Date
28/6/19

Unit-IV Interpolation

UNIT-IV

*** Introduction :-**

Since $y = f(x)$ be the given function. the given function defined in the interval (a, b) then it is called "interpolation".

Consider 'x' takes the values $x_0, x_1, x_2, x_3, x_4 \dots, x_n$ the corresponding y-values are $y_0, y_1, y_2, y_3, y_4 \dots, y_n$ respectively. And the differences of x are is 'h' then

$x_1 - x_0 = h,\ x_2 - x_1 = h,\ x_3 - x_2 = h, \dots, x_n - x_{n-1} = h$

$\Rightarrow x_1 = x_0 + h$

$\Rightarrow x_2 = x_1 + h \Rightarrow x_2 = (x_0 + h) + h$

$$\boxed{x_2 = x_0 + 2h}$$

$\Rightarrow x_3 = x_2 + h \Rightarrow x_3 = (x_0 + 2h) + h$

$$\boxed{x_3 = x_0 + 3h}$$

$\Rightarrow x_n - x_{n-1} = h \Rightarrow \boxed{x_n = x_0 + nh}$

Given, $y = f(x)$

$y_0 = f(x_0)$

$y_1 = f(x_1)$

$y_1 = f(x_0 + h)$

$y_2 = f(x_2)$

$= f(x_0 + 2h)$

$y_3 = f(x_3)$

$= f(x_0 + 3h)$

$y_n = f(x_n)$

$$\boxed{y_n = f(x_0 + nh)}$$

The differences

$y_1 - y_0,\ y_2 - y_1,\ y_3 - y_2,\ y_4 - y_3, \dots$ are represented by

$\Delta y_0, \Delta y_1, \Delta y_2, \Delta y_3, \ldots$ respectively are called first order forward differences and Δ is called forward difference operator.

The differences

$\Delta y_1 - \Delta y_0, \Delta y_2 - \Delta y_1, \Delta y_3 - \Delta y_2, \ldots$ are represented by $\Delta^2 y_0, \Delta^2 y_1, \Delta^2 y_2, \ldots$ are called second order forward differences

The differences

$\Delta^2 y_1 - \Delta^2 y_0, \Delta^2 y_2 - \Delta^2 y_1, \Delta^2 y_3 - \Delta^2 y_2, \ldots$ are represented by $\Delta^3 y_0, \Delta^3 y_1, \Delta^3 y_2, \ldots$ respectively are called third order forward differences

The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$ are represented by $\nabla y_1, \nabla y_2, \nabla y_3, \nabla y_4, \ldots$ respectively are called first order backward differences and ∇ is called Backward difference operator.

The differences $\nabla y_2 - \nabla y_1, \nabla y_3 - \nabla y_2, \nabla y_4 - \nabla y_3, \ldots$ are represented by $\nabla^2 y_2, \nabla^2 y_3, \nabla^2 y_4, \ldots$ respectively are called second order backward differences

The differences $\nabla^2 y_3 - \nabla^2 y_2, \nabla^2 y_4 - \nabla^2 y_3, \ldots$ are represented by $\Delta^3 y_3, \Delta^3 y_4, \nabla^3 y_5, \ldots$ respectively are called third order backward differences.

Date 10/07/18

The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$ are represented by small (d) $\delta y_{1/2}, \delta y_{3/2}, \delta y_{5/2}, \delta y_{7/2}, \ldots$ respectively are called central differences and δ is called Central difference operator.

The differences $\delta y_{3/2} - \delta y_{1/2}, \delta y_{5/2} - \delta y_{3/2}, \delta y_{7/2} - \delta y_{5/2}, \ldots$ are represented by $\delta^2 y_1, \delta^2 y_2, \delta^2 y_3, \ldots$ respectively are called second order central differences

Similarly $\delta^2 y_2 - \delta^2 y_1, \delta^2 y_3 - \delta^2 y_2, \delta^2 y_4 - \delta^2_3 \ldots$ are represented by $\delta^3 y_{3/2}, \delta^3 y_{5/2}, \delta^3 y_{7/2} \ldots$ respectively are called the third order central differences

Shifting Operator *

Since 'E' is called shifting operator. It shifts the given function into the next level.

Cons Therefore

$$Ey_0 = y_1 \Rightarrow Ef(x_0) = f(x_1)$$

$$\boxed{Ef(x_0) = f(x_0+h)}$$

$$Ey_1 = y_2 \Rightarrow Ef(x_1) = f(x_2)$$

$$Ef(x_0+h) = f(x_0+2h)$$

$$E \cdot Ef(x_0) = f(x_0+2h)$$

$$\boxed{E^2 f(x_0) = f(x_0+2h)}$$

$$\therefore E^n f(x_0) = f(x_0+nh)$$

Similarly $E^3 f(x_0) = f(x_0+3h)$

Therefore $\boxed{E^n f(x) = f(x+nh)}$ **

Note

Since $E^n f(x) = f(x+nh)$

put $n = -n \Rightarrow E^{-n} f(x) = f(x+(-n)h)$

$\therefore$ ** $\boxed{E^{-n} f(x) = f(x-nh)}$

Book Work

Since we know the $y_1 - y_0 = \Delta y_0 \rightarrow ①$

and $Ey_0 = y_1 \rightarrow ②$

from ① & ②

$$Ey_0 - y_0 = \Delta y_0$$

$$(E-1)y_0 = \Delta y_0$$

$$E - 1 = \Delta$$

$$\boxed{E = 1 + \Delta}$$ *

Relation between S.O and forward difference

Since we know that $y_1 - y_0 = \nabla y_1 \rightarrow ①$

we know and $Ey_0 = y_1$

$$\Rightarrow y_0 = E^{-1}y_1 \rightarrow ②$$

from ① & ②

$$y_1 - E^{-1}y_1 = \nabla y_1$$

$$y_1(1 - E^{-1}) = \nabla y_1$$

$$1 - E^{-1} = \nabla$$

$$\boxed{E^{-1} = 1 - \nabla}$$ *

Relation between shifting operator and backward differences

Since we know that

$$y_1 - y_0 = \delta y_{1/2} \rightarrow$$

$$\Rightarrow y_{\frac{1}{2}+\frac{1}{2}} - y_{\frac{1}{2}-\frac{1}{2}} = \delta y_{1/2}$$

$\because$ $E^{1}y_3 = y_{3+}$

$E^{1}y_0 = y_{0+}$

$$E^{1/2}y_{1/2} - E^{-1/2}y_{1/2} = \delta y_{1/2}$$

$$y_{1/2}\left[E^{1/2} - E^{-1/2}\right] = \delta y_{1/2}$$

$$\boxed{E^{1/2} - E^{-1/2} = \delta}$$ *

Relation between central difference and shifting operator

Average Operator

'μ' is called Average operator such that

$$\mu y_n = \frac{y_{n+1/2} + y_{n-\frac{1}{2}}}{2}$$

$$\mu y_n = \frac{E^{1/2} y_n + E^{-1/2} y_n}{2}$$

$$\mu y_n = \left[\frac{E^{1/2} + E^{-1/2}}{2}\right] y_n$$

$$\boxed{\mu = \frac{E^{1/2} + E^{-1/2}}{2}}$$

The above equation is the relation between Average operator and shifting operator

Pascal's Triangle.

0

1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$\Delta^4 y_0 = 1y_1 - 4y_2 + 6y_3 - 4y_4 + 1y_5$

Date 26/07/13

Newtons forward interpolation formulae

Consider $y = f(x)$ be the given function. x creates the values $x_0, x_1, x_2, \ldots x_n$ and the common difference between 'x' is 'h'.

The corresponding 'y' values are $y_0, y_1, y_2, \ldots y_n$ respectively then

$$y_n = f(x_0 + nh)$$
$$= E^n f(x_0)$$

$$(1+\Delta)^n y_0 = y_n$$

$$\because (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+ - - -$$

$$y_n = (1+\Delta)^n = \left[1+n\Delta+\frac{n(n-1)}{2!}\Delta^2+\frac{n(n-1)(n-2)}{3!}\Delta^3+ - - \right]y_0$$

$$y_n = y_0+n\Delta y_0+\frac{n(n-1)}{2!}\Delta^2 y_0+\frac{n(n-1)(n-2)}{3!}\Delta^3 y_0+ - -$$

Newton's Backward Interpolation Formulae

At arbitrary value $x=x_n$ the corresponding y values is y_n then $y_n = f(x_n)$

$$\Rightarrow y_n = f(x_0+nh)$$
$$= E^n f(x_n)$$
$$= (E^{-1})^{-n} f(x_n)$$
$$= (1-\nabla)^{-n} y_n$$

$$\because (1-x)^{-n} = \left[1+nx+\frac{n(n+1)}{2!}x^2+\frac{n(n+1)(n+2)}{3!}x^3+\frac{n(n+1)(n+2)}{4!}x\right.$$

$$y_n = (1-\nabla)^{-n} = \left[1+n\nabla+\frac{n(n+1)}{2!}\nabla^2+\frac{n(n+1)(n+2)}{3!}\nabla^3+\frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4\right.$$

$$y_n = y_n+n\nabla y_n+\frac{n(n+1)}{2!}\nabla^2 y_n+\frac{n(n+1)(n+2)}{3!}\nabla^3 y_n+\frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$

