

# Approximation as Differentiation notes

## 1 OVERVIEW

Some notes on the key constructions:

- Categories of families, special case of Grothendieck construction

Set will often be **Setoid** in the Agda code but I'm glossing that here.

## 2 DEFINITIONS

### 2.1 Indexed families of objects

For any set  $I$  and category  $C$  write  $\mathbf{Fam}(I, C)$  for the category where:

- the objects are the  $I$ -indexed families of objects of  $C$ ;
- a morphism from  $X$  to  $Y$  is a family of morphisms  $f_i : X_i \rightarrow Y_i$  in  $C$  for any  $i \in I$ .

Equivalently,  $\mathbf{Fam}(I, C)$  is the functor category  $[I, C]$ .

Suppose  $x \in \mathbf{Fam}(I, C)$  and  $y \in \mathbf{Fam}(J, C)$ . Define  $y_f$  for any  $f : I \rightarrow J$  to be the  $I$ -indexed family of objects of  $C$ .

### 2.2 Category of families

For a functor  $F : C \rightarrow \mathbf{Set}$ , we have the category where:

- objects are pairs  $(I, x)$  with  $I$  an object of  $C$  and  $x \in FI$
- morphisms from  $(I, x)$  to  $(J, y)$  are morphisms  $f : I \rightarrow J$  in  $C$  where  $(Ff)(x) = y$

This is the Grothendieck construction for a functor  $F : C \rightarrow \mathbf{Cat}$ , in the special case where  $F : C \rightarrow \mathbf{Set}$ . (Where we read a **Set**-valued functor as a **Cat**-valued functor restricted to discrete categories.)

In our code we give a slightly different construction. For a category  $C$ , define the category where:

- objects are pairs  $(I, x)$  with  $I$  a set and  $x$  an  $I$ -indexed family of objects of  $C$
- morphisms from  $(I, x)$  to  $(J, y)$  are functions  $f : I \rightarrow J$  paired with morphisms

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