Analyzing the Effect of Data Revisions on Predictive Densities Using a Small-Scale DSGE Model

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Abstract

We evaluate the impact of data revisions on predictive densities using a small-scale, New Keynesian, DSGE forecasting model. We generate predictive densities for output growth, inflation, and the nominal interest rate at multiple horizons using initially released data. These densities are then compared to those obtained when the model is estimated and forecasted using revised data. We document significant differences in predictive densities depending on data vintage.

The main revisions that affect the predictive densities are the annual revisions to the data that occur once a year when the past three years of data are revised, or the benchmark revisions that occur about every five years. We illustrate how the contours of the predictive densities change at each annual revision. We run a variety of experiments that suggest that data revisions should be a concern for policymakers and forecasters.

1 Introduction

The literature on real-time data analysis contains a rich description of the impact of data revisions on point forecasts, as shown by Croushore (2006). But little is known about the impact of data revisions on forecast densities. Our goal is to begin filling this gap by using a standard, small-scale, New Keynesian, DSGE model to generate forecast densities. We then examine how revisions affect the predictive densities from our model.

We begin with a counter-factual experiment. We first estimate the structural parameters of the model and then feed in initial-release data to back out the implied structural shocks. Then, keeping the same shocks to total factor productivity and government spending, suppose that the Fed, followed an exact Taylor rule thus assuming (counterfactually) that there is no monetary-policy shock. Under this scenario we generate a new path for output growth, inflation, and the interest rate. Next, we repeat this experiment using revised data, to assess how different monetary policy would have been and to examine the implications for output growth and inflation. The results suggest that the economy would have behaved very differently had policymakers had revised data at hand instead of the initially released data.

Next, to illustrate the impact of revisions on the model's estimated shocks, we show scatter plots that compare estimates of the demand and supply shocks made using revised data to those made using initially released data. Again, we find large differences. Most *disturbing*, there is a tendency for shocks that are estimated to be large using initially released data to be revised down. Similarly, shocks that are estimated to be small using initially released data tend to be revised up. Confusion over whether a shock is a supply shock or a demand shock may lead to uncertainty about the appropriate policy response.

Our next exercise is to examine how predictive densities change when revisions occur, especially benchmark revisions that change the methods of data construction and that incorporate census information. We estimate the model first using initially released data, and then using revised data. We generate predictive densities at multiple horizons (in output, inflation, and the nominal interest rate). We document significant differences in predictive densities depending on data vintage. Not only are the forecast means different between initially released and revised data, so are the standard deviations of the data, as are the predictive correlations between variables. However, these changes in densities do not appear to be systematic.

In our final exercise, we examine the point forecasts generated by the model using using initially released and revised data. We compare the monetary policy paths in each forecast and examine the differences in those paths from using the different vintages of data. We find that in some years, the differences in the policy paths are substantial, suggesting that policy could be significantly different if policymakers had more accurate data.

Density forecasting is occupying an increasingly important role in macroeconomic analysis and policymaking. Central banks, most notably in Europe, have been illustrating their forecast densities with fan charts for over a decade now, as the survey article Tay and Wallis (2002) describes. Tay and Wallis note that the first survey to incorporate density forecasts was the precursor of the Survey of Professional Forecasters run by the Federal Reserve Bank of Philadelphia (see Croushore (1993)). The Bank of England was the first central bank to release the density forecasts of policymakers, doing so since 1996, and developing what are now known as fan charts. Recently, researchers have begun devising tests for accuracy for density forecasts, such as Diebold, Gunther, and Tay (1998), Amisano and Giacomini (2007), and Geweke and Amisano (2010).

The impact of data revisions on density forecasts has not been studied in detail. In fact, the only papers that we know of that examine density forecasts in real time are those of Clark (2011) and Herbst and Schorfheide (2011). Clark uses real-time data in density forecasting, but his goal is to show that incorporating stochastic volatility in forecasting models helps reduce forecast errors, so he does not investigate the impact of data revisions on the forecasts. Herbst and Schorfheide develop posterior predictive checks to evaluate conditional and unconditional density forecasts. They use real-time data to find that the medium-scale Smets and Wouters (2007) model does not lead to uniform improvement in the quality of density forecasts compared with the simpler three-equation New Keynesian DSGE benchmark model.

Our choice of the small-scale NKDSGE model used in e.g., Del Negro and Schorfheide (2004), is motivated by our ultimate goal, which is to help policymakers understand how to account for the possibility of data revisions in the policy-making process. Del Negro and Schorfheide show how to analyze the impact of policy interventions, but do so using final data, assuming for simplicity that data are not revised significantly. However, the evidence from the real-time literature, described in Croushore (2011), suggests that such revisions may be crucial for forecasting and policy analysis. So, to understand appropriate policy responses, we need to first examine the significance of data revisions for density forecasts.

2 The DSGE Model

We use the model described in Del Negro and Schorfheide (2004). Agents are comprised of households, firms, the government, and the monetary policymaker. The key frictions in the model are habit persistence in consumption by households and menu costs faced by firms when they adjust prices, leading to sticky prices. Households maximize utility from consumption, leisure and real balances. Monopolistically competitive firms choose their quantity of labor input, setting prices to maximize profits. There is no capital in the model. The government spends according to an autoregressive process plus a shock. The monetary policymaker follows a Taylor rule, setting the nominal interest in response to deviations of output and inflation from target, plus a shock.

