

# UNIT-5

## SEMICONDUCTOR PHYSICS

### Introduction:

On the basis of conduction mechanism solids are classified in to three categories. Conductors, semiconductors and insulators. In conductors' large no. of free electrons are available which use ful for more electrical conduction mechanism. These materials have very low resistivity and very high conductivity. In insulators no free electrons are available, hence there are not useful for the electrical conduction mechanism. These materials have very high resistivity and almost zero conductivity.

The semiconductors have intermediate properties of conductors and insulators. These materials behave as insulators at low temperatures and as conductors at high temaprature.more over these materials have two types of charge carriers.i.e, electrons and holes. The inversion of semiconductors open a new branch of technology "electronics" this leads to the development of ICs, microprocessors, computers and supercomputers. These materials play a vital role in all most all advanced electronic devices.

Semiconductors are divided into two types.

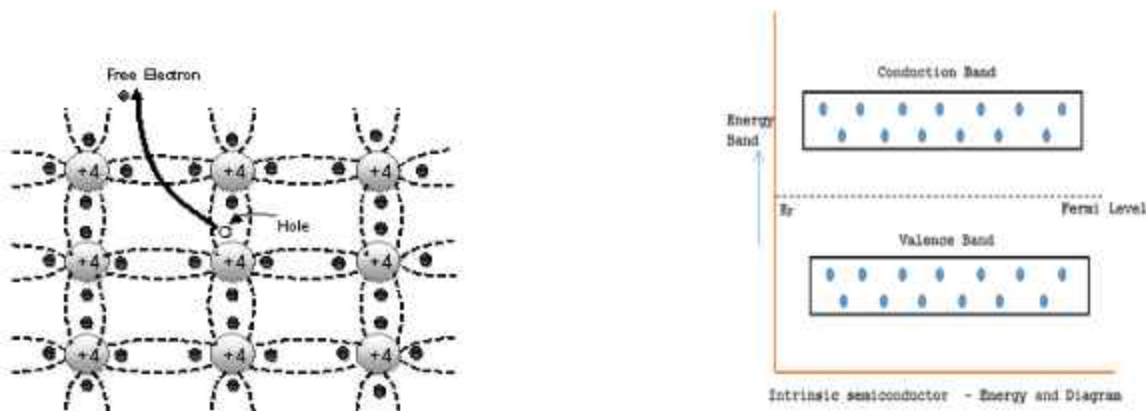
1. Intrinsic semiconductors.
2. Extrinsic semiconductors.

### INTRINSIC SEMICONDUCTORS

Intrinsic semiconductors are those in which impurities are not present and are therefore called pure semiconductors. In these semiconductors few crystal defects may be present Fermi level exists exactly at mid way of the energy gap. When a semiconductor is at 0 k.then it behaves as an insulator and conduction occurs at high temperatures.due to thermal excitation of electrons from the valence band to the conduction band.

Ex:-germanium, silicon etc.

In order to get in sight view of an intrinsic semiconductor, let us consider silicon, which has four valence electrons. In order to gain stability it has to make four covalent bonds with four other silicon atoms as shown in fig.



- In electrons which are participating in the covalent bonds are known as valance electrons.
- If some energy is supplied then covalent bonds break electrons will come out and move freely resulting in the formation of covalent sites in the covalent bond these are known as +ve charge carrier named as hole.
- The electrons which come out of the valence band move freely without any constraints and have more energy then the electrons in the covalent bonds or valence bond.
- The no. of conduction electrons will be equal to the number of vacant sites or hole in the valence band.
- The no. of holes in the valence band is equal to the no. of electrons in the conduction band. So the Fermi energy level lays mid way in the forbidden gap.

## EXTRINSIC SEMICONDUCTORS

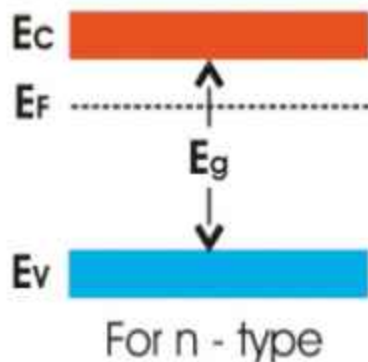
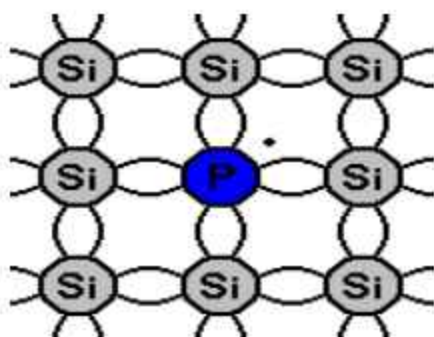
In intrinsic (or) pure semiconductors the carrier concentration of both electrons and holes at normal temperatures is very low, hence to get appreciable current density through the semiconductors, a large electric field should be applied this problem can overcome by adding suitable impurities into the intrinsic semiconductors.

The extrinsic semiconductors are those in which impurities can be either III group elements (or) V group elements. Based on the impurities present in the extrinsic semiconductors. They are classified into two categories.

1. N-type semiconductor.
2. P-type semiconductor.

### N-Type Semiconductor:

When a pentavalent impurities (or) V group element are doped into a intrinsic semiconductor then four valance electrons of impurities atom make four covalent bond with four intrinsic atoms and fifth electrons is left free as shown in fig.

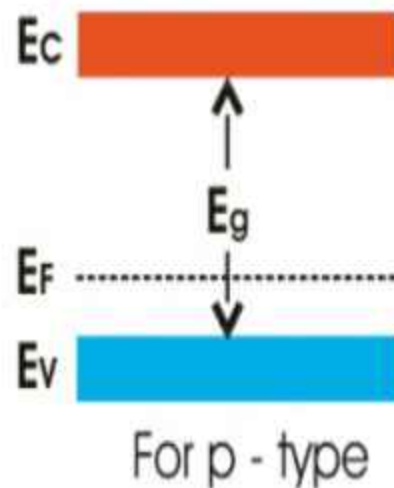
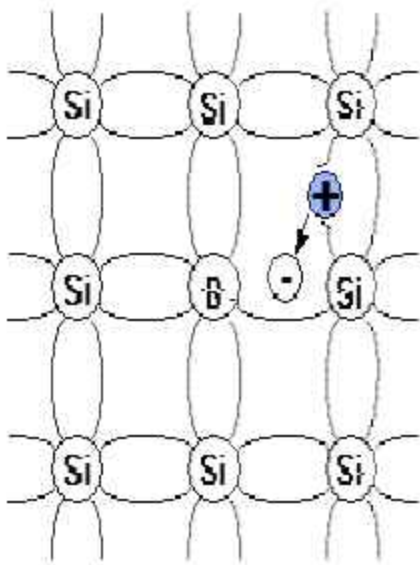


This electron is free to move anywhere in the crystal and is known as a conduction electron which is donated by a pentavalent impurity. In this way, how many donor impurities are doped into an intrinsic semiconductor that many electrons are donated to the conduction band.

The no. of electrons is more than the number of holes in N-type semiconductor. Hence electrons are majority charge carriers, whereas holes are the minority charge carriers. So the Fermi energy level shifts towards the conduction band.

### **P-Type Semiconductor:**

When a trivalent impurity (or) III group element is doped into an intrinsic semiconductor, then three valence electrons of the impurity atom make three covalent bonds with three intrinsic atoms, and in the fourth bond, one position is vacant. It means a trivalent impurity is donating a hole to the valence of the intrinsic semiconductor (or) accepting an electron from the valence band of the intrinsic semiconductor. The trivalent impurity is known as an acceptor. In this way, how many acceptor impurities are doped into an intrinsic semiconductor that many holes are donated to the valence band.



