On a Liquidity Premium in the Forward Premium and Implications for Interest Parity Theory

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Abstract

Forward contracts do not provide a hedge against exchange risk if putatively covered arbitragers *ex ante* face the prospect of liquidating their assets prior to the maturity date. Covered interest parity then does not represent a zero-arbitrage condition since investors do not receive a liquidity premium as compensation for the liquidity risk, though lags in the underlying flow of funds may cause dealers to erroneously conclude that it does. The theoretical model predicts an empirical regularity between the modulus of international interest rate differentials and conditional forecasts of exchange rate volatility, a new result in the literature. Accounting for a liquidity premium in the forward premium is also shown to provide a new avenue of explaining the forward premium puzzle: the stylized fact that currencies with higher interest rates often appreciate is consistent with the derived null hypothesis of this theory. Conventional empirical tests of uncovered interest parity are shown to be invalid on both theoretical and econometric grounds and an alternative procedure is presented. These propositions are supported by empirical tests using a sample of five major currencies over some three decades.

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1. Introduction

The notion that a forward contract enables the international arbitrager to hedge his risk is axiomatic, but misplaced if a particular state of the world – a 'liquidity event' – reveals itself and induces the covered investor to liquidate his foreign currency assets immediately to raise funds in domestic currency. He obtains liquidity by placing the foreign security as collateral subject to a discount and selling the foreign currency at the prevailing spot rate prior to the maturity of the forward contract. However, he remains contractually obliged to re-buy foreign currency in the future, prior to the contract's maturity date, and deliver it to the counter-party upon its expiration at the stipulated forward rate. He will consequently confront an exchange risk conditional on the liquidity event though he is otherwise 'covered' in the forward market. The presence of this liquidity risk inevitably has implications for both covered (CIP) and uncovered (UIP) interest parities which, in the present framework, are expressions of equilibria in financial markets *conditional* on the non-occurrence of the liquidity event. One purpose of this paper is to characterize the *unconditional* covered and uncovered interest parities, or *u*-CIP and *u*-UIP, respectively.

That CIP is demonstratively evident in the data imposes a burden to immediately provide an argument for why it may nonetheless be a reflection of disequilibrium in financial markets. In his early definitive work on forward markets, Einzig [8, p. 170-71] asks: "[D]o Interest Parities affect forward rates through psychological or mechanical channels?" and notes that among dealers "the belief has arisen that the adjustment of forward rates is necessarily automatic and instantaneous" but that "[i]n reality the adjustment is far from being so rapid and complete." Einzig's observation is salient. Though in practice the forward premium does not meaningfully depart from the corresponding international interest rate differential, this fact does not invariably imply that the underlying flow of transactions is also in equilibrium. Dealers may price the forward premium consistent with its equilibrium value conditional on the non-occurrence of the liquidity event – call it the *observed* forward premium – because the received wisdom does not account for the model of investor behavior described here. This value of the forward premium will normally differ from its unconditional equilibrium value – call that the *shadow* forward premium – but if there are lags in the response of the underlying flow of funds to the disequilibrium, the observed forward rate will not change quickly, causing dealers to erroneously conclude that a forward transaction priced at its conditional equilibrium, or observed, value is in fact priced at its unconditional equilibrium, or shadow, value.

Given CIP, the persistent failure of the forward premium to provide an unbiased forecast of spot rate appreciation has been labeled as the forward premium puzzle and interpreted as the failure of UIP [6]. We propose instead that the finding of a 'puzzle' results from the implicit failure of CIP and argue that the observed forward premium varies in response to changes in both the rational forecast of the spot rate and the liquidity premium – which in turn is affected by exchange rate volatility – and the mixing-up of the two effects can hopelessly weaken the power of the observed forward premium to accurately predict spot rate changes.

The re-examination of CIP and its restatement as the *u*-CIP provides the point of departure. Using the theoretical framework in section 2 we derive the zero-arbitrage conditions yielding unconditional covered and uncovered interest parities. We show that the modulus of the observed forward premium should vary in a systematic way with the conditional forecast of exchange rate volatility if conventional CIP does not represent equilibrium in financial markets. We argue that the *shadow* forward premium is an unbiased forecast of the change in the spot rate, and that the putative failure of UIP in the data is fully consistent with the implied null of *u*-UIP, a result that points to a new avenue for the resolution of the forward premium puzzle. Empirical evidence using roughly three decades of data on five major currencies is provided in section 3. Concluding comments are in Section 4.

2. Theoretical Framework

Consider an interest arbitrager who can buy a default-free Home country security and a Foreign security for which he simultaneously obtains forward cover at the prevailing forward rate. *Ex ante* he faces two possible states of the world: a loss state in which a liquidity event occurs with probability *p*, or a no-loss state in which such an event does not occur. In equilibrium the marginal investor is indifferent between the Home and Foreign assets: his expected utilities from both are equal. Assume that both Home country and Foreign country investors have identical preferences. Utility conditional on the state of the world is $u(y)$; $u(0) = 0, u'(y) > 0, u''(y) < 0$ and y is the conditional return on the asset. If the conditional returns in the loss and no-loss states for a given asset are y_1 and y_2, respectively, then the unconditional expected utility from that asset, Home or Foreign, is

(1) $$E(u(y)) = pu(y_1) + (1-p)u(y_2)$$

2.1 *Conditional Returns to the Home Country Investor*

The default-free interest rates on the Home and Foreign securities are r_t and r_t^*, respectively. Define the exchange rate as the Home currency price of a unit of Foreign currency; *t* is the time label. The spot rate is S_t. The prevailing rate for forward cover is the *shadow* rate F_t which subsumes consideration of liquidity risk; the shadow forward premium is $x_t = \ln(F_t) - \ln(S_t)$. The *observed* forward rate and premium are F_t^o and $x_t^o = \ln(F_t^o) - \ln(S_t)$. In the no-loss state the returns on the Home and Foreign securities are r_t and $r_t^* + x_t$, respectively.

In the loss state, assume that the liquidity event requires the investor to liquidate his entire holding of the security. Let the liquidity event, were it to occur, occur only with the smallest possible lapse of time after the investor buys the security in a covered transaction. The time lapse is small enough to preclude the spot rate from changing so it is assumed that the investor knows the spot price of the currency is the same in both states of the world. Assume also that in the face of a systemic liquidity event the supply of liquidity is not infinitely elastic; the investor obtains liquidity against placement of security which is discounted. Let the discounts be a known proportion (k) of interest rates: kr_t and kr_t^* ($k > 0$) for Home and Foreign assets, respectively.[1] Therefore the return on the Home security in the loss state is $-kr_t$. The return on the Foreign security must, additionally, take account of exchange risk because, while he sells his foreign currency proceeds forthwith without loss or gain (at the period-*t* spot rate as assumed above) the investor must still deliver foreign currency one period hence in the amount stipulated in his forward contract. This we assume he facilitates by purchasing the requisite amount of foreign currency at the yet unknown spot rate prevailing at time $t+1$, the contracted date of delivery, which causes a rate of gain or loss of approximately $(F_t - S_{t+1})S_t^{-1}$. The total conditional return on the Foreign security then is $-kr_t^* + (F_t - S_{t+1})S_t^{-1}$, or approximately $-kr_t^* + (x_t - \Delta\ln(S_{t+1}))$. It is a random variable in period *t* whose distribution reflects the evolution of $\Delta\ln(S_{t+1})$ and is a source of risk. Thus we make the case that a forward cover on arbitrage transactions does not insulate the investor from exchange risk. The summary of conditional returns on each asset for the Home investor is provided in Table 1a where all values are known in period *t* except that in the lower left cell.

