UNIT– V

NP HARD & NP– COMPLETE :-

We can categorize the problems as

1. P– class:-

→ The class P consist of those problems that are solvable in polynomial time. $O(n^k)$ → worstcase.

→ These problems are called tractable.

→ Formally an algorithm is polynomial time algorithm, if there exist a polynomial $p(n)$ such that the algorithm can solve any instance of size n in a time $O(p(n))$.

2. NP– class:-

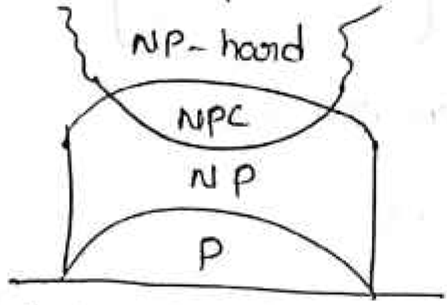
→ The class NP consist of those problems that are verifiable in polynomial time.

→ NP is the class of decision problems for which it is easy to check the correctness of claimed answer, with the aid of little extra information.

→ Hence we are not asking for a way to find a solution but only to verify that an alleged solution is really correct.

Definition of NP-class Problem:- The set of all decision-based problems came into the division of NP Problems, who can't be solved an output within polynomial time but verified in the polynomial time. NP class contains P class as a subset.

Definition of P-class Problem:- The set of decision-based problems come into the division of P Problems who can be solved (or) produced output with in polynomial time.



* string matching:-

→ A string matching automation is a very useful tool which is used in string matching algorithm.

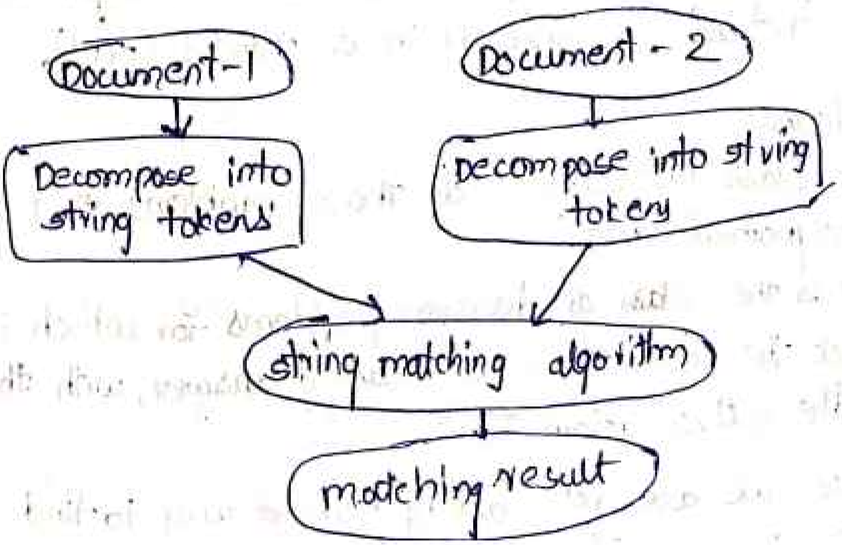
→ It examines every character in the text exactly once & reports all the valid shifts in O(n) time.

→ The good string matching helps in performing time-efficient tasks in multiple domains

Applications of string matching algorithm:-

→ Plagiarism Detection:-

→ The documents to be compared are decomposed into string tokens & compared using string matching algorithm. It is used to find similarities b/w them.



→ Bioinformatics & DNA sequencing:-

Bioinformatics involves applying information technology & computer science to problems involving genetic sequences to find DNA patterns. String matching algorithms and DNA analysis are both collectively used for finding the occurance of the pattern set

→ Digital Forensics:-

STA are used to locate specific text strings of interest in the digital forensic text.

→ Spelling checker:-

Trie is built based on a predefined set of pattern. Then this trie is used for string matching

→ Spam filters:-

Spam filters use string matching to discard the spam.

