Sales and the (Mis)Measurement of Price Level Fluctuations

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Using a large set of scanner data from grocery and drug stores located throughout the US, I show that temporary price reductions ("sales") have a large impact on the growth rate of average price paid. The month-to-month change in an index of average price paid is four times as volatile as the month-to-month change in an index of the "regular" price because sale frequency, depth, and quantity response vary over time. There is a positive relationship between the frequency of sales and the local unemployment rate. The CPI does not fully capture the effect of variations in sale frequency and depth so the use of CPI item indexes to deflate nominal magnitudes results in errors that are correlated with the business cycle.

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1. Introduction

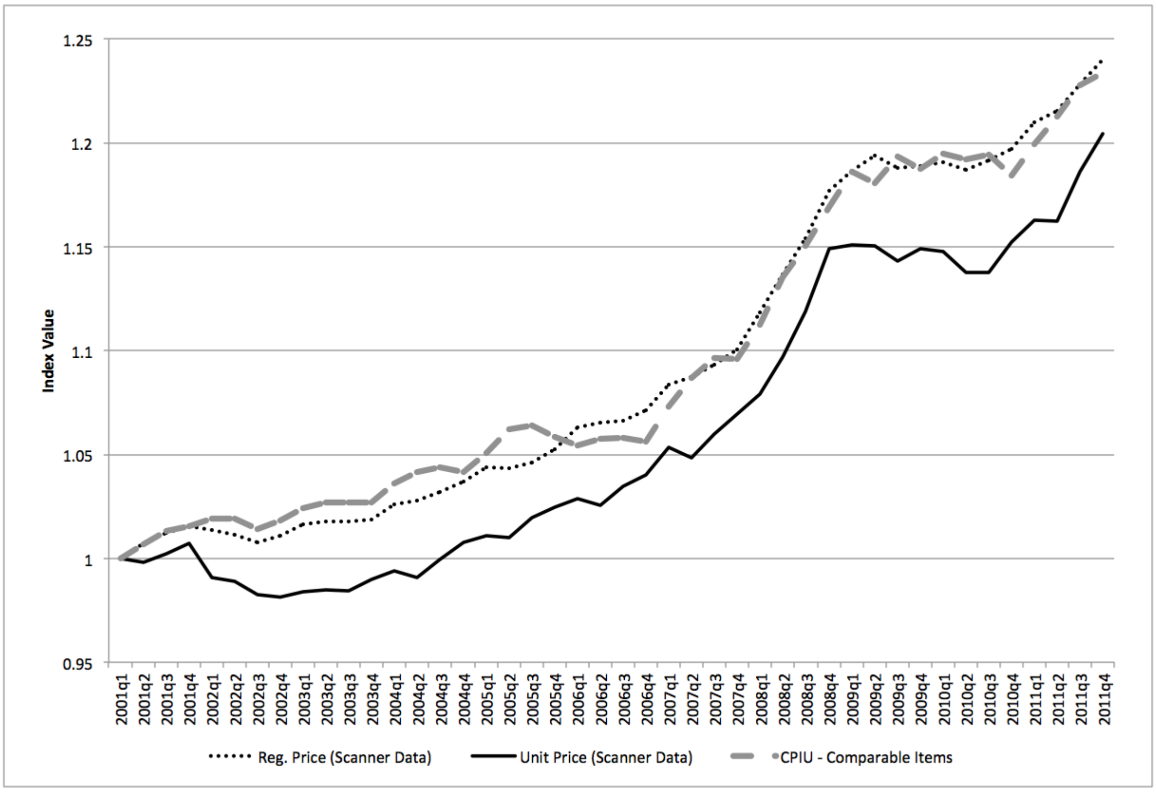
Temporary price reductions ("sales" hereafter) are an important source of price variation for roughly 40 percent of non-shelter consumption spending (Nakamura and Steinsson, 2008) Although several studies successfully reconcile the prevalence of sales in the data with the fundamental importance of sticky prices in New Keynesian macroeconomic models, an important empirical question has yet to be addressed: to what extent do sales influence the dynamics of the aggregate price level?

The main contribution of this paper is to document the aggregate behavior of sales over the two most recent US business cycles. Using a large sample of scanner data, show that 1) sales result in an 8% to 10% gap between average price paid and the "regular" price 2) roughly one third of the seasonally adjuested quarterly variation in average price paid is due to variation in sales, and 3) the cross-sectional relationship between sales and unemployment is time-varying and positive when the national unemployment rate is relatively high.

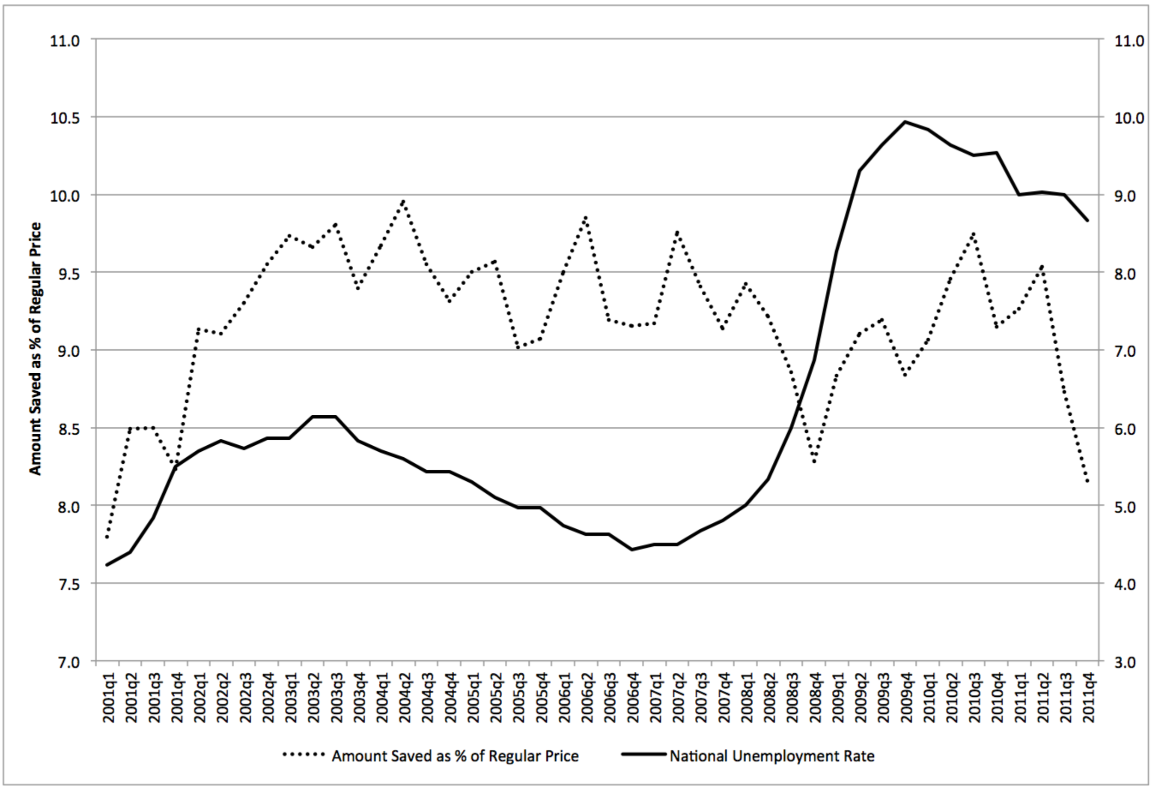
The most important result of this paper can be observed by comparing an index of the "regular price" to an index of the average price paid. These two price indices will run parallel to each other when the size, frequency, and quantity response to sales are fixed. I construct these indices and plot them along with the comparable components of the CPI in Panel A of Figure 1. First, note first how closely the index of regular price and the CPI track each other. This is largely because CPI item indexes are not constructed in a way that will fully reflect variation in the amount saved from sales. Second, following the recession of 2001, average unit price fell more quickly and sharply than did the regular price. This is mainly the result of a five percentage point increase in the fraction of items on sale. Finally, Panel B of Figure 1 suggests that the gap between the regular price and average price paid varies with the business cycle.

Why do these results matter for macroeconomics? First, CPI item indexes are used to deflate nominal indexes into indexes of real variables. My results indicate that because sales vary over time in a way that is correlated with the business cycle, the computation of real consumption growth, for example, will be subject to error that is correlated with

Figure 1: Average Regular Price, Average Unit Price, and the CPI



Panel A: Regular Price, Unit Price, the CPI



Panel B: Regular Price Index minus Unit Price Index and Unemployment

Panel A plots quarterly indexes of unit price and regular price over the full sample. The CPI for comparable items is also included. Panel B plots the difference between total revenue assuming the regular price and actual revenue expressed as a percent of regular price revenue.

the business cycle. Second, macro models aiming to incorporate price setting behavior that mimics retail pricing should result in fluctuations in the effect of sales on average price paid.