Problems

1. find $\Delta f(x)$, $f(x) = x^3-x^2+x+10$, $h=1$

Solⁿ Since we know that

$$\Delta f(x) = f(x+h)-f(x)$$
$$= f(x+1)-f(x)$$
$$= (x+1)^3-(x+1)^2+(x+1)+10-[x^3-x^2+x+10]$$
$$= x^3+1^3+3x^2 1+3x-[x^2+1+2x]+x+1$$
$$+10-x^3+x^2-x-10$$
$$= \cancel{x^3}+1+3x^2+3x-\cancel{x^2}-\cancel{1}-2x+\cancel{x}+\cancel{1}+\cancel{10}$$
$$-\cancel{x^3}+\cancel{x^2}-\cancel{x}-\cancel{10}$$
$$= 3x^2+3x+1-2x$$

$\therefore \Delta f(x) = 3x^2+x+1$

2 find $\Delta^2 f(x)$, given $f(x) = e^{2x}$, $h=1$

solu) Since $\Delta f(x) = f(x+h) - f(x)$

we know that

$$\Delta f(x) = f(x+1) - f(x)$$
$$= e^{2(x+1)} - e^{2x}$$
$$= e^{2x+2} - e^{2x}$$
$$= e^{2x} e^2 - e^{2x}$$
$$\Delta f(x) = e^{2x}[e^2 - 1]$$
$$\Delta e^{2x} = e^{2x}(e^2-1) \rightarrow ①$$
$$\Delta^2 f(x) = \Delta[\Delta f(x)]$$
$$= \Delta[e^{2x}(e^2-1)]$$
$$= (e^2-1)[\Delta e^{2x}]$$
$$= (e^2-1)[e^{2x}(e^2-1)] \text{ from } ①$$
$$\therefore \Delta^2 f(x) = (e^2-1)^2 e^{2x}$$

3 If $f(x) = \frac{10}{x!}$ find $\Delta f(x)$, and $h=1$

solu) $\Delta f(x) = f(x+h) - f(x)$

$$= f(x+1) - f(x)$$
$$= \frac{10}{(x+1)!} - \frac{10}{x!} \Rightarrow \frac{10}{(x+1)!\,x!} - \frac{10}{x!}$$
$$= \frac{10 - 10(x+1)}{(x+1)!\,x!}$$
$$= \frac{10[1-x-1]}{(x+1)!}$$
$$= \frac{-10x}{(x+1)!}$$

4 find Δ^2

7 Show that $\delta^2 E = \Delta^2$

<u>Solu</u> $\delta^2 E = \Delta^2$

$\delta = E^{1/2} - E^{-1/2}$

$\Delta = E - 1$

$L.H.S = (E^{1/2} - E^{-1/2})^2 E$

$= \left((E^{1/2})^2 + (E^{-1/2})^2 - 2E^{1/2}E^{-1/2}\right) E$

$= [E + E^{-1} - 2] E$

$= E^2 + E^{-1} \cdot E - 2E$

$= [E^2 + 1 - 2E \cdot 1]$

$= [E-1]^2$

$= \Delta^{2-1} = R.H.S$

$L.H.S = R.H.S$

Hence proved.

8 Show that $\mu\delta = \frac{E-1}{2}$

<u>Solu</u> $\mu = \frac{E^{1/2} + E^{-1/2}}{2}$ $\delta = E^{1/2} - E^{-1/2}$

$L.H.S = \left[\frac{E^{1/2} + E^{-1/2}}{2}\right][E^{1/2} - E^{-1/2}]$

$= \frac{(E^{1/2})^2 - (E^{-1/2})^2}{2}$

$= \frac{E^1 - E^{-1}}{2}$

$= R.H.S$

[illegible]

9 Show that $\Delta = \nabla(1-\nabla)^{-1}$

<u>Solu</u> $\Delta = \nabla(1-\nabla)^{-1}$

$1 - \nabla = E^{-1}$

$R.H.S = \nabla(E^{-1})^{-1}$

$= \nabla E$, $\nabla = 1 - E^{-1}$

$= (1-E^{-1})E$

$= E - E^{-1}E$

$= E - 1$

$= \Delta$

$=$ R.H.S

10. Write forward difference table for

x : 10 20 30 40

y : 1.1 2.0 4.4 7.9

Solu) Forward Difference Table

x	y	Δ	Δ^2	Δ^3
10	1.1			
		$= 2.0 - 1.1$ $= 0.9$		
20	2.0		$= 2.4 - 0.9$ $= 1.5$	
		$= 4.4 - 2.0 = 2.4$		$= 1.1 - 1.5$ $= -0.4$
30	4.4		$= 3.5 - 2.4$ $= 1.1$	
		$= 7.9 - 4.4$ $= 3.5$		
40	7.9			

11. Construct the difference table for the given data and evaluate $\Delta^2 f(2)$

x : 0 1 2 3 4

$f(x)$: 1.0 1.5 2.2 3.1 4.6

Solu) Difference table

x	$f(x)$	1st	2nd	3rd	4th
0	1.0				
		$1.5 - 1.0$ $= 0.5$			
1	1.5		$0.7 - 0.5$ $= 0.2$		
		$2.2 - 1.5$ $= 0.7$		$0.2 - 0.2$ $= 0.0$	
2	2.2		$0.9 - 0.7$ $= 0.2$		$0.4 - 0.0$ $= 0.4$
		$3.1 - 2.2$ $= 0.9$		$0.6 - 0.2$ $= 0.4$	
3	3.1		$1.5 - 0.9$ $= 0.6$		
		$4.6 - 3.1$ $= 1.5$			
4	4.6				

why we should do forward

In the above question Δ is given so that

from the difference table $\Delta^2 f(2) = 0.6$

Forward starts with y_0

Note in backward starts y_0, y_1, y_2, y_3

from the difference table $\nabla^2 f(2) = 0.2$

12. find the missing value of the following data.

x :	1	2	3	4	5
f(x) :	7	−	13	21	37

Solu Difference table

x	f(x)	1st	2nd	3rd	4th
1	7				
		$y-7$			
2	y		$(13-y)(y-7) = 20-2y$		
		$13-y$		$y-5-20+2y$ $= 3y-25$	
3	13		$8-(13-y) = y-5$		$13-y-$ $+25$ $=38-4$
		$21-13$ 8		$8-y+5$ $=13-y$	
4	21		$16-8 = 8$		
		$37-21$ 16			
5	37				

[(13 − y)(y − 7)

$13y - y^2 + 7y - 91$

13 $6y - y^2 - 91$

$y^2 - 6y + 91$]

From the Difference table

$38 - 4y = 0$

$38 = 4y$

$y = 9.5$

4)38(
36
20

13. Prove that $U_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$

Solu

$$R.H.S = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$$
$$= u_3 + \Delta u_2 + \Delta^2 u_1 + (\Delta^2 u_2 - \Delta^2 u_1)$$
$$= u_3 + \Delta u_2 + \cancel{\Delta^2 u_1} + \Delta^2 u_2 - \cancel{\Delta^2 u_1}$$
$$= u_3 + \Delta u_2 + \Delta^2 u_2$$
$$= u_3 + \cancel{\Delta u_2} + (\Delta u_3 - \cancel{\Delta u_2})$$
$$= u_3 + \Delta u_3$$
$$= \cancel{u_3} + u_4 - u_3$$
$$= u_4 = R = L.H.S$$

14. Evaluate $u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

Solu

$$[= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$$
$$= u_0 + 4(u_1 - u_0) + 6(\Delta u_0 - \Delta u_{-1}) + 10(\Delta^2 u_0 - \Delta u_{-1})$$
$$= u_0 + 4u_1 - 4u_0 + 6\Delta u_0 - 6\Delta u_{-1} + 10\Delta^2 u_0 - 10\Delta u^2_{-1}$$

$= u_0 + 6\Delta u_0 + 10\Delta^2 u_0 + 4u_1 - 6\Delta u_{-1} - 10\Delta u_{-1}$

$= u_0 + 6\Delta u_0 + 10\Delta^2 u_0 + 4u_1 - 10\Delta u_{-1}^2 - 6\Delta u_{-1}]$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10[\Delta^2 u_0 - \Delta^2 u_{-1}]$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^2 u_0 - 10\Delta^2 u_{-1}$

$= u_0 + 4\Delta u_0 + 10\Delta^2 u_0 - 4\Delta^2 u_{-1}$

$= u_0 + 4\Delta u_0 + 10\Delta^2 u_0 - 4(\Delta u_0 - \Delta u_{-1})$

$= u_0 + \cancel{4\Delta u_0} + 10\Delta^2 u_0 - \cancel{4\Delta u_0} + 4\Delta u_{-1}$

$= u_0 + 10\Delta^2 u_0 + 4\Delta u_{-1}$

$= u_0 + 10[\Delta u_1 - \Delta u_0) + 4(u_0 - u_{-1})$

$= u_0 + 10\Delta u_1 - 10\Delta u_0 + 4u_0 - 4u_{-1}$

$= u_0 + 10[u_2 - u_1] - 10[u_1 - u_0] + 4u_0 - 4u_{-1}$

$= 10u_2 - 20u_1 + 15u_0 - 4u_{-1}$

15. Evoluate $\Delta(e^{ax}\log(bx))$

Soln $\Delta f(x) = f(x+h) - f(x)$

$= e^{a(x+h)}\log b(x+h) - e^{ax}\log(bx)$

16. u_x is a function of x for which 5^{th} differences are constant and $u_1 + u_7 = -786$; $u_2 + u_6 = 686$; $u_3 + u_5 = 1088$ find u_4

Soln Since given that 5^{th} differences are constants

$\therefore \Delta^6 u_1 = 0$.