Naive-String-Matching:-

The naive approach tests all the possible placement of pattern P[1.. m] relative to text T[1.. n], we try shifts s = 0, 1, ... n−m, successively & for each shift s. Compare T[s+1.... s+m] to P[1...m].

Algorithm.

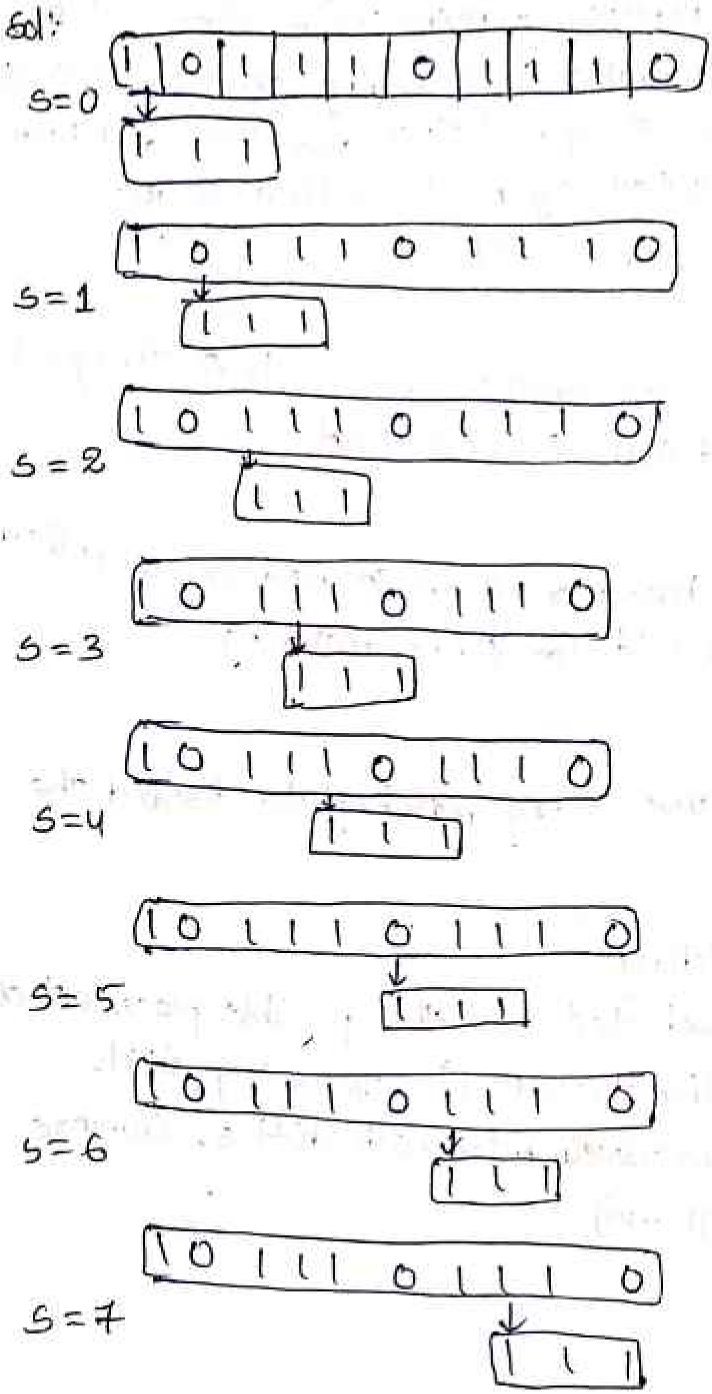
1. n ← length[T]
2. m ← length[P]
3. for s ← 0 to n − m
4. do if P[1... m] = T [s+1 .... s+m]
5. then print "Pattern occurs with shift" s.

Analysis:-

∴ Total complexity is $O(n-m+1)$.

Example: Text - 1011101110   P - 111

Sol:



∴ s2 & s6 are valid shift.