Background and Relevant Literature

High frequency (weekly) variation in the price of individual items poses challenges for the measurement of the price level and for modeling price setting behavior in macroeconomic models. In addressing either issue, the first step is to decide between two fundamentally different motivations for the existence of sales. One approach for generating sale like behavior in the macro literature has been to include purely exogenous, idiosyncratic, large, and transient cost shocks as in Eichenbaum, Jaimovich, and Rebelo (2011); Kehoe and Midrigan (2010). The other approach, which is more common in microeconomic literature on sales is to treat sales as a form of price dispersion. Guimaraes and Sheedy (2011) is the only macroeconomic study in which sales arise as a form of price descrimination.

All of the items studied below are storable for at least several weeks and many could be stored indefinitely (does laundry detergent ever expire?). The point is that a gallon of Tide available for purchase today is in most cases a very close substitute for a gallon of Tide available for purchase a week from today. Since the amount that can be saved by buying on sale often quite large (20% or more is common) and some items are on sale quite often, I submit that the appropriate way to model sales is as a form of price dispersion arising out of consumer heterogeneity in preferences, information, and/or search costs. From this point of view, the frequency and size of sales, as well as search intensity are all important factors that determine the price level.

Several macroeconomic studies have reconciled the existence of frequent price changes with the critical New Keynesian assumption of sticky prices by way of more nuanced notions of nominal rigidity (Eichenbaum, Jaimovich, and Rebelo 2011; Kehoe and Midrigan 2010; Guimaraes and Sheedy 2011). These studies uniformly conclude that sales have nothing to do with aggregate price adjustment to nominal shocks and are thus of little interest to macroeconomists. However, these models do not generate fluctuations in the aggregate behavior of sales. I show that sales do substantially contribute to non-seasonal aggregat price variablity and conclude that sales are important for macroeconomics.

One practical implication of my results is that price indexes used to deflate nominal magnitudes into real magnitudes should incorporate information on quantity sold to the extent possible. This is important because as demonstrated above, indexes of unit price and posted price (eg the CPI) do not move in parallel. For example, if during an episode of downward pressure on prices the amount saved from sales rises, inflation measured by the CPI will tend to be overstated and real GDP will tend to be understated. This source of measurement error could be problematic for a monetary policy rule that relies on accurate decomposition of NGDP growth into inflation and real growth (eg a Taylor Rule).

My results and conclusions are similar to those in Stigler and Kindahl (1970), which emphasizes the importance of identifying prices paid rather than the menu price, in the context of producer prices. The analysis in Chevalier and Kashyap (2011) is closely related to this work and reaches similar conclusions. Chevalier and Kashyap (2011) focus on average unit price and the "best price" within very narrow product categories. In this paper, I summarize the aggregate effect of sales on the price level across 24 categories. We reach the same conclusion that sales (best prices) have an important effect on the average price paid over time.

A related line of research explores cost of living measurement issues created by high frequency price changes. Feenstra and Shapiro (2003) develop the theoretically correct cost of living index using a model that allows consumers to buy for storage. The model presumes that households have perfect foresight of prices during a fixed planning period over which they optimize purchases and consumption. Using scanner data on canned tuna, it is shown that a fixed base Törnqvist index is the best approximation of the theoretically correct index. The data also support the hypothesis that a substantial amount of items purchased at sale prices is stored for future consumption.

Griffith, Leibtag, Leicester, and Nevo (2008) use household purchasing data from the UK to show that an average of 29.5% of consumption expenditure is on items that are on sale. This results in an average amount "saved" of 6.5% of total expenditure. These measures vary substantially across households with some saving as much as 21% of annual expenditure from buying on sale. The contribution of the present study to the empirical research on cost of living measurement is to quantify the gap between unit prices and posted prices caused by high frequency price changes and the associated shopper behavior. This gap widened by nearly two percentage points within a single year and it is not uncommon for a full percentage point change to occur from one quarter to the next.

2. A Model of Sale Frequency and Size

The purpose of the following model is to sketch a plausible way in which sales could vary with macroeconomic conditions under the presumption that the primary reason sales exist is to price discriminate between different types of consumers. In this model, sales are used to attract a particular type of consumer (a "shopper"). I assume that a firm can attract more shoppers to the store (who will buy a basket of goods) by increasing the fraction of items on sale and/or by increasing the depth of sales. The idea that sales are used to attract shoppers is supported by several of empirical studies(Chevalier, Kashyap, and Rossi, 2003; DeGraba, 2006; Hosken and Reiffen, 2004). The cost of increasing the frequency or depth of sales is that the firm's loyal customers, who are perfectly willing to pay the "regular price", end up paying the sale price some of the time. Firms choose the frequency and depth of sales to maximize profits.

I do not explicitly model consumers' behavior. Instead, I simply assume that some consumers do not react to sale prices, while others do. This assumption is motivated by the theoretical literature on price dispersion, which relies on consumer heterogeneity and search costs as key ingredients. In the model below, as in other models of price dispersion, sales are a way for stores to price discriminate.[1]

[1] For some examples of this literature, see Burdett and Judd (1983); Conlisk, Gerstner, and Sobel (1984); Lal (1990); Reinganum (1979); Sobel (1984); Varian (1980)

2.1. Model Setup

A representative firm sells a continuum of items with measure one. For simplicity, I assume that a unit of each item has the same marginal cost C. Consumers wish to purchase an average of one unit during the shopping period as long as the price is not more than their maximum willingness to pay, R.

There are two types of consumers: Buyers and Shoppers. There is one buyer per store and a total of S Shoppers (who may visit any store). Firms cannot distinguish Buyers from Shoppers. Buyers visit a predetermined store and purchase one unit of each item as long as its price is no larger than R. Therefore, firms set the "regular price" equal to R.[2] In order to attract Shoppers, firms must put some fraction of items $f \in [0,1]$ on sale at a price of $R(1-d)$. Note that d is the "discount" versus regular price and will always be between 0 and 1. For tractability, I assume that the discount is the same for all items.

The effect of shopper behavior on a firm's revenue is summarized by two functions. First, the fraction of Shoppers who visit the representative store is given by the function $\Phi(f,d) \in [0,1]$, which maps the fraction of items on sale, f and the discount, d onto some number between zero and one. I also assume that more items on sale and deeper discounts attracts a larger share of the Shoppers (Φ_1, $\Phi_2 \geq 0$).[3] Second, I assume that Shoppers purchase a larger fraction of items at the sale price than f, the fraction of items on sale. This assumption captures the idea that Shoppers are more attentive to sale prices and may substitute across similar items or make multiple trips to obtain a larger share of items at a sale price. This behavior is summarized by $\mu(f)$, the fraction of items that Shoppers obtain at the sale price. By assumption, $1 \geq \mu(f) \geq f > 0$, and $\mu'(f) \geq 0$.

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[2] I choose not to model how R is determined in order to focus on sales. Any price above R would result in zero purchases and anything less would result in no additional units sold, so the "regular price" is R.

[3] Φ_1 is the partial derivative of Φ with respect to the first argument, f.

Since Buyers are passive and buy one unit of everything, they get a fraction f of their items at the sale price.

Stores select the fraction of items to put on sale, f, and the discount d to maximize profits.

$$\max_{f,d} Rf(1-d) + R(1-f) - C + \Phi(f,d)S\left(R\mu(f)(1-d) + R\left(1-\mu(f)\right) - C\right)$$

which can be written:

$$\max_{f,d} R\left[(1-fd-c) + \Phi S\left(1-\mu d - c\right)\right]$$

where $c \equiv \frac{C}{R}$ is the marginal cost expressed as a fraction of the regular price, R. Also, for notational convenience, I drop the arguments of Φ and μ.

The first order condition (for an interior solution) to the profit maximization problem with respect to f is:

$$\Phi_1 S(1-\mu d - c) = d\left(1 + \Phi S \mu'\right)$$

This says that the marginal profit from additional Shoppers must be equal to the marginal cost of putting more items on sale. The marginal cost of putting more items on sale (increasing f) is due to the reduction in average price paid, which occurs at a rate of d for Buyers and $d\Phi S\mu'$ for Shoppers. The term μ' is the rate at which Shoppers increase the proportion of units purchased on sale.

The first order condition (for an interior solution) with respect to d is:

$$\Phi_2 S(1-\mu d - c) = f + S\Phi\mu$$

The interpretation of this condition is similar. The marginal profit of increasing d is equal to the additional Shoppers times the profit per unit from Shoppers. The marginal cost of increasing d is the reduction in revenue from Buyers, f, and from Shoppers $S\Phi\mu$.

The point of this model is to illustrate that multiproduct retailers might adjust

f and d in response to changes in the number of Shoppers (relative to the number of Buyers). Without making specific assumptions about the functional forms of Φ and μ, little can be said about the optimal choice of d and f. An increase in the proportion of Shoppers, S, would seem to increase the benefit to having sales, but changing f affects the marginal benefit of changing d and vice versa. The model formalizes the the idea that store managers may opt to adjust average price by way of a change in either the frequency of sales f, or the depth of sales d if the number of customers who are responsive to sales changes. Below I present numerically obtained results of a specific version of the model.