The corresponding conditional expected utilities of the Home investor are in Table 1b. The argument of utility in the lower left cell is the certainty equivalent of the uncertain conditional return on the Foreign asset: the foreign exchange risk premium is $\pi_t = 0.5\lambda\, h_{t+1} \big| I_t$ where $h_{t+1} \big| I_t$ is the variance of $\Delta\ln(S_{t+1})$ conditional on the

[1] The unit of *k* is time; if the frequency of interest rates is monthly (annual), *k* is the number of months (years).

information set at time t and λ is the measure of constant absolute risk aversion. The Home investor's unconditional expected utilities from the Home security $E(u^H)$ and Foreign security $E(u^F)$, respectively, are: [2]

(2) $$E(u^H) = pu(-kr_t) + (1-p)u(r_t)$$

(3) $$E(u^F) = pu(-kr_t^* + x_t - E\Delta\ln(S_{t+1}) - \pi_t) + (1-p)u(r_t^* + x_t)$$

2.2 *Zero-arbitrage Conditions for the Home and Foreign Investors*

The zero-arbitrage condition leaves the Home country investor indifferent between the Home and Foreign securities, i.e., $E(u^H) - E(u^F) = 0$. Equations (2) and (3) therefore imply

(4) $$p[u(-kr_t) - u(-kr_t^* + x_t - E\Delta\ln(S_{t+1}) - \pi_t)] + (1-p)[u(r_t) - u(r_t^* + x_t)] = 0$$

Turning to the Foreign investor, denote his utility by u^*. With identical preferences, symmetry with equations (2) and (3) yields the corresponding expressions $E(u^{*F})$ and $E(u^{*H})$:

(5) $$E(u^{*F}) = pu(-kr_t^*) + (1-p)u(r_t^*)$$

(6) $$E(u^{*H}) = pu(-kr_t - x_t + E\Delta\ln(S_{t+1}) - \pi_t) + (1-p)u(r_t - x_t)$$

The resulting zero-arbitrage condition, ensuring $E(u^{*F}) - E(u^{*H}) = 0$ for the Foreign investor, is

(7) $$p[u(-kr_t^*) - u(-kr_t - x_t + E(\Delta S_{t+1}) - \pi_t)] + (1-p)[u(r_t^*) - u(r_t - x_t)] = 0$$

2.3 *Rational Expectations and u-CIP*

The international financial market is in equilibrium when equations (4) and (7) hold simultaneously. Observe that conditional on the no-loss state $(p = 0)$ equation (4) reduces to $u(r_t) = u(r_t^* + x_t)$ and equation (7) to $u(r_t^*) = u(r_t - x_t)$; both are satisfied when the shadow forward premium is $x_t = r_t - r_t^*$ so CIP is subsumed in this general framework as a special case that obtains conditional on the no-loss state of the world. However, by

[2] The Home investor will invest in the Home asset only if $E(u^H(r_t)) > u^H(0)$. In equation (2), for instance, noting that $u(0) = 0$, linearize $u(\cdot)$ around 0 to obtain $E(u^H) = (1-p)r[u'(0) + ru''(0)] - pkr[u'(0) - kru''(0)]$. Since $\lambda = -u''(0)/u'(0)$ the condition for $E(u^H) > 0$ then is $pk(1-p)^{-1} < (1-\lambda r)(1+\lambda kr)^{-1}$. The same condition applies to equation (5) for the Foreign investor.

inspection of equations (4) and (7) we see under *u*-CIP $(p > 0)$ that $x_t \neq r_t - r_t^*$.

To derive the *u*-CIP condition consider these first-order linear approximations of the two zero-arbitrage conditions: in equation (4) linearize $u(-kr_t^* + x_t - E\Delta \ln(S_{t+1}) - \pi_t)$ around $-kr_t$ and $u(r_t^* + x_t)$ around r_t, and in equation (7) linearize $u(-kr_t - x_t + E\Delta \ln(S_{t+1}) - \pi_t)$ around $-kr_t^*$ and $u(r_t - x_t)$ around r_t^*. On simplification equations (4) and (7), respectively, yield the equilibrium conditions for the Home and Foreign investors:

(8) $$(r_t - r_t^*) - x_t = \frac{p}{1-p}\Psi(r_t)[k(r_t - r_t^*) + (x_t - E\Delta \ln(S_{t+1})) - \pi_t]$$

(9) $$(r_t - r_t^*) - x_t = \frac{p}{1-p}\Psi(r_t^*)[k(r_t - r_t^*) + (x_t - E\Delta \ln(S_{t+1})) + \pi_t] \quad \text{where}$$

(10) $$\Psi(y_t) \equiv \frac{u'(-ky_t)}{u'(y_t)} > 1 \quad (y_t = r_t, r_t^*)$$

As defined in (10), $\Psi(\cdot)$ in equations (8) and (9) is the investors' marginal utility of own-country asset return in the loss- relative to the no-loss state. It may be interpreted as a pseudo-MRS term which has the form of a marginal rate of substitution but contains no choice variable in the function.[3] Assume that the market determined shadow forward premium x_t embodies all relevant information available in time *t* and is a rational forecast of $\Delta \ln(S_{t+1})$, i.e.:

(11) $$x_t = E\Delta \ln(S_{t+1})$$

Then in both equations (8) and (9) the term $(x_t - E\Delta \ln(S_{t+1})) = 0$, and these two equations together yield the *u*-CIP condition for financial market equilibrium given below:

(12) $$x_t = (r_t - r_t^*) - \ell_t(\cdot)$$

where the term ℓ_t is the *liquidity risk premium* defined as

(13) $$\ell_t \equiv \frac{p}{1-p}\left[\frac{\Psi(r_t)\Psi(r_t^*)}{\Psi(r_t) - \Psi(r_t^*)}\right]\lambda h_{t+1} \Big| I_t$$

[3] Note that $\Psi(y_t) > 1$ since $u'(\cdot) > 0$ and $u''(\cdot) < 0$, and that while $u'(-ky_t) > 0$, $\partial u(-ky_t)/\partial y_t = -ku'(-ky_t) < 0$.

Several aspects of the *u*-CIP relationship in equation (12) are notable. Firstly, the shadow forward premium x_t is decomposed into an interest rate differential term and a liquidity risk premium term ℓ_t. Secondly, because $\mathrm{sgn}[\Psi(r_t) - \Psi(r_t^*)] = \mathrm{sgn}(r_t - r_t^*)$, ℓ_t may be positive or negative,[4] but since $(r_t - r_t^*)$ and ℓ_t are of the same sign, x_t and $(r_t - r_t^*)$ may be of opposite signs. Therefore and thirdly, given (11), $(r_t - r_t^*)$ and $E\Delta\ln(S_{t+1})$ may *also* be of opposite signs.[5] Fourthly, an important property of equation (12) is that it holds simultaneously from the perspectives of both Home and the Foreign investors. Lastly, $x_t \rightarrow (r_t - r_t^*)$ as $(p, \lambda) \rightarrow 0$.

Since $\ell_t = (r_t - r_t^*) - x_t$, substituting for ℓ_t from (13) on the LHS of either equation (8) or (9) yields:

(14) $$(r_t - r_t^*) = \left\{ \frac{\Psi(r_t) + \Psi(r_t^*)}{k[\Psi(r_t) - \Psi(r_t^*)]} \right\} \frac{\lambda}{2} h_{t+1} \big| I_t$$

It is shown in *Appendix 2* that to an approximation, equation (14) can be written as:

(15) $$\left| r_t - r_t^* \right| = \kappa \sqrt{h_{t+1}} \big| I_t$$

where $\kappa \equiv \sqrt{k(1+k)}$ and $\sqrt{h_{t+1}} \big| I_t$ is the positive root. Equation (15) describes the existence of a systematic relationship between the modulus of the interest rate differential and the conditional forecast of exchange rate volatility which we believe to be a new result. The *u*-CIP proposition in equation (12) cannot be directly tested because x_t is unobserved, but failure to empirically refute its testable implication in equation (15) would be grounds for indicting CIP as an invalid characterization of financial market equilibrium. If $\mathrm{sgn}(\ell_t) = \mathrm{sgn}(r_t - r_t^*) > 0$ for instance, then by equation (12) we know that $r_t > r_t^* + x_t$ and that investors deem the Home asset to have a higher liquidity risk. But under CIP the observed forward premium x_t^o is:

(16) $$x_t^o = r_t - r_t^*$$

So pricing the forward premium per equation (16) rather than (12) implies excess risk-adjusted returns on ‘covered’ positions in the Foreign asset, and an excess supply (demand) of Foreign currency in the forward (spot)

[4] Note from the definition in (13) that $\mathrm{sgn}(\ell_t) = \mathrm{sgn}[\Psi(r_t) - \Psi(r_t^*)]$ and $\mathrm{sgn}[\Psi(r_t) - \Psi(r_t^*)] = \mathrm{sgn}(r_t - r_t^*)$ since, as shown in *Appendix 1*, $\Psi'(y_t) > 0$.

