

# Approximation as Differentiation notes

## 1 OVERVIEW

Some notes on the key constructions:

- Categories of families, special case of Grothendieck construction

## 2 DEFINITIONS

### 2.1 Category of families

For a functor  $F : C \rightarrow \mathbf{Set}$ , we have the category where:

- objects are pairs  $(A, x)$  where  $x \in FA$
- morphisms from  $(A, x)$  to  $(B, y)$  are morphisms  $f : A \rightarrow B$  in  $C$  where  $(Ff)(x) = y$

This is the Grothendieck construction for a functor  $F : C \rightarrow \mathbf{Cat}$ , in the special case where  $F : C \rightarrow \mathbf{Set}$ . (Where we read a  $\mathbf{Set}$ -valued functor as a  $\mathbf{Cat}$ -valued functor restricted to discrete categories.)

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