. Idea: **Increase the number of interactions** to make it homogeneous, but **reject** some of the interactions $\implies$ **null-collisions**

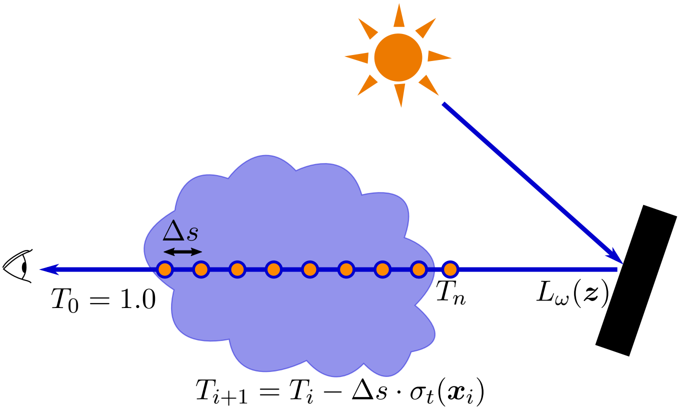
$$L(\boldsymbol{x},\boldsymbol{\omega}) = \int_0^\infty T_{\bar{\sigma}}(\boldsymbol{x},\boldsymbol{y}) \Big[\underbrace{\sigma_s(\boldsymbol{y}) L_s(\boldsymbol{y},\boldsymbol{\omega})}_{\text{in-scatter}} + \underbrace{\sigma_a(\boldsymbol{y}) L_e(\boldsymbol{y},\boldsymbol{\omega})}_{\text{emission}} + \underbrace{\sigma_n(\boldsymbol{y}) L(\boldsymbol{y},\boldsymbol{\omega})}_{\text{null-collision}} \Big] d\boldsymbol{y} \tag{11}$$

$$T_{\bar{\sigma}}(\boldsymbol{x},\boldsymbol{y}) = e^{-\int_0^y \sigma_s(\boldsymbol{s}) + \sigma_a(\boldsymbol{s}) + \sigma_n(\boldsymbol{s}) d\boldsymbol{s}} \tag{12}$$

$$\sigma_n(\boldsymbol{x}) = \bar{\sigma} - \sigma_t(\boldsymbol{x}) \tag{13}$$

$$\bar{\sigma} = \sigma_s(\boldsymbol{x}) + \sigma_a(\boldsymbol{x}) + \sigma_n(\boldsymbol{x}) \tag{14}$$

Transmittance Estimation
Ray Marching



Acceleration Data Structures

- Spatially-varying properties
- Data access usually dominates the render time
  $\Longrightarrow$ data structures are key for achieving good performance
- Volume data can quickly grow into hundreds of gigabytes for production

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