There are three sources of uncertainty in the model:

1. A (supply) shock to total factor productivity, $\epsilon_{z,t}$
2. A shock to monetary policy, $\epsilon_{R,t}$
3. A (demand) shock to government spending, $\epsilon_{g,t}$

The percentage deviation of a variable (x_t) from its steady-state value (x) is defined as:

$$\hat{x_t} = ln(x_t/x).$$

The variables in the model are output (y_t), inflation (π_t), and the nominal interest rate (R_t). The model can be approximated by an intertemporal Euler equation, a New Keynesian Phillips curve, and an interest-rate feedback rule. In log-linear form, we have:

$$\hat{y}_t = E_t[\hat{y}_{t+1}] - \tau^{-1}(\hat{R}_t - E_t[\hat{\pi}_{t+1}]) + (1-\rho_g)\hat{g}_t + \rho_z \tau^{-1}\hat{z}_t$$
$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa[\hat{y}_t - \hat{g}_t]$$
$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_1 \hat{\pi}_t + \psi_2 \hat{y}_t) + \epsilon_{R,t}$$

The parameter τ is the inverse of the intertemporal elasticity of substitution and β is the household's discount factor. The parameter κ captures the slope of the Phillips curve.

We complete the model with three measurement equations:
Output growth:

$$YGR_t = \bar{\gamma} + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$$

Inflation:

$$INF_t = \bar{\pi} + 4\hat{\pi}_t$$

Monetary policy:

$$FFR_t = \bar{\pi} + \bar{r} + 4\gamma + 4\hat{R}_t$$

The linearized model is put into state-space form:

$$S_t = TS_{t-1} + Re_t$$

$$X_t = D + ZS_t$$

We solve the model using Sims's Gensys algorithm and estimate it using Bayesian methods, as described by An and Schorfheide (2007) using the same priors as in Lubik and Schorfheide (2005). A random-walk Metropolis algorithm is used to generate draws from the posterior distribution of the structural parameter vector.

3 Data

We use real GDP as the output variable, the GDP deflator as the price index used to determine the inflation rate, and the federal funds rate as the interest rate, which is the indicator of monetary policy. Both GDP and the GDP deflator are subject to data revisions, so we use data from the Federal Reserve Bank of Philadelphia's Real-Time Data Set for Macroeconomists (see Croushore and Stark (2001)).

We estimate the model using quarterly data on output growth, inflation, and the federal funds rate beginning in 1984Q1 . The starting date is determined by the desire to choose a period over which the Fed's inflation target was relatively stable.

Research in the real-time literature, described by Croushore (2011), suggests that the most important revision to the data occurs in the annual revision each year. So, we consider sample ending dates at the end of January each year from 2000 to 2011, when the government provides the initial release of the data for GDP and the GDP deflator for the prior year. We use both the initially released data and annual revision data that are released later in the year (usually at the end of July) that revise the data for the previous several years or more. From 2000 to 2011, these annual revisions occur each year and revise the preceding three years of data, except in 2003 and 2009, when the revisions go back longer in time and are called benchmark revisions.

A key question is whether the revisions to the data are economically significant enough to matter for forecasts and monetary policy. Most studies of point forecasts find significant changes in forecasts based on data revisions (see examples in Croushore (2011)). In terms of the exercise we conduct in this study, a look at the data suggests that the revisions may significantly affect forecast densities. Every annual revision affects the contour of the recent several years of data in some way. Rather than illustrate each revision for each of the three variables, we will simply show graphs of three revisions that appear on the surface to matter for forecasts. They are: (1) the benchmark revision of 2003 on inflation; (2) the benchmark revision of 2009 on output growth; and (3) the annual revision of 2010 on the inflation rate.

Figure 1 shows the initially released quarterly data on inflation (using the GDP deflator) from 1998 to 2002, released by the government at the end of January 2003. It also shows the results of the benchmark revision released in December 2003 over the same sample period. Note in particular a very large revision in the fourth quarter of 2001, which was initially released as -0.5 percent per year but revised up to +1.6 percent in the benchmark revision. In the initially released data, inflation appears to be rising sharply at the end of 2002, but in the revised data, inflation is fairly stable at the end of 2002. These significant differences in recent history could have a strong impact on the forecasts.

Figure 2 shows data on output growth based on the initial release in January 2009 compared with the benchmark revision of July 2009. The revision to output growth was substantial for the period of the financial crisis that began in 2008, with the downward revision to the four quarters of 2008 averaging over 1.5 percentage points per quarter. Perhaps policymakers would have responded even more vigorously than they did, had they known the depths of the recession in terms of lost output.

Benchmark revisions are not the only ones that lead to significant changes in recent data. The annual revision in 2010, shown in Figure 3 for inflation, led to a much more volatile recent pattern in the data and a very different