[5] The implication of this observation for the forward premium puzzle is explored in the following section.

market. Though the international flow of funds is in disequilibrium at x_t^o, if dealers believe that “the adjustment of forward rates is necessarily automatic and instantaneous” but as mentioned in section I, the resulting “adjustment is far from being so rapid and complete,” [8, pp. 170-171], then owing to lags in the flow of funds they may erroneously conclude that the spot-forward swap priced at x_t^o is in fact consistent with equilibrium since there is no immediate pressure on prices.

2.4 *Forward Premium Puzzle and* **u**-*UIP*

The literature on the forward premium puzzle – that currencies with higher interest rates tend to appreciate – is broad in its sweep. The multiple approaches to resolving the puzzle include rejection of rational expectations, frictions, a variety of econometric issues and the existence of a time-varying risk premium; Engel [9] offers a comprehensive survey. Evans and Lewis [11], Chinn and Frankel [7] and Chakraborty and Evans [5] utilize Bayesian learning, survey data and discounted least-squares learning, respectively, to argue that uncovered interest parity may not be rejected if rational expectations do not hold. A wide variety of consumption-based studies attempt to model the existence of risk-premia by introducing rigidities and transactions cost frictions. Bacchetta and van Wincoop [1] argue that infrequent portfolio adjustments lead to predictable excess returns on currencies that are not exploited. Baillie and Bollerslev [3] note that non-linearity between spot rates and forward premia render standard econometric tests unsuitable.

It is instructive to relate our theory to the seminal work of Fama [13] which posited the existence of a time-varying risk premium (though it does not consider liquidity risk) and where the “market determined certainty equivalent of the future spot exchange rate” (p. 320) is what we call the *observed* forward rate F_t^o, as distinct from the *shadow* forward rate F_t. In the Fama decomposition, $\ln(F_t^o) = E\ln(S_{t+1}) + m_t$ or:

$$E\Delta\ln(S_{t+1}) = x_t^o - m_t \tag{17}$$

where m_t is the Fama risk premium, distinct from the liquidity risk premium ℓ_t, and $x_t^o \equiv \ln(F_t^o) - \ln(S_t)$.

In the present framework the substitution of equation (12) in (11) yields $E\Delta\ln(S_{t+1}) = x_t^o - \ell_t$ and, therefore, given the Fama equation (17), we infer that $m_t = \ell_t$: if x_t is a rational forecast of $\Delta\ln(S_{t+1})$ then the Fama risk premium m_t is entirely the liquidity premium ℓ_t. This result is appealing because with both the uncovered and the putatively covered transactions being risky, investor indifference between them requires their risk-adjusted returns to be equal, or that $r_t - m_t = r_t^* + E\Delta\ln(S_{t+1}) = r_t - \ell_t$, respectively. Therefore, by locating the source of the Fama risk premium in liquidity risk, this analysis re-interprets the Fama equation and extends it in a new direction.

To further characterize the liquidity risk premium, linearize to the first order $\Psi(r_t)$ around r_t^*. Using the result from *Appendix 1* that $\Psi'(r_t) = \lambda(1+k)\Psi(r_t)$ then yields $[\Psi(r_t) - \Psi(r_t^*)] = \lambda(1+k)\Psi(r_t^*)(r_t - r_t^*)$. Additionally, squaring both sides of equation (15) implies $(r_t - r_t^*)^{-1} h_{t+1} \big| I_t = k(1+k)(r_t - r_t^*)$. By iteratively making these two substitutions, equation (13) reduces to the following expression for the liquidity premium:

(18) $$\ell_t = \frac{pk}{(1-p)}\Psi(r_t)(r_t - r_t^*)$$

In equation (18) $\mathrm{sgn}(\ell_t) = \mathrm{sgn}(r_t - r_t^*)$ and notably, the liquidity premium is functionally related to both the interest rate differential and the level of the Home interest rate. The term $pk(1-p)^{-1}\Psi(r_t) > 0$ and since $\Psi(r_t) > 1$ by the definition in (10), it may exceed unity. Substituting from equation (18) for ℓ_t in the *u*-CIP equation (12) yields

(19) $$x_t = \left[1 - \frac{pk}{1-p}\Psi(r_t)\right](r_t - r_t^*)$$

On the LHS $x_t = E\Delta\ln(S_{t+1})$ by equation (11), and on the RHS $(r_t - r_t^*) = x_t^o$ from (16). Therefore, we obtain

(20) $$E\Delta\ln(S_{t+1}) = \left[1 - \frac{pk}{1-p}\Psi(r_t)\right]x_t^o$$

Equation (20) is the expression of unconditional uncovered interest parity *u*-UIP. Conditional on r_t the coefficient of x_t^o is less than one since $pk(1-p)^{-1}\Psi(r_t) > 0$, and it may be positive or negative so even if $x_t^0 = (r_t - r_t^*) > 0$ the Home currency may appreciate. As examples, consider three scenarios where a representative investor with exponential preferences $u(y) = 1 - \exp(-\lambda y)$ envisages a liquidity event resulting in either 5%, 30% or 50% discounts on bonds placed in exchange for liquidity. At a monthly Home interest rate

$r = 0.005$, the loss from obtaining liquidity under these scenarios is $k = 10, 60, 100$ months of interest.[6] To satisfy the parameter restriction in *n*.2 let $pk(1-p)^{-1} = 0.99(1-\lambda r_t)(1+\lambda k r_t)^{-1}$; $\Psi(r_t) = \exp(\lambda(1+k)r_t)$. The implied coefficient of x_t^0 in equation (20) conditional on $r_t = 0.005$ under each k scenario for $\lambda = 2, 4$, is presented in Table 2. The coefficient value range in the simulation, +0.01 to -1.44, is very typical of empirical coefficient estimates in numerous studies that regress $\Delta \ln(S_{t+1})$ on x_t^o, including equation (26) below in this one.

This analysis points to a new understanding that the 'forward premium puzzle' may not at all be a puzzle. If investors require a liquidity premium and the CIP fails to reflect zero-arbitrage, then a putative negative 'bias' in the coefficient of x_t^o in the amount $-pk(1-p)^{-1}\Psi(r_t)$ should be expected. Regressions of $\Delta \ln(S_{t+1})$ on x_t^o as implemented in conventional tests of UIP should, in the *u*-UIP scheme of equation (20), be tested under the null that the coefficient on x_t^o *is less than one.* The null of unity correctly applies only if the regressand is the shadow rather than the observed forward premium, x_t rather than x_t^o, though the former is unobserved and precludes a direct test.

Conventional tests of UIP are rendered invalid on empirical grounds as well. Firstly, omission from the estimating equation of the term $pk(1-p)^{-1}\Psi(r_t)x_t^o$ results in a specification bias. Secondly, the omitted term, included in the error term, is a function of x_t^o so that the error term is not orthogonal to x_t^o, resulting in invalid parameter estimates. Though regressing $(\Delta \ln(S_{t+1}) - x_t^o)$ on x_t^o as is often done circumvents the non-orthogonality issue and is to be preferred for this reason, the specification error persists due to the exclusion of the pseudo-MRS variable $\Psi(r_t)$, mitigated as this error is in inverse proportion to the variance of r_t.

