

Approximation as Differentiation notes

1 OVERVIEW

Some notes on the key constructions:

- Categories of families, special case of Grothendieck construction

Set will often be **Setoid** in the Agda code but I'm glossing that here.

2 DEFINITIONS

2.1 Indexed families of objects

For any set I and category C write $\mathbf{Fam}(I, C)$ for the category where:

- the objects are the I -indexed families of objects of C ;
- a morphism from X to Y is a family of morphisms $f_i : X_i \rightarrow Y_i$ in C for any $i \in I$.

Equivalently, $\mathbf{Fam}(I, C)$ is the functor category $[I, C]$.

Suppose $X \in \mathbf{Fam}(I, C)$ and $Y \in \mathbf{Fam}(J, C)$. For any $f : I \rightarrow J$ define $Y[f] \in \mathbf{Fam}(I, C)$ where $Y[f]_i = Y_{f_i}$.

2.2 Category of families

For a functor $F : C \rightarrow \mathbf{Set}$, we have the category where:

- objects are pairs (I, x) with I an object of C and $x \in FI$
- morphisms from (I, x) to (J, y) are morphisms $f : I \rightarrow J$ in C where $(Ff)(x) = y$

This is the Grothendieck construction for a functor $F : C \rightarrow \mathbf{Cat}$, in the special case where $F : C \rightarrow \mathbf{Set}$. (Where we read a **Set**-valued functor as a **Cat**-valued functor restricted to discrete categories.)

In our code we give a slightly different construction. For a category C , define the category where:

- objects are pairs (I, x) with I a set and x an I -indexed family of objects of C
- morphisms from (I, x) to (J, y) are functions $f : I \rightarrow J$ paired with morphisms

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Manuscript submitted to ACM

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