* The Rabin-Karp Algorithm:-

→ The rabin karp string matching algorithm calculates a hash value for the pattern, as well as for each m-character subsequences of text to be compared.

→ If the hash values are unequal the algorithm will determine the hash value for next m-character sequence.

→ If the hash values are equal the algorithm will analyze the pattern & the m-character sequence

→ In this way, there is only one comparison per text subsequence & character matching is only required when the hash values match.

Algorithm:

$n \leftarrow length[T]$
$m \leftarrow length[P]$
$h \leftarrow d^{m-1} \bmod q$
$p \leftarrow 0$
$t_0 \leftarrow 0$
for $i \leftarrow 1$ to m
do $p \leftarrow (dp + P[i]) \bmod q$
$t_0 \leftarrow (dt_0 + T[i]) \bmod q$
for $s \leftarrow 0$ to $n-m$
do if $p = t_s$
then if $P[1 \ldots m] = T[s+1 \ldots s+m]$
then "Pattern occurs with shift" s
If $s < n-m$
then $t_{s+1} \leftarrow (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q$

Example:
$T = 314159\,26535\ldots$. $P = 26$, $T.length = 11 \Rightarrow Q = 11$
$P \bmod Q = 26 \bmod 11 = 4$

Solution:

T → | 3 1 4 1 5 9 2 6 5 3 5 |

P → | 2 6 |

S1: | 3 | 1 | 4 1 5 9 2 6 5 3 5 |

31 mod 11 = 9 ≠ 4

| 3 | 1 | 4 | 1 5 9 2 6 5 3 5 |

14 mod 11 = 3 ≠ 4

| 3 1 | 4 | 1 | 5 9 2 6 5 3 5 |

41 mod 11 = 8 ≠ 4

| 3 1 4 | 1 | 5 | 9 2 6 5 3 5 |

15 mod 11 = 4 = 4 SPURIOUS HIT

3 1 4 1 | 5 | 9 | 2 6 5 3 5

59 mod 11 = 4 = 4 SPURIOUS HIT

3 1 4 1 5 | 9 | 2 | 6 5 3 5

92 mod 11 = 4

3 1 4 1 5 9 | 2 | 6 | 5 3 5

26 mod 11 = 4 Exact match

3 1 4 1 5 9 2 | 6 | 5 | 3 5

65 mod 11 = 10 $\neq$ 4

3 1 4 1 5 9 2 6 | 5 | 3 | 5

53 mod 11 = 9 $\neq$ 4

3 1 4 1 5 9 2 6 5 | 3 | 5

35 mod 11 = 2 $\neq$ 4

∴ The pattern occurs with shift 6.

Analysis:-

for small problems $O(1)$

large " $O(n+m)$

worst case $O((n-m+1)m)$

The Knuth-Morris-Pratt Algorithm:-

→ It introduce a linear time algorithm for the string matching problem.

→ A matching time of O(n) is achieved by avoiding comparison with an element of 's' that have previously been involved in comparison with some element of the pattern 'p' to be matched.

→ components:-

The prefix Function (π):-

→ It encapsulates knowledge about how the pattern matches against the shift of itself.

→ This info can be used to avoid a useless shift of the pattern p.

Algorithm:-

$m \leftarrow$ length[P].

$\pi[1] \leftarrow 0$

$k \leftarrow 0$

for $q \leftarrow 2$ to m

do while $k > 0$ & $P[k+1] \neq P[q]$

do $k \leftarrow \pi[k]$

If $P[k+1] = P[q]$

then $k \leftarrow k+1$

$\pi[q] \leftarrow k$

Return π.

The algo placed on proper prefix & suffix

The KMP matcher:-

With string 's', pattern 'p' & prefix function π as inputs find the occurance of 'p' in 's' & returns the number of shifts of 'p' after which occurances are found.