3. Estimation and Results

The *u*-CIP and *u*-UIP relationships in equations (15) and (20), respectively, are tested using a monthly sample of five major currencies for the pre-global financial crisis period 1980:01-2007:12. The sample includes the Canadian Dollar CAD, the Deutschemark DEM, the Yen JPY, the Swiss Franc CHF and the Pound GBP. Spot rates are monthly end-of-period US Dollar USD prices of foreign currency from *International Financial*

[6] $r = 0.005$ is the sample mean of monthly U.S. interest (Eurodollar) rate for the sample in section 3.

Statistics [14]; for 1999:01-2007:12 the DEM series is obtained by converting the USD/EUR rates at the legacy rate 1.95583 DEM/EUR [10]. Monthly Eurocurrency rates are Datastream EC…1M series,[7] used in computing the observed forward premium using $x_t^{oj} = r_t^{USA} - r_t^j$ for currency *j*, where the U.S. is the Home country. Summary statistics are in Table 3. All estimation was conducted in EViews 7. The conditional variance of returns on the spot rate is obtained from the GARCH (1,1) process:

$$
\begin{aligned}
&\Delta \ln(S_t) = c + u_t \\
&u_t \big| I_{t-1} \sim N\left(0,\ h_t\right) \qquad (21)\\
&h_t = w + a u_{t-1}^2 + b h_{t-1}
\end{aligned}
$$

In equation (21) h_t is the variance of u_t conditional on available information in period $t-1$; the conditional forecast $\sqrt{h_{t+1}}\,\big|I_t$ is its estimated one-period lead value which uses only period *t* information . Both the ADF and Phillips Perron tests (not presented) reject the presence of unit roots in each $\Delta \ln(S_t)$ series. The resulting estimates provided in Table 4 indicate significant GARCH effects in each of the five currencies and significant ARCH effects in CAD and GBP using Bollerslev-Wooldridge heteroskedasticity consistent coefficient covariances. For all currencies $\hat{a} + \hat{b} < 1$. Diagnostic checks of the residuals indicate equation (21) to be a good specification for both the mean and the variance equations.[8]

3.1 *Unconditional Covered Interest Parity* (u-*CIP*)

The estimating model for the proposition in equation (15) is in equation (22) where $x_t^o = r_t - r_t^*$:

$$
\begin{aligned}
&\left|x_t^{oj}\right| = \beta_0^j + \beta_1 \sqrt{h_{t+1}^j}\,\big|I_t + \varepsilon_t^j \\
&\varepsilon_t^j = \rho \varepsilon_{t-1}^j + \eta_t^j \qquad (22)\\
&E(\varepsilon_t^j \varepsilon_t^k) = \sigma^{jk},\ E(\varepsilon_s^j \varepsilon_t^k) = 0
\end{aligned}
$$

The joint null is $H_0 : (\beta_1 > 0, \beta_0^j = 0 \ \forall\ j)$. There are several issues of estimation to be addressed in regard to model specification. Since equation (15) denotes equilibrium in international financial markets it is not

[7] The original monthly data on annual Eurocurrency currency rates per cent (r_a) are converted into monthly rates (r_m) using $r_m = (1 + 0.01 r_a)^{1/12} - 1$.

[8] In each regression, low values of the estimated Ljung-Box *Q*-statistic for all lags up to 36 lags indicate the absence of serial correlation in standardized residuals at 5% significance except that for JPY the *p*-value=0.04 at lag 11. The estimated Ljung-Box Q^2-statistics for serial correlation in standardized squared-residuals at all lags up to 36 lags as well as ARCH LM tests confirm the absence of any remaining ARCH effects in the variance equations.

appropriate to treat $\sqrt{h_{t+1}}\,|I_t$ as exogenous. Monetary authorities may base current, or period *t*, interest rate decisions on the conditional forecast of exchange rate volatility; likewise investors may condition such volatility forecast on the current interest rate differential x_t^o. Specifically, in the estimated forecast of the conditional standard deviation implied in equation (21), $\sqrt{h_{t+1}}\,|I_t = \sqrt{w + au_t^2 + bh_t}$, we must allow for the possibility that new information contained in the residual u_t is partly based on observations on $|x_t^o|$.

The correct specification requires the use of an appropriate instrument for $\sqrt{h_{t+1}}\,|I_t$, a candidate for which is its own lagged value; $\sqrt{h_t}\,|I_{t-1}$ is obviously correlated with $\sqrt{h_{t+1}}\,|I_t$. However, correlograms (not presented) of all $|x_t^o|$ series suggest classic first-order autocorrelation processes with autocorrelations decaying slowly, spikes at lag one in partial autocorrelations which then collapse at lag two. Consequently ε_t in equation (22) is not orthogonal to $\sqrt{h_t}\,|I_{t-1}$ resulting in invalid OLS estimates. However, Fair [12, p. 508] has noted that such AR(1) models can be consistently estimated using 2SLS if the instruments are uncorrelated with η_t and the set of instruments includes at least $|x_{t-1}^o|$ and $\sqrt{h_t}\,|I_{t-1}$, and any other relevant exogenous and lagged endogenous variables. The absence of exogenous variables does not present identification problems because equation (15) is an expression of an equilibrium condition: our purpose is not to identify either the monetary authorities' or the investors' 'reaction functions' relating moduli of interest rate differentials and conditional exchange rate volatility forecasts.

Further, since $|x_t^{oj}| = |r_t^{USA} - r_t^j|$ for currency *j*, it is reasonable to assume that shocks in r_t^{USA} will propagate contemporaneously into the moduli of all observed forward premiums so $E(\varepsilon_t^j \varepsilon_t^k) \neq 0$. To utilize the information in the contemporaneous covariance structure of errors and capture currency specific effects, the model in equation (22) is estimated using the SUR methodology with currency fixed effects. Finally, β_1 was restricted for efficiency reasons since the variance of $\sqrt{h_{t+1}}\,|I_t$ in the stacked data (7.4x10^{-5}) was larger than those of CAD, DEM and JPY by factors of four to nine.

We also estimate the following log transformation of equation (15), $\ln|x_t^o| = \ln\kappa + \ln\sqrt{h_{t+1}}\,|I_t$:

$$
\begin{aligned}
&\ln\left|x_t^{o\,j}\right| = \beta_0^j + \beta_1 \ln\sqrt{h_{t+1}^j}\,\left|I_t\right. + \varepsilon_t^j \\
(23)\qquad &\varepsilon_t^j = \rho^j \varepsilon_{t-1}^j + \eta_t^j \\
&E(\varepsilon_t^j \varepsilon_t^k) = \sigma^{jk},\ E(\varepsilon_s^j \varepsilon_t^k) = 0
\end{aligned}
$$

The null suggested here by equation (15) is $H_0 : \beta_1 = 1$. Estimates were obtained using 2SLS with $\ln\left|x_{t-1}^o\right|$, $\ln\sqrt{h_t}\,\left|I_{t-1}\right.$ and fixed effects as instruments. Because four of the 336 observations on $\left|x_t^{o\,CAD}\right|$ in the sample were zero, CAD was excluded from the cross-sections in implementing equation (23). The estimation procedures and error structures in equations (22) and (23) are otherwise no different.

Figure 1 shows the scatterplot of $\left|x_t^{o\,j}\right|$ against $\sqrt{h_{t+1}^j}\,\left|I_t\right.$. Panel unit root tests of $\left|x_t^{o\,j}\right|$ and $\sqrt{h_{t+1}^j}\,\left|I_t\right.$ series (and their logs) with individual fixed effects and individual unit root processes decisively rejected the null of non-stationarity in both pool series[9] and the residuals were clearly stationary on inspection.