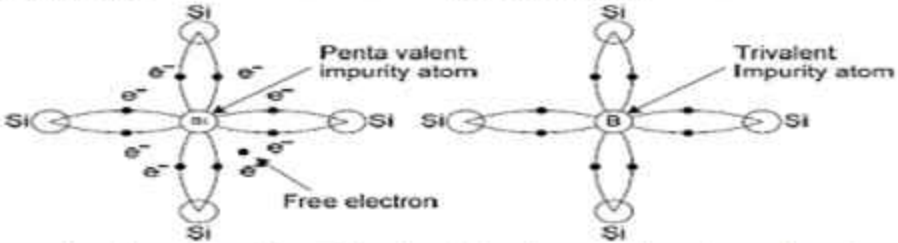
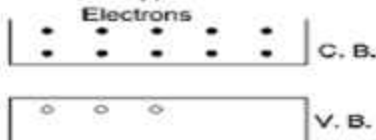
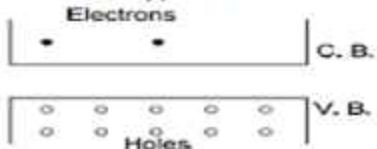
The no. of holes is more than the no. of electrons in p-type semiconductors. Hence holes are the majority charge carriers and electrons are the minority charge carriers. So the Fermi energy level shifts towards the valence band.



### DISTINGUISH BETWEEN INTRINSIC AND EXTRINSIC SEMICONDUCTORS

INTRINSIC SEMICONDUCTORS	EXTRINSIC SEMICONDUCTORS
1. It is pure semi-conducting material and no impurity atoms are added to it.	1. It is prepared by doping a small quantity of impurity atoms to the pure semi-conducting material.
2. Examples: crystalline forms of pure silicon and germanium.	2.Examples: silicon "Si" and germanium "Ge" crystals with impurity atoms of As, Sb, P etc. or In, B, Al etc.
3. The number of free electrons in the conduction band and the no. of holes in valence band is exactly equal and very small indeed.	3. The number of free electrons and holes is never equal. There is excess of electrons in n-type semi-conductors and excess of holes in p-type semi-conductors.
4. Its electrical conductivity is low.	4. Its electrical conductivity is high.
5. Its electrical conductivity is a function of temperature alone.	5. Its electrical conductivity depends upon the temperature as well as on the quantity of impurity atoms doped the structure.

### DISTINGUISH BETWEEN N-TYPE AND P-TYPE SEMICONDUCTORS

S.No.	N-type semiconductors	P-type semiconductors
1.	In these the impurity of some pentavalent element like P, As, Sb, Bi, etc. is mixed	In these, the impurity of some trivalent element like b, Al, In, Ga etc. is mixed
2.		
3.	In these the impurity atom donates one electrons, hence these are known as donor type semiconductors	In these, the impurity atom can accept one electron, hence these are known as acceptor type semiconductors.
4.	In these the electrons are majority current carriers and holes are minority current carriers. (i.e. the electron density is more than hole density $n_n \gg n_p$ )	In these the holes are majority current carriers and electrons are minority current carriers i.e. $n_p \gg n_n$
5.	In these there is majority of negative particles (electrons) and hence are known as N-type semiconductors 	In these there is majority of positive particles (catters) and hence are known as P-type semiconductors. 
6.	In these the donor energy level is close to the conduction band and far away from valence band.	In these the acceptor energy level is close to the valence band and far away from conduction band.

## CARRIER CONCENTRATION IN INTRINSIC SEMICONDUCTOR

Let  $dn$  be the number of electrons available between energy interval  $E$  and  $E + dE$  in the conduction band

$$dn = Z(E)F(E)dE \text{----- (1)}$$

Where  $Z(E)dE$  = density of states in the energy interval  $E$  and  $E + dE$  and  $F(E)$  = probability of an electron occurring in an energy state  $E$ .

But we know that 
$$Z(E)dE = \frac{4\pi}{h^3} (2m_e)^{\frac{3}{2}} E_f^{\frac{1}{2}} dE$$

Since  $E$  starts from the bottom of the conduction band  $E_c$

$$Z(E)dE = \frac{4\pi}{h^3} (2m_e)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} dE \text{----- (2)}$$

Probability of an electron occupying an energy state  $E$  is given by

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{KT}\right)}$$

For all temperatures  $E - E_f \gg KT$  then  $\exp\left(\frac{E - E_f}{KT}\right) \gg 1$

$$\therefore F(E) = \frac{1}{\exp\left(\frac{E - E_f}{KT}\right)} = \exp\left(\frac{E_f - E}{KT}\right) \text{----- (3)}$$

From (1), (2) and (3)

$$\begin{aligned} n &= \int dn = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2m_e)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} e^{\left(\frac{E_f - E}{KT}\right)} dE \\ &= \frac{4\pi}{h^3} (2m_e)^{\frac{3}{2}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} e^{\left(\frac{E_f - E_c}{KT}\right)} e^{\left(\frac{E_c - E}{KT}\right)} dE \end{aligned}$$

$$\text{Put } \frac{E_f - E_c}{KT} = x \Rightarrow E_f - E_c = KTx$$

$$dE = KTx \text{ and } (E - E_c)^{\frac{1}{2}} = x^{\frac{1}{2}} (KT)^{\frac{1}{2}}$$

$$n = \frac{4\pi}{h^3} (2m_e)^{\frac{3}{2}} \int_0^{\infty} x^{\frac{1}{2}} (KT)^{\frac{1}{2}} e^{-x} dx$$

$$\therefore n = \frac{4\pi}{h^3} (2m_e KT)^{\frac{3}{2}} e^{\left(\frac{E_f - E_c}{KT}\right)} \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx$$

Using gamma function it can be shown that

$$\therefore n = \frac{4\pi}{h^3} (2m_e KT)^{\frac{3}{2}} e^{\left(\frac{E_f - E_c}{KT}\right)} \frac{\sqrt{\pi}}{2}$$

$$\text{i.e. } \boxed{n = 2 \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_f - E_c}{KT}\right)}}$$

### CALCULATION OF DENSITY OF HOLES:

Let  $dP$  be the number of holes in the energy interval  $E$  and  $E+dE$  in the valence band

$$dP = Z(E)(1 - F(E))dE$$

We know that

$$Z(E)dE = \frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE$$

$$1 - F(E) = 1 - \frac{1}{1 + e^{\left(\frac{E - E_f}{KT}\right)}} = 1 - \left(1 + e^{\left(\frac{E - E_f}{KT}\right)}\right)^{-1} = 1 - \left(1 - e^{\left(\frac{E - E_f}{KT}\right)}\right) = e^{\left(\frac{E - E_f}{KT}\right)}$$

Therefore the number of holes in the valence band per unit volume is given by

$$p = \int dp = \int_{-\infty}^{E_v} \frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} e^{\left(\frac{E - E_f}{KT}\right)} dE$$

$$p = \frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\left(\frac{E - E_f}{KT}\right)} dE$$

$$p = \frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\left(\frac{E}{KT}\right)} e^{\left(\frac{-E_f}{KT}\right)} dE$$

$$p = \frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} e^{\left(\frac{-E_f}{KT}\right)} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\left(\frac{E}{KT}\right)} dE$$

$$\text{Let } \frac{E_v - E}{KT} = x \Rightarrow E_v - E = xKT$$

$$\text{i.e. } \Rightarrow dE = -KTdx$$

$$\frac{E_v}{KT} - \frac{E}{KT} = x \Rightarrow \frac{E}{KT} = \frac{E_v}{KT} - x$$

$$\therefore p = -\frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} e^{\left(\frac{-E_f}{KT}\right)} \int_{\infty}^0 x^{\frac{1}{2}} (KT)^{\frac{1}{2}} e^{\left(\frac{E_v}{KT}\right)} e^{-x} dx$$

$$\therefore p = \frac{4\pi}{h^3} (2m_h)^{\frac{3}{2}} e^{\left(\frac{E_v - E_f}{KT}\right)} (KT)^{\frac{3}{2}} \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx$$

$$\therefore p = \frac{4\pi}{h^3} (2m_h KT)^{\frac{3}{2}} e^{\left(\frac{E_v - E_f}{KT}\right)} \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx$$

$$p = \frac{4\pi}{h^3} (2m_h KT)^{\frac{3}{2}} e^{\left(\frac{E_v - E_f}{KT}\right)} \frac{\sqrt{\pi}}{2}$$

$$p = 2 \left( \frac{2\pi}{h^3} m_h KT \right)^{\frac{3}{2}} e^{\left( \frac{E_v - E_f}{KT} \right)}$$