Since we know that $\Delta = E - 1$

$\therefore (E-1)^6 u_1 = 0$

$[1.E^6 - 6c_1 E^5 + 6c_2 E^4 - 6c_3 E^3 + 6c_4 E^2 - 6c_5 E + 6c_6 1] u_1 = 0$

$E^6 u_1 - 6Eu_1^5 + \frac{6\times5}{1\times2} Eu_1^4 - \frac{6\times5\times4}{1\times2\times3} Eu_1^3 + \frac{6\times5\times4\times3}{1\times2\times3\times4} Eu_1^2 - \frac{6\times5\times4\times3\times2}{1.2.3.4.5} Eu_1 + u_1 = 0$

$E^6 u_1 - 6E^5 u_1 + 15E^4 u_1 - 20E^3 u_1 + 15E^2 u_1 - 6Eu_1 + u_1 = 0$

$u_7 - 6u_6 + 15u_5 - 20u_4 + 15u_3 - 6u_2 + u_1 = 0$

$(u_7 + u_1) - 6(u_6 + u_2) + 15(u_5 + u_3) - 20u_4 = 0$

$-786 - 6(686) + 15(1088) - 20u_4 = 0$

$-786 - 4116 + 16320 - 20u_4 = 0$

$-4902 + 16320 = 20u_4$

$20u_4 = 11418$

$\therefore u_4 = \frac{11418}{20}$

$u_4 = 570.9$

Note:-

$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots\ldots+$

Since we know that $Ef(x) = f(x+h)$ by Taylor's series formula

$= f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \frac{h^4}{4!} f''''(x) \ldots$

$= f(x) + h \frac{d}{dx} f(x) + \frac{h^2}{2!} \frac{d^2}{dx^2} f(x) + \frac{h^3}{3!} \frac{d^3}{dx^3} f(x) + \frac{h^4}{4!} \frac{d^4}{dx^4} f(x)$

$= f(x) \left[1 + h\frac{d}{dx} + \frac{h^2}{2!}\frac{d^2}{dx^2} + \frac{h^3}{3!}\frac{d^3}{dx^3} + \frac{h^4}{4!}\frac{d^4}{dx^4} + \ldots\right]$

$= f(x) \left[1+hD + \frac{h^2D^2}{2!} + \ldots\right]$

$= f(x) \left[1+hD+\frac{(hD)^2}{2!}+\frac{(hD)^3}{3!}+\frac{(hD)^4}{4!}+\ldots\right]$

$\left[\text{Here } \frac{d}{dx} = \right.$

$\therefore Ef(x) = f(x) \cdot e^{hD}$

$\boxed{\therefore E = e^{hD}}$ (or) $E = 1+\Delta \Rightarrow 1+\Delta = e^{hD}$

$\Delta = e^{hD} - 1$

17. Show that $\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n! h^n}{x(x+h)(x+2h)\ldots(x+nh)}$

Soln $\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n! h^n}{x(x+h)(x+2h)\ldots(x+nh)}$

$\therefore \Delta f(x) = f(x+h) - f(x)$

Now $n=1$

$\Delta\left[\frac{1}{x}\right] = \frac{1}{x+h} - \frac{1}{x}$

$= \frac{x-(x+h)}{x(x+h)}$

$= \frac{x-x-h}{x(x+h)}$

$= \frac{(-1)h}{x(x+h)} \longrightarrow ①$

$n=2$

$\Delta^2\left[\frac{1}{x}\right] = \Delta\left[\Delta\left[\frac{1}{x}\right]\right]$

$= \Delta\left[\frac{(-1)h}{x(x+h)}\right]$

$= h(-1)\left[\Delta\frac{1}{x(x+h)}\right]$

$= h(-1)\left[\frac{1}{(x+h)(x+h+h)} - \frac{1}{x(x+h)}\right]$

$$= (-1)h\left[\frac{x-(x+2h)}{x(x+h)(x+2h)}\right]$$

$$= (-1)h\left[\frac{x-x-2h}{x(x+h)(x+2h)}\right]$$

$$= \frac{(-1)^2 2\, h^2}{x(x+h)(x+2h)}$$

$$= \frac{(-1)^2 2!\, h^2}{x(x+h)(x+2h)}$$

$$= \frac{(-1)^2 2!\, h^2}{x(x+h)(x+2h)} \longrightarrow ②$$

If $n=3$

$$\Delta^3\left[\frac{1}{x}\right] = \Delta\left[\Delta^2\left[\frac{1}{x}\right]\right]$$

$$= \Delta\left[\frac{(-1)^2 2!\, h^2}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2 2!\, h^2\left[\Delta\left(\frac{1}{x(x+h)(x+2h)}\right)\right]$$

$$= (-1)^2 2!\, h^2\left[\frac{1}{(x+h)(x+h+h)(x+h+2h)} - \frac{1}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2 2!\, h^2\left[\frac{1}{(x+h)(x+2h)(x+3h)} - \frac{1}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2 2!\, h^2\left[\frac{x-(x+3h)}{x(x+h)(x+2h)(x+3h)}\right]$$

$$= (-1)^2 2!\, h^2\left[\frac{x-x-3h}{x(x+h)(x+2h)(x+3h)}\right]$$

$$= \frac{(-1)^2 2!\, h^2 (-1) 3h}{x(x+h)(x+2h)(x+3h)}$$

$$= \frac{(-1)^3\, 1\times 2\times 3\; h^3}{x(x+h)(x+2h)(x+3h)}$$

$$\therefore \Delta^3\left[\frac{1}{x}\right] = \frac{(-1)^3\, 3!\, h^3}{x(x+h)(x+2h)(x+3h)} \longrightarrow ③$$

Hence from ①, ② & ③

$$\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n\, n!\, h^n}{x(x+h)(x+2h)\cdots(x+nh)}$$

Date 5/8/18

18. Given, $u_0+u_8=1.9243$; $u_1+u_7=1.9590$ $u_2+u_6=1.9823$ $u_3+u_5=1.9956$ then find u_4.

soln Since

$\Delta^8 u_0=0$

$(E-1)^8 u_0=0$

$[1\cdot E^8-8c_1E^7+8c_2E^6+8c_3E^5+8c_4E^4+8c_5E^3+8c_6E^2+8c_7E+8c_8]$

$u_0E^8-8E^7u_0+\frac{8\times7}{1\times2}E^6u_0+\frac{8\times7\times6}{1\times2\times3}E^5u_0+\frac{8\times7\times6\times5}{1\times2\times3\times4}E^4u_0$

$+\frac{8\times7\times6\times5\times4}{1\times2\times3\times4\times5}E^3u_0+\frac{8\times7\times6\times5\times4\times3}{1\times2\times3\times4\times5\times6}E^2u_0+\frac{8\times7\times6\times5\times4\times3\times2}{1\times2\times3\times4\times5\times6\times7}E$

$+8u_0=0$

$u_8-8u_7+28u_6-56u_5+70u_4-56u_3+28u_2-8u_1+u_0=0$

$70u_4(u_0+u_8)-8(u_1+u_7)+28(u_2+u_6)-56(u_3+u_5)=0$

$70u_4+1.9243-8(1.9590)+28(1.9823)-56(1.9956)=0$

$70u_4+1.9243-15.672+55.5044-111.7536=0$

$69.9969=70u_4$

$u_4=\frac{69.9969}{70}$

$u_4=0.99995374$

$\therefore u_4=1$

19. find the missing term in the following

x:	0	5	10	15	20	25
y:	6	10	—	17	—	31

soln Consider $\Delta^4 y_0=0\rightarrow①$ $\Delta^4 y_1=0\rightarrow②$ and we know that

$\Delta=E-1$ sub in ① & ②

$(E-1)^4y_0=0$ and $(E-1)^4y_1=0$

$\Rightarrow[1\cdot E^4-4c_1E^3+4c_2E^2+4c_3E+4c_4]=0$ &

$[1\cdot E^4-4c_1E^3+4c_2E^2+4c_3E+4c_4]y_1=0$

$[1\cdot E^4-4c_1E^3+4c_2E^2+4c_3E+4]y_0=0$ &]

$\Rightarrow[u_4-4u_3+\frac{4\times3}{1\times2}u_2+\frac{4\times3\times2}{1\times2\times3}u_1+4]$

$\Rightarrow E^4y_0-4E^3y_0+\frac{4\times3}{1\times2}y_0E^2-\frac{4\times3\times2}{1\times2\times3}Ey_0+1\cdot y_0=0$ &

$E^4y_0-4E^3y_1+\frac{4\times3}{1\times2}y_1E^2-\frac{4\times3\times2}{1\times2\times3}Ey_1+1\cdot y_1=0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0 \rightarrow ③$

$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0 \rightarrow ④$

from ③

$[17 - 4y_3 + 6(10) - 4(6) + 7]$

$y_4 - 4(17) + 6y_2 - 4(10) + 6 = 0$

$\Rightarrow y_4 - 4(17) + 6y_2 - 4(10) + 6 = 0$

$\Rightarrow y_4 + 6y_2 - 68 - 40 + 6 = 0$

$\Rightarrow y_4 + 6y_2 - 102 = 0$

$\Rightarrow y_4 + 6y_2 = 102 \rightarrow ⑤$

from ④

$31 - 4y_4 + 6(17) - 4y_2 + 10 = 0$

$31 + 102 + 10 = 4y_4 + 4y_2$

$143 = 4y_4 + 4y_2 \rightarrow ⑥$

$\left.\begin{array}{l} 6y_2 + y_4 - 102 = 0 \\ 4y_2 + 4y_4 - 143 = 0 \end{array}\right\}$ By 2×2