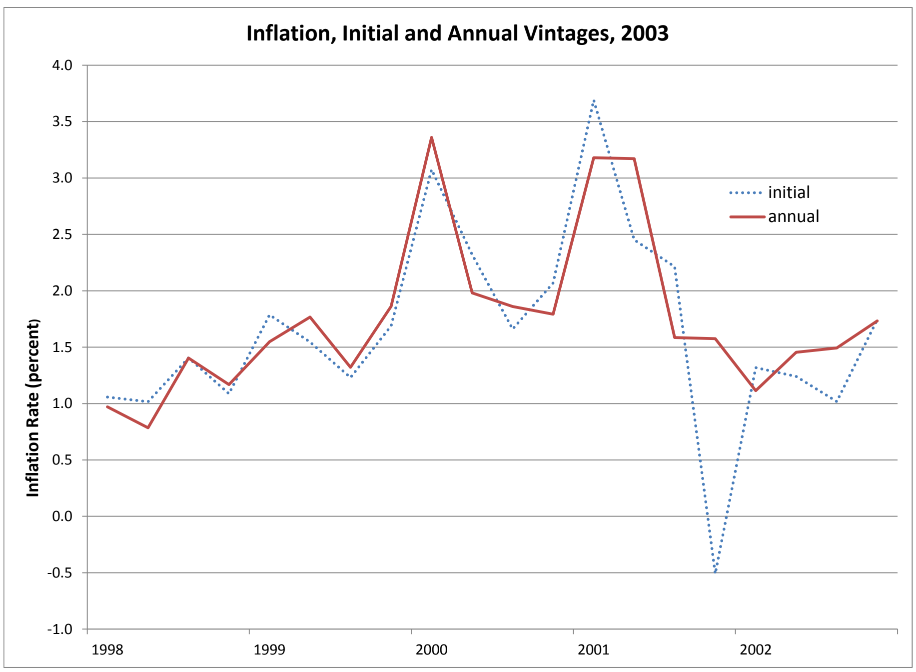

Figure 1: Inflation, Initial and Annual Vintages, 2003

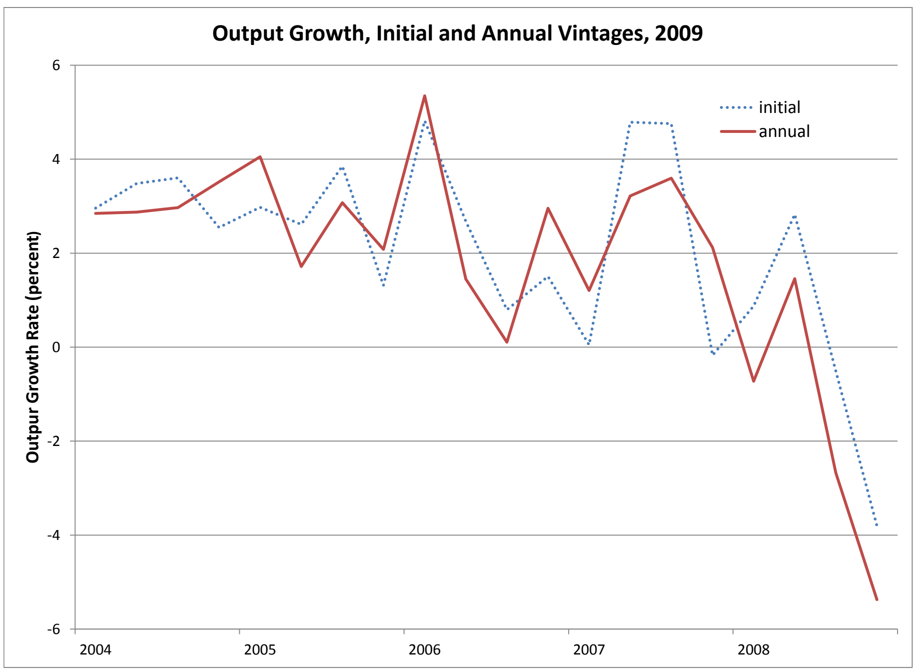

Figure 2: Output Growth, Initial and Annual Vintages, 2009

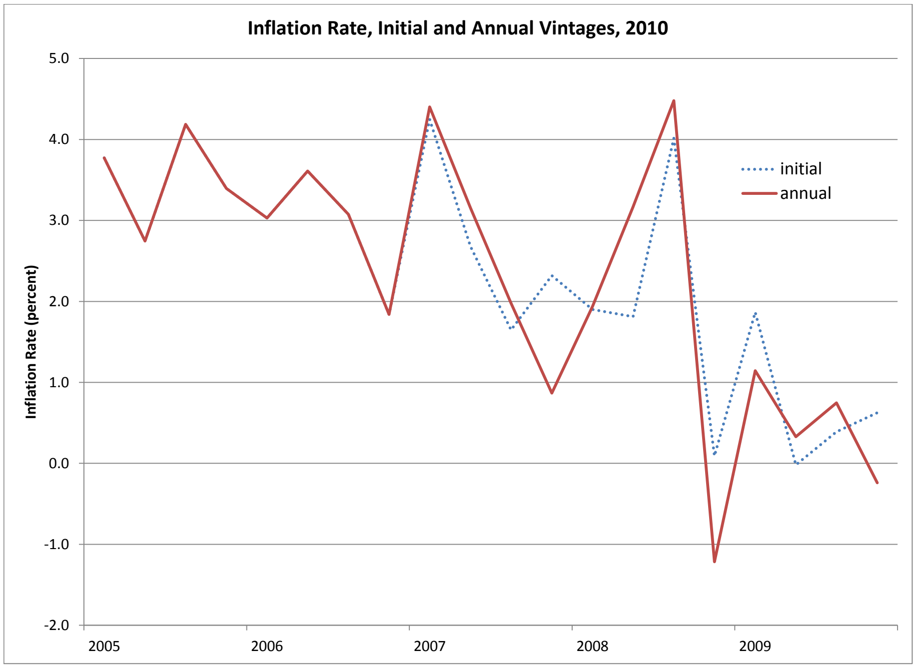

Figure 3: Inflation, Initial and Annual Vintages, 2010

jumping-off point for the forecasts. Inflation in the fourth quarter of 2009 was 0.6 percent and rising in the initially released data, but was revised to -0.2 percent and declining in the annual revision.

Given these examples, there appears a reasonable case to suggest that the revisions to the data may lead to significant changes in forecasts and policy reactions. We investigate those changes next.

4 Exercises

In this section, we report the results of four exercises: (1) a counter-factual experiment that shows how different monetary policy would have been with revised compared with initially released data; (2) a comparison of estimates of the structural shocks of the DSGE model when estimated with initially released data compared with revised data; (3) an examination of density forecasts and how the data revisions affect their distributions; (4) a comparison of forecasts of the short-term interest rate and how they are affected by data revisions.

4.1 A Counterfactual Experiment

We begin with a counter-factual experiment. First, we estimate the structural parameters of the model using initial release data for each year and back out the estimated structural shocks. Keeping the sequence of shocks to total factor productivity and government spending intact, we then assume that the Fed follows the Taylor rule exactly (thus counterfactually assuming that the monetary-policy shock is always zero). Based on these shocks, we generate an alternative path for output growth, inflation, and the interest rate. Next, we repeat this experiment using revised data.