Since  $n = p = n_i$

$$n_i^2 = np = 4 \left( \frac{2\pi KT}{h^2} \right)^3 (m_h m_e)^{\frac{3}{2}} e^{\left( \frac{-E_g}{KT} \right)}$$

$$\Rightarrow n_i = 2 \left( \frac{2\pi KT}{h^2} \right)^{\frac{3}{2}} (m_h m_e)^{\frac{3}{4}} e^{\left( \frac{-E_g}{2KT} \right)}$$

**Show that in an intrinsic semiconductor Fermi level is at middle of valance band and conduction band:**

In an intrinsic semiconductor

$$n = p$$

$$2 \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{2}} e^{\left( \frac{E_f - E_c}{KT} \right)} = 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} e^{\left( \frac{E_v - E_f}{KT} \right)}$$

$$m_e^{\frac{3}{2}} \left( \frac{2\pi KT}{h^2} \right)^{\frac{3}{2}} e^{\left( \frac{E_f - E_c}{KT} \right)} = m_h^{\frac{3}{2}} \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} e^{\left( \frac{E_v - E_f}{KT} \right)}$$

$$\left( \frac{m_h}{m_e} \right)^{\frac{3}{2}} = \exp\left( \frac{E_f - E_c - E_v + E_f}{KT} \right) = \exp\left( \frac{2E_f - E_v - E_c}{KT} \right)$$

Taking logarithm on both sides we get

$$\frac{3}{2} \log \frac{m_h}{m_e} = \frac{2E_f - E_v - E_c}{KT}$$

$$\Rightarrow 2E_f = E_v + E_c + \frac{3}{2} KT \log \frac{m_h}{m_e}$$

$$i.e. E_f = \frac{E_v + E_c}{2} + \frac{3}{4} KT \log \frac{m_h}{m_e}$$

But  $m_h = m_e$

$$\Rightarrow E_f = \frac{E_v + E_c}{2}$$

i.e. Fermi level is located half way between valance band and conduction band and the position is independent of temperature.

### CONDUCTIVITY IN INTRINSIC SEMICONDUCTORS:

Since  $n = p = n_i$

$$n_i^2 = np = 4 \left( \frac{2\pi KT}{h^2} \right)^3 (m_h m_e)^{\frac{3}{2}} e^{\left( \frac{-E_g}{KT} \right)}$$

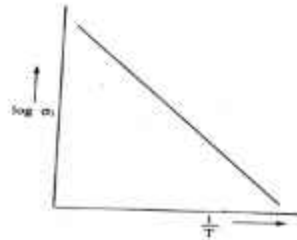
$$\Rightarrow n_i = 2 \left( \frac{2\pi KT}{h^2} \right)^{\frac{3}{2}} (m_h m_e)^{\frac{3}{4}} e^{\left( \frac{-E_g}{2KT} \right)}$$

Where  $E_g$  is the energy gap and it is equal to  $E_g = E_c - E_v$

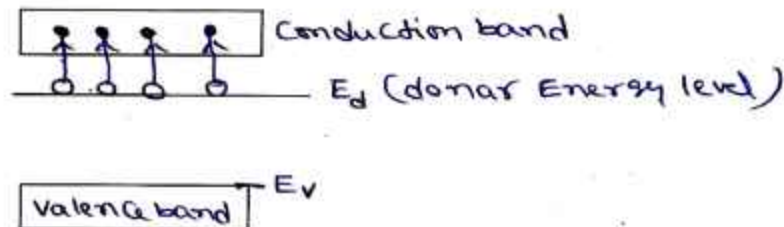
Therefore electrical conductivity  $\sigma = ne\mu_e + pe\mu_h = n_i e(\mu_e + \mu_h)$

$$\sigma = A e^{\left( \frac{-E_g}{2KT} \right)}$$

And as T increases  $\sigma$  increases.



### CARRIER CONCENTRATION IN N-TYPE SEMICONDUCTORS.



The energy level diagram of an N-type semiconductor is shown in the fig. At very low temperatures all the donor levels are filled with electrons. With increase of temperature more and more atoms get excited and the density of electrons in the conduction band increases.

Density of electrons in the conduction band

$$n_e = 2 \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{2}} e^{\left( \frac{E_f - E_c}{KT} \right)} \text{----- (1) but the number of vacancies per unit volume in the donor}$$

$$\text{level is given by } N_d(1 - F(E)) = N_d \left[ 1 - \frac{1}{e^{\left( \frac{E_d - E_f}{KT} \right)} + 1} \right] = N_d \left[ 1 - (1 - e^{\left( \frac{E_d - E_f}{KT} \right)}) \right] = N_d e^{\left( \frac{E_d - E_f}{KT} \right)} \text{----- (2)}$$



But the number of electrons in the conduction band is equal to the number of vacancies in the donor level.

$$2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}} e^{\left(\frac{E_f - E_c}{K T}\right)} = N_d e^{\left(\frac{E_d - E_f}{K T}\right)}$$

Taking log on both sides

$$\log 2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}} + \frac{E_f - E_c}{K T} = \log N_d + \frac{E_d - E_f}{K T}$$

$$\frac{E_f - E_c - E_d + E_f}{K T} = \log N_d - \log 2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}}$$

$$2E_f - (E_c + E_d) = K T \left[ \log \frac{N_d}{2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}}} \right]$$

$$E_f = \frac{(E_c + E_d)}{2} + \frac{K T}{2} \left[ \log \frac{N_d}{2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}}} \right] \text{----- (3)}$$

$$\text{At 0K } E_f = \frac{(E_c + E_d)}{2}$$

**DENSITY OF ELECTRONS IN CONDUCTION BAND:** We know that  $n = 2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}} e^{\left(\frac{E_f - E_c}{K T}\right)}$

On substituting the value of  $E_f$  from equ. (3) In the above equation we get

$$n = 2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}} e^{\left(\frac{\frac{(E_c + E_d)}{2} + \frac{K T}{2} \left[ \log \frac{N_d}{2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}}} \right] - E_c}{K T}\right)}$$

$$n = 2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}} \exp\left(\frac{(E_c + E_d)}{2 K T} + \frac{1}{2} \log \frac{N_d}{2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}}} - \frac{E_c}{2 K T}\right)$$

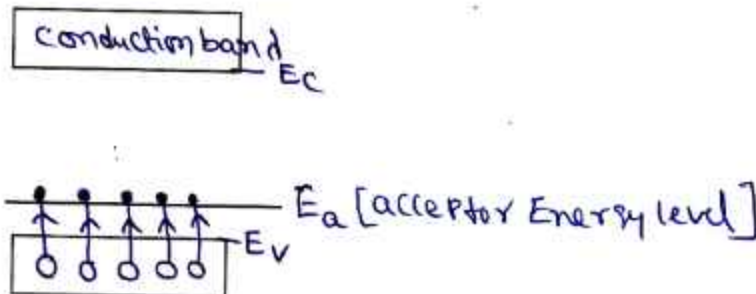
$$n = 2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}} \exp\left(\frac{(E_d - E_c)}{2 K T} + \frac{1}{2} \log \frac{N_d}{2\left(\frac{2\pi m_e K T}{h^2}\right)^{\frac{3}{2}}}\right)$$

$$n = 2 \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{2}} \exp\left(\frac{(E_d - E_c)}{2KT}\right) + \log \frac{N_d^{\frac{1}{2}}}{2^{\frac{1}{2}} \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{4}}}$$

$$n = 2 \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{2}} \frac{N_d^{\frac{1}{2}}}{2^{\frac{1}{2}} \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{4}}} \exp\left(\frac{(E_d - E_c)}{2KT}\right)$$

$$\boxed{n = (2N_d)^{\frac{1}{2}} \left( \frac{2\pi m_e KT}{h^2} \right)^{\frac{3}{4}} \exp\left(\frac{(E_d - E_c)}{2KT}\right)}$$

### CARRIER CONCENTRATION IN P-TYPE SEMICONDUCTORS:



The energy level diagram of a p-type semiconductor is shown in the fig.