$\therefore y_2 = 13.25 \;,\; y_4 = 22.5$

$y_2 \quad y_4$

$1 \quad -102 \quad 1$

$4 \quad -143 \quad 4 \quad 4$

$\frac{y_2}{265} = \frac{y_4}{450} = \frac{1}{20}$

$y_2 = \frac{265}{20} = 13.25 \;;\; y_4 = \frac{450}{20} = 22.5$

20: find the missing value of the following table

x	1	2	3	4	5
y	7	?	13	21	37
	y_0	y_1	y_2	y_3	y_4

sol:

$\Delta^4 y_0 = 0$ since we know that

$\therefore E = 1 + \Delta$

$\Delta = E - 1$

$(E-1)^4 y_0 = 0$

$[1 \cdot E^4 - 4c_1 E^3 + 4c_2 E^2 - 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + \frac{4\times3}{1\times2} E^2 y_0 - \frac{4\times3\times2}{1\times2\times3} E\, y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$37 - 4(21) + 6(13) - 4x + 4(7) = 0$

$37 - 84 + 68 - 4x + 28 = 0$

$65 + 68 - 84 = 4x$

$3 - 81 = 4x$

$-78 = 4x$

$\left[\begin{array}{l} 7 - 4x = \\ 7 = 4x \\ x = \frac{7}{4} \end{array}\right]$

$37 - 4 \times 21 + 6 \times 13 - 4y_1 + 7 = 0$

$-84 \quad -4y_1 + 37 + 7 + 78 = 0$

$-4y_1 + 38 = 0$

$4y_1 = 38$

$y_1 = 9.5$

21. Estimate the missing term in the following table

x	1	2	3	4	5	6	7
y	2	4	8	—	32	64	128
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

<u>soly</u> Since we know that

$\Delta^6 y_0 = 0 \qquad E = 1+\Delta$

$\qquad\qquad\quad \Delta = E-1$

$(E-1)^6 y_0 = 0$

$[E^6 \cdot 1 \mp 6c_1 E^5 + 6c_2 E^4 \mp 6c_3 E^3 + 6c_4 E^2 \mp 6c_5 E + 6c_6] y_0 = 0$

$E^6 y_0 \mp 6E^5 y_0 + 15E^4 y_0 \mp \frac{6\cdot5\cdot4}{1\cdot2\cdot3} E^3 y_0 + \frac{6\cdot5\cdot4\cdot3}{1\cdot2\cdot3\cdot4} E^2 y_0 \mp \frac{6\cdot5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4\cdot5} E y_0 + y_0$

$y_6 \mp 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$

$128 - 6(64) + 15(32) - 20(y_3) + 15(8) - 6(4) + 2 = 0$

$128 - 384 + 480 - 20y_3 + 120 - 24 + 2 = 0 \qquad 322 = 20y_3$

$730 - 284 - 24 - 20y_3 = 0 \qquad y_3 = \frac{322}{20}$

$730 - 308 = 20y_3$

$422 = 20y_3 \qquad y_3 = 16.1$

$y_3 = 21.1$

384
80
304

480
120
128
12
738
308
422

1.5×4
120

321
15
160
37
41

22. Given $\log^{100} = 2$; $\log^{101} = 2.0043$; $\log^{103} = 2.0128$; $\log^{104} = 2.0170$ and find log 102.

<u>soly</u> Here given

x	100	101	102	103	104
y = log x	2	2.0043	—	2.0128	2.0170
	y_0	y_1	y_2	y_3	y_4

Since we know that

$\Delta^4 y_0 = 0$

$\Delta = E-1$

From $E = 1+\Delta$

$(E-1)^4 y_0 = 0$

$[1.E^4 \mp 4c_1 E^3 + 4c_2^2 E - 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + \frac{4\times3}{1\times2} E^2 y_0 - \frac{4\times3\times2}{1\times2\times3} E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$2.0170 - 4(2.0128) + 6y_2 - 4(2.0043) + 2 = 0$

$2.0170 - 8.0512 + 6y_2 - 8.0172 + 2 = 0$

$4.0170 - 16.0684 + 6y_2 = 0$

$6y_2 = \frac{12.0514}{6}$

$\therefore y_2 = 2.0086$

$\therefore \log_{102} = 2.0086$

2.0170
2
4.0170
2.0340
8.0512
8.0172
0.0340
16.0684

23. find the missing values of the following

x	10	15	20	25	30	35
y	43 y_0	— y_1	29 y_2	32 y_3	— y_4	77 45

Solu) Since we know that

$\Delta^4 y_0 = 0 \quad ; \quad E = 1+\Delta \Rightarrow \Delta = E-1$

$(E-1)^4 y_0 = 0$

$[1.E^4 - 4c_1 E^3 + 4c_2 E^2 + 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$y_4 - 4(32) + 6(29) - 4y_1 + 43 = 0$

$y_4 - 128 + 174 - 4y_1 + 43 = 0$

$y_4 - 4y_1 = 128 - 174 - 43$

$y_4 - 4y_1 = 128 - 217$

$y_4 - 4y_1 = -89 \rightarrow ①$ (or) $4y_1 - y_4 = 89 \rightarrow ①$

174
43
217

217
128
89

$\Delta^4 y_1 = 0$ $= (-y_4 + 4y_1) = 89$ $4y_1 - y_4 = 89$

$(E-1)^4 y_1 = 0$

$[1.E^4 - 4c_1 E^3 + 4c_2 E^2 \mp 4c_3 E + 4c_4] y_1 = 0$

$E^4 y_1 - 4E^3 y_1 + 6E^2 y_1 - 4E y_1 + y_1 = 0$

$y^5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$

$77 - 4y_4 + 6(32) - 4(29) + y_1 = 0$

$y_1 - 4y_4 + 77 + 192 - 116 = 0$

$y_1 - 4y_4 = 116 - 192 - 77$

$y_1 - 4y_4 = 116 - 269$

$y_1 - 4y_4 = -153 \rightarrow ②$

192
77
269
116
153

153
4
612

89
4
356

356
153
509

89
612
1502

$y_1 \quad y_4 \quad 1$

$-1 \quad -89 \quad 4 \quad -1$

$-4 \quad 153 \quad 1 \quad -4$

$$\frac{y_1}{-153-356} = \frac{y_4}{-89-612} = \frac{1}{-16+1}$$

$$\frac{y_1}{-509} = \frac{y_4}{-701} = \frac{1}{-15}$$

$$y_1 = \frac{-509}{-15} \; ; \; y_4 = \frac{-701}{-15}$$

$y_1 = 33.9334 \; ; \; y_4 = 46.7334$

Date 6/8/18

24. Estimate the production for 1964 and 1966 from the following data.

years (x)	1961	1962	1963	1964	1965	1966	1967
production (y)	200	220	260	—	350	—	430
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Sol) Given that years x = 1961, 1962, 1963, 1964, 1965, 1966, 1967

y = 200, 220, 260, —, 350, —, 430

$E = 1 + \Delta$

$\Delta^5 y_0 = 0 \rightarrow ①$

$(E-1)^5 y_0 = 0$

$[1.E^5 - 5c_1E^4 + 5c_2E^3 - 5c_3E^2 + 5c_4E - 5c_5]\, y_0 = 0$

$y_0E^5 - 5y_0E^4 + 5c_2E^3y_0 - 5c_3E^2y_0 + 5c_4Ey_0 - 5c_5\, y_0$

$$y_5 - 5y_4 + \frac{5\times4}{1\times2}y_3 - \frac{5\times4\times3}{1\times2\times3}y_2 + \frac{5\times4\times3\times2}{1\cdot2\cdot3\cdot4}y_1 + y_0 = 0$$

$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 + y_0 = 0$

$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) + 200 = 0$

$y_5 - 1750 + 10y_3 - 2600 + 1100 + 200 = 0$

$y_5 + 10y_3 - 3050 = 0 \rightarrow ③$

3050

$\Delta^5 y_1 = 0 \rightarrow ③$

$y_6 \cdot (E-1)^5 y_1 = 0$

$[1 \cdot E^5 \mp 5c_1 E^4 + 5c_2 E^3 \mp 5c_3 E^2 + 5c_4 E \mp 5c_5] y_1 = 0$

$E^5 y_1 \mp 5c_1 E^4 y_1 + 5c_2 E^3 y_1 \mp 5c_3 E^2 y_1 + 5c_4 E\, y_1 \mp 5c_5 y_1 = 0$