2.2. The Behavior of Average Unit Price

To get some idea of how f and d are related to S, I analyze a numerical example. Before doing so, it will be useful to describe a measure of sales activity that summarizes the effects of f, d, and the average response to sales. First, note that the average price paid is:

$$U \equiv \frac{R\left(\left(1 - fd\right) + \Phi S\left(1 - \mu d\right)\right)}{1 + \Phi S}$$

Rearranging terms yields the following expression:

$$1 - \frac{U}{R} = fd\left(w_b + w_s\frac{\mu}{f}\right) \tag{1}$$

where $w_b = \frac{1}{1+\Phi S}$ and $w_s = \frac{\Phi S}{1+\Phi S}$ indicate the fraction of the firm's customers that are Buyers and Shoppers respectively. I refer to the LHS of equation 1 as the "realized discount" (RD). The last term on the RHS of 1 can be thought of as the average quantity response to a sale. It is the quantity sold at a sale price relative to the average quantity sold (at any price). Buyers buy the same quantity of a sale item as a non-sale item. Shoppers buy proportionately more items on sale, as indicated by the ratio $\frac{\mu}{f} > 1$. In Section 4, I derive this relationship using a different approach that may make it more intuitive.

The model predicts that the realized discount will depend on the number of Shoppers S, and the marginal cost divided by the regular price, c. I explore the model further by selecting the following functional forms: $\Phi(f, d) = f^{\alpha_1} d^{\alpha_2}$, and $\mu(f) = f^{\beta}$. With these functional forms, the optimization problem must be solved numerically, which I do using a grid search over the domain of f and d. I selected the parameters ($\alpha_1 = .25$, $\alpha_2 = .5$, $\beta = .38$) that yield results similar to those found in the data ($d \approx .20$ and $f \approx .15$).

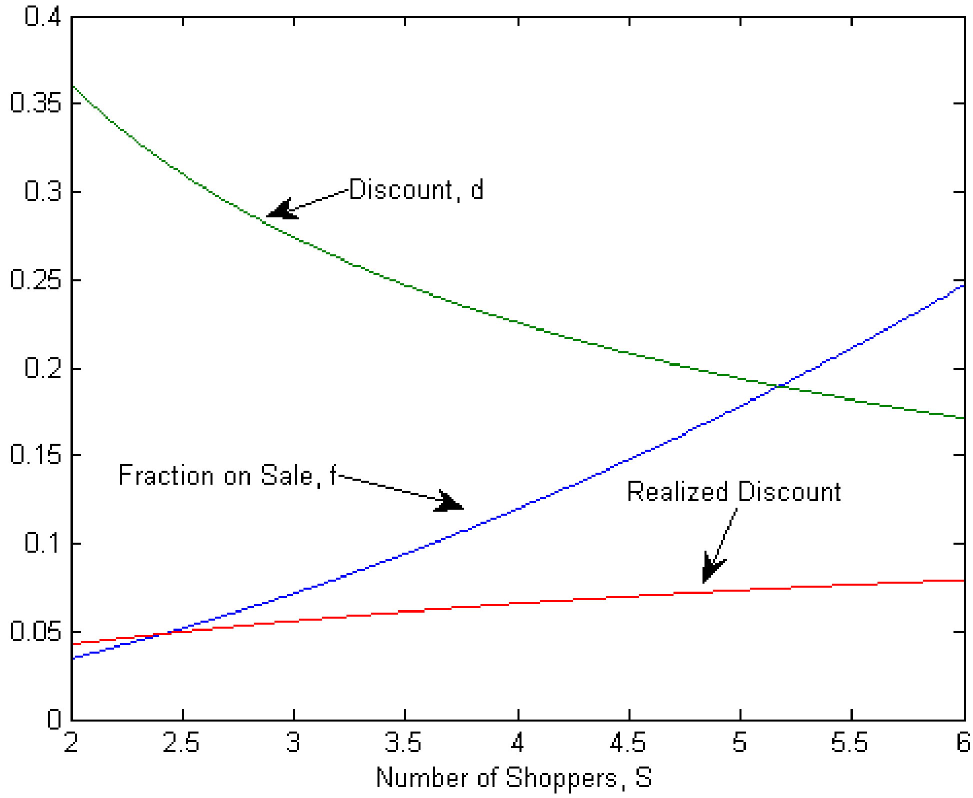
Figure 2 plots the optimal f and d as well as the realized discount (RD) against S, the fraction of consumers that are Shoppers. From this chart we can see that as the proportion of Shoppers increases, the fraction of items on sale increases; but the average discount actually falls. The net effect of an increase in the number of Shoppers is an increase in the realized discount. As we will see in Section 4.1, these predictions are qualitatively consistent with the data. The correlation between f and d is negative and the correlation between f and RD is positive.

3. Data

In the analysis that follows, I use the Symphony IRI Scanner Data research database (Bronnenberg, Kruger, and Mela, 2008). Scanner data is highly disaggregated and has three advantages over Bureau of Labor Statistics survey data for studying pricing behavior.[4] First, quantity sold is available which indicates how "important" a particular price is. Second, scanner data contain weekly observations which allows researchers to observe almost all movements in prices since intra-week price movements are rare. BLS survey data are sampled monthly or bimonthly. Finally, with scanner data we observe the price and quantity sold of *all* products available within a category-store. The main disadvantage of this particular scanner data set is that it covers a small subset of the universe of consumption goods contained in the CPI.

[4] Scanner data contains an observation for each store-UPC-week. Each observation contains the quantity sold and total dollar sales as well as information about feature and display for the store selling the item.

Figure 2: Optimal Frequency and Discount of Sales



The figure above plots the optimal choice of f and d, as well as the resulting realized discount, RD associated with various levels of S, the number of Shoppers. I use the following functional forms for this example: $\Phi(f,d) = f^{.50}d^{.25}$ and $\mu(f) = f^{.38}$.

The IRI data cover 31 categories in 50 different markets and contain both grocery stores and drug stores from several different chains. From this data set, I select the data from that provided observations for for all 11 years (2001-2011) so that all of the analysis is done with a balanced panel of stores and categories. I end up with 24 categories sold in each of 451 stores located in 40 different markets. The data contains weekly observations for the calendar years 2001 through 2011. This is important because it spans the recession of 2001 and the recession of 2008/09.

Each observation (a store-product-week) contains total revenue and units sold. This allows me to compute the average price for each week in the data. For over 95% of the observations, it appears that a single price was charged during the week (based on the fact that the average price doesn't contain a fraction of a penny). In cases where there were two or more prices charged during the week, I simply treat the observed "average price" as the posted price for that week.[5]

3.1. Identifying a Sale and the Regular Price

To proceed, I need to construct two variables: 1) a binary variable, $sale_{it}$, that indicates whether item i was on sale during week t and 2) the non-sale or "regular" price (which is different from the observed price when an item is on sale). Ideally these variables would come straight from the store, but unfortunately they do not.[6] Instead, I construct them using an algorithm discussed in detail in the appendix. The intuation behind the algorythm is as follows. The regular price equal to the observed price whenever it is greater than or equal to the 13 week centered moving average price. Spells of five weeks or more of the same price are considered to be regular price weeks. Any weeks with unidentified regular prices are then set to be equal to the most recently observed regular price. Any week in which the observed price is 5% or more below the newly constructed

[5]This issue introduces a potential measurement error into my analysis. If the fraction of multiprice weeks increases, the likelihood that I identify the correct regular price goes down and I will tend to understate the regular price and therefore the realized disocunt. Given how closely the CPI tracks the index of regular price I have computed, I believe that this measurement error is small.

[6]The IRI data contain a price reduction indicator that is computed using IRI's proprietary algorithm for identifying sale prices. For the sake of transparency I chose to create my own sale indicator. I use the IRI sale flag as a benchmark for evaluating the algorithm.

regular price is flagged as a sale week.

3.2. Summary Statistics

The three characteristics of sales that deserve our attention are defined as follows:

1. Frequency: f (measured by the fraction of items on sale per week)

2. Size: d (percentage discount off of the regular price)

3. Quantity Ratio: z (quantity sold per sale week relative to average quantity sold)

Frequency and size are often referred to in the marketing literature as the breadth and depth of price promotions and are determined by the store's managers in order to maximize profit given expectations about shopper behavior (Blattberg, Briesch, and Fox, 1995). The quantity ratio, z, is the quantity sold per sale week relative to the average weekly quantity sold. It measures consumer response to sales.

To see how these three variables are related, suppose we observe that for a given time period, the regular price for an item is R and that the unit price (average price paid) is U. The unit price is simply a weighted average of the regular price and the sale price S:

$$U = (1 - w)\, R + wS$$

where w is the quantity sold at a sale price divided by the total quantity sold. We can rearrange this equation the following way:

$$1 - \frac{U}{R} = wd \tag{2}$$

Here, $d \equiv \frac{R-S}{R}$ is the average sale discount expressed as a percent of the regular price. Just as we did in Section 2, let us call the LHS of equation 2 the realized discount, RD. This is the amount saved from buying on sale expressed as a percentage of regular price. Note also that we can write $w = zf$, where z is the quantity response to a sale (the ratio of units sold per sale week to units sold per week) and f is the fraction of weeks on sale. Thus, we have the following expression relating the realized discount and the three characteristics of sales:

$$1 - \frac{U}{R} = zfd \tag{3}$$

Table 1 contains sample averages by category of the following measures of sales: fraction of weeks on sale, fraction of revenue from sales, fraction of units sold at a sale price, the median discount, and the median quantity ratio. I also provide the number of store-products and average annual revenue for the entire sample. There are several things worth mentioning about these statistics.