$N_a$  is the acceptor concentration i.e. the number of acceptor atoms per unit volume of the material and  $E_a$  is the acceptor energy level. At very low temperatures all the acceptor levels are empty. With increase of temperature the electrons move from valance band and occupy the vacant sites in the acceptor energy level, thereby leaving holes in the valance band.

The density of holes in the valance band is given by

$$p = 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} e^{\left( \frac{E_v - E_f}{KT} \right)} \text{----- (1)}$$

Since  $E_f$  lies below acceptor level then density of ionized acceptor is given by

$$N_a F(E) = N_a e^{\left( \frac{E_f - E_a}{KT} \right)}$$

Since the density of holes in the valance band is equal to the density of ionized acceptor

$$\text{i.e. } 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} e^{\left( \frac{E_v - E_f}{KT} \right)} = N_a e^{\left( \frac{E_f - E_a}{KT} \right)}$$

$$\exp\left(\frac{E_v - E_f - E_f + E_a}{KT}\right) = \frac{N_a}{2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}}}$$

Taking logarithm on both sides we get

$$\frac{E_v - E_f - E_f + E_a}{KT} = \log \frac{N_a}{2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}}}$$

$$\text{i.e. } E_f = \frac{E_v + E_a}{2} - \frac{KT}{2} \log \frac{N_a}{2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}}} \text{-----(2)}$$

$$\text{At 0K } E_f = \frac{E_v + E_a}{2}$$

i.e. at 0K Fermi level will be exactly at the middle of acceptor level

The density of holes in the valance band is given by

$$p = 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} e^{\left( \frac{E_v - E_f}{KT} \right)}$$

On substituting the value of  $E_f$  from equation (2) we get

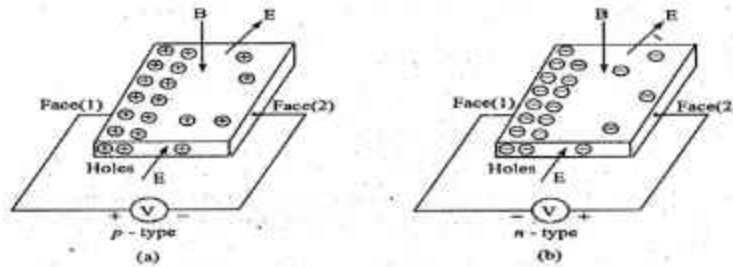
$$p = 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} \exp \left( \frac{E_v}{KT} - \frac{1}{KT} \left( \frac{E_v + E_a}{2} - \frac{KT}{2} \log \frac{N_a}{2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}}} \right) \right)$$

$$p = 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} \exp \left( \frac{2E_v}{2KT} - \frac{E_v + E_a}{2KT} \right) \frac{N_a^{\frac{1}{2}}}{\left[ 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} \right]^{\frac{1}{2}}}$$

$$p = 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} \frac{N_a^{\frac{1}{2}}}{\left[ 2 \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{2}} \right]^{\frac{1}{2}}} \exp \left( \frac{E_v - E_a}{2KT} \right)$$

$$\boxed{p = (2N_a)^{\frac{1}{2}} \left( \frac{2\pi}{h^2} m_h KT \right)^{\frac{3}{4}} \exp \left( \frac{E_v - E_a}{2KT} \right)}$$

## HALL EFFECT:



When a piece of conductor (metal or semiconductor) carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and magnetic field. This phenomenon is called Hall Effect and the generated voltage is known as Hall-voltage.

Let  $B$  be the applied magnetic field at right angle to the direction of the current flow. If the material is N-type and  $v$  is the velocity of the electrons then the electrons will experience a force of  $Bev$  due to magnetic field at right angle to both the direction and  $B$ . This causes the electron current to be deflected causing a negative charge to accumulate on one face of the slab, a PD is therefore established across the faces (1) and (2). This field giving rise to a force  $eE_H$  on electrons in the opposite directions.

$$\text{At equilibrium } eE_H = Bev$$

$$E_H = Bv$$

$$\text{But } J = nev \Rightarrow v = \frac{J}{ne}$$

$$E_H = \frac{BJ}{ne} = R_H BJ \text{ where } R_H = \frac{1}{ne} = \frac{E_H}{BJ}$$

Since all the three quantities  $E_H$ ,  $B$  and  $J$  are measurable then Hall coefficient  $R_H$  and carrier density can be calculated.

$$\text{For N-type material } R_H = -\frac{1}{ne}$$

$$\text{For p-type material } R_H = \frac{1}{ne}$$

Based on this we can identify the type of charge.

### Determination of hall coefficient $R_H$

If  $b$  is the width of the sample across the Hall voltage  $V_H$  is measured then

$$E_H = \frac{V_H}{b}$$

$$\text{Hence } R_H = \frac{E_H}{BJ} = \frac{V_H}{BJb}$$

$$V_H = R_H BJb$$

If  $t$  is the thickness of the sample then its cross section is  $bt$  and current density

$$J = \frac{I}{bt}$$

$$\text{Hence } V_H = \frac{R_H BIb}{t}$$

$$\text{i.e. } \boxed{R_H = \frac{V_H t}{IB}}$$

$V_H$  Will be opposite for N-and P-type semiconductors.

### Applications of Hall Effect

1. The sign of charge carriers can be determined.
2. The carrier density can be estimated.
3. The mobility of charge carriers can be measured directly.
4. It can be used to determine whether the given material is a metal, an insulator or a semiconductor.
5. The magnetic field can be measured by knowing the values of Hall voltage and Hall coefficient.

### Drift current or conduction current in semiconductors:

In semiconductors the electrical conductivity is due to the concentration of electrons and holes. When an electric field is applied across the semiconductor the applied electric field causes the free electrons to drift towards positive direction and holes to drift towards negative direction of electric field.

The total current due to holes and electrons in the presence of electric field is called drift current or conduction current in semi conductors.

$$\text{Total drift current } I_{drift} = I_{electrons} + I_{holes}$$

$$\text{i.e. } I_{drift} = ne\mu_e EA + pe\mu_h EA = eEA(n\mu_e + p\mu_h)$$

in case of intrinsic semiconductors  $n = p = n_i$

$$I_{drift} = n_i eEA(\mu_e + \mu_h)$$

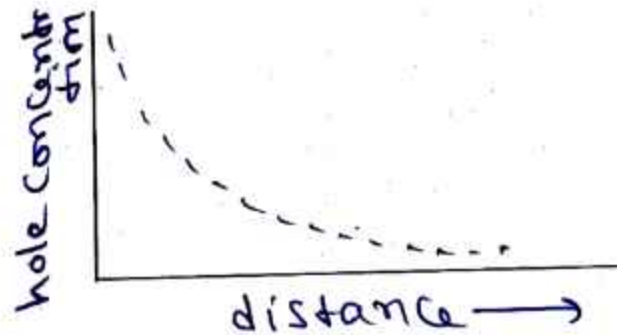
$$J_{drift} = n_i eE(\mu_e + \mu_h)$$

Therefore electrical conductivity of intrinsic semiconductor due to drifting action of electrons and holes is given by

$$\boxed{\sigma_{drift} = n_i e(\mu_e + \mu_h)}$$



### Diffusion current in semiconductors:



The figure shows the variation of holes density with distance in semiconductors. There exists a concentration gradient  $\frac{dp}{dx}$