$y_6 - 5y_5 + 10y_4 - 10y_3 + 5y_2 - y_1 = 0$

$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$

$430 - 5y_5 + 3500 - 10y_3 + 1300 - 220 = 0$

$5010 - 5y_5 - 10y_3 = 0$

$5y_5 + 10y_3 = 5010 \rightarrow ④$

From ② and ④

$y_5 + 10y_3 = 3450$

$5y_5 + 10y_3 = 5010$

$-4y_5 = -1560$

$y_5 = \frac{-1560}{-4} = 390$

$y_5 + 10y_3 = 3450$

$390 + 10y_3 = 3450$

$10y_3 = 3450 - 390$

$10y_3 = 3060$

$y_3 = 306$

From ① & ②,

$6 \quad -706 \quad 20 \quad 6$

$15 \quad -1196 \quad 15 \quad 15$

$\frac{y_3}{-7176+10590} = \frac{y_5}{-10590+239} = \frac{1}{300-90}$

$\frac{y_3}{3414} = \frac{y_5}{13330} = \frac{1}{210}$

$y_3 = \frac{3414}{210}$; $y_5 = \frac{1330}{210}$

$y_3 = 16.2571$ $y_5 = 6.33$

63.47

31. Fit a polynomial of degree 3 and hence determine $y(3.5)$ for the following data

x: 3 4 5 6

y: 6 24 60 120

Difference table

x	y	1st	2nd	3rd
3	6			
		18		
4	24		18	
		36		6
5	60		24	
		60		
6	120			

By Newton's forward Interpolation formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$\frac{18}{2}$ 34

$$n = \frac{x - x_0}{h} = \frac{x-3}{1} = x-3$$

$x = x$ $x_0 = 3$ $h = 1$

$$y(3.5) = 6 + (x-3)18 + \frac{(x-3)(x-3-1)}{2!}18 + \frac{(x-3)(x-3-1)(x-3-2)}{3!}6$$

$$= 6 + 18x - 54 + \frac{(x-3)(x-4)}{\cancel{2}}\cancel{18}^{9} + \frac{(x-3)(x-4)(x-5)}{\cancel{6}}\cancel{6}$$

$$= 6 + 18x - 54 + (x^2 - 3x - 4x + 12)9 + [x^2 - 3x - 4x + 12](x-5)$$

$$= 6 + 18x - 54 + 9x^2 - 27x - 36x + 108 + x^3 - 3x^2 - 4x^2 + 12x - 5x^2 + 15x + 20x - 60$$

12 -36 -23 15 12 35 65 54 49 36 $+11$ -54 -9

$$= x^3 - 3x^2 + 2x$$

put x 3.5

$$y(3.5) = (3.5)^3 - 3(3.5)^2 + 2(3.5)$$

$$= 42.875 - 3(12.25) + 7$$

$$= 42.875 - 36.75 + 7$$

42.875 36.75 6.125

$$\therefore y(3.5) = 13.125$$

32. Find the cubic polynomial which takes the following values

$y(0) = 1,\ y(1) = 0,\ y(2) = 1,\ y(3) = 10.$

Hence obtain $y(4)$

$y(0) = 1, \quad y(1) = 0 \quad y(2) = 1 \quad y(3) = 10$

Difference table

x	y	1st	2nd	3rd
0	1			
		-1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

Newtons forward interpolation formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)\Delta^2 y_0}{2!} + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

$x = x \quad x_0 = 0, \quad h = 1$

$$y_n = 1 + x(-1) + \frac{x(x-1)2}{2} + \frac{x(x-1)(x-2)}{6}6$$

$$= 1 - x + x^2 - x + (x^2 - x)(x-2)$$

$$= 1 - x + x^2 - x + x^3 - x^2 - 2x^2 + 2x$$

$$= x^3 - 2x^2 + 1$$

put $x = 4$

$$y(4) = 4^3 - 2(4)^2 + 1$$

$$= 64 - 32 + 1$$

$y(4) = 33$ $[0, 3]$ interval '4' is out of interval so it is called extrapolation

33 find the polynomial interpolating the data

$x : 0 \quad 1 \quad 2$

$y : 0 \quad 5 \quad 2$ Difference Table

x	y	1st	2nd
0	0		
		5	
1	5		-8
		-3	
2	2		

Newton's forward Interpolation formula

$$y_n = y_n + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

$x = x \quad x_0 = 0 \quad h = 1$

$$\left(y_n = x + x(0) + \frac{x(x-1)}{2!}5 + \frac{x(x-1)(x-2)}{\cancel{6}\,3}\left(\overset{4}{-\cancel{8}}\right)\right.$$

$$= x + 0 + \frac{(x^2-x)5}{2} + \frac{(x^2-x)(x-2)}{3}(4)$$

$$= x + \frac{5x^2-5x}{2} + \frac{(x^3-x^2-2x^2+2x)}{3}(-4)$$

$$= x + 5x^2 - 5x\,]$$

$$y_n = 0 + x\,5 + \frac{x(x-1)}{\cancel{2}}\,x(-\cancel{8})4$$

$$= 5x - (x^2-x)4$$

$$= 5x - 4x^2 + 4x$$

$$\therefore y_n = -4x^2 + 9x$$

34 Find the polynomial of deg(4) which takes the following Values

x :	2	4	6	8	10
y :	0	0	$\underline{9}$	0	0

35 Use Newton's forward Difference formula to obtain the interpolating polynomial $f(x)$ satisfying the following data

x :	1	2	3	4
y :	26	18	4	1

and find and $x = 5$

$\frac{18}{2}$ $\begin{array}{r}26\\-18\\\hline 8\end{array}$

soln Form the Difference table

39

x	y	1st	2nd	3rd
1	26			
		−8		
2	18		−6	
		−14		16
3	4		11	
		−3		
4	1			

12 mistake [illegible]

From Newton's Interpolation forward formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x-x_0}{h} = \frac{x-1}{1} = x-1$$

$$x = x;\ x_0 = 1;\ h = 1$$

$$y_n = 26 + (x-1)(-8) + \frac{(x-1)(x-1-1)}{2}\overset{3}{(-6)} + \frac{(x-1)(x-1-1)(x-2-1)}{\cancel{6}\,3}$$

$$= 26 - 8x + 8 + (3x-3)(x-1-1) + \frac{(x-1)(x-2)(x-3)8}{3}$$

$$= 26 - 8x + 8 - (3x-3)(x-2) + \frac{[x^2 - x - 2x + 2](x-3)}{3} \times 8$$

$$= 26 - 8x + 8 - [3x^2 - 3x - 6x + 6] + [x^3 - 3x^2 - x^2 + 2x - 3x^2 + 3x + 6x - 6] \times \frac{8}{3}$$

$$= 26 - 8x + 8 - 3x^2 + 9x - 6 + [x^3 - 6x^2 + 11x - 6] \times \frac{8}{3}$$

$$= 26 - 8x + 8 - 3x^2 + 9x - 6 + [x^3 - 6x^2 + 11x - 6] \times \frac{8}{3}$$

$$= 78 - 24x + 24 - 9x^2 + 27x - 18 + (x^3 - 6x^2 + 11x - 6)8$$

$$= 84 - 24x - 9x^2 + 27x + 8x^3 - 48x^2 + 88x - 48$$

$$y_n = 8x^3 - 57x^2 + 91x + 36$$

84
48
36

put $x = 5$

$$y(5) = 8(5)^3 - 57(5)^2 + 91(5) + 36$$

$$= 8(125) - 57(25) + 455 + 36$$

$$= 1000 - 1425 + 455 + 36$$

$$\therefore y(5) = 66$$

3) Forming the difference table

x	y	1st	2nd	3rd	4th
2	0				
		0			
4	0		1		
		1		-3	
6	1		-2		+6
		-1		3	
8	0		1		
		0			
10	0				

From Newtons forward interpolation formula

$$y_n = y_n + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta$$

$$n = \frac{x-x_0}{h} = \frac{x-2}{2}$$

$$y_n = 0 + \frac{x-2}{2}0 + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)}{2}1 + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)\left(\frac{x-2}{2}-2\right)}{6}(-3$$

$$+ \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)\left(\frac{x-2}{2}-2\right)\left(\frac{x-2}{2}-3\right)}{24}6$$

$$y_n = \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)}{2} \mp \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)}{2} + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)}{4}$$

$$y_n = \frac{(x-2)(x-4)}{8} - \frac{(x-2)(x-4)(x-6)}{16} + \frac{(x-2)(x-4)(x-6)(x-}{64}$$

$$y_n = \frac{x^2-2x-4x+8}{8} - \frac{[x^2-2x-4x+8][x-6]}{16} + \frac{[x^2-2x-4x+8][x^2-6x-8x+48]}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{[x^3-2x^2-4x^2+8x-6x^2+12x+24x-48]}{16}$$

$$+ \frac{x^4-6x^3-8x^3+48x^2-2x^3+12x+16x^2-96x-4x^3+24x^2+32x^2+192x+8x^2-48x-64x+404}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{[x^3-12x^2+44x-48]}{16} + \frac{x^4-20x^3+44x+404}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{x^3+12x^2-44x+48}{16} + \frac{x^4-20x^3+44x+404}{64}$$

$$y_n = \frac{8x^2-48x+64-4x^3+48x^2-176x+192+x^4-20x^3+44x+404}{64}$$

$$y_n = x^4-16x^3+48x^2-180x+660$$

192
36
156
176
180

Date: 10/1/18 i) find the no. of students from the following data who secured marks not more than 45