First, although sales account for a small fraction of price quotes (16.7 percent), they account for a relatively large fraction of revenue (34.2 percent). Two to three times as many units are sold during a sale week compared to the average (as measured by the quantity ratio). The spike in quantity sold during a sale week is much higher than can be explained by a simple model of supply and demand (Hendel and Nevo, 2006; Feenstra and Shapiro, 2003). This fact, along with the storeability of the items in this sample strongly suggest that sales are primarily a mechanism for price discrimination.

Second, the larger categories tend to have sales more frequently. Figure 3 illustrates this by plotting annual revenue against fraction of items on sale for each category in the sample. There is a positive relationship between a category's share of expenditure and its frequency of sales.

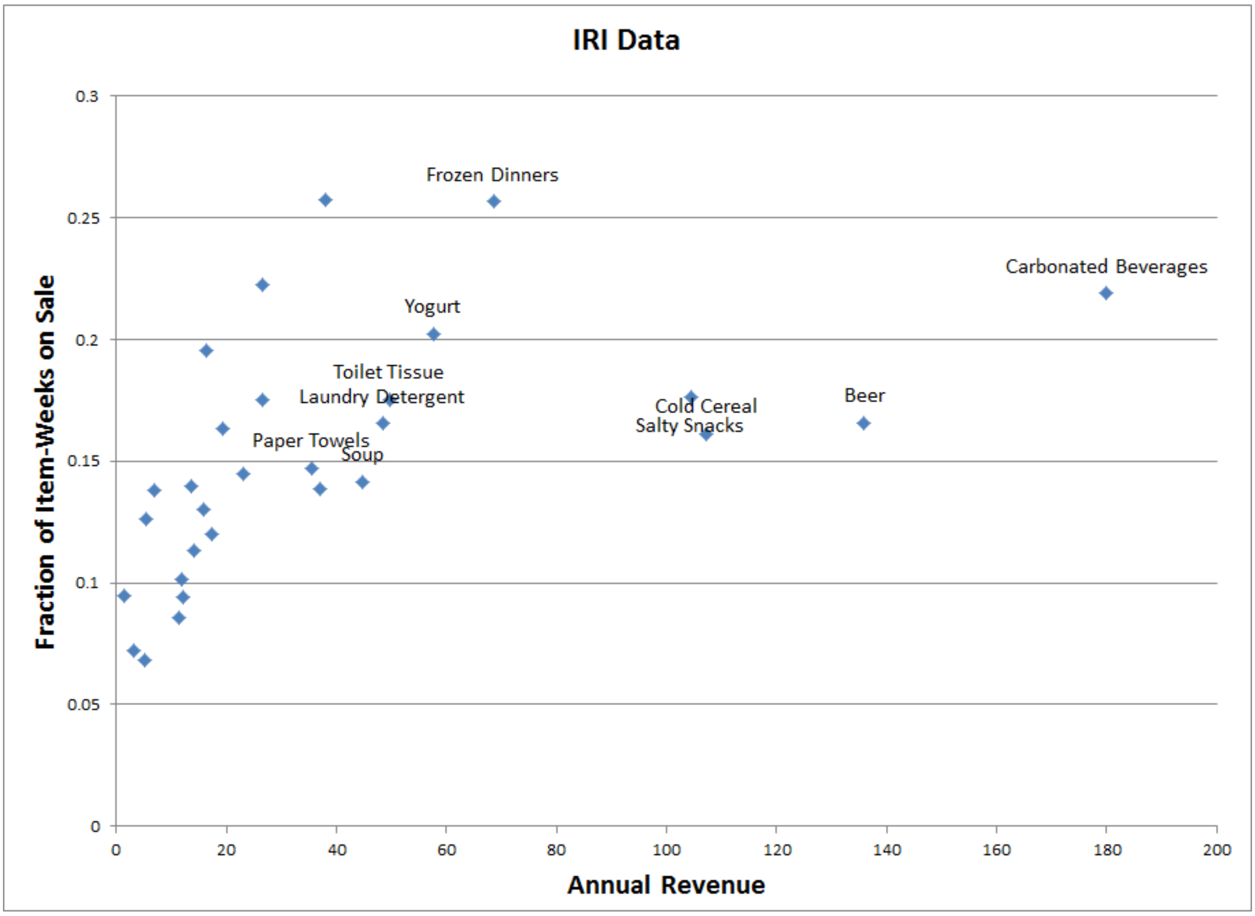
The relationship between expenditure and frequency of sale holds at the UPC level as well. To show this, I calculate average weekly revenue for each UPC in the sample and separate UPCs into within store revenue deciles. I plot the average fraction of weeks on sale by within store revenue decile in Figure 4. There is a strong positive relationship between an item's *long run* revenue share and the frequency with which it is on sale. This relationship is consistent with the idea that sales are often used to attract trips to the store (Chevalier, Kashyap, and Rossi, 2003; Hosken and Reiffen, 2004; Lal and Matutes, 1994).

Table 1: IRI Summary Statistics by Category

Category	Store x Products	Annual Revenue ($Millions)	Sales Fraction of Total: Weeks	Revenue	Units	Median Discount	Median Quantity Ratio
Carbonated Beverages	225,408	180.0	21.9%	43.0%	44.5%	22.1%	1.9
Beer	125,690	136.0	16.6%	35.0%	30.1%	11.7%	1.8
Cold Cereal	160,894	107.3	16.1%	31.8%	41.5%	30.3%	2.9
Salty Snacks	317,814	104.5	17.6%	32.1%	35.8%	21.4%	2.0
Frozen Dinners*	237,906	68.7	25.7%	42.5%	48.1%	28.3%	2.6
Yogurt	147,471	57.7	20.2%	28.0%	39.3%	23.9%	2.0
Toilet Tissue	35,925	49.7	17.5%	39.6%	35.3%	23.7%	3.2
Laundry Detergent	87,963	48.5	16.5%	35.7%	44.8%	24.1%	3.1
Soup	138,809	44.8	14.1%	29.6%	36.9%	27.1%	3.0
Frozen Pizza	81,255	38.0	25.7%	42.7%	49.6%	24.1%	2.7
Coffee	103,386	37.0	13.8%	32.2%	39.4%	19.3%	2.7
Paper Towels	31,729	35.5	14.7%	36.4%	31.0%	22.6%	2.6
Hot dogs	28,152	26.7	22.2%	36.8%	48.3%	29.8%	2.4
Spaghetti Sauce	78,634	26.5	17.5%	33.5%	41.9%	22.7%	2.7
Diapers*	80,693	23.2	14.5%	23.8%	27.3%	15.4%	2.5
Margarine and Butter	31,970	19.3	16.3%	22.6%	29.0%	22.7%	1.7
Mayonnaise	24,292	17.3	12.0%	26.7%	31.1%	21.6%	2.1
Facial Tissue	28,251	16.3	19.6%	31.7%	38.8%	23.6%	2.1
Toothpaste	122,630	16.0	13.0%	27.6%	34.3%	22.4%	2.7
Shampoos	223,410	14.1	11.4%	21.6%	29.0%	23.0%	3.0
Peanut Butter	21,693	13.6	14.0%	24.6%	34.0%	19.2%	2.2
Mustard and Ketchup	42,846	12.1	9.4%	23.2%	30.5%	19.1%	2.3
Deodorant	213,178	12.0	10.1%	21.3%	28.1%	25.1%	3.1
Blades	58,811	11.4	8.6%	13.3%	18.7%	20.2%	2.3
Household Cleaners*	33,456	6.9	13.8%	22.4%	27.7%	19.5%	2.1
Toothbrushes*	76,042	5.5	12.6%	25.8%	32.1%	25.1%	3.1
Sugar Substitutes	12,911	5.1	6.8%	9.9%	11.9%	13.1%	1.7
Photo Supplies*	14,725	3.2	7.2%	20.0%	23.0%	21.3%	2.7
Razors	16,179	1.5	9.5%	21.2%	22.9%	16.7%	2.6
All Categories	**3,012,670**	**1,380.5**	**16.7%**	**34.2%**	**39.5%**		