Thus the motion of charge carriers from the region of higher concentration leads to a current called diffusion current.

$$\text{Hole current density } J_p \propto \frac{dp}{dx} \propto e$$

$$\text{i.e. } J_p = -eD_p \frac{dp}{dx}$$

$$\text{Electron current density } J_n \propto \frac{dn}{dx}$$

$$J_n \propto e$$

$$J_n = eD_n \frac{dn}{dx}$$

The total diffusion current density  $J_{\text{diffusion}} = J_p + J_n$

$$\text{i.e. } J_{\text{diffusion}} = -e(D_p \frac{dp}{dx} + D_n \frac{dn}{dx})$$

Therefore current density in semiconductors  $J_{\text{diffusion}} + J_{\text{drift}}$

$$\text{ie } J = e\{n_i E(\mu_e + \mu_h) - D_p \frac{dp}{dx} + D_n \frac{dn}{dx}\}$$

### EINSTEIN RELATION:

Semiconductor devices in general operate under non-equilibrium conditions. For example when a bar of n-type germanium acting as a photo detecting device, is illuminated with a light of sufficient energy, excess charge carriers are produced in the exposed region of the material. Hence the semiconductor is not in thermal equilibrium. This leads to a diffusion current creating an internal electric field and drift current balances the diffusion component.

Let  $\Delta n$  be the number of charge carriers increased in a particular region then

$$\Delta n e E \mu_e = e D_n \frac{\partial \Delta n}{\partial x} \text{ --- (1)}$$

The force  $F$  on excess carriers restoring equilibrium is given by

$$F = \Delta n e E = \frac{e D_n}{\mu_e} \frac{\partial \Delta n}{\partial x} \text{ ----- (2)}$$

According to kinetic theory of gasses increased pressure =  $\Delta n K T$

$$\text{i.e. Force} = K T \frac{\partial \Delta n}{\partial x} \text{ ----- (3)}$$

from (2) and (3) we get

$$K T \frac{\partial \Delta n}{\partial x} = \frac{e D_n}{\mu_e} \frac{\partial \Delta n}{\partial x}$$

$$\frac{e D_n}{\mu_e} = K T \Rightarrow \boxed{D_n = \frac{\mu_e K T}{e}}$$

$$\text{Similarly } \boxed{D_p = \frac{\mu_h K T}{e}}$$

These are called Einstein Relations.

## QUESTION BANK

### UNIT-VII

1. a. Derive an expression for the number of electrons per unit volume in the conduction band of an N-type semiconductor.  
b Explain the effect of temperature and dopant on Fermi level in N-type semiconductor
2. a Explain Hall effect. Derive the expression for Hall coefficient of N-type semiconductor.  
b Explain the applications of hall effect.
3. a what is intrinsic semiconductor? Derive an expression for electron concentration in an intrinsic semiconductor.  
b Distinguish between direct and indirect band gap semiconductors.
4. a What is intrinsic semiconductor? Derive an expression for hole concentration in an intrinsic semiconductor  
b Explain drift and diffusion currents.
5. a Derive an expression for the number of holes per unit volume in the valence band of P-type semiconductors  
b Derive Einstein relation.

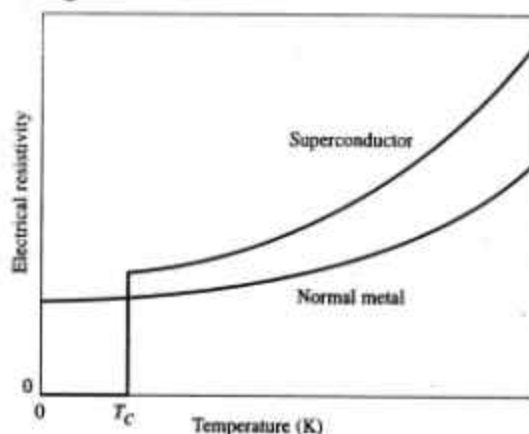
## SUPERCONDUCTIVITY

The sudden disappearance of electrical resistance in materials below a certain temperature is known as superconductivity. The materials that exhibit superconductivity and which are in superconducting state are called superconductors. The temperature at which a normal material turns into a superconductor is called critical temperature  $T_c$ .

Metals are good conductors of electricity as they have plenty of free electrons. However they offer resistance to the flow of current. Because the scattering of conduction electrons by thermal vibrations of atoms and the presence of impurities or imperfections in crystal.

When temperature increases the amplitude of lattice vibrations increases. This leads to more scattering of electrons and to more resistance.

Fig. shows the resistance of the metal varies with temperature. Even at 0k metals offer some resistance.



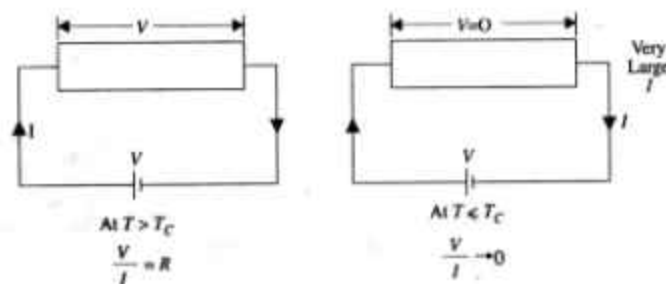
The curve B of fig. shows the behavior of a superconductor at high temperature. Its behavior is same as that of metals. At a particular temperature called critical temperature  $T_c$  the resistance drops suddenly to zero. The value of critical temperature differs from material to material.

### PROPERTIES OF SUPERCONDUCTORS:

#### 1. Zero Electrical Resistance:

Superconductor possesses zero electrical resistance below the transition temperature. Fig.a shows when the superconductor is in normal state. When the material is in normal state a voltage drop is observed across its ends. When

the material is cooled below its critical temperature  $T_c$  the potential difference disappears, as shown in fig.b



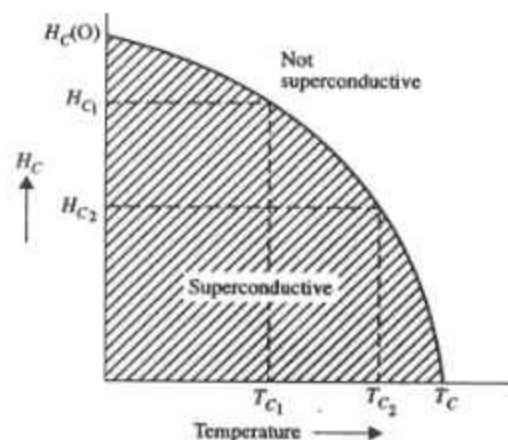
#### 2. Effect of Temperature:

When the temperature of a superconductor is increased, the material transforms into a normal material above the critical temperature  $T_c$ . The transition is reversible. when the material is cooled below the  $T_c$ . It regains its resistivity and goes into the superconducting state.

### 3. Effect of Magnetic Field ( $H_c$ ):

Krammerlingh ones observed that superconductivity vanishes if strong magnetic field is applied. The minimum magnetic field to which the superconductivity vanishes is called critical magnetic field  $H_c$ .

When the applied magnetic exceeds the critical value  $H_c$  the superconducting state is destroyed and the material goes into the normal state. At any temperature  $T < T_c$  the material remains superconducting state until a corresponding critical magnetic field is applied.



