1. (a.)

$$
\begin{aligned}
H_0 &: \hat{\beta}_1 + \hat{\beta}_2 = 0 \\
H_1 &: \hat{\beta}_1 + \hat{\beta}_2 \neq 0
\end{aligned} \tag{1}
$$

(b.)

To test single linear hypothesis, we could do t-test by constructing test statistic:

$$T = \frac{c'\hat{\beta}}{s\sqrt{c'(x'x)^{-1}c}} \sim t(6)$$

where c' is $[0 \quad 1 \quad 1 \quad 0]$, and $s = \frac{\sum \hat{\epsilon}^2}{6} = \frac{\hat{\epsilon}'\hat{\epsilon}}{6}$

For $|T| > 2.45$, null hypothesis is rejected. Thus, effects of increased expenditure of candidate A or B do not cancel out.

(c.)

If null hypothesis rejected, then at least one candidate's expenditure will have effect on the outcome.

$$
\begin{aligned}
H_0 &: \hat{\beta}_1 = \hat{\beta}_2 = 0 \\
H_1 &: \hat{\beta}_1 \neq 0 \text{ or } \hat{\beta}_2 \neq 0
\end{aligned} \tag{2}
$$

(d.)

To perform an joint hypotheses F-test, construct test statistics:

$$F = \frac{(R\hat{\beta} - r)'(R(X'X)^{-1}R')^{-1}(R\hat{\beta} - r)/2}{\hat{\epsilon}'\hat{\epsilon}/6} \sim F(2, 6) \tag{3}$$

where $R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, r = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

For $F > 5.14$, null hypothesis is rejected. At least one candidate's expenditure have effect on the outcome.

2. (a.) t-statistic of `branchWest` = -1.29 with degree of freedom = 1261. p-value = 0.20. Null hypothesis is not rejected under $\alpha = 0.05$

   (b.) t-statistic of `branchWest` = -1.29 with degree of freedom = 1261. Since t distribution is symmetric, p-value of one-sided test is half of two-sided test = 0.10. Null hypothesis is still not rejected under $\alpha = 0.05$

   (c.) This is an overall F-test. F-statistic = 97.9 follows F distribution with degree of freedom (4, 1261). p-value $< 2 \times 10^{-16}$. Null hypothesis is rejected under $\alpha = 0.05$

3. (a.) Let `schyr` = X, `cog` = Z

$$
\begin{aligned}
\hat{\beta}_1 &= \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} \\
&\xrightarrow{prob.} \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}(X)} \\
&= \frac{\mathrm{Cov}(X, \beta_0 + \beta_1 X + \beta_2 Z + \epsilon)}{\sigma_1^2} \\
&= \frac{\beta_1 \mathrm{Var}(X) + \beta_2 \mathrm{Cov}(X, Z) + \mathrm{Cov}(X, \epsilon))}{\sigma_1^2} \\
&= \frac{\beta_1 \sigma_1^2 + \beta_2 \rho \sigma_1 \sigma_2}{\sigma_1^2} \\
&= \beta_1 + \beta_2 \rho \frac{\sigma_2}{\sigma_1}
\end{aligned}
$$

Thus, $\hat{\beta}_1$ is inconsistent estimator for β_1.

(b.)

$$
\hat{\beta}_1 - \beta_1 \xrightarrow{prob.} \beta_2 \rho (\sigma_2 / \sigma_1) > 0, \text{for } \beta_2 > 0 \tag{4}
$$

$\hat{\beta}_1$ is upward biased.

(c.)

$$
\sqrt{n}(\hat{\beta}_1 - \beta_1) \xrightarrow{p} \sqrt{n}(\beta_2 \rho (\sigma_2 / \sigma_1)) \to \infty \text{ as n} \to \infty \tag{5}
$$