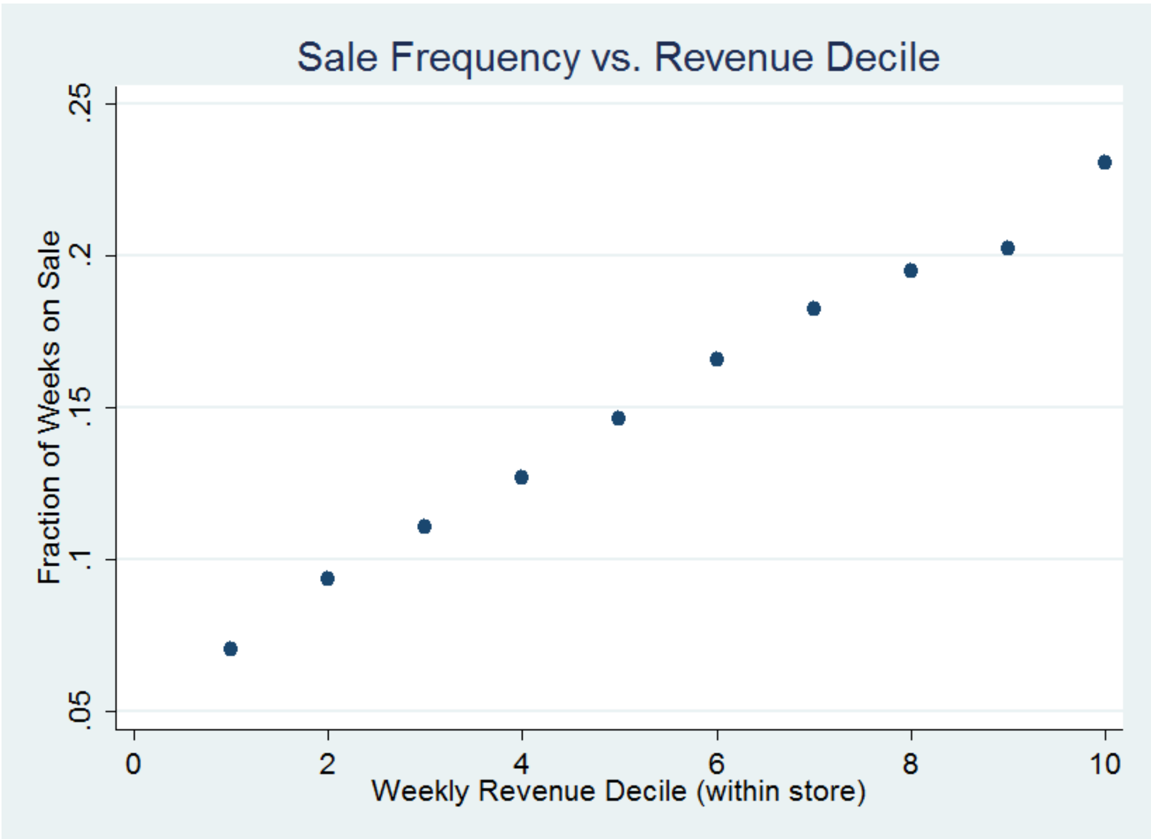
Notes: Column 2 contains the number of store-UPC combinations in the data (larger than the number of UPCs since most UPCs are sold at several stores). *These categories were excluded from all time series analysis due primarily to an unusually large spike in the number of items included in the data set that occurred entirely during the first week of 2007.

Figure 3: Annual Revenue versus Fraction of Items on Sale by Category



Notes: Each point represents a category. The vertical axis is fraction of item-weeks on sale and the horizontal axis is average annual revenue (as reported in Table 1).

Figure 4: Fraction of Weeks on Sale by Average Weekly Revenue Decile



Notes: This figure plots average fraction of weeks on sale by within store revenue decile.

Finally, the discount is often quite large. In the IRI data, the median discount versus regular price is over 20 percent for 21 of the 29 categories listed and over 25 percent for six of them. If shoppers are willing to search for deals, hold some inventory, and selectively substitute, substantial savings are available from buying on sale. One study has found that households in the UK save an average of 6.5 percent of annual expenditure by buying on sale (Griffith, Leibtag, Leicester, and Nevo, 2008).

To summarize, sale prices occur most frequently on popular items, discounts are often 20% or more, and a disproportionate share of purchases occur at sales prices. These facts indicate that sales are an important determinant of average price paid (unit price) and also nominal consumption expenditure. Chevalier and Kashyap (2011) suggest that average unit price (or more practically, an average of best price and regular price) provides a better picture of the price consumers are paying over time than any individual price series. In the next section I quantify the effect of sales on the *dynamics* of average unit price.

4. Empirical Analysis

In this section, I wish to answer two questions: 1) How much do sales contribute to variation in the growth rate of average price paid? 2) Is the effect of sales on average unit price related to macroeconomic conditions?

I address the first question first by simply graphing averages of the realized discount, sale frequency, sale depth, and the quantity response over time. I also quantify how much of the variation in the price level can be attributed to changes in the regular price and changes in the characteristics of sales. Sale activity is at least as important as regular price changes in terms of explaining monthly variation in the price level.

As for the second question, recall that the model presented in Section 2 predicts that the realized discount depends on the proportion of consumers who respond to sales. I test this hypothesis via panel regressions in which unemployment is a proxy for the proportion of consumers who are Shoppers. The idea is that as the unemployment rate rises, more people will have the time and incentive to shop for better prices. I show that the unemployment rate is positively related to the realized discount. Relatively high unemployment rates are associated with more sales activity in this sample.

4.1. Regular Price and Unit Price

To see whether (and how) sales vary in the aggregate, I calculate the revenue weighted mean of $\left(1 - \frac{U}{R}\right)$, z, f, and d (across store-UPCs) for each quarter and plot the results in Figure 5. The average realized discount $\left(1 - \frac{U}{R}\right)$ increased rapidly over the course of 2002, after which it remained fairly high (with seasonal fluctuation) until about the end of 2004. By the end of 2008, the average realized discount had fallen back to its initial level of just over nine percent.

The cause of the rapid rise in realized discount was evidently a large increase in the frequency of sales that occurred during 2002. The frequency of sales continued its growth, but at a much slower rate until the middle of 2006, after which it began to decline slowly. The average quantity response (z_t) and average discount (d_t) fell sharply along with the initial jump in the frequency of sales that occurred in the first quarter of 2002. The rapid rise in the realized discount that began in the third quarter of 2002 resulted from all three components (frequency, discount, and quantity response) rising together. The effect of sales on unit price shrinks (with some seasonal fluctuations) for the rest of the sample period due primarily to decreases in the average discount and quantity response.

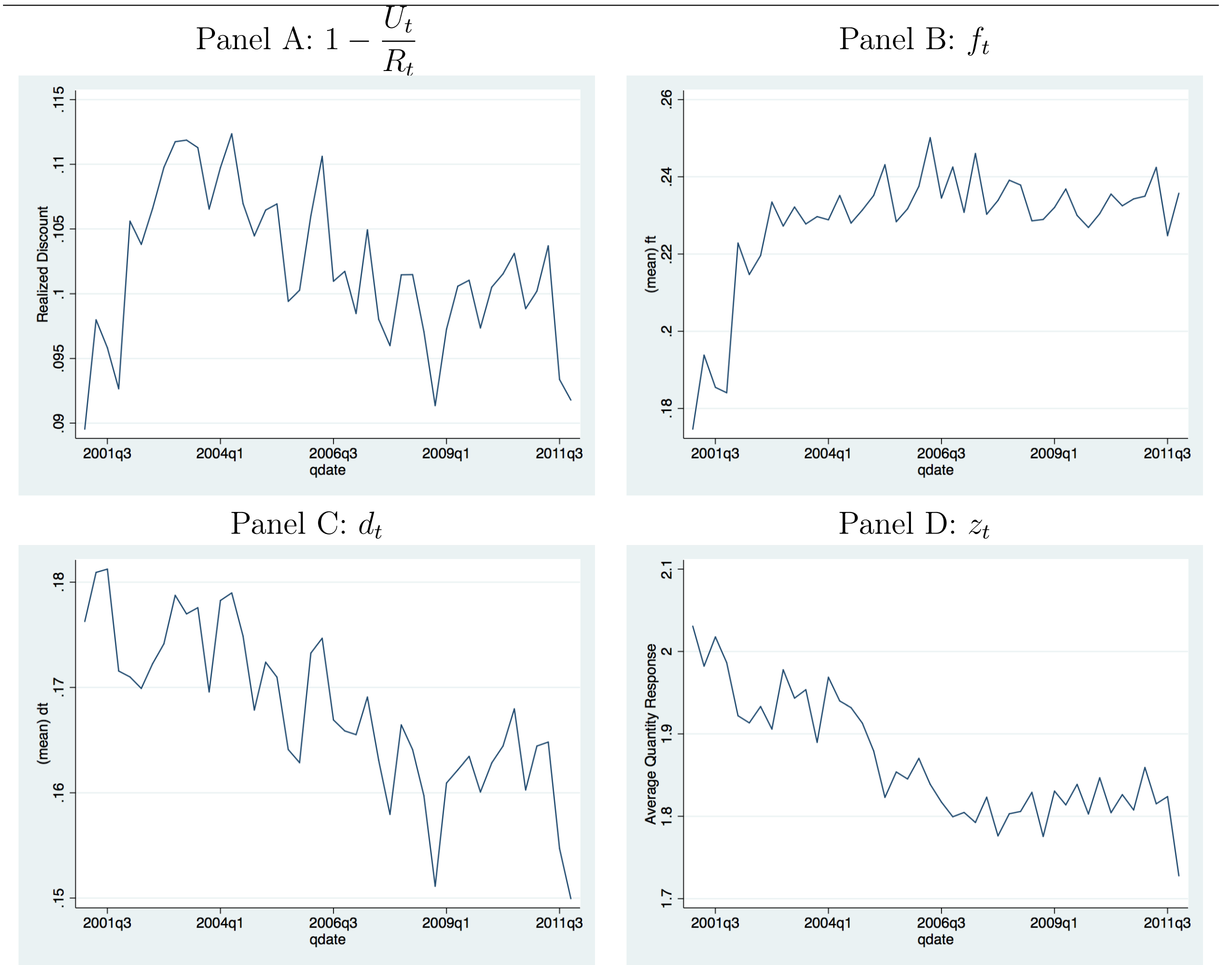
4.2. Sales and Aggregate Price

In this section, I analyze the impact of sales on the price level over time by comparing an index of regular price to an index of average unit price. This allows us to quantify the effect of sales and regular price on the growth rate of average unit price over time.