When the magnetic field exceeds the critical value the material goes into the normal state. The critical field required to destroy the superconducting state decreases progressively with increasing temperature. For example a magnetic field of 0.004 Tesla will destroy the superconductivity of mercury at  $T \approx 0\text{K}$  where as a field of 0.02 Tesla is sufficient of destroy its superconductivity at about 3K. The dependence of critical field on temperature is governed by the following relation.

$$H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right] \text{----- (1)}$$

Where  $H_c(0)$  is critical magnetic field at 0K

### 4. Critical Current

An electric current is always associated with a magnetic field hence when current flows in a superconductor magnetic field induced and if the magnetic field produced by this current is equal to critical magnetic  $H_c$ , the superconducting state disappears.

*The current density at which superconductivity disappears is called the critical current density.*

$$I_c = 2 \pi r H_c$$

Where 'r' is radius of the specimen,  $H_c$  is critical magnetic field.

This rule is known as silsbee's rule.

### 5. Persistent current:

Once current is set up in a closed loop of superconductor it will continue to keep to flow. Such a steady current which flows with undiminishing strength is called persistent current. Calculations show that once the current flow is initiated it persists more than  $10^5$  years.

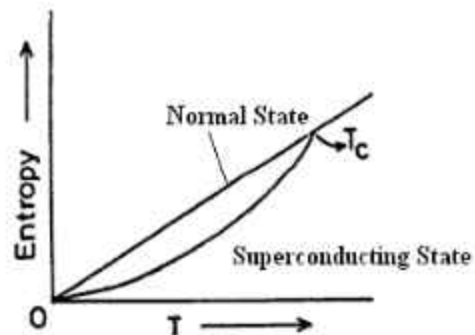
## Thermal Properties:

### 6. Entropy:

Entropy is the measure of disorder of a system.

In superconductors the entropy decreases below the critical temperature. This means the superconducting state is more ordered than the normal state.

Figure shows the entropy versus temperature for aluminium. For normal aluminium entropy varies linearly with temperature, while for superconducting aluminium, it decreases below the critical temperature.

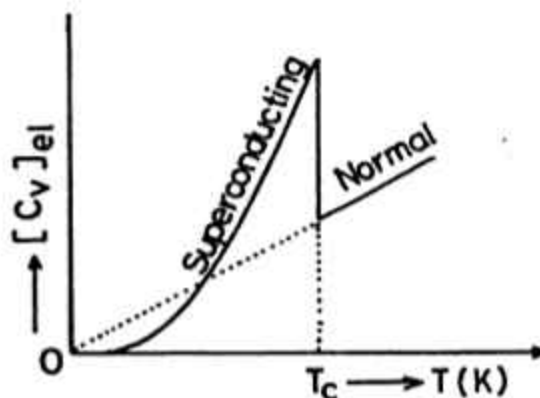


### 7. Specific Heat:

The specific heat of the normal metal is of the form

$$C_v(T) = \gamma T + \beta T^3 \text{ -----(1)}$$

The first term in the above equation gives specific heat of *electrons* in the metal and the second term gives specific heat due to *lattice vibrations* at low temperatures.



Lattice vibrations part remains unaffected in superconducting state i.e., it has the same value  $\beta T^3$  in the normal and superconducting states. The specific heat of the superconductor shows a jump at the transition temperature  $T_c$  as shown in fig. because the superconductivity affects electrons mainly.

So electronic specific heat varies exponential manner.

$$C_{v,el}(T) = A \exp(-\Delta/K_B T) \text{ -----(2)}$$

Where  $\Delta = bK_B T_c$  is called energy gap and

A and b are constants.

This exponential form is an indication of the existence of a finite gap in the energy spectrum of electrons in superconducting state.

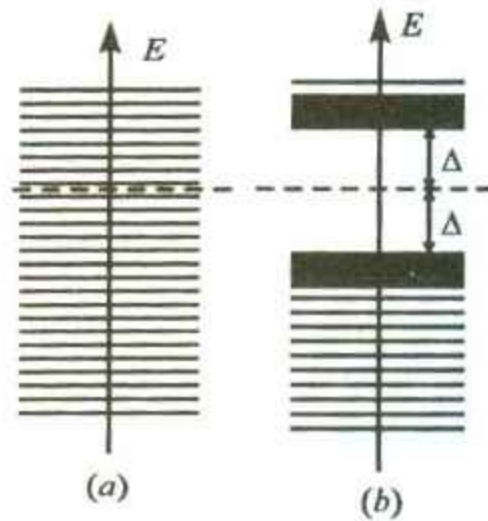


### ENERGY GAP:

The electrons of a Cooper pair have a lower energy than two unpaired electrons in normal state. The energy distribution (energy spectrum) of electrons exhibits an energy gap, as shown in fig.b.

Cooper pairs occupy the lower state. The finite energy  $2\Delta$  must be expended to dissociate a Cooper pair. For a superconductor at 0k, the width of the gap is proportional to the critical temperature. Thus,

$$E_g = 2\Delta = 2b(K_B T_c) = 3.52 K_B T_c$$



### ISOTOPE EFFECT ON SUPERCONDUCTIVITY:

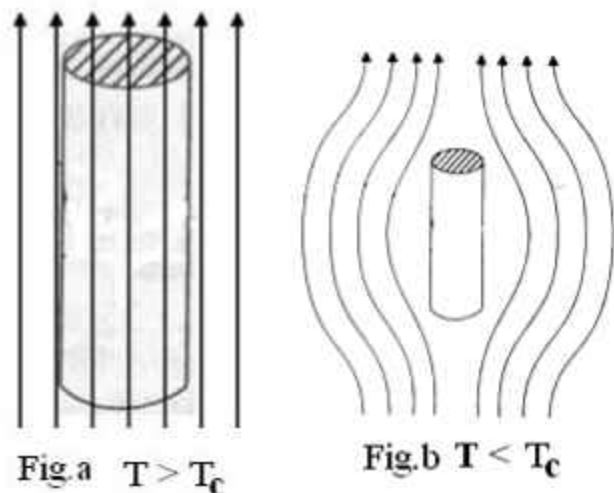
In isotope effect the critical temperature of the superconductor varies with the isotopic mass. The transition temperature  $T_c$  is found to be proportional to the reciprocal of the square root of their respective isotopic masses i.e.,

$$T_c \propto \frac{1}{\sqrt{M}} \quad \text{or} \quad T_c M^{1/2} = \text{constant}$$

### MEISSNER'S EFFECT OR PERFECT DIAMAGNETISM:

Definition: "Expulsion of magnetic flux from a superconducting material when it is placed in magnetic field is called Meissner's Effect".

In fig.a the sample is in its normal state and the magnetic lines of force pass through the specimen. If specimen is cooled below the transition temperature  $T_c$  magnetic lines of force expelled out of the specimen as shown in fig.b. What happens is when the magnetic field is applied to the superconductor this magnetic field generates currents on the surface of the superconductor. These surface currents generate magnetic field inside the superconductor which is exactly opposite to the direction of external magnetic field.



So the external magnetic field cannot penetrate through the specimen.

The effect is reversible if the temperature is increased ( $T > T_c$ ) the magnetic flux suddenly penetrate the specimen and the substance is in normal state. The magnetic induction inside the specimen is given by

$$B = \mu_0(H + I) \text{ ----- (1)}$$

Where  $\mu_0$  is permeability of air or free space, 'I' is intensity of magnetization

H is magnetic field intensity.

In superconducting state inside the material no magnetic flux is stored so  $B = 0$  inside the specimen.

$\therefore$  from (1),  $H = -I$  that is the magnetization I is equal and opposite to the applied field H.

the medium is therefore diamagnetic and susceptibility  $\chi = \frac{I}{H} = -1$

From the above condition susceptibility is negative only for diamagnetic materials. So superconductor acts as a perfect diamagnet.