36/f

Marks	30-40	40-50	50-60	60-70	70-80
no of students	35 +	48 +	70 +	40 +	22

Difference table

Marks (x)(below)	No of Students (y)	1st	2nd	3rd	4th
40	35	48	22	-52	64
50	83	70	-30	12	
60	153	40	-18		
70	193	22			
80	215				

from Newton's forward interpolation formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$$y_n \quad n = \frac{x - x_0}{h} \quad ; x = 45, \; x_0 = 40, \; h = 10$$

$$n = \frac{45-40}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$y(45) = 35 + (0.5)(48) + \frac{(0.5)(0.5-1)}{2!} \overset{11}{22} + \frac{(0.5)(0.5-1)(0.5-2)}{6_3} \times (-52)$$

$$+ \frac{(0.5)\overset{(0.5-1)}{(0.5-2)}(0.5-3)}{24_{12}} \times \overset{2}{\cancel{64}}$$

$$y(45) = 35 + 24 - 2.75 + \frac{(0.5)(-0.5)(-1.5)}{3}(26)$$

$$+ \frac{(0.5)(0.5)(-1.5)(-2.5)}{3} \times 8$$

$$y(45) = 35 + 24 - 2.75 - 3.25 - 2.5$$

$\therefore y(45) = 50.5$ ∴ below 45 marks = 50.5

∴ No. of students who secured below 45 marks = 51 (approximate)

ii) No.of students in between 40and 45 =
No.of students secured 45 marks - No.of students secured 40 marks

$= 51 - 35$

$= 16$

above 45

$= 215 - 51$

$= 164$

3) find the no.of men getting the wages between Rs.10 and Rs.15 from the following table

wages	0-10	10-20	20-30	30-40
Frequency	9 +	30 +	35 +	42

(a) Difference Table

x (below)	y	1st	2nd	3rd
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

From Newtons Forword interpolation formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

$$n = \frac{x - x_0}{h} \quad x = 15 \ ; \ x_0 = 10 \ , \ h = 10$$

$$n = \frac{15-10}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$y(15) = 9 + 30(0.5) + \frac{(0.5)(0.5-1)}{2} 5 + \frac{(0.5)(0.5-1)(0.5-2)}{\cancel{6}3} \cancel{2}$$

$$= 9 + 15.0 + \frac{(0.5)(-0.5)}{2}.5 + \frac{(0.5)(-0.5)(-1.5)}{3}$$

$$= 9 + 15 - 0.625 + 0.125$$

$y(15) = 23.5$

∴ No.of men got the wages below Rs.15 = 23.5
= 24 (approximately)

the wages in between Rs.10 and Rs.15
No.of men who got below Rs.15 - below Rs.10
$= 24 - 9 = 15$

$= 9+42+210+105 \quad = 366$

40 Using Newtons Backward interpolation formula, find $e^{-1.9}$ from the following table

x :	1	1.25	1.5	1.75	2
ye^{-x} :	0.3679	0.2865	0.2231	0.1738	0.1353

Soln Difference table

x	ye^{-x}	1st	2nd	3rd	4th
1	0.3679				
		~~-0.0321~~ -0.0814			
1.25	0.2865		0.018 ~~0.5647~~		
		-0.0634		-0.0039	
1.5	0.2231		~~0.0161~~ 0.1237		0.0006
		-0.0493		-0.0033	
1.75	0.1738		0.0108		
		-0.0385			
2	0.1353				

From Newton's Backward Interpolation formula

$$y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}$$

$[y_n = 0.1353 +]$ $\quad n = \frac{x - x_0}{h} \quad h = 0.25;\ x = 1.9;\ x_0 = 2$

$$n = \frac{1.9-2}{0.25} = \frac{-0.1}{0.25} = -0.4$$

$$y_{1.9} = 0.1353 + (-0.4)(-0.0385) + \frac{(-0.4)(-0.4+1)(0.0108)}{1.2}$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)}{1.2.3}(-0.0033) + \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{1.2.3.4} \times 0.0006$$

$[y_n = 0.1353 + 0.0154 + 6.264 \times 10^{-3} - 8.448 \times 10^{-4}$

$= 3.6756]$

$y_{2.9} = 0.1353 + 0.0154 - \underset{1296}{0.00734} + 0.0002112 + 0.00002496$

$y_{2.9} = 0.13797614 = 0.138$

41. Find the cos(25°) and cos(75°) from the following data.

h=10								
x	10	20	30	40	50	60	70	80
y = cos x	0.9848	0.9397	0.866	0.766	0.6428	0.5	0.3420	0.1727

42. Using Newton's formulae find the value of y and x = 36 from the following data:

x :	21	25	29	33	37
y :	18.4	17.8	17.1	16.3	15.5

Soln	x	y	1st	2nd	3rd	4th	5th
41	10	0.9848					
			-0.0451				
	20	0.9397		-0.0286			
			-0.0737		0.0023		
	30	0.866		-0.0263		0.0008	
			-0.1		0.0031		-0.003
	40	0.766		-0.0232		0.0005	
			-0.1232		0.0036		0.0003
	50	0.6428		-0.0196		0.0008	
			-0.1428		0.0044		-0.0013
	60	0.5		-0.0152		-0.0005	
			-0.158		0.0039		
	70	0.3420		-0.0173			
			-0.1093				
	80	0.1727					

7th
-0.0016

Newton's Forward Interpolation formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\Delta^5 y_0 + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)\Delta^6 y_0}{6!}$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)\Delta^7 y_0}{7!}$$

$n = \frac{x-x_0}{h}$; $x = 25$; $x_0 = 10$; $h = 10$. $n = \frac{25-10}{10} = \frac{15^3}{10_2} = 1.5$

$$y_n = 0.9848 + 1.5(-0.0451) + \frac{1.5(1.5-1)}{2}(-0.0286) + \frac{(1.5)(1.5-1)(1.5-2)}{6}\,0.002$$

$$+ \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)}{24}\,0.0008 + \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5\,(-0.0003)}{120}$$

$$+ \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)}{720}(0.0006)$$

$$+ (1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(1.5-6)/5040\,(-0.0016)$$

$$y_n = 0.9848 - 0.06765 - \frac{0.02145}{2} - 0.0625 \times 0.0023$$
$$+ 0.0234375 \times 0.0008 + 7.8125 \times 10^{-4} \times 0.0003 +$$
$$+ 6.510416667 \times 10^{-5} \times 0.0006 + 4.650297619 \times 10^{-6} \times 0.0016$$

$$y_n = 0.9848 - 0.06765 - 0.010725 - 0.00014375 + 0.00001875$$
$$+ 0.000000234375 + 0.0000000390625 + 0.00000000744047619$$

$y(25) = 0.9063002809$

Newtons Backward Interpolation formulae

$$y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$
$$+ \frac{n(n+1)(n+2)(n+3)(n+4)}{5!}\nabla^5 y_n + \frac{n(n+1)(n+2)(n+3)(n+4)(n+5)}{6!}\nabla^4 y_n$$
$$+ \frac{n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}{7!}\nabla^7 y_n$$

$n = \frac{x - x_0}{h}$; $x = 75$, $x_0 = 80$; $h = 10$ $n = \frac{75-80}{10} = \frac{-5}{10} = -0.5$

$$\boxed{y_n} = ~~-0.5~~$$
$$0.9848 + (-0.5)(-0.0451) + \frac{(-0.5)(-0.5+1)}{2!}(-0.0286)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{6} \times 0.0023 + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{24} 0.009$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{120}(-0.0003)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{720} \times 0.0001$$

wrong within the brackets

$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)(-0.5+6)}{5040}(-0.0014)$$

$$y_n = 0.9848 + 0.02255 + \frac{0.00715}{2} + \frac{0.0008625}{6} - \frac{0.00075}{24} + \frac{0.00098437}{120}$$
$$- \frac{0.008859375}{720} + \frac{0.1299375}{5040}$$

$$\cos(75) = 0.9848 + 0.02255 + 0.003575 + 0.00014375 - 0.00003125$$
$$+ 0.000008203125 - 0.0000123046875 + 0.000025178125$$

$\cos(75) = 1.01105918$]]

$$= 0.1727 + (-0.5)(-0.1693) + \frac{(-0.5)(-0.5+1)}{2}(-0.0113)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(0.0039) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(0.0005)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{5!}(-0.0018)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{6!}(-0.001)$$

Date 13/7/18

Legranges Interpolation Formula

Consider $y = f(x)$ be the given function. x takes the values $x_0, x_1, x_2, x_3, x_4, \ldots$ the corresponding y values are $y_0, y_1, y_2, y_3, y_4, \ldots$ respectively. then

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

Date 14/7/18

1 using Legranges formula to find $f(6)$ from the following table.

	x_0	x_1	x_2	x_3	x_4
x	2	5	7	10	12
$f(x)$	18	180	448	1210	2028
	y_0	y_1	y_2	y_3	y_4