Price Index Construction

In order to construct indexes of regular price and unit price, I employ the same approach used by the BLS to calculate the consumer price index with one important difference.

Figure 5: The Characteristics of Sales Over Time



Panel A: Revenue weighted mean of the realized discount: $1 - \frac{U_t}{R_t}$. Panel B: Revenue weighted mean of f_{it}, the fraction of weeks that all items i is on sale during quarter t. Panel C: Revenue weighted mean of $d_{it} = \frac{R_{it} - S_{it}}{R_{it}}$, the average percent discount as a percent of regular price, *conditional on at least one sale* during quarter t. Panel D: Revenue weighted mean of the quantity response, z_{it} which is the average units sold per sale week relative to average units sold in all weeks.

The BLS samples a single price quote each month. I have weekly price and quantity data and wish to make use of this extra information. Rather than randomly selecting a single price from each month (as the BLS does), I take an average of all prices within a month. For the index of regular price, I take a simple average of regular price for the month (usually there are 4 and they are often all the same). For the index of unit price, I take total revenue divided by units sold to get the average price paid for a given item during a given month.

I aggregate across store-UPC's the same way that the BLS aggregates sampled prices. I calculate a price relative for each store-UPC-month (the ratio of current price to last month's price) and then compute the geometric average of price relatives within each store-category using fixed weights.[7] Additional details can be found in the Appendix.

Decomposing Unit Price Volatility

The first thing worth noting is that unit price is substantially more volatile than regular price. Depending on the market, the standard deviation of the month-to-month log change of unit price is two to four times that of regular price (see columns 4 and 5 of Table 2). This alone is evidence that changes in regular price only explain a fraction of the variation in unit price. Of course much of this volatility may be due to seasonal variation in sales.

To control for seasonal variation in the growth of unit price, I project the log difference of the unit price index, r_t^u, onto the log change of the regular price index r_t^r and a vector of monthly dummies $\mathbf{m_t}$:

$$r_t^u = \mathbf{m_t}\lambda + \alpha_1 r_t^r + e_t \tag{4}$$

The purpose of estimating equation 4 is to see how much of the variation in r_t^u is explained by seasonal variation (captured by the term $\mathbf{m_t}\lambda$) and variation in r_t^r. Whatever variation remains unexplained is due to non-seasonal variation in sales. Table 2 contains

[7] I use the store-UPC's average weekly revenue as the weight.

Table 2: **OLS Projection:** $r_t^u = \mathbf{m_t}\lambda + \alpha_1 r_t^r + e_t$

Monthly Index (t indexes months).

	α_1	R-Squared	$\sigma_{Unit\,Price}$	$\sigma_{Reg\,Price}$
New York	0.16	0.31	1.1%	0.3%
Los Angeles	0.33	0.39	1.2%	0.4%
San Francisco	0.36	0.51	1.7%	0.5%
Dallas	0.40	0.39	1.1%	0.6%
Houston	0.41	0.44	1.0%	0.4%
All Markets	0.51	0.44	0.8%	0.2%

Quarterly Index (t indexes quarters)

	α_1	R-Squared	$\sigma_{Unit\,Price}$	$\sigma_{Reg\,Price}$
New York	0.31	0.71	1.2%	0.4%
Los Angeles	0.55	0.49	1.5%	0.7%
San Francisco	0.34	0.50	1.5%	0.6%
Dallas	0.80	0.66	1.2%	0.8%
Houston	0.87	0.73	1.4%	0.8%
All Markets	0.80	0.66	0.6%	0.4%

Notes: Each row contains the of an OLS projection of the log difference of unit price (r_t^u) on the log difference of regular price (r_t^r) and a vector of month dummies ($\mathbf{m_t}$). Both $r_t^u$and r_t^r are standardized (demeaned and divided by the standard deviation) prior to the estimation. The top and bottom panels use respectively monthly and quarterly aggregated time periods. For example, the top panel uses the log change of a monthly price index.

OLS estimates of Equation 4 for monthly and quarterly frequencies. The R-squared statistics indicate that roughly 30 to 50 percent of the month-to-month variation in aggregate price can be explained by variation in regular price and seasonal variation in sales. Aggregating to a quarterly index increases the explanatory power of regular price, but still leaves about a third of the variation in unit price to be explained by non-seasonal variation in (the frequency, size, and quantity ratio of) sales. I conclude that *non-seasonal* variation in the *aggregate* behavior of sales contributes an economically significant amount of variation to the growth rate of average unit price.

4.3. Sales and Unemployment

To begin this section, I refer the reader back to Figure 1. This graph suggests that regular price grows faster than average unit price when unemployment is relatively high. The relatively short sample period limits my ability to draw strong conclusions about the relationship between sales and unemployment over time. However, with 450 individual stores located across 40 markets, we do have a fairly large cross section to work with.

To get a sense of whether there is any relationship between the unemployment rate and sales, I begin with a non-parametric test. Specifically, I compute the rank correlation coefficient (Kendall's tau) between the local unemployment rate and several measures of sale activity for each quarter. The idea is to test for a statistical relationship in the cross-section without making any particular assumptions about its structure. I plot Kendall's tau for each quarter in Figure 6.

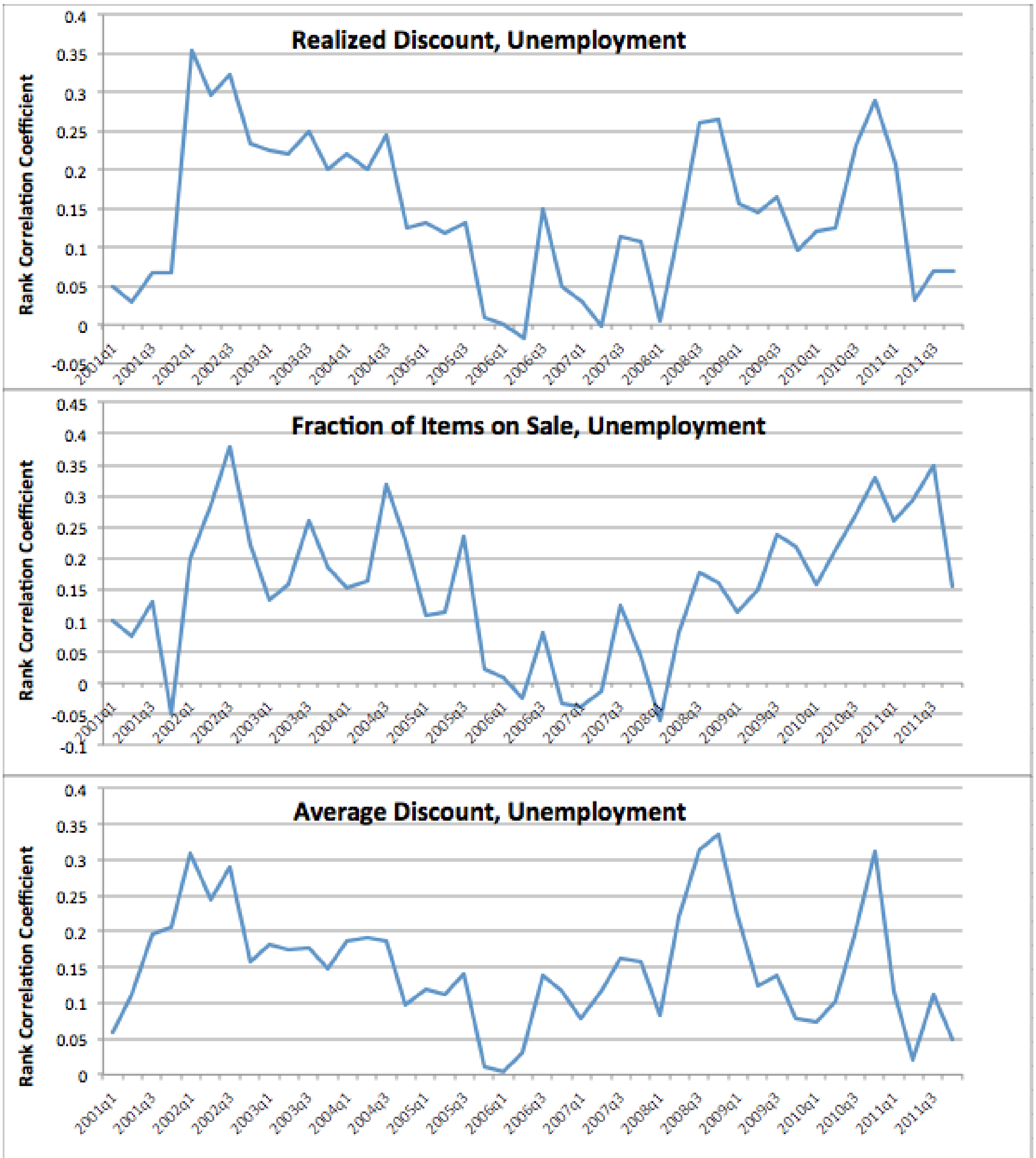
Early in the sample period (2001), when the national unemployment rate is relatively low, the rank correlation coefficient of realized discount and the local unemployment rate is statistically indistinguishable from zero. Once the national unemployment rate rises, we see a strong positive relationship between the realized discount and local unemployment (Kendall's tau of about .30). As the national unemployment rate falls back to pre-recession levels (by the end of 2005), the cross-sectional relationship diminishes. The cross-sectional relationship pops up again, though somewhat inconsistenly at the onset of the most recent recession and returns to zero by the end of the sample period (2011).