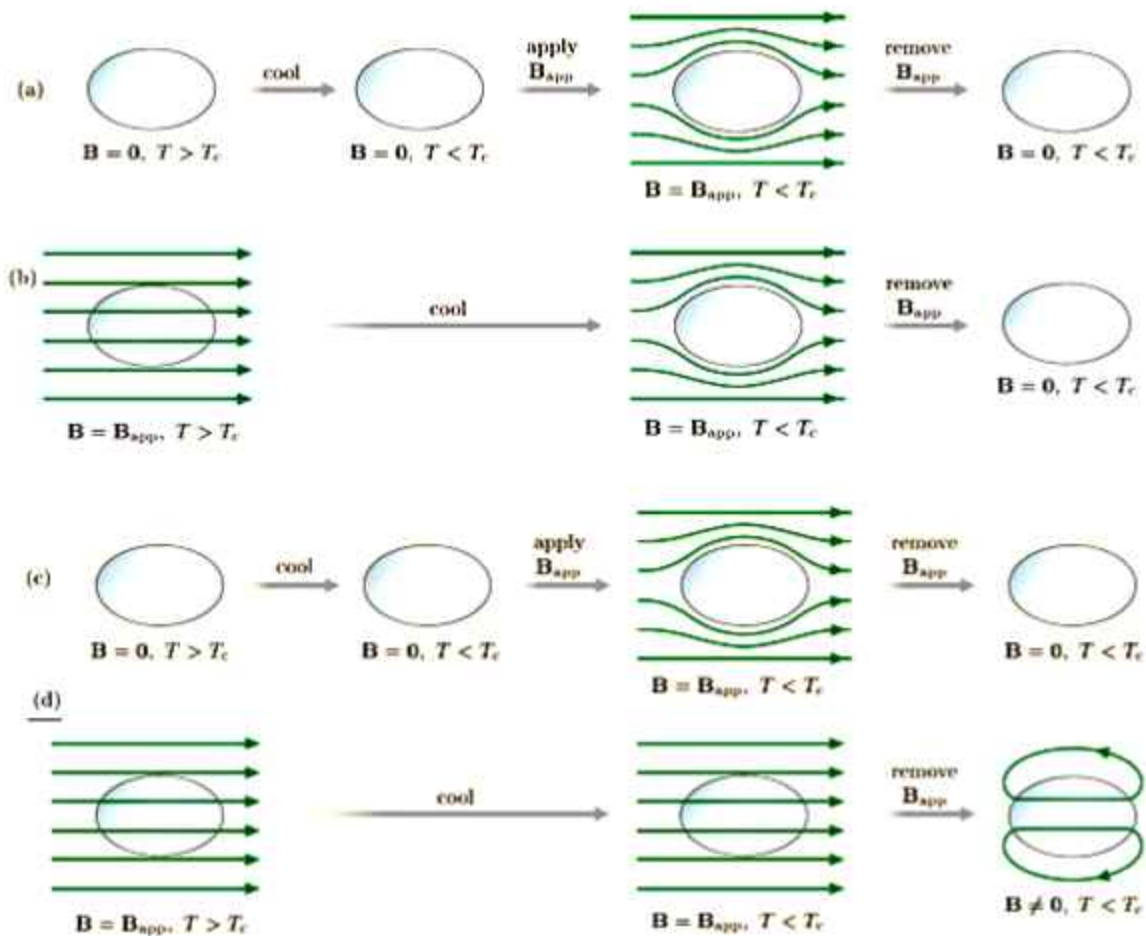


Fig.a&b show the behavior of superconductor in magnetic field.

Fig.c&d show the behavior of perfect conductor in magnetic field.

When the perfect conductor is cooled in presence of the external magnetic field, some part of magnetic field is frozen inside the material.

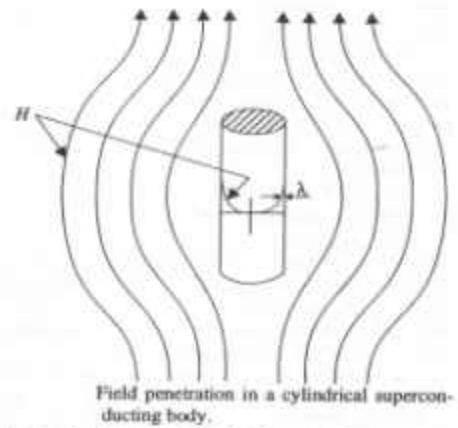
The difference between a superconductor and a perfect conductor is that the perfect conductor is only an ideal conductor while superconductor is both an ideal conductor and ideal diamagnet.

## LONDON PENETRATION DEPTH:

When the magnetic field is applied to the superconductor, the magnetic field enters at the surface of the superconductor and decreases exponentially as shown in figure.

Due to the existence of surface currents the magnetic field penetrates some distance into superconductor and decaying exponentially to zero over length ' $\lambda$ '. The length  $\lambda$  is called London penetration depth.

It may be defined as the effective depth to which a magnetic field penetrates a superconductor. The decrease of magnetic field penetration is given by London equation proposed by F.London and H.London in 1935



$$H(x) = H(0) e^{-\frac{x}{\lambda}}$$

Where  $H(0)$  is field on the surface at  $x = 0$

$\lambda$  is order of  $10^3$  to  $10^4 \text{ \AA}$ . It is independent of frequency of the magnetic field but it strongly depends on temperature, is given by

$$\left(\frac{\lambda_T}{\lambda_0}\right)^2 = \frac{1}{1 - \left(\frac{T}{T_c}\right)^4}$$

Where  $\lambda_T$  and  $\lambda_0$  are penetration depths at  $T$  and  $0\text{K}$ .

## TYPE I & TYPE II SUPERCONDUCTORS:-

Based on the magnetic behavior superconductors are classified into two categories namely soft or type I, hard or type II superconductors.

Type I or Soft: They exhibit complete Meissner effect. This is they are completely diamagnetic.

In type I transition from superconducting to normal state proceeds when the applied magnetic field exceeds  $H_c$ . Let us consider a cylindrical specimen with its axis coinciding with the direction of magnetic field. In type I materials as  $H_c$  is reached the entire specimen enters the normal state as a result

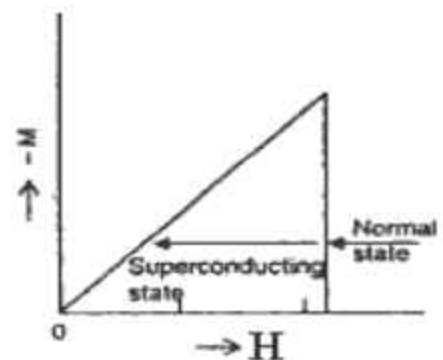


Fig. a

resistivity returns and the superconductor is no longer diamagnetic.

*Al, lead, indium are the examples for type I superconductors.* The critical field relatively low for type I superconductors. The critical field relatively low for type I superconductors. They would generate fields of about 0.01 tesla to 0.2 tesla.

Type II or Hard: In type II superconductors the transition to a completely normal state occurs gradually. For applied field below  $H_{c1}$  the material is diamagnetic and hence field is completely repelled,  $H_{c1}$  is called lower critical field. At  $H_{c1}$  the flux begins to penetrate the specimen and penetration increases until  $H_{c2}$  is reached. At  $H_{c2}$  the magnetization vanishes and the specimen becomes normal.  $H_{c2}$  is called upper critical field. Thus the magnetization of type II superconductor vanishes gradually as the field is increased but in type I magnetization vanishes suddenly.

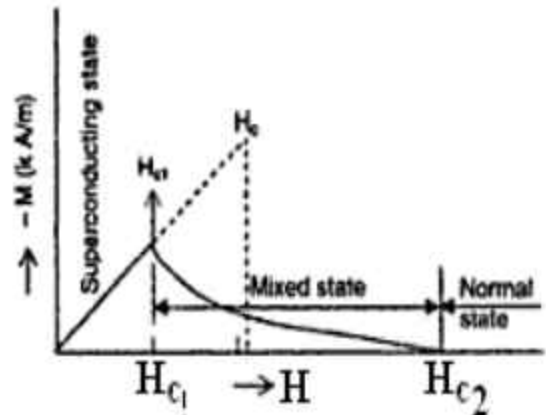


Fig b

In type II superconductor due to arising of super currents, the magnetic field can flow not only over the surface of a conductor but also in its bulk. Between  $H_{c1}$  and  $H_{c2}$  the state of the material is called mixed state or vortex state. In this state even though the material has zero resistance, flux penetration takes place via normal filaments of the material (normal filaments mean magnetic field is wrapped around the +ve ions of the material.) As the field strength increased these normal filaments no. increases. At  $H=H_{c2}$  these normal filaments spread over the entire body and the whole material changes over to the normal state.