Time complexity:

O(m) for prefix function m → times of execution

O(n) for kmp matcher n → runs

Ex: Compute π for the 'p' below

p: a b a b a c a

Sol: Initially $m = length[P] = 7$

$\pi[1] = 0$ & $k = 0$

$q = 2, k = 0, \pi[2] = 0$

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
π	0	0					

$q = 3 \quad k = 0 \quad \pi[3] = 1$

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
π	0	0	1				

$q = 4 \quad k = 1 \quad \pi[4] = 2$

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
π	0	0	1	2			

$q = 5 \quad k = 2 \quad \pi[5] = 3$

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
π	0	0	1	2	3		

$q = 6, k = 3, \pi[6] = 0$

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
π	0	0	1	2	3	0	

$q = 7 \quad r = 1 \;, \; \pi[7] = 1$

q	1	2	3	4	5	6	7
P	a	b	a	b	a	c	a
π	0	0	1	2	3	0	1

The prefix function computation is comple.

Text b a c b a b a b a b a c a c a

P a b a b a c a

Prefix function

q	1	2	3	4	5	6	7
p	a	b	a	b	a	c	a
π	0	0	1	2	3	0	1.

→ Initially n = size of T = 15

m = size of p = 7

S1:- $i = 1, q = 0$, comparing P[1] with T[1]

T b a c b a b a b a b a c a c a

P a b a b a c a

P[1] does not match with T[1] 'p' will be shifted one position to right.

T b a c b a b a b a b a c a c a

P a b a b a c a

S2:- $i = 2, q = 0$.. comparing P[1] with T[2].

T b a c b a b a b a b a c a c a

P a b a b a c a

P[1] matches T[2]. since there is a match p is not shifted.

s=3 i = 3 , q = 1

comparing P[2] with T[3] , P[2] doesn't match with T[3]

T b a c b a b a b a b a c a c a

P a b a b a c a

Back tracking on p , comparing P[1] and T[3]

s=4 i = 4 q = 0

comparing P[1] with T[4] , P[1] doesn't match with T[4]

T b a c b a b a b a b a c a c a

b a b a b a c a

S=5 i=5 q = 0

comparing P[1] with T[5] , P[1] match with T[5]

T b a c b a b a b a b a c a c a

b a b a b a c a

S=6 i=6 q = 1

comparing P[2] with T[6] , P[2] matches with T[6]

T b a c b $\boxed{a}$ b a b a b a c a c a

P $\boxed{a}$ b a b a c a

S=7 i = 7 q = 2

comparing P[3] with T[7] , P[3] matches with T[7]

T b a c b $\boxed{a\ b}$ a b a b a c a c a

P $\boxed{a\ b}$ a b a c a

S=8 i = 8 q = 3

comparing P[4] with T[8] , P[4] matches with T[8]

T b a c b $\boxed{a\ b\ a}$ b a b a c a c a

P $\boxed{a\ b\ a}$ b a c a

i = 9 , q = 4

comparing P[5] with T[9] , P[5] matches with T[9].

step 9:- i=9 q=4

comparing P[5] with T[9] , P[5] matches with T[9]

T b a c b a b a b a b a c a a b

P a b a b a c a

step 10:- i=10, q=5

comparing P[6] with T[10], P[6] doesn't match with T[10]

T :- b a c b a b a b a b a c a a b

P :- a b a b a c a

step 11:- i=11, q=4

comparing P[5] with T[11], P[5] match with T[11]

T : b a c b a b a b a b a c a a b

P :- a b a b a c a

step 12: i=12, q=5

comparing P[6] with T[12] , P[6] matches with T[12]

T : b a c b a b a b a b a c a a b

P :- a b a b a c a

step 13:- i=3 q=6

comparing P[7] with T[13], P[7] matches with T[13]

T :- b a c b a b a b a b a c a a b

P :- a b a b a c a

Pattern 'P' has been found to complexity occur in a string 'T'. The total number of shifts that took place for the match to be found is i-m = 13-7 = 6 shifts