Figure 8 suggests that the relationship between the realized discount and unemployment varies over time and is positive when unemployment is high relative to recent experience. I proceed with panel regressions in which I assume (initially) that the effect of unemployment is fixed and then for robustness I separate the sample into two time periods (pre-2007 and post-2007). In all cases I test the hypothesis that unemployment is related to each of the following measures of sales: the realized discount, fraction of revenue occuring at a sale price, and the revenue weighted average discount. Recall that the model presented in Section 2 predicts that the realized discount varies positively with the number of Shoppers and the ratio of marginal cost to regular price.

The baseline specification that estimate is the following:

$$RD_{ijt} = \beta_1 Unemp_{jt} + \beta_2 c_{ijt} + u_i + e_{it}$$

Figure 6: Cross Sectional Rank Correlation of Unemployment and Realized Discount



Notes: I compute the Kendall's tau (rank correlation coefficient) of the unemployment rate and several measures of sale activity for each quarter in the data set. There are 40 observations (markets) in each quarter.

Table 3: Panel Regressions (Annual Observations)

Dependent Variable	Realized Discount		Sale Fraction of Revenue		Discount	
	Levels	First Diff	Levels	First Diff	Level	First Diff
$Unemp_{jt}$	0.16***	0.13***	0.13**	0.28***	0.20***	0.08***
	(0.019)	(0.014)	(0.050)	(0.042)	(0.022)	(0.016)
$PPI_{jt}/RegPrice_{ijt}$	-0.13***	-0.09***	0.09***	0.21***	-0.25***	-0.19***
	(0.010)	(0.014)	(0.03)	(0.034)	(0.012)	(0.023)
$Constant$	0.23***	0.001***	.22***	0.005***	0.42***	0
	(0.010)	(0.0001)	(.038)	(0.0002)	(0.011)	0
Groups	451	451	451	451	451	451
Time Periods	11	10	11	10	11	10
R-Squared (within)	0.11	0.02	0.01	0.01	.24	0.03

The dependent variable *Realized Discount* is described in Section 4.1. Sales fraction of revenue is the fraction of total revenue that occurred at a sale price. The discount is the revenue weighted discount off of regular price. $Unemp_{jt}$ is the average unemployment rate for the BLS MSA that corresponds to IRI market j during year t. The variable $PPI_{jt}/RegPrice_{ijt}$ is a proxy for c in the model. It is the average of the PPI across all of the relevant categories divided that by the index of the regular price for store i. Standard errors are robust to arbitrary serial correlation.

where RD_{ijt} is the revenue-weighted average of the realized discount in store i in market j, during year t, and c_{ijt} is an average of the relevant Producer Price Indexes divided by an index of store $i's$ regular price. This variable controls for variation in the marginal cost relative to the regular price. Using BLS data, I match each store to a city (MSA) and use the annual average of the local unemployment rate, $Unemp_{jt}$, as the primary independent variable of interest. The reason for aggregating to the year is to avoid issues with seasonality in both the unemployment rate and the sales variables. I also estimate the same model using the fraction of revenue at a sale price and the weighted average discount as dependent variables. To control for the possibility of a unit specific trend, I also estimate each regression in first differences. In all cases, I use the fixed effects estimator.

The results of the baseline model are reported in Table 3. The point estimates of β_1 suggest that the realized discount increases by 13 to 16 basis points for a one percentage point increase in the unemployment rate. Although the parameter β_1 is estimated farily precisely, it is on the low end of what most would consider economically significant. Nevertheless, the basic idea of the model, that stores may vary the frequency and depth of sales in response to macroeconomic conditions, seems to hold up at first pass.

Table 4: Panel Regressions (Time Subsets)

Dependent Variable	Realized Discount		Sale Fraction of Revenue		Discount	
	(2001-2006)	(2007-2011)	(2001 - 2006)	(2007-2011)	(2001 - 2006)	(2007-2011)
$Unemp_{jt}$	0.38***	0.17***	1.32***	0.16***	0.05	0.20***
	(0.044)	(0.011)	(0.14)	(0.030)	(0.05)	(0.012)
$PPI_{jt}/RegPrice_{ijt}$	-.056***	-0.139***	0.44***	-0.08**	-0.25***	-0.20***
	(0.009)	(0.008)	(0.031)	(0.023)	(0.011)	(0.010)
Groups	451	451	451	451	451	451
Time Periods	6	5	6	5	6	5
R-Squared (within)	.05	0.13	0.10	0.02	0.18	0.17

Panel regressions from Table 3 estimated for subsets of the data.

As a robustness check, I repeat the estimations for two different subsets of the data: 2001-2006 and 2007 to 2011. The basis for this separation comes from the rank correlation analysisi in which it appears that there are two distinct episodes to consider. The results of this exercise are presented in Table 4. Here it appears that the relationship between sales and unemployment is quite different during the two time periods. Although the sign on the coefficeint estimates are the same, the magnitudes differ substantially. The results can be summarized as follows. Overall, the sale response to unemployment was positive throughout the sample but larger in the first half of the decade. In the first half of the sample, stores tended to increase the frequency of sales in respons to higher unemployment. In the second half of the sample, stores tended to increase the discount in response to higher unemployment.

5. Conclusion

In the analysis above, I provided evidence that sales have a large influence on the average price paid and the rate at which it changes over time. In particular, indexes of unit price and regular price diverged substantially (up to 3 index points) following the recession of 2001. I presented a model of how a multiproduct firm might choose the frequency and depth of sales when the primary objective is to attract price sensitive shoppers. Certain parameterizations of the model predict that an increase in the number of consumers who respond to sales results in a higher frequency of sales and a reduction in the average unit price. Panel regressions support this prediction. Unit price falls relative to the regular price when the unemployment rate rises, after controlling for changes in the margin between regular price and wholesale cost.

I have also shown that the CPI is less volatile than an index of unit price. One task often fulfilled by the CPI is to deflate nominal magnitudes into real magnitudes. The intent of this paper is not to recommend specific changes to CPI methodology, but my results raise two important questions.[8] 1) Does the CPI do a reasonable job of deflating nominal magnitudes into real magnitudes? 2) How accurately is inflation measured over the course of the business cycle? The characteristics of sales change over time and have an economically significant effect on unit price that is not fully reflected in the CPI. Since it appears that fluctuations in effect of sales on unit price are correlated with macroeconomic conditions, the GDP deflator is subject to measurement error that is correlated with the business cycle.

To see just how important this is from a policy and measurement perspective, let us consider what happened during the 9 months beginning in the fall of 2008. During this episode (in which there was an historically large decline in NGDP) the CPI item indexes studied here rose by 2.4 percentage points. This stands in stark contrast to the 0.6 percentage point *decline* in average price paid. Since the CPI item indexes are used to deflate nominal indexes of consumption growth (which are then aggregated into real GDP growth), we might reasonably conclude that during this period, real consumption growth of the the 24 categores studied here was understated by 3 percentage points, and that the CPI overstated growth rate of actual prices paid by an equal amount. This highlights just how difficult deflating nominal magnitudes over the short run can be. I leave for future reseach the question of what type of monetary policy is best suited for a world in which deflating nominal magnitudes is subject to error that is correlated with the business cycle.

[8]For a lengthy discussion of scanner data and the CPI, see the book that contains the article by Feenstra and Shapiro (2003).

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A. Sale Price Filter

There are several operational definitions of sale and regular prices used in the literature. I find that they are not well suited for this study because they tend to miss sales that are more complicated than a one or two week drop in price followed by a return to the previous price.