When we estimate the model, we obtain estimates of the structural parameter estimates. Collecting estimates of the key parameters in vector θ, we can generate measures of the structural shocks using the Kalman smoother. The state-space representation of the model is:

$$S_t = T(\theta)S_{t-1} + R(\theta)\hat{e}_t, \quad \hat{e}_t \sim N(0, \Sigma)$$

$$X_t = D + ZS_t$$

and where Σ is assumed diagonal.

By substitution, we see that the current state observation can be expressed as a function of the estimated parameters and structural shocks:

$$S_t = R(\theta)\hat{e}_t + T(\theta)R(\theta)\hat{e}_{t-1} + ...$$

For a given initial state, S_0, we can feed in alternative sequences of structural shocks to generate counterfactuals. Our experiment will consist of replacing the monetary policy shocks with zeroes to see what would happen in the model if the Fed were to set policy according to the Taylor rule exactly. We do this first with initially released data and repeat the exercise with revised data; then we compare the alternative policy paths.

Our experiment consists of feeding in the initial release data on output growth, inflation, and the interest rate into the estimated equations and then backing out the structural shocks:

$$[\hat{e}_t^{i,R} \hat{e}_t^{i,g} \hat{e}_t^{i,z}]$$

Then, set $\hat{e}_t^{i,R} = 0$ for all t. Now, use the same shocks for government spending and productivity, but with the new monetary policy shocks (i.e., setting $\hat{e}_t^i = 0$) and run them through the structural equations to determine the alternative paths for output growth, inflation, and the interest rate. Of course, the model allows for a feedback response from shocks to policy and vice versa, but the fundamental structural shocks are exogenous. We then repeat this exercise using revised data. The outcome of the experiments are, for each year, counterfactual paths for output growth, inflation, and the interest rate, for both initially released data and revised data, which we can compare with the actual paths of the variables. We generate figures showing the results for each year from 2000 to 2011, but show only several years here.

Figure 4 shows the results of the counterfactual exercise for the model estimated and simulated on a history up to and including the 2005 revisions. This is an interesting case because in 2005 the government only released an annual revision, which did not appear to change the data by a large amount. However, the revision did cause real GDP growth for 2002 to 2004 to be revised down, while inflation was revised up for most of the period. The revision implies a new set of structural shock estimates that in turn lead to a somewhat different policy response and a different counterfactual for the data. The most notable result is the change in the path of the interest rate, with a somewhat easier policy, on average, using the revised data than using the initially released data. Thus, had policymakers known how the data were going to be revised, they would have set the policy rate at a lower level over 2004-2005. Of course, in fact the Fed set policy much easier than the Taylor rule in the model suggested, as the figure shows, presumably because of the headwinds faced by the economy in the aftermath of the 2001 recession.

Figure 5 shows the same type of exercise for 2009. In this case, the main result of the revision to the data was a large downward revision to output growth for 2008. However, earlier data showed mixed revisions, so the end result is that monetary policy over the period from 2005 to 2007 was generally tighter (higher interest rates) using revised data than using initially released data.

The results of the counterfactual exercise suggest that data revisions are substantial enough to lead to potentially large changes in the implied policy

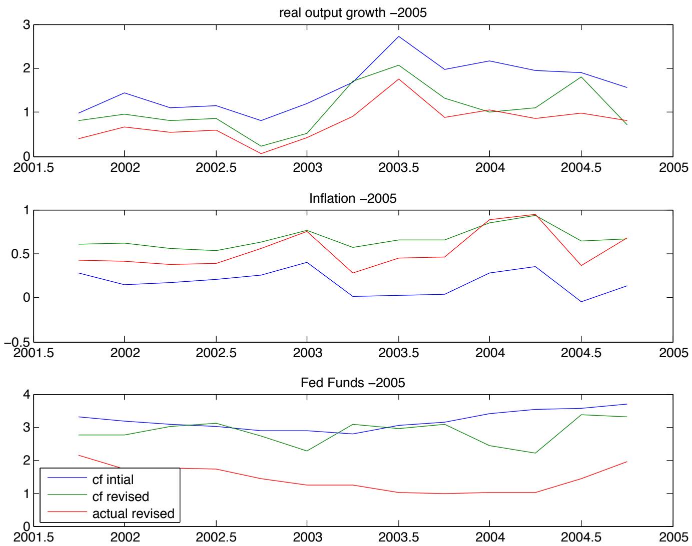

Figure 4: Counterfactual Experiment, 2005

interest. If policymakers were credibly committed to following a simple Taylor rule, their policy prescriptions would change significantly if they had revised data rather than initially released data.[1]

4.2 How Do Revisions Affect Model Variables?

To illustrate the impact of revisions on the model's estimated shocks, we show scatter plots of demand and supply shocks, as well as output growth and inflation, in the space of initial and annual revision data. We are looking for some systematic component to the revisions.

We take the government spending shock as a demand shock and the TFP shock as a supply shock. The question is: do our estimates of demand and supply shocks vary substantially when we use initially released data compared with revised data? Figure 6 shows the results, plotting the shocks based on initial data on the horizontal axis, while the shocks estimated using revised data are shown on the vertical axis. If data revisions were inconsequential for

[1]Of course, if we were to estimate the model without a monetary policy shock, the estimated structural shocks would likely look quite a bit different than what we have estimated, including a monetary policy shock in the model.