Tables 5 and 6 provide the estimates of equations (22) and (23) respectively. In both sets of results, in addition to 2SLS estimates, GLS estimates are also provided for comparison. In Table 5, the 2SLS estimate $\hat{\beta}_1 = 0.12$ is more than two standard errors from zero. With none of the fixed effects significantly different from zero, for the current sample we infer that $\left|x_t^o\right| = 0.12\sqrt{h_{t+1}}\,\left|I_t\right.$; this systematic positive relationship between the modulus of the observed forward premium and the conditional forecast of exchange rate volatility is a new empirical regularity reported in this paper.[10] The evidence affirms support in the data for the *u*-CIP proposition in equation (12) and indirectly of the proposition that the shadow forward premium is an unbiased predictor of returns on future spot rates: $E(\Delta S_{t+1}) = x_t$. The 2SLS estimates of the log specification in Table 6 corroborate the

---

[9] Unit root test statistics for the two pooled series $\left|x_t^{o\,j}\right|$ and $\sqrt{h_{t+1}^j}\,\left|I_t\right.$, respectively, with all five currencies in the pool, are: Im-Pesaran-Shin *W*-statistic (-6.03 and -4.40), ADF-Fischer χ^2 – statistic (79.4 and 40.5), the Phillips-Perron χ^2 – statistic (82.6 and 62.7) all of which have *p*-values < 0.00 under the null that a unit root exists. For the log-log model, CAD was omitted from the pool. The corresponding panel unit root test statistics for the natural logs of the two variables are: Im-Pesaran-Shin *W*-statistic (-3.9 and -6.0), ADF-Fischer χ^2 – statistic (38.0 and 56.9), the Phillips-Perron χ^2 – statistic (40.1 and 55.1) all of which also have *p*-values < 0.00 under the null that the series are non-stationary.

[10] The striking differences between the 2SLS and GLS results in Table 5 are notable. In contrast to 2SLS, the GLS estimate of β_1 is practically zero and not statistically different from zero, while all fixed effects are statistically significant.

findings. No fixed effect is statistically significant. $\hat{\beta}_1 = 2.07$ and three standard errors from zero, and with $p = 0.105$ the Wald test cannot reject the null $\beta_1 = 1$: while the point estimate of the elasticity of the modulus forward premium with respect to the conditional forecast of exchange rate volatility is 2.07, the hypothesized null of unity cannot be rejected. Thus, the failure to refute the proposition of equation (15) in the data in both estimations provides an empirical basis for the claim that *u*-CIP – not conventional CIP – is the valid characterization of financial market equilibrium.

3.2 *Unconditional Uncovered Interest Parity* (u-*UIP*)

$\Psi(r_t)$ is a non-linear function of unknown form which makes it difficult to implement a test of the *u*-UIP hypothesis of equation (20). However, since $\Psi'(r_t) > 0$, we note that the term in brackets in that equation varies inversely with r_t. The intuition is that when $\Psi(r_t)$ is larger, investors place a larger weight on liquidity risk and seek a higher liquidity premium ℓ_t. This increases the divergence of the observed interest rate differential x_t^o from the shadow forward premium x_t where the latter is the efficient forecast of $\Delta \ln(S_{t+1})$, resulting in a larger deviation from unity of the coefficient of x_t^o. On this prior we create a dummy variable D_t such that:

(24) $$D_t = \begin{cases} 1 & if\ r_t^{USA} \geq Mean(r_t^{USA}) \\ 0 & otherwise \end{cases}$$

We include slope and fixed effects dummies in the estimating specification of equation (20) in an attempt to capture the effects of $\Psi(r_t)$. The estimating model is:

(25) $$\Delta \ln(S_{t+1}^j) - x_t^{o\,j} = \alpha^j + \alpha_D^j D_t + (\beta - 1) x_t^{o\,j} + \beta_D D_t x_t^{o\,j} + \upsilon_{t+1}^j$$

The joint null is $H_0 : (\beta - 1 < 0, \beta_D < 0, \alpha^j = \alpha_D^j = 0) \quad \forall\ j$ since in periods of high U.S. interest rates $(D_t = 1)$ we expect the coefficient of x_t^o to be smaller than in periods of low U.S. interest rates $(D_t = 0)$. Though we have argued earlier that the standard test of UIP suffers from specification bias under present assumptions, for comparison we also implement that test by regressing $(\Delta \ln(S_{t+1}) - x_t^o)$ on x_t^o without the dummy variable. In contrast to the above *u*-UIP formulation, the joint null now is $H_0 : (\beta - 1 = 0, \alpha^j = 0\ \forall\ j)$ in the specification:

(26) $$\Delta \ln(S_{t+1}^{j}) - x_t^{o\,j} = \alpha^{j} + (\beta - 1)x_t^{o\,j} + \upsilon_{t+1}^{j}$$

The coefficients of $x_t^{o\,j}$ in both are restricted to be equal across cross-sections as a practical necessity [4] since the standard deviations of $\Delta(\ln S_{t+1}^{j})$ are 12.3 to 15.6 times those of $x_t^{o\,j}$ in the sample . Correlograms showed no evidence of serial correlation with all autocorrelations up to 36 lags well within two standard errors of zero.[11] Panel unit root tests (not presented) left no doubt regarding the stationarity of these variables. White robust coefficient covariances have been used in the diagnostics. All spot rates are USD prices of foreign currencies so we assume that $E(\upsilon_t^{j}\upsilon_t^{k}) = \sigma^{jk}$. Equations (25) and (26) are estimated using SUR.

Estimates of equation (25) in Table 7 indeed broadly support the *u*-UIP hypothesis: the implicit effect of $\Psi(r)$ is notably evident: $\hat{\beta} = -0.47$ and $\hat{\beta}_D = -2.17$, both more than two standards errors from zero. These estimates imply that the coefficient on x_t^{o} is -0.47 (-2.64) when the U.S. interest rate is low (high), in accord with the implication of equation (20) since $\Psi^{'}(r) > 0$. All constants are statistically insignificant when $D_t = 0$. When $D_t = 1$ the constants for JPY and CHF are significantly different from zero suggesting either that D_t as constructed fails to adequately capture the variation of $\Psi(r)$ or possibly that $x_t \neq E(\Delta \ln S_{t+1})$. Finally, the conventional test of UIP in equation (26) rejects $H_0 : \beta = 1$: at seven standard errors from zero, $\hat{\beta} = -1.47$, an unsurprising result in keeping with the literature on the forward premium puzzle.

Taken as a whole, these empirical findings affirm support in the data for the claim that the source of the bias characterizing the forward premium puzzle is in the liquidity risk premium component of the observed forward premium which, in turn, reflects the failure of CIP rather than the failure of UIP.

4. Concluding Comments

In this paper we have argued that if international arbitragers confront a non-diversifiable liquidity risk then covered interest parity does not represent the unconditional equilibrium in financial markets. A covered arbitrage transaction will be subject to potential losses from liquidation of assets prior to maturity as well as to foreign currency risk conditional on the liquidity event revealing itself. Since covered interest parity theory does not take into account this possibility, forward transactions priced in accordance with it leave the underlying flow

[11] Autocorrelations at lag 1 for $\Delta(\ln S_t^{DEM})$ and $\Delta(\ln S_t^{CHF})$, respectively, are 0.091 (*p*=0.093) and 0.098 (*p*=0.072).

of funds in disequilibrium. If the flow of funds responds to the disequilibrium with lags, then there will not be an immediate pressure on forward rates, leading dealers to conclude that the forward market is in equilibrium when in fact it is not. We derive the unconditional covered interest parity relationship and present evidence from five major currencies in support of the refutable hypothesis of this proposition that there exists a systematic positive relationship between the modulus of the interest rate differential, or the observed forward premium, and the conditional forecast of exchange rate volatility.