By Legranges interpolation formula

soln

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$y(6) = \frac{(6-5)(6-7)(6-10)(6-12)}{(2-5)(2-7)(2-10)(2-12)} \cdot 18 + \frac{(6-2)(6-7)(6-10)(6-12)}{(5-2)(5-7)(5-10)(5-12)} \times$$

$$+ \frac{(6-2)(6-5)(6-10)(6-12)}{(7-2)(7-5)(7-10)(7-12)} \times 448 + \frac{(6-2)(6-5)(6-7)(6-12)}{(10-2)(10-5)(10-7)(10-12)} \times$$

$$+ \frac{(6-2)(6-5)(6-7)(6-10)}{(12-2)(12-5)(12-7)(12-10)} \times 2028$$

$$y(6) = \frac{1(-1)(-4)(-6)}{(-3)(-5)(-8)(-10)} \times 18 + \frac{4(-1)(-4)(-6)}{3(-2)(-5)(-7)} \times 180$$

$$+ \frac{4(1)(-4)(-6)}{5(2)(-3)(-5)} \times 448 + \frac{(4)(1)(-1)(-6)}{8 \cdot 5 \cdot 3 \cdot 2} \times 1210 + \frac{4 \cdot 1 \cdot (-1)(4}{10 \cdot 7 \cdot 5 \cdot}$$

$$y(6) = \frac{-\cancel{2}}{1980}\times 187 - \frac{72}{-770}180 + \frac{72}{150}\times 448 - \frac{24}{800}\times 1620 + \frac{16}{200}$$

$$y(6) = \frac{-9}{25} + \frac{6480}{15} + \frac{72}{150}\times 448 - 121 + \frac{16\times 2028}{700}$$

$$y(6) = -0.36 + 432 + 215 - 121 + 46.354$$

$$y(6) = 572.7147$$

$$y(6) = \frac{-9}{25} + \frac{576}{7} + \frac{7168}{25} - 121 + \frac{8112}{175}$$

$$y(6) = -121 + \frac{-9+7168}{25} + \frac{576}{7} + \frac{8112}{175}$$

$$y(6) = -121 + \frac{7159}{25} + \frac{576}{7} + \frac{8112}{175}$$

$$y(6) = -121 + 286.36 + 82.2857 + 46.3542$$

$$\therefore y(6) = 293.9999 \approx 294$$

2. Using the Lagranges interpolation formulae find the value of y(10) from the following table

x: 5 6 9 11

y: 12 13 14 16

Sol: By Lagranges interpolation formulae

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

100

$$y(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}13 + \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}.16$$

1210

$$y(10) = \frac{4\cdot 1\cdot(-1)}{(-1)(-4)(-6)}\cdot 12 + \frac{5\cdot 1\cdot(-1)}{1(-3)(-5)}13 + \frac{5(4)(-1)}{4(3)(-2)}14 + \frac{5(4)(1)}{6\cdot 5\cdot 2}16$$

$$y(10) = \frac{+2}{1} - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = \frac{6-13+35+16}{3}$$

$y(10) = \frac{44}{3}$

$\therefore y(10) = 14.667$

3. Find the cubic Lagranges Interpolating polynomial from the following data.

x :	$\overset{x_0}{0}$	$\overset{x_1}{1}$	$\overset{x_2}{2}$	$\overset{x_3}{5}$
$f(x)$:	$\underset{y_0}{2}$	$\underset{y_1}{3}$	$\underset{y_2}{12}$	$\underset{y_3}{147}$

sol) The Lagranges Interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} 2 + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} 3$$

$$+ \frac{(x-0)(x-2)(x-5)}{(2-0)(2-1)(2-5)} 12 + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} 147$$

$$= \frac{(x-1)(x-2)(x-5)}{(-1)(-2)(-5)} 2 + \frac{x(x-2)(x-5)}{1(-1)(-4)} 3$$

$$+ \frac{x(x-1)(x-5)}{2 \cdot 1 \cdot (-3)} 12 + \frac{x(x-1)(x-2)}{5(4)(3)} 147$$

$$= -\frac{(x-1)(x-2)(x-5)}{5} + \frac{x(x-2)(x-5)}{4} \times 3$$

$$- \frac{x(x-1)(x-5)}{} \times 2 + \frac{x(x-1)(x-2)}{60} \times 147$$

$$= -\frac{(x^2-x-2x+2)(x-5)}{5} + \frac{(x^2-2x)(x-5)}{4} \times 3$$

$$-(x^2-x)(x-5)2 + \frac{(x^2-x)(x-2)}{60} \times 147$$

$$= -[x^3-x^2-2x^2+2x-5x^2+5x+10x-10]/5$$

$$+ \frac{[x^3-2x^2-5x^2+10x]3}{4} - [x^3-x^2-5x^2+5x]2$$

$$+ \frac{[x^3-x^2-2x^2+2x]}{60} \times 147$$

$= -[x^3 - 7x^2 + 10x]$

$= -\frac{[x^3 - 8x^2 + 17x - 10]}{5} + \frac{[x^3 - 7x^2 + 10x]3}{4} - [x^3 - 6x^2 + 5x]2$

$+ \frac{[x^3 - 3x^2 + 2x] + 49}{\cancel{40}\ 20}$

$= \frac{-x^3 + 8x^2 - 17x + 10}{5} + \frac{3x^3 - 21x^2 + 30x}{4} - 2x^3 + 12x^2 - 10x$

$+ x^3 - 3x^2 + 2x + \frac{49x^3 - 147x^2 + 98x}{20}$

$= \frac{-4x^3 + 32x^2 - 68x + 40 + 15x^3 - 105x^2 + 150x - 40x^3 + 240x^2 - 200x + 49x^3 - 147x^2 + 98x}{20}$

$= \frac{20x^3 + 20x^2 - 20x + 40}{20}$

$= \frac{20(x^3 + x^2 - x + 20)}{20}$

$\therefore f(x) = x^3 + x^2 - x + 20$

4. find the Legranges interpolating polynomial for the given data :

$x : 1^{x_0} \quad 2^{x_1} \quad 3^{x_2} \quad 4^{x_3}$

$f(x) : 1_{y_0} \quad 8_{y_1} \quad 27_{y_2} \quad 64_{y_3}$

Solu) The Legranges interpolation formulae

$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$

$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$

$f(x) = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)} 1 + \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)} 8$

$+ \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)} 27 + \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)} 64$

$f(x) = \frac{(x-2)(x-3)(x-4)}{(-1)(-2)(-3)} + \frac{(x-1)(x-3)(x-4)}{1(-1)(-2)} \cancel{8}\ 4$

$+ \frac{(x-1)(x-2)(x-4)}{2(1)(-1)} 27 + \frac{(x-1)(x-2)(x-3)}{3(\cancel{2})(1)} \overset{32}{\cancel{64}}$

$= \frac{(x^2 - 2x - 3x + 6)(x-4)}{-6} + \frac{(x^2 - x - 3x + 3)(x-4)}{\cancel{2}} 4$

$+ \frac{(x^2 - x - 2x + 2)(x-4)}{-2} 27 + \frac{[x^2 - x - 2x + 2](x-3)}{3} 32$

$$= \frac{-(x^2-5x+6)(x-4)}{6} + (x^2-4x+3)(x-3)\,24$$

$$- \frac{(x^2-3x+2)(x-4)}{2}\,27 + \frac{[x^2-3x+2][x-3]}{3}\,32$$

$$[= -\frac{[x^3-5x^2+6x-4x^2+20x-24]}{6} + [x^3-4x^2+3x-3x^2+12x-9]\,4$$

$$- \frac{[x^3-3x^2+2x-4x^2+12x-8]}{2}\,27 + \frac{[x^3-3x^2+2x-3x^2+9x-}{3}$$

$$= \frac{-x^3+5x^2-6x+4x^2-20x+24}{6} + 4x^3-16x^2+12x-12x^2+48x-36$$

$$- \frac{27x^3+81x^2\pm54x+108x^2-324x+216}{2} + \frac{32x^3-96x^2+64x-96x^2+288x-192}{3}$$

wrong

$$= \frac{-x^3+5x^2-6x+4x^2-20x+24+24x^3-96x^2+72x-72x^2+288x+216-81x^3+243x^2-162x+324x^2-972x+648+64x^3-192x^2+128x-192x^2+576x-384}{6}$$

$$= 6x^3]$$

$$= \frac{-[x^3-7x^2+12x-2x^2+14x-24]+24[x^3-7x^2+12x-x^2+7x-12]-27\times3(x^3-6x^2+8x-x^2+6x-8]+64[x^3-5x^2+6x-x^2+5x-6]}{}$$

$$= \frac{-[x^3-9x^2+26x-24]+24[x^3-8x^2+19x-12]-27[x^3-7x^2+14x-8]+64[x^3-6x^2+11x-6]}{6}$$

$$= \frac{+[x^3+9x^2-26x+24]+24x^3-192x^2+456x-288-81x^3+567x^2-1134x+648+64x^3-384x^2+704x-384}{6}$$

$$= \frac{1}{6}[6x^3+0+0+0] = x^3$$

5. Using Lagranges Interpolation formula to fit a polynomial to the following data

	x_0	x_1	x_2	x_3
x :	-1	0	2	3
	u_0	u_1	u_2	u_3
u_x :	-8	3	1	12

And also find the value u_1

Sol: By Lagranges interpolation formulae

$$u_x = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} u_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} u_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} u_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} u_3$$

$$u_x = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-3) + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)} 3$$

$$+ \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)} 1 + \frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)} 12$$

$$\Big[u_x = \frac{(x-0)(x-2)(x-3)}{-1\times-3\times-4}(-3) + \frac{(x+1)(x-2)(x-3)}{-2\times-3\times1}\;12\;3$$

$$+ \frac{(x+1)x(x-3)}{3\cdot2(-1)} + \frac{(x+1)(x)(x-2)}{4\cdot3\cdot1} 12^{3}$$

$$u_x = \frac{2x[x^2-2x-3x+6]}{3} + \frac{x[x^2+x-2x-2]}{4}$$

wrong

$$+ \frac{[x^2+x-3x-3]x}{-6} + [x^2+x-2x-2]x$$

$$u_x = \frac{2x^3-4x^2-6x^2+12x}{3} + \frac{x^3+x^2-2x^2-2x}{4}$$

$$- \frac{x^3+x^2-3x^2-3x}{6} + x^3+x^2-2x^2-2x$$

$$u_x = \frac{2x^3-10x^2+12x+x^3-5x^2}{3}\Big]$$

$$u_x = \frac{x(x^2-5x+6)}{-1\times3\times-4}\times-3 + \frac{(x+1)(x^2-5x+6)}{1\times-2\times-3}\times3$$