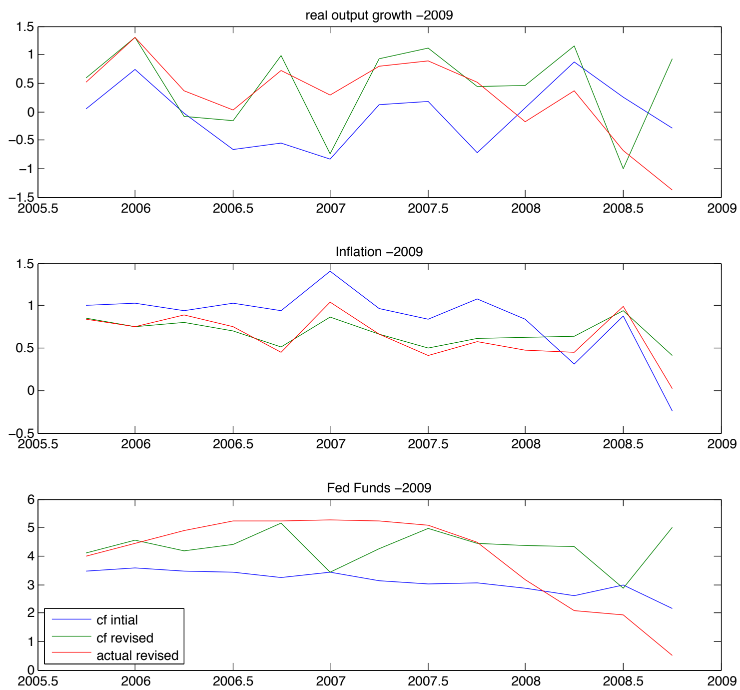

Figure 5: Counterfactual Experiment, 2009

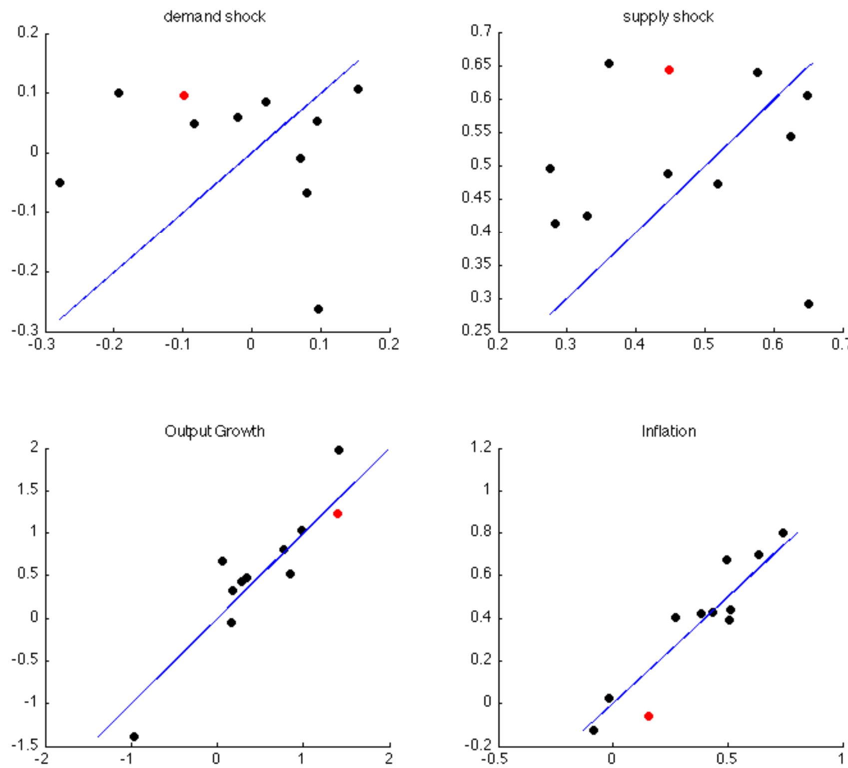

Figure 6: State Variable Scatter Plots

the measurement of shocks, we would expect the data plotted in the figure to line up closely along the 45-degree line.

Figure 6 shows the measures of the shocks to output growth and inflation and their sensitivity to revision. Also plotted is a 45-degree line along which the observations under the initial revision and the annual revision are identical. While the shocks show dispersion around the 45 degree line, the best-fitting line through the points in a least-squares sense is not statistically different from the 45-degree line. However, it looks like in years when demand shocks are low in the initially released data (x-axis), there is a tendency for them to be raised in the annual revision data (y-axis) And when demand shocks are measured as high in the initially released data, there is a tendency for them to be revised down, so the points lie below the 45-degree line. Similarly for supply shocks – when they are small in the initial release data they tend to be revised up in the final release data. Given that the optimal response of policy depends on whether a shock is thought to be a demand shock or a supply shock, the systematic tendency in the revisions to the shock measures has significant implications. Policymakers may thus want to consider the possibility that data will be revised and thus not react too strongly to a demand or supply shock, given uncertainty about its magnitude.

4.3 How Data Revisions Affect Predictive Densities

Our next exercise is to examine how predictive densities change when revisions occur, especially benchmark revisions that change the methods of data construction and that incorporate census information. We estimate the model first using initially released data, and then using revised data. We generate predictive densities at multiple horizons (in output, inflation, and the nominal interest rate). We look at the three revisions that we examined earlier in Figures 1, 2, and 3: The benchmark revision of 2003, the benchmark revision of 2009, and the annual revision of 2010.

The benchmark revision in 2003 revised the past 75 years of data, with especially large effects from 1993 to 2002. The Bureau of Economic Analysis made methodological changes concerning the output of insurance companies and banks, and the purchases of government services. So, consider a forecaster using our DSGE model in February 2003, who has at hand the initial data release for the fourth quarter of 2002; then compare those DSGE model forecasts with those made at the same date but based on the benchmark revised data. The results for inflation forecasts are shown in Figure 7.