We examine the implications of liquidity risk for uncovered interest parity by decomposing the observed forward premium into a shadow forward premium and a liquidity risk premium. We derive the unconditional uncovered interest parity relationship, extending the important work of Fama (1984) by placing the source of his risk premium in the liquidity risk premium. This relationship provides new insight into the cause of the widely reported forward premium puzzle. Because the observed forward premium is affected both by changes in expected returns on spot rates and by changes in conditional forecasts of exchange rate volatility, we show that the observed forward premium will *normally* provide biased forecasts of changes in spot rates. In our framework, the extent of the putative bias is a direct function of the liquidity risk premium.

Data on five major currencies provide affirmation for the hypotheses that the conventional covered interest parity is a conditional and not an unconditional characterization of international financial market equilibrium; that the shadow forward premium is an unbiased forecast of future changes in the spot rate; that the Fama risk-premium is entirely the liquidity premium in the forward contract; that conventional tests of uncovered interest parity are in fact conditional tests that – conditional on the absence of a liquidity event; and that the anomalous empirical finding that the forward premium is a biased predictor of changes in spot rates is the expected finding of the unconditional uncovered interest parity theory which subsumes consideration of liquidity risk. Our framework suggests a new way to resolve the trenchant forward premium puzzle by accounting for the presence of a liquidity premium in the observed forward premium; the cost of doing so is to call into question the faithful application of conventional covered interest parity in economic analyses.

The empirical estimates in this study are based on the pre-global financial crisis sample of currencies because liquidity shocks during the crisis would likely have caused significant parameter shifts, especially since *ex ante* perceptions of liquidity shocks that are assumed in the theoretical model may have been substantially modified post-crisis. Moreover, Bansal and Dahlquist [2] have found that developed and developing countries broadly exhibit cross-sectional structural differences with regard to violation of uncovered interest parity.

Consequently, empirical studies based on different sample periods and currency samples, including forward contracts of different maturities, remain subjects of further research.

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Table 1a: *Conditional returns on Home and Foreign securities for the Home investor*

	Loss State: $\Pr. = p$	**No-Loss State**: $\Pr. = 1-p$
Home Security	$-kr_t$	r_t
Foreign Security	$-kr_t^* + (x_t - \Delta \ln(S_{t+1}))$	$r_t^* + x_t$

Table 1b: *Conditional expected utilities of the Home investor from Home and Foreign securities*

	Loss State: $\Pr. = p$	**No-Loss State**: $\Pr. = 1-p$
Home Security	$u(-kr_t)$	$u(r_t)$
Foreign Security	$u(-kr_t^* + x_t - E\Delta \ln(S_{t+1}) - \pi_t).$	$u(r_t^* + x_t)$

Table 2: *Implied coefficient of* x^o *in equation* (20) *under three liquidity event scenarios*

Scenario: ***k* =**	**λ=2**	**λ=4**
10 months	+0.01	-0.01
60 months	-0.13	-0.49
100 months	-0.35	-1.44

Notes:

- The coefficient of x^o is $\left[1 - \frac{pk}{1-p}\Psi(r)\right]$.
- Each coefficient value is conditional on monthly Home interest rate $r = 0.005$.
- $\Psi(r)$ is defined in equation (10).
- If $u(y) = 1 - \exp(-\lambda y)$ then $\Psi(r) = \exp(\lambda(1+k)r)$ for conditional returns $y = r, -kr$.
- The parameter restriction in *n*.2 is met by setting $pk(1-p)^{-1} = 0.99(1-\lambda r)(1+\lambda kr)^{-1}$: given *r*, *p* implicitly satisfies this equation for different values of k and λ.

Table 3: *Summary statistics for sample currencies*
1980:01 – 2007:12

		CAD	**DEM**	**JPY**	**CHF**	**GBP**	**USA**
Spot Rate Appreciation: $\Delta \ln(S_t)$	Mean	0.000474	0.000804	0.002207	0.001097	-0.000371	
	Median	0.000133	0.000284	-0.002250	-0.000964	-0.000135	
	Max	0.058574	0.093176	0.150092	0.116418	0.131350	
	Min	-0.052198	-0.121737	-0.113918	-0.109297	-0.127690	
	Standard Deviation	0.016234	0.031292	0.032415	0.033945	0.029859	
	T	335	335	335	335	335	
Observed Forward Premium: $x_t^o = r_t^{USA} - r_t^*$	Mean	-0.000564	0.001011	0.002573	0.002222	-0.001482	
	Median	-0.000507	0.001478	0.002543	0.002516	-0.001399	
	Max	0.003411	0.008570	0.009875	0.011436	0.005923	
	Min	-0.003999	-0.005037	-0.003095	-0.004322	-0.005858	
	Standard Deviation	0.001318	0.002281	0.002072	0.002482	0.001908	
	T	336	336	336	336	336	
Absolute observed forward premium: $\lvert x_t^o \rvert$	Mean	0.001126	0.002063	0.002789	0.002739	0.001859	
	Median	0.000869	0.001755	0.002634	0.002657	0.001528	
	Max	0.003999	0.008570	0.009875	0.011436	0.005923	
	Min	0.000000	0.000013	0.000024	0.000051	0.000012	
	Standard Deviation	0.000887	0.001399	0.001770	0.001893	0.001542	
	T	336	336	336	336	336	
Conditional Forecast of the Standard Deviation of Spot Rate Appreciation: $\sqrt{h_{t+1}} \lvert I_t$	Mean	0.015786	0.031266	0.032231	0.035002	0.029176	
	Median	0.014931	0.030317	0.032192	0.033392	0.029722	
	Max	0.031873	0.052378	0.041725	0.057520	0.049140	
	Min	0.010632	0.026303	0.027012	0.025597	0.019336	
	Standard Deviation	0.003493	0.004068	0.003035	0.007800	0.006841	
	T	335	335	335	335	335	
Monthly interest rate: r_t	Mean	0.005827	0.004241	0.002675	0.003026	0.006750	0.005259
	Median	0.004744	0.003725	0.002034	0.002732	0.005813	0.004602
	Max	0.016492	0.009817	0.010933	0.008545	0.014067	0.016448
	Min	0.001639	0.001677	-0.000078	0.000130	0.002745	0.000855
	Standard Deviation	0.003180	0.002004	0.002598	0.002081	0.002896	0.002957
	T	336	336	336	336	336	336

Table 4: *Estimates of equation* (21)

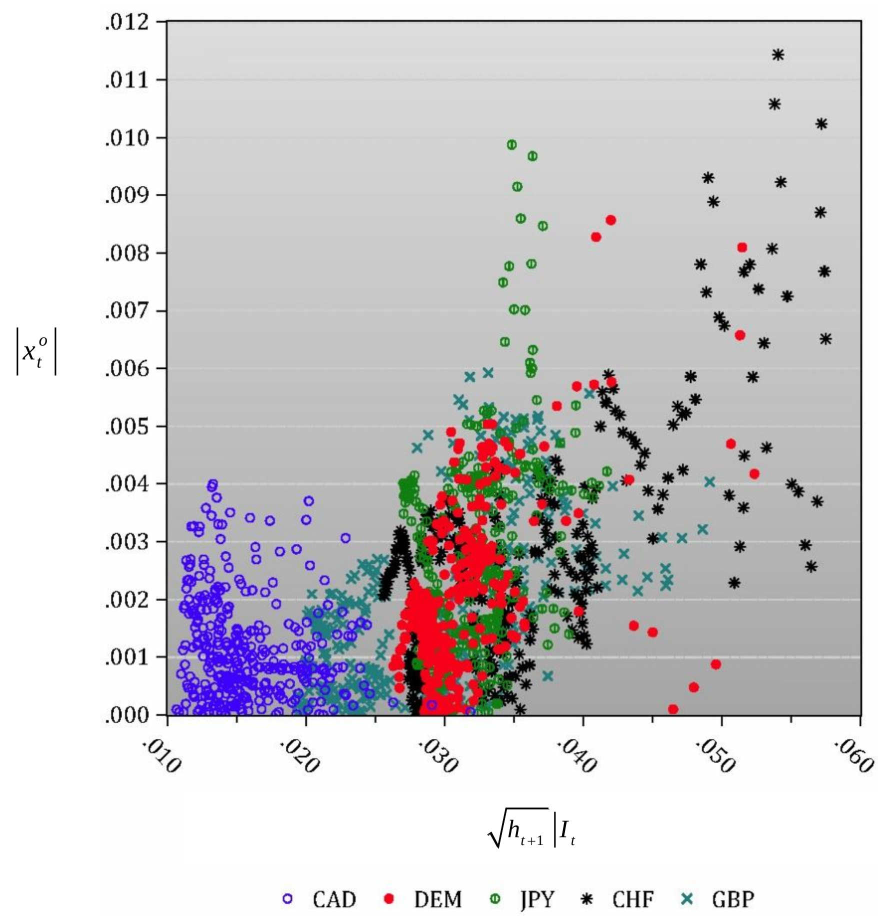
Equation (21):
$$\Delta \ln(S_t) = c + u_t$$
$$u_t \big| I_{t-1} \sim N\left(0,\ h_t\right)$$
$$h_t = w + a u_{t-1}^2 + b h_{t-1}$$