$$+ \frac{(x+1)(x^2-3x)}{3\times2\times-1} + \frac{(x+1)(x^2-2x)}{4\times3\times1}\times12$$

$$u_x = \frac{2x^3-10x^2+12x}{3} + \frac{x^3-5x^2+6x+x^2-5x+6}{2} - \frac{(x^3-3x^2+x^2-3x)}{6}$$

$$+ (x^3-2x^2+x^2-2x)$$

$$u_x = \frac{2(2x^3-10x^2+12x)+3(x^3-4x^2+x+6)-(x^3-3x^2+x^2-3x)+6(x^3-x^2-2x)}{6}$$

$$u_x = \frac{1}{6}\{4x^3-20x^2+24x+3x^3-12x^2+3x+18-x^3+2x^2+3x+6x^3-6x^2-12x\}$$

$$= \frac{12x^3-36x^2+18x+18}{6} = 2x^3-6x^2+3x+3 \quad \text{If } x=1$$

$$u_1 = 2(1)^3-6(1)^2+3(1)+3 = 2 \,//$$

Date
17/7/18

Central Differences

Gauss - Forward Interpolating Formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}$$

$$\frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \cdots$$

Gauss - Backward Interpolating Formulae

$$y_n = y_0 + n\Delta y_{-1} + \frac{(n+1)n}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2} + \frac{(n+2)(n+1)n(n-1)}{4!}$$

$$\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-3} + \cdots$$

1 find F(2.5) using the following table

x : 1 2 3 4

f(x) : 1 8 27 64 Difference table

solu

x	f	1st	2nd	3rd
1 x_{-1}	1 y_{-1}			
		7 y_{-1}		
2 x_0	8 y_0		12 y_{-1}	
		19 y_0		6 y_{-1}
3 x_1	27 y_1		18 y_0	
		37 y_1		
4 x_2	64 y_2			

Gauss forward interpolating formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1}$$

$n = \frac{x - x_0}{h}$; $x_0 = \boxed{2.0}$ $x = 2.5$; $h = 1$

$n = \frac{2.5 - 2}{1} = 0.5$

$$y_n = 8 + (0.5)(19) + \frac{(0.5)(0.5-1)}{2!}12 + \frac{(0.5+1)(0.5)(0.5-1)}{3!}6$$

$$= 8 + 9.5 + \frac{(0.5)(-0.5)}{\cancel{2}}\cancel{12}^{6} + \frac{(1.5)(0.5)(-0.5)}{\cancel{6}\,3} \times \cancel{6}^{2}$$

$$= 8 + 9.5 - \underset{1.5}{\cancel{7.5}} - 0.375$$

$y_{(2.5)} = 15.625$

2. from the following table find y when $x = 38$

x : 30 35 40 45 50

y : 15.9 14.9 14.1 13.3 12.5

Difference table

x y 1st 2nd 3rd 4th

30 x_{-1} 15.9 y_{-1}

-1 y_{-1}

35 x_0 14.9 y_0 0.2 y_{-1} -0.2 y_{-1} 0.2 y_{-1}

-0.8 y_0

40 14.1 y_1 0 y_0

-0.8 y_1 0 y_0

45 13.3 y_2 0 y_1

-0.8 y_2

50 12.5 y_3

By applying Gauss forward interpolating formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$n = \frac{x - x_0}{h}$; $x = 38$ $x_0 = 35$; $h = 5$

$n = \frac{38-35}{5} = \frac{3}{5} = 0.6$

$$y_n = 14.9 + (0.6)(-0.8) + \frac{(0.6)(0.6-1)}{2!}(0.2) + \frac{(0.6+1)(0.6)(0.6-1)}{3!}(-0.2)$$

$$\left[+ \frac{(0.6+1)(0.6)(0.6-1)(0.6-2)}{4!}(0.2)\right]$$

$y_{(38)} = 14.9 - 0.48 - 0.024 + 0.0128 + (0.001$

$y_{(38)} = 14.4038$

3 using Gauss forward interpolating formulae we find $f(3.3)$ from the following data

x 1 2 3 4 5

f(x) 15.3 15.1 15. 14.5 14

Difference table

x y 1st 2nd 3rd 4th

1 x_{-2} 15.3 y_{-2} -0.2 y_{-2} -0.3 y_{-2} -0.5 y_{-2} -2.5

2 x_{-1} 15.1 y_{-1} 0.9 y_{-2}

-0.1 y_{-1}

3 x_0 15 y_0 -0.4 y_{-1} +0.4 y_{-1}

+0.5 y_0

4 14.5 -0.0 y_0

-0.5

5 14

By applying Gauss forward interpolating formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$n = \frac{x-x_0}{h}$ $x = 3.3$; $x_0 = 3$; $h = 1$; $n = \frac{3.3-3}{1} = 0.3$

$$y(3.3) = 15 + 0.3(-0.5) + \frac{(0.3)(0.3-1)}{2!}(-0.4) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(0.4)$$

$$+ \frac{(0.3+1)(0.3)(0.3-1)(0.3-2)}{4!} \times 0.9$$

$y(3.3) = 15 - 0.15 + 0.042 - 0.0286 + 0.02016) - 0.0182 + 0.0580125$

$y(3.3) \equiv 14.88456$ $14.9318125 \equiv$

$= 15 - 0.15 + 0.042 - 0.182 + 0.01740375 = 14.89120375$

4. Find the polynomial which quit the data in the following table using ~~Gauss~~ Newtons forward formula

x	3	5	7	9	11
y	6	24	58	108	174

solu) Difference table

x	y	1st	2nd	3rd	4th
3	6				
		18	16		
5	24			0	
		34			0
7	58		16		
		50		0	
9	108		16		
		66			
11	174				

By applying Newtons forward formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 +$$

$n = \frac{x-x_0}{h}$ $x_0 = 3$; $x_0 = x$ $h = 2$ $n = \frac{3x}{2}$

$n = \frac{x-3}{2}$

$$y_n = 6 + \left(\frac{x-3}{2}\right)18 + \frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{2} 16$$

$$= 6 + 9x - 27 + \left(\frac{x-3}{2}\right)\left(\frac{x-5}{2}\right) \times 16$$

$$= 6 + 9x - 27 + [x-3][x-5]2$$

$$= 6 + 9x - 27 + [x^2 - 3x - 5x + 15]2$$

$$= -21 + 9x + 2x^2 - 6x - 10x + 30$$

$y_n = 2x^2 - 7x + 9$

[illegible] By using Gauss Backward interpolating formulae find the value of y and x = 3.3 from the following data

x	1	2	3	4	5
y	15.3	15.1	15	14.5	14

Ans) Difference table

x	y	1st	2nd	3rd	4th
1 x_{-2}	15.3 y_{-2}	-0.2 y_{-2}	-0.2 y_{-2}	-0.5 y_{-2}	0.9 y_{-2}
2 x_{-1}	15.1 y_{-1}	-0.1 y_{-1}	-0.4 y_{-1}	+0.4 y_{-1}	
3 x_0	15 y_0	0.5 y_0	0 y_0		
4	14.5	-0.5			
5	14				

By using newton's Gauss Backward formula

$$y_n = y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+2)(n+1)n(n-1)}{4!}\Delta^4 y$$

$h = 1$; $n = \frac{3.3-3}{1} = 0.3$

$n = \frac{x - x_0}{h}$; $x = 3.3$; $x_0 = 3$;

$$y(3.3) = 15 + (0.3)([illegible]) + \frac{(0.3+1)(0.3)}{2!}(-0.4) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(-0.$$

$$+ \frac{(0.3+2)(0.3+1)(0.3)(0.3-1)}{4!}(40.9)$$

$[y_n = 15 + 4.53 - 0.0195 + 0.0183]$

$y(3.3) = 15 - 0.03 - 0.078 + 0.02275 - 0.02354625$

$y_{3.3} = 14.89120375$

$y(3.3) = 14.8912$

6 From the following table find the value of y when x = 1.35

x :	1	1.2	1.4	1.6	1.8	2
y :	0.0	-0.112	-0.016	0.336	0.992	2

why we used Gauss backward

x	y	1st	2nd	3rd	4th	
1	0.0					
		-0.112 Δy_{-1}				
1.2 x_0	-0.112 y_0		0.208 y_{-1}			
		0.096		0.048 y_{-1}		
1.4	-0.016		0.256		0	
		0.352		0.048		0
1.6	0.336		0.304		0	
		0.656		0.048		
1.8	0.992		0.352			
		-0.992				
2	2					

By applying Gauss backward interpolating formula

$$y_n = y_0 + n\Delta y_{-1} + \frac{(n+1)n}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2}$$

$n = \frac{x-x_0}{h}$ $x = 1.35$ $x_0 = 1.2$; $h = 0.2$; $n = \frac{1.35-1.2}{0.2} = 0$

$$y(1.35) = (-0.112) + (0.75)(-0.112) + \frac{(0.75)(0.75+1)}{2!}(0.208)$$

$$= -0.112 - 0.084 + 0.1365$$

$$\therefore y(1.35) = -0.0595$$