Figure 7 shows some changes in the distribution of the inflation forecasts that occur because of the benchmark revision. Even though the jumping-off point of the forecasts is the same for the initially released and revised data and the median forecast (solid line) is about unchanged, the revisions lead to a widening of the 95th percentile and the 5th percentile of the density forecasts, which are shown as dotted lines.

The benchmark revision in 2009 made no major definitional changes, but recent data from the period of the financial crisis were revised substantially. The revisions were especially large in 2008, when output growth was revised down sharply. Results of making DSGE model forecasts of output growth using initially released versus revised data are shown in Figure 8.

In this case, the revision affects the jumping-off point for the forecasts,

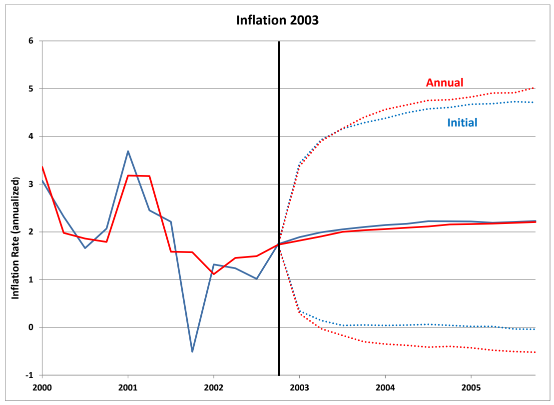

Figure 7: Distribution Chart, Inflation, 2003

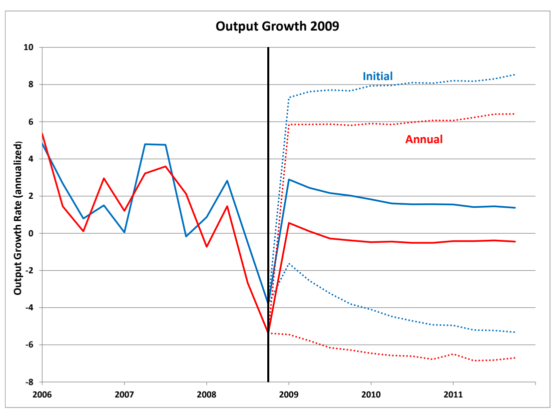

Figure 8: Distribution Chart, Output Growth, 2009

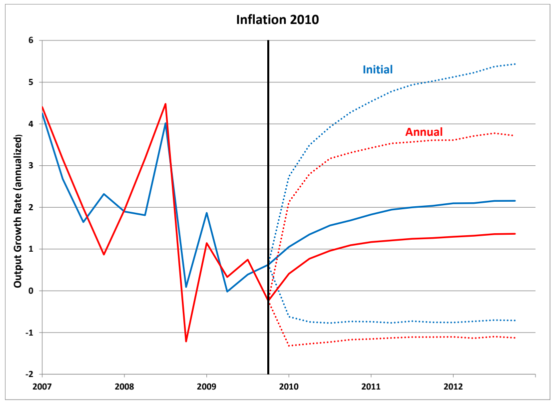

Figure 9: Distribution Chart, Inflation, 2010

so the median forecast changes substantially for output growth. With the revision having led to a downward revision in output growth of nearly two percentage points, the forecasts are similarly revised downward. But the contours of the 95th and 5th percentiles of the distribution also change shape because of the revision.

The 2010 annual revision made no changes in method and just revised the last three years of data. But it did revise down the inflation rate throughout the financial crisis period, as shown in Figure 9. The revision leads the DSGE forecasts to change considerably, as the figure shows. The downward revision to the inflation numbers at the end of 2009 leads to a downward revision in the median forecast, as the solid line shows. But the spread of the forecasts between the 95th and 5th percentiles of the distribution also declines significantly.

To generalize these results, let's look at how the first two moments of the predictive densities change for different forecast horizons. Consider a plot of the first or second moment of the forecast for a given horizon, h. If we plot the moments based on using initially released data in each of the 11 years and compare them to the moments based on using revised data, we can create plots like those in Figures 10, 11, and 12 for h = 1, 4, and 8 quarters.

In each of the plots, if the points were close to the 45-degree line, data revisions would have little impact on the moments of the forecast distribution.