	CAD	**DEM**	**JPY**	**CHF**	**GBP**
Mean Equation					
$\hat{c}$	-0.0005 (-0.64)	0.0009 (0.53)	0.0024 (1.36)	0.0014 (0.76)	0.0005 (0.27)
Variance Equation					
$\hat{w}$	0.00001 (1.15)	**0.00001** (2.05)	0.00004 (0.74)	0.00000 (0.93)	0.00000 (0.90)
$\hat{a}$	**0.1084** (2.81)	0.0298 (1.19)	0.0306 (1.16)	0.0095 (0.78)	**0.0531** (2.44)
$\hat{b}$	**0.8406** (12.12)	**0.9134** (23.46)	**0.9271** (12.56)	**0.9808** (97.64)	**0.9360** (40.71)
$\overline{R}^2 =$	-0.003	-0.000	-0.000	-0.000	-0.000
Log L =	922	689	676	663	716
Q (18)=	24.3	17.2	22.7	18.4	15.1
$Q^2(18)$=	8.8	10.8	15.8	10.3	6.8
ARCH LM F-stat	0.08	0.66	0.25	0.00	0.18
Pr F(1, 332)	0.78	0.42	0.62	0.96	0.67
T=	335	335	335	335	335

Notes:

- GARCH (1,1) estimates from EViews 7.
- S is the spot rate; *z*-statistics are in parentheses; boldface denotes significance at 5% or better;.
- Coefficient covariances are Bollerslev-Wooldridge heteroskedasticity consistent, and $(\hat{a} + \hat{b}) < 1$ in each regression.
- $Q(18)$ is the estimated Ljung-Box statistic for up to 18th order serial correlation in standardized residuals. The results are consistent with the absence of ARCH effects in the mean equations.
- $Q^2(18)$ is the is the estimated Ljung-Box statistic for up to 18th order serial correlation in standardized squared-residuals. No remaining ARCH effects are indicated in the results in any variance equation.
- *ARCH LM* is the estimated *F*-statistic for the null hypothesis of no ARCH effects, and *Pr F*(1, 332) is the corresponding *p*-value. No remaining ARCH effects are indicated in the results in any variance equation.
- *T* is the number of observations before exclusions.

Figure 1: *Scatterplot of* $\left|x_t^o\right|$ *against* $\sqrt{h_{t+1}}\left|I_t\right.$ *for five currencies*
1980:01 – 2007:12



Notes:

- $\left|x_t^o\right| = \left|r_t^{USA} - r_t^{\,j}\right|$ for j = CAD, DEM, JPY, CHF, GBP.
- $\sqrt{h_{t+1}}\left|I_t\right.$ is the one-period ahead value of the conditional standard deviation estimated from equation (21) (see Table 4).

Table 5: *2SLS and GLS estimates of equation* (22)

Equation (22): $\left|x_t^{oj}\right| = \beta_0^j + \beta_1\sqrt{h_{t+1}^j}\left|I_t + \varepsilon_t^j\right.$

$\varepsilon_t^j = \rho^j\varepsilon_{t-1}^j + \eta_t^j$

$E(\varepsilon_t^j\varepsilon_t^k) = \sigma^{jk},\ E(\varepsilon_s^j\varepsilon_t^k) = 0$

	2SLS	GLS
$\hat{\beta}_1$	**0.12**	0.01
	(2.17)	(0.87)
Fixed Effects $\hat{\beta}_0^j$: $j =$		
CAD	-0.001	**0.001**
	(-0.95)	(3.16)
DEM	-0.002	**0.002**
	(-1.20)	(2.52)
JPY	-0.001	**0.002**
	(-0.71)	(2.30)
CHF	-0.002	**0.002**
	(-0.95)	(3.34)
GBP	-0.002	**0.001**
	(-1.05)	(2.18)
Autocorrelations		
$\hat{\rho}^{CAD}$	**0.87**	**0.85**
	(28.0)	(28.7)
$\hat{\rho}^{DEM}$	**0.92**	**0.93**
	(28.0)	(31.4)
$\hat{\rho}^{JPY}$	**0.95**	**0.96**
	(50.0)	(55.4)
$\hat{\rho}^{CHF}$	**0.89**	**0.92**
	(28.9)	(33.8)
$\hat{\rho}^{GBP}$	**0.88**	**0.93**
	(18.0)	(36.1)
$\overline{R}^2$	0.89	0.89
F	**1392**	**1394**
Pr F	0.00	0.00
NxT	1670	1670
Instrument Rank	11	

Notes:
- 2SLS and GLS estimates from EViews 7 using Seemingly Unrelated Regression and cross-section SUR weights.
- $\left|x_t^{oj}\right|$ is the absolute value of the observed forward premium; $\sqrt{h_{t+1}^j}\left|I_t\right.$ is the one-period ahead forecast of the conditional standard deviation of the appreciation of the spot rate. $\sqrt{h_{t+1}^j}\left|I_t\right.$ was instrumented by $\sqrt{h_t^j}\left|I_{t-1}\right.$. See Section 3.1 for discussion.
- Bold face denotes significance at 5% or better; *t*-statistics in parentheses utilize White robust standard errors.
- *N* is the number of cross sections (5) and *T* is the number of observations per cross-section after exclusions (334). Cross sections include CAD, DEM, JPY, CHF and GBP.

Table 6: *2SLS and GLS estimates of equation* (23)

Equation (23): $\ln \left| x_t^{oj} \right| = \beta_0^j + \beta_1 \ln \sqrt{h_{t+1}^j} \left| I_t \right. + \varepsilon_t^j$

$\varepsilon_t^j = \rho^j \varepsilon_{t-1}^j + \eta_t^j$

$E(\varepsilon_t^j \varepsilon_t^k) = \sigma^{jk},\ E(\varepsilon_s^j \varepsilon_t^k) = 0$

	2SLS Estimates	GLS Estimates
$\hat{\beta}_1$	**2.07** (3.15)	**1.17** (3.50)
Fixed Effects $\hat{\beta}_0^j$: j =		
DEM	0.69 (0.29)	**-2.44** (-1.99)
JPY	0.94 (0.41)	-2.15 (-1.76)
CHF	0.80 (0.35)	-2.25 (-1.86)
GBP	0.56 (0.24)	**-2.64** (-2.18)
Autocorrelations		
$\hat{\rho}^{DEM}$	**0.87** (12.9)	**0.87** (12.9)
$\hat{\rho}^{JPY}$	**0.89** (16.4)	**0.89** (16.5)
$\hat{\rho}^{CHF}$	**0.91** (19.4)	**0.92** (20.4)
$\hat{\rho}^{GBP}$	**0.79** (17.9)	**0.81** (19.5)
$\overline{R}^2$	0.82	0.82
F	**748**	**751**
Pr F	0.00	0.00
NxT	1336	1336
Instrument Rank	9	