Part of the difficulty in choosing an operational definition of a sale is that there is no widely accepted theoretically based definition. My starting point for settling this issue is the fact that an item's price is usually one of two frequently observed prices (Eichenbaum, Jaimovich, and Rebelo, 2011). In Glandon (2011), I present a simple model in which stores select randomly between two prices, the higher of which is the "regular price" and the lower of which is the sale price. Thus, I use the following algorithm to create a series of regular prices for each product.

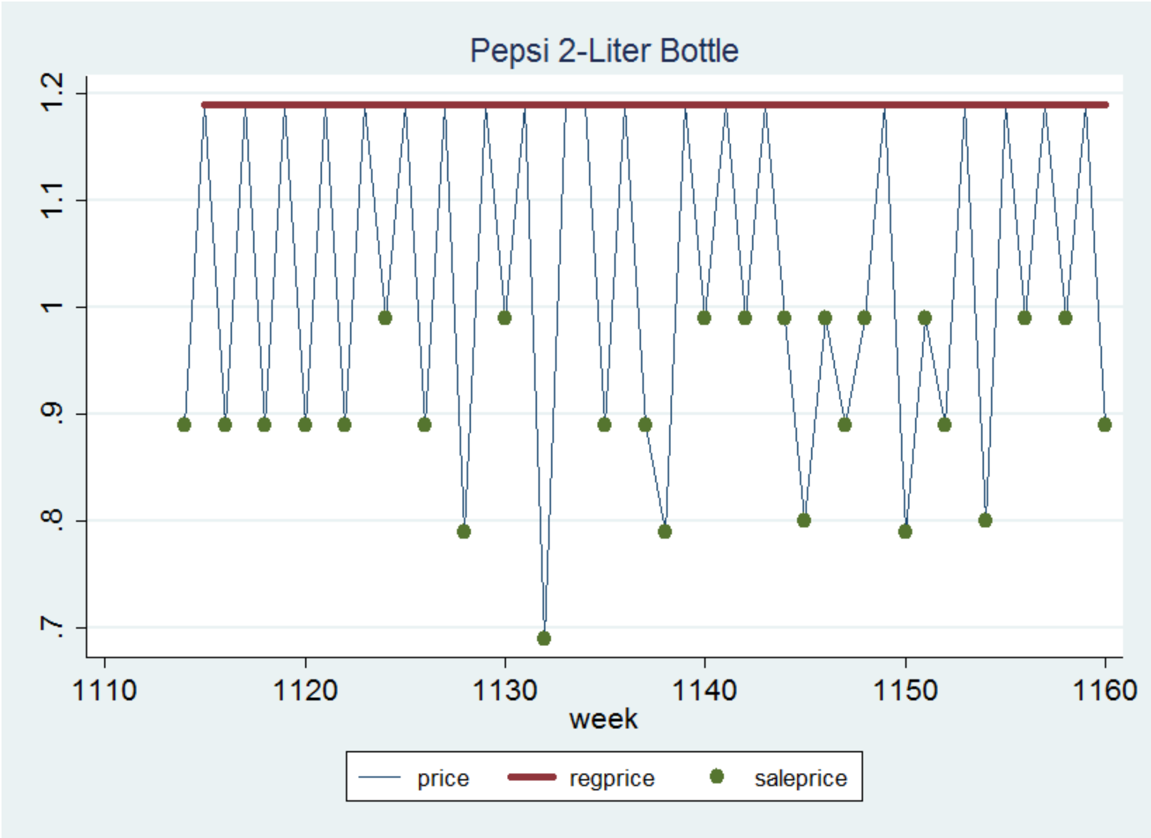
I start by setting the regular price equal to the observed price whenever one of the following two conditions holds:

1. The observed price is larger than or equal to its 13 week centered moving average.

2. The observed price does not change for six weeks or more.

Next, if the the regular price is identified for week t and week t is part of a price spell (consecutive weeks of identical prices), I set the regular price for the entire spell to the observed price. The purpose of this is to ensure that regular price changes in the week that we observe the change. It is needed when regular price falls because the moving average is slower to fall. I also do not allow the regular price to drop for one week and return to the previous price. Instead, I set the regular price in all three periods to be equal to each other. Following Kehoe and Midrigan (2010), I set the remaining unspecified regular prices equal to the previous period's regular price. In these instances the observed price is likely to be a sale price. I now have an observed price series and a regular price series. An item is considered to be on sale if the observed price is at least five percent below the regular price. Chevalier and Kashyap (2011) use a similar tolerance to allow for the fact that there are occasional small price measurement errors. These errors result from the

Figure 7: Regular Price Filter Example from the Data



Notes: This is an example (Pepsi 2-Liter bottles at a store in New York City) from the IRI data set. The thin red line represents the scanned price and the thick blue line is the regular price, as determined by the regular price algorithm described in the text. All prices that are 5% or more below the regular price line are considered sale prices, which are represented by green dots.

fact that price is calculated from total dollar sales and total unit sales.

Figure 7 shows an example of the regular price series that I constructed using the algorithm described above. The thin lines connect the observed price series. This particular example is Pepsi 2 Liter Bottles from a store in New York City. It happens to be the number one revenue generating store-UPC in the sample of carbonated beverages sold in New York City. Notice that there are several instances in which the price drops in one week and then drops further the next week before returning to the regular price. A sale is often more complicated than a simple price drop followed by a return to the previous price.

B. Aggregating Regular Price and Unit Price over Time

Here I provide additional details of the unit price versus regular price calculations discussed in Section 4.1:

I define unit price of item i during quarter t as U_{it}.

$$U_{it} = \frac{1}{q_t} \sum_{w \in t} q_{iw} p_{iw}$$

Where q_{iw} and p_{iw} are the quantity and price of item i during week w. Let $\mathcal{R}(t)$ represent the set of weeks during which an item was not on sale in quarter t and $\mathcal{S}(t)$ be the set of weeks during which the item was on sale in quarter t. Let q_t^r and q_t^s represent the quantity sold during weeks in $\mathcal{R}(t)$ and $\mathcal{S}(t)$ respectively. Note that $q_{it} = q_{it}^s + q_{it}^r$. Let us define the average regular price R_{it} and average sale price S_{it}:

$$R_{it} = \frac{1}{q_t^r} \sum_{w \in \mathcal{R}(t)} q_{iw} p_{iw}$$

$$S_{it} = \frac{1}{q_t^s} \sum_{w \in \mathcal{S}(t)} q_{iw} p_{iw}$$

Let $w_{it} = \frac{q_{it}^s}{q_{it}} \Rightarrow 1 - w_{it} = \frac{q_{it}^r}{q_{it}}$. Thus we can write unit price as the weighted average of sale price and regular price.

$$U_{it} = (1 - w_{it}) R_{it} + w_{it} S_{it}$$

Rearranging, we have:

$$1 - \frac{U_{it}}{R_{it}} = w_{it} \left(1 - \frac{S_{it}}{R_{it}}\right)$$

We can decompose the term w_{it} into the product of the fraction of weeks on sale, f_{it} and the quantity response to a sale, z_{it}. Let $s(t)$ and $r(t)$ represent the number of weeks in quarter t that an item was on sale, and not on sale, respectively. By definition,

$f_{it} = \frac{s(t)}{s(t)+r(t)}$ and $z_{it} = \frac{\bar{q}_{it}^{s}}{\bar{q}_{it}}$.

$$w_{it} = \left(\frac{s(t)}{s(t) + r(t)} \right) \left(\frac{q_{it}^{s}}{s(t)} \frac{s(t) + r(t)}{q_{it}} \right) = f_{it} z_{it}$$

C. Calculation of Price Indexes

To analyze the importance of sales on the dynamics of prices, I calculate two different price indexes using the scanner data. The only difference between these two indexes occurs in the aggregation across weeks within a store-product-month cell. The equations that follow show how I construct the average price for each store-product-month. Denote the average price of UPC j in month t as $P_{j,t}$ with a superscript to denote the two different averages: r and u indicating *regular*, and *unit* respectively:

Average regular price:

$$P^r_{j,t} = \frac{1}{n_{j,t}} \sum_{w(t)} p^r_{j,w(t)} \tag{5}$$

Average unit price:

$$P^u_{j,t} = \frac{1}{q_{j,t}} \sum_{w(t)} q_{j,w(t)} p_{j,w(t)} \tag{6}$$

where $w(t)$ indexes the week of month t, $n_{j,t}$ is the total number of price observations for UPC j in month t (stores $\times$ weeks), and $q_{j,w(t)}$ is the quantity sold. The average regular price is the simple average of regular price. The average unit price is revenue divided by quantity sold.

Since we have several thousand UPCs in our sample, we need to aggregate across products into a single price index. To do so, I use the same technique as the BLS, which is to take the geometric average of monthly price relatives (using the UPC's average share of revenue as the weight). Importantly, I use the same aggregation procedure for both price indexes. The month t price index with base period 0 is:

$$I_{0,t-1} = \exp\left(\sum_j w_j \ln \frac{P_{j,t}}{P_{j,0}}\right) \tag{7}$$