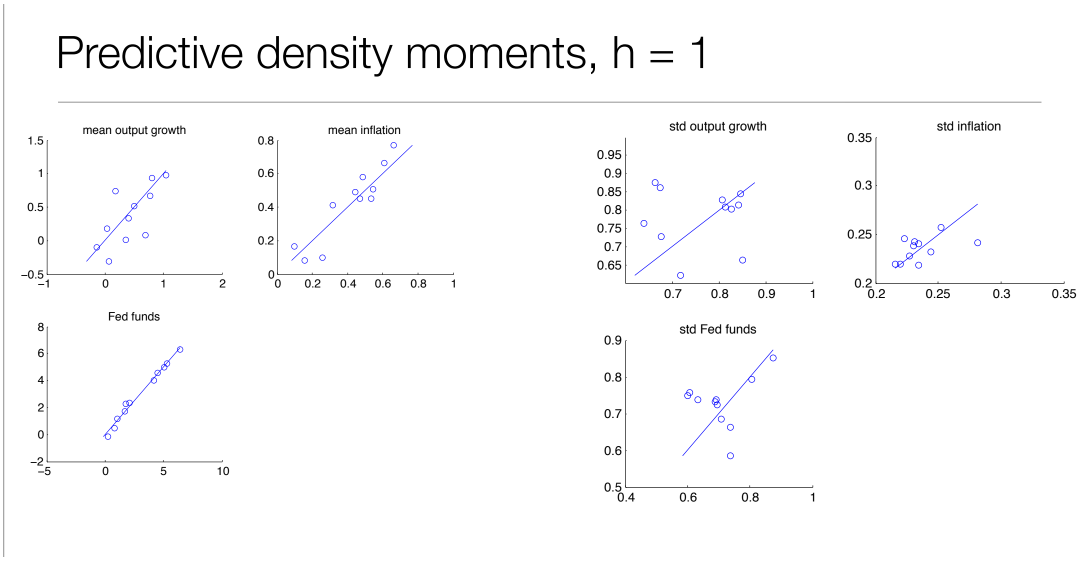

Figure 10: Predictive Density Moments, h=1

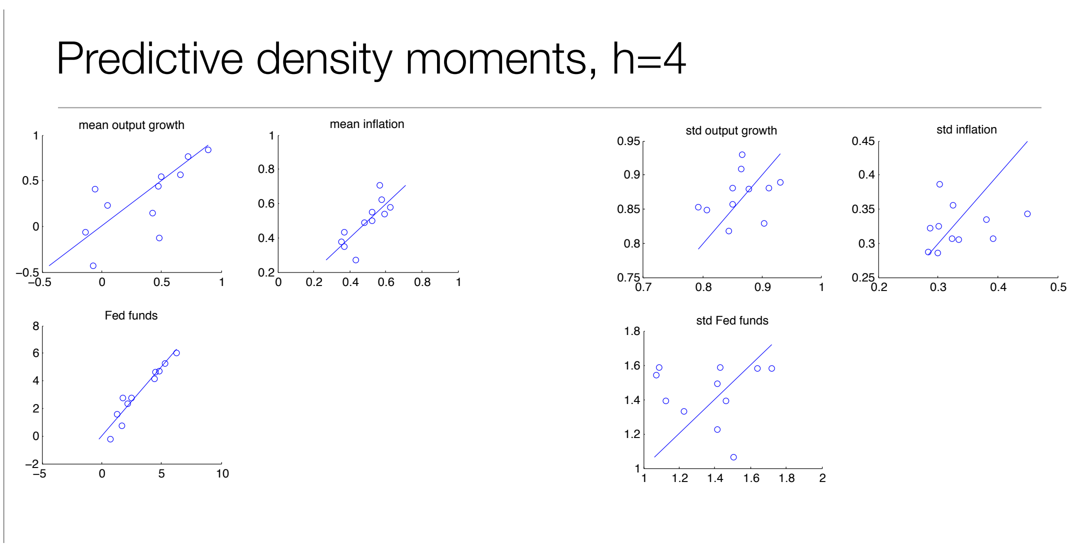

Figure 11: Predictive Density Moments, h=4

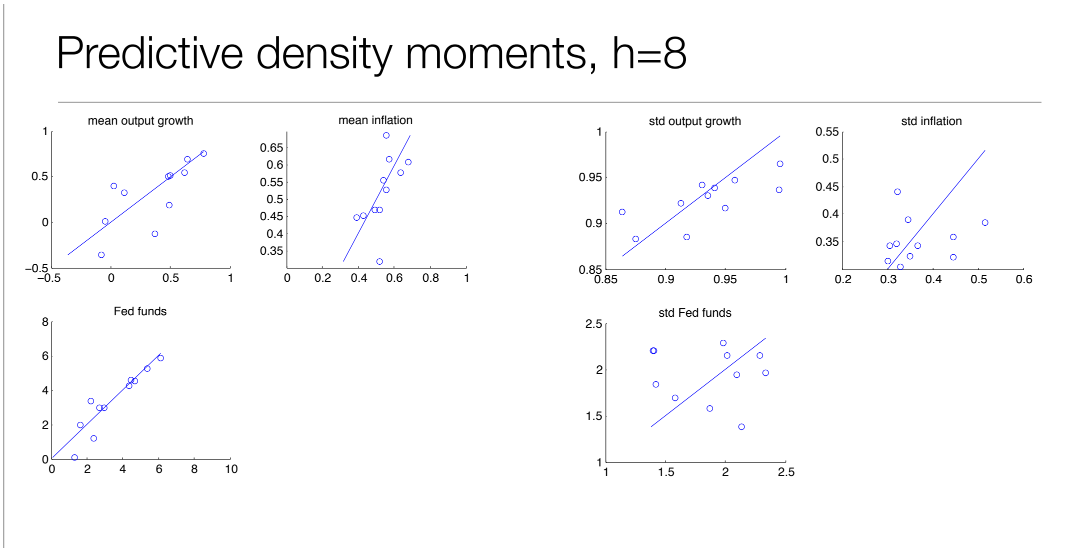

Figure 12: Predictive Density Moments, h=8

For the most part, the various plots are fairly symmetric about the 45-degree line, but the deviations from that line can be large. In a few cases, such as the standard deviation of the fed funds forecasts at horizons 1 and 4, there is a tendency for large values based on initially released data to be revised down, and for small values to be revised up. But there do not seem to be any other systematic tendencies.

As a final comparison, we look at the correlations between output growth and inflation forecasts, to see how they change when data are revised. In this experiment, we run 10,000 stochastic simulations of the model, capture the forecasts, and look at the correlation between the forecasts of output growth and inflation. The results of this exercise are shown in Figures 13, 14, and 15 for a forecast horizon of 4 quarters, for both initially released data and revised data.