Notes:

- 2SLS and GLS estimates from EViews 7 using Seemingly Unrelated Regression and cross-section SUR weights.
- $\left| x_t^{oj} \right|$ is the absolute value of the observed forward premium; $\sqrt{h_{t+1}^j} \left| I_t \right.$ is the one-period ahead forecast of the conditional standard deviation of the appreciation of the spot rate. $\ln \left| x_{t-1}^{oj} \right|$ was used as the instrument for $\ln \left| x_t^{oj} \right|$ and $\ln \sqrt{h_t^j} \left| I_{t-1} \right.$ for $\ln \sqrt{h_{t+1}^j} \left| I_t \right.$. See Section 3.1 for discussion.
- Bold face denotes significance at 5% or better; *t*-statistics in parentheses utilize White robust standard errors.
- *N* is the number of cross sections (4) and *T* is the number of observations per cross-section after exclusions (334). Currency cross sections include DEM, JPY, CHF and GBP.
- In the 2SLS estimate, the Wald test fails to reject the null that $\beta_1 = 1$; the associated *p*-value is 0.105.

Table 7: *Regression estimates of equations* (25) *and* (26)

Equation (25): $\Delta \ln(S_{t+1}^{j}) - x_t^{o\,j} = \alpha^{j} + \alpha_D^{j} D_t + (\beta - 1) x_t^{o\,j} + \beta_D D_t x_t^{o\,j} + \upsilon_{t+1}^{j}$

Equation (26): $\Delta \ln(S_{t+1}^{j}) - x_t^{o\,j} = \alpha^{j} + (\beta - 1) x_t^{o\,j} + \upsilon_{t+1}^{j}$

	Equation (25)	Equation (26)
$\hat{\beta}$	**-0.47** (-2.18)	**-1.47** (-6.87)
$\hat{\beta}_D$	**-2.17** (-2.62)	
Constants		
$\hat{\alpha}^{CAD}$	0.001 (0.51)	-0.000 (-0.39)
$\hat{\alpha}_D^{CAD}$	-0.003 (-1.73)	
$\hat{\alpha}^{DEM}$	0.001 (0.44)	0.002 (1.31)
$\hat{\alpha}_D^{DEM}$	0.007 (1.73)	
$\hat{\alpha}^{JPY}$	0.002 (0.82)	**0.006** (3.07)
$\hat{\alpha}_D^{JPY}$	**0.010** (2.29)	
$\hat{\alpha}^{CHF}$	0.001 (0.52)	**0.004** (2.11)
$\hat{\alpha}_D^{CHF}$	**0.010** **(2.24)**	
$\hat{\alpha}^{GBP}$	- 0.000 (-0.19)	- 0.003 (-1.58)
$\hat{\alpha}_D^{GBP}$	-0.005 (-1.45)	
$\bar{R}^2$	0.032	0.027
F	6.1	10.4
Pr F	0.00	0.00
NxT	1675	1675

Notes:

- $\Delta \ln(S_{t+1}^{j})$ is the one-period ahead change in the log of the spot rate; x_t^{j} is the observed forward premium. GLS estimates assume $E(\upsilon_t^{j}\upsilon_t^{k}) = \sigma^{jk}$. Estimated in EViews 7 using cross-section SUR weights.
- Bold face denotes significance at 5% or less; t-statistics in parentheses use White robust standard errors.
- N is the number of cross sections (5), T is the number of observations per cross-section after exclusions (335). Cross sections include CAD, DEM, JPY, CHF and GBP.
- $D_t = 1$ if $r_t^{USA} \geq Mean(r_t^{USA})$, 0 otherwise. $\sum_1^{335} D_t = 136$ in the sample.
- In equation (25), the estimated Ljung-Box statistic for up to 18th order serial correlation in standardized residuals, $Q(18)$, for CAD, DEM, JPY, CHF and GBP respectively, are 19.9, 16.7, 24.7, 16.1 and 23.3. All indicate absence of serial correlation. Residuals in equation (26) are qualitatively no different.

Appendix 1: *Derivation of* $\Psi'(y_t)$

Equation (10) in the text implies:

$$\Psi'(y_t) = -k\left(\frac{u''(-ky_t)}{u'(y_t)}\right) - \Psi(y_t)\left(\frac{u''(y_t)}{u'(y_t)}\right)$$

$$= k\left(-\frac{u'(-ky_t)}{u'(y_t)}\frac{u''(-ky_t)}{u'(-ky_t)}\right) + \Psi(y_t)\left(-\frac{u''(y_t)}{u'(y_t)}\right)$$

Since the Arrow-Pratt measure of constant absolute risk aversion is: $\lambda = -\frac{u''(-ky_t)}{u'(-ky_t)} = -\frac{u''(y_t)}{u'(y_t)}$

$$\Psi'(y_t) = k\Psi(y_t)\lambda + \Psi(y_t)\lambda$$

Thus: $\Psi'(y_t) = \lambda(1+k)\Psi(y_t)$

Appendix 2: *Derivation of Equation* (15)

The first-order linearization of $\Psi(r_t^*)$ around r_t is:

(A2.1) $\Psi(r_t^*) = \Psi(r_t) + \Psi'(r_t)(r_t^* - r_t)$

From *Appendix 1*:

(A2.2) $\Psi'(r_t) = \lambda(1+k)\Psi(r_t)$

In (A2.1) substitute for $\Psi'(r_t)$ from (A2.2) and then re-arrange to obtain:

(A2.3) $\Psi(r_t) - \Psi(r_t^*) = \lambda(1+k)\Psi(r_t)(r_t - r_t^*)$

Substituting the RHS of (A2.3) for the term $[\Psi(r_t) - \Psi(r_t^*)]$ in the denominator in equation (14) yields:

(A2.4) $(r_t - r_t^*) = \left\{\frac{1 + \Psi(r_t^*)\Psi(r_t)^{-1}}{k\lambda(1+k)(r_t - r_t^*)}\right\}\frac{\lambda}{2}h_{t+1}\Big| I_t$

Now, in respect to the numerator of the term in brackets in (A2.4), using (A2.1) we obtain:

(A2.5) $\Psi(r_t^*)\Psi(r_t)^{-1} = 1 + \Psi'(r_t)\Psi(r_t)^{-1}(r_t^* - r_t)$

Substituting the expression for $\Psi'(r_t)$ from (A2.2) in (A2.5) gives

(A2.6) $\Psi(r_t^*)\Psi(r_t)^{-1} = 1 - \lambda(1+k)(r_t - r_t^*)$

Substituting the RHS of (A2.6) for the term $\Psi(r_t^*)\Psi(r_t)^{-1}$ in brackets in (A2.4) yields:

(A2.7) $$(r_t - r_t^*) = \left\{ \frac{2 - \lambda(1+k)(r_t - r_t^*)}{\lambda(1+k)(r_t - r_t^*)} \right\} \frac{\lambda}{2k} h_{t+1} \big| I_t$$

For monthly data of order 10^{-3} or 10^{-4} for $(r_t - r_t^*)$, the numerator of the term in brackets is approximately 2 for reasonable values of parameters λ and k, and (A2.7) can be written as:

(A2.8) $$(r_t - r_t^*) = \left\{ \frac{2}{\lambda(1+k)(r_t - r_t^*)} \right\} \frac{\lambda}{2k} h_{t+1} \big| I_t$$

or

(A2.9) $$(r_t - r_t^*)^2 = \frac{h_{t+1} \big| I_t}{k(1+k)}$$

Defining $\kappa \equiv \sqrt{k(1+k)}$ and taking positive roots of each side yields:

(A2.10) $$\left| r_t - r_t^* \right| = \kappa \sqrt{h_{t+1}} \big| I_t$$

Equation (A2.10) is equation (15) in the text.