In Figure 13, based on 2004 data, there is not much change in the correlations between initially released data and revised data. But using the 2005 data, shown in Figure 14, we see a negative correlation between output growth and inflation based on initially revised data, but a positive correlation based on revised data. The opposite pattern is seen in 2006 data, as shown in Figure 15; in that case, there is a positive correlation between output growth

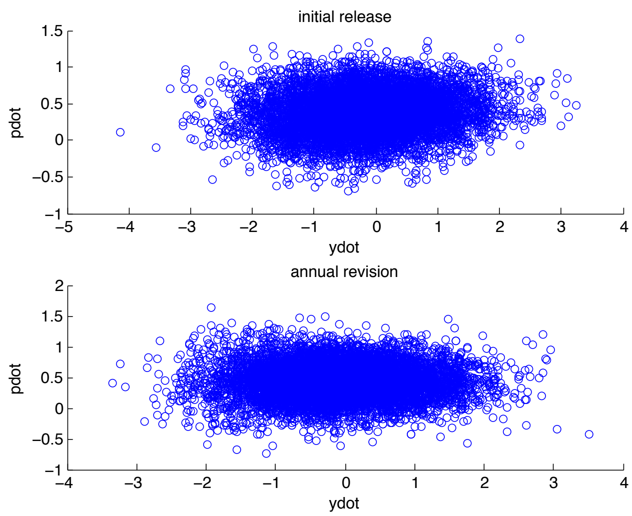

Figure 13: Output-Inflation Predictive Correlation, 2004

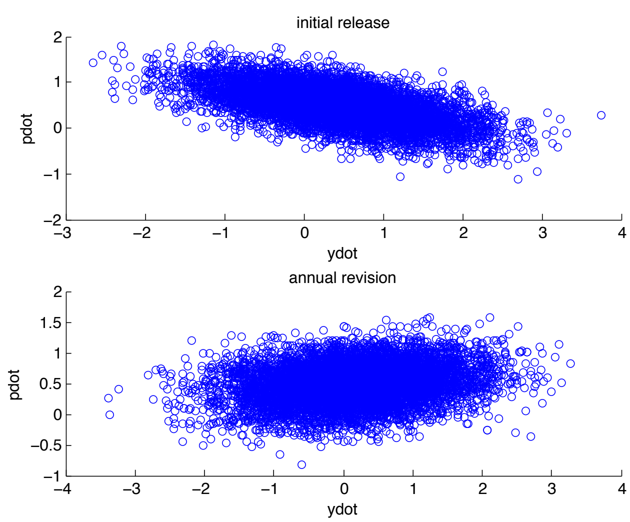

Figure 14: Output-Inflation Predictive Correlation, 2005

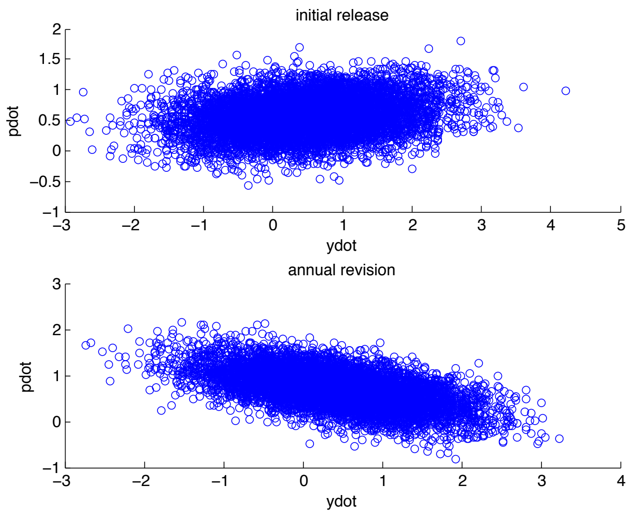

Figure 15: Output-Inflation Predictive Correlation, 2006

and inflation based on the initially released data, but a negative correlation based on revised data. Clearly the data revisions have an impact on the forecasts, though it is difficult to figure out why the revisions generate the observed pattern.

These examples suggest that data revisions can have a significant impact on density forecasts. Changes in the jumping-off point of the forecasts clearly influence the median forecast and the location of other percentiles of the distribution. But the revisions also lead to subtle changes in the distribution of the forecasts.

4.4 A Policy Experiment

As a final experiment, consider the following forecasting exercise. First, using the model and the initially released data each year, generate a forecast for the future path of the interest rate. Second, repeat that forecasting exercise each year, using revised data. Third, look at the differences in the forecasted path of the interest rate each year to see how large an impact revisions have on policy. The results are shown in Figure 16. In the figure, there are 11 lines, one for each year. Each line represents the difference in the policy forecast from using revised data compared with initially released data. While in most years the differences are modest, in three years the differences are substantially, nearly as large as 1.5 percentage points on the federal funds interest rate.

Thus, nearly 30 percent of data revisions lead to very large differences in forecasts of policy. So, revisions matter substantially for forecasts.

5 Conclusions

The real-time literature has suggested that data revisions are not trivial and may have significant effects on forecasts. This paper is the first to illustrate how data revisions affect predictive densities. In our four sets of experiments, we have demonstrated that the differences between initially released data and data that have undergone an annual revision create substantial changes in predictive densities, with significant implications for policy.

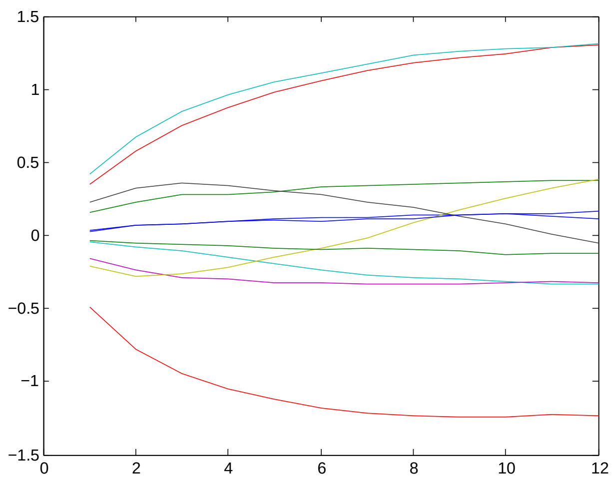

Figure 16: Fed Funds Forecast Experiment

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