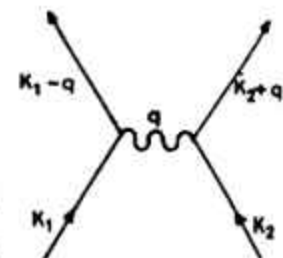
*Niobium, silicon, vanadium are the examples for type II.*

## BCS THEORY OF SUPERCONDUCTIVITY:

The fruitful approach to a microscopic theory of superconductivity has been given by

Bardeen, cooper, Schrieffer in 1957. The basic information behind the BCS theory is isotopic effect and specific heat of superconductor.

Suppose an electron with wave vector  $K_1$  approaches a positive ion core, it suffers attractive coulomb interaction. Due to this attraction ion core is set into vibration and consequently distorts the lattice. This





distortion in the lattice then travels away as mechanical wave called phonon  $q$ . In this process electron 1 loses its momentum to lattice. So momentum of first electron decreases. Another electron with wave vector  $K_2$  absorbs energy from the phonon and gains momentum. Thus two electrons interact via the mechanical wave phonon and these two electrons pair up by means of phonon called cooper pair.

This interaction is called electron-lattice-electron interaction. This interaction is strongest when the two electrons have equal and opposite momenta and spins. As a result of interaction, we have two electrons with wave vectors  $K_1 - q$  and  $k_2 + q$  shown in fig.

Explanation for superconductivity: when current flows in a normal conductor total linear momentum of free electrons is transferred to the lattice via electron-ion collisions.

The electrical resistivity of superconductor is zero because cooper pairs lose no energy to the lattice due to no collisions between the cooper pair and lattice. All cooper pairs move with identical velocity in superconducting state.

When energy is supplied to the superconductor these cooper pairs gain enough energy and break up. So material offers resistivity as the temperature increases.

BCS theory gives two important results, namely existence of *energy gap* and *flux quantization*.

## FLUX QUANTIZATION:

Superconductor in the form of a ring is placed in the magnetic field, the magnetic flux that passes through the ring is the sum of the flux from the applied field and the flux from the superconducting currents which flow in the ring. However the total flux is quantized.

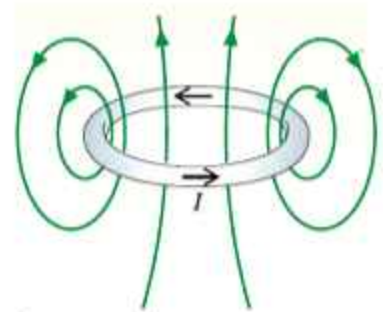
Flux enclosed by a superconducting ring  $\Phi$  is given by

$$\Phi = n \frac{h}{2e} \quad n = 1, 2, 3, \dots$$

or  $\Phi = n\Phi_0$

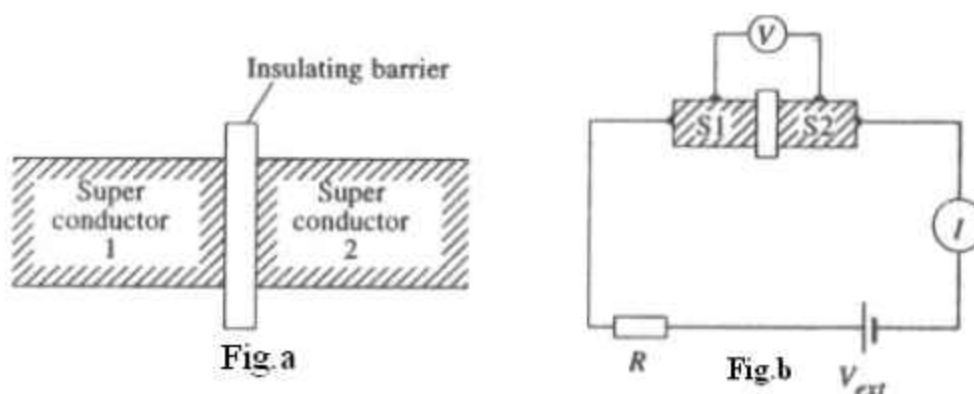
where  $\Phi_0 = \frac{h}{2e}$  is called **fluxon**. Its value is  $\Phi_0 = 2.07 \times 10^{-15}$  weber

Here ' $2e$ ' is charge of an electron pair. It confirms that the existence of electron pair in superconductor.





## JOSEPHSON EFFECT:



Consider a sandwich arrangement consisting of two superconducting metal films separated by a thin oxide layer of  $10\text{-}20\text{\AA}$  thick. This arrangement is known as Josephson's junction.

### *D.C. Josephson effect:*

When no external voltage is applied to the sandwich arrangement, a direct current flows through the superconductor Josephson junction and the potential difference at its ends is zero. That is voltmeter shows no voltage drop. This is known as *D.C. Josephson effect*. This is due to tunneling of cooper pairs from one film of the superconductor into the other through the narrow insulator.

The current through the junction is given by

$$I = I_c \sin \phi \text{ ----- (1)}$$

Where  $\phi$  is the phase difference between the wave functions describing cooper pairs on both sides of barrier.

$I_c$  is the critical current which the junction can support, it depends on the thickness and width of the insulation layer. It is usually very small.

### *A.C. Josephson effect:*

When  $I$  exceed  $I_c$ , large currents through the junction give rise to a potential difference  $V$  between the superconducting films. And an electromagnetic radiation appears across the junction. In this case the energies of cooper pairs on both sides of the barrier differ by  $2eV$ . It causes a frequency difference  $\nu = E/h = 2eV/h$  and phase difference between the two waves is

$$\phi = 2\pi\nu t = 2\pi t \left( \frac{2eV}{h} \right) \text{ ----- (2)}$$

From (1) and (2)  $I = I_c \sin\left(\frac{4\pi e V t}{h}\right)$  ----- (3)

This current is alternating current. Thus when a d.c. voltage exists across a Josephson junction, an A.C. current is produced. This is known as **A.C. Josephson** effect.

At  $V=1\mu\text{V}$ , a.c current 483.6 MHz is produced.

#### **Applications of Josephson Effect:**

1. Josephson effect is used to generate microwave radiation with frequency  $\omega = \frac{2eV}{h}$
2. A.C Josephson effect is used to define standard Volt.
3. A.C Josephson effect is also used to measure very low temperature based on the variation of frequency of the emitted radiation with temperature.
4. A.C Josephson effect is used for switching of signal from one circuit to other. The switching time is the order of 1ps and hence very useful in high speed computers.

#### **APPLICATIONS OF SUPERCONDUCTIVITY:**

1. *Transmission losses can be reduced:* when the power is transmitted through the ordinary cables, 1/5 th of the power generated is lost due to  $I^2R$  losses.

When superconductors are used as cables, losses are avoided and electrical power transmission can be done at low voltage level.

2. *Size of the machines can be reduced:* superconducting coils in transformers and electrical machines generate much stronger magnetic fields than magnetic circuits of ferromagnetic materials. Therefore the size of motors and generators will be drastically reduced.

3. *High efficiency & low cost:* high magnetic fields are required in many areas of research and diagnostic equipments in medicine. The electromagnets are very big size, demand large electrical power and require continuous cooling.

Superconducting solenoids produce very strong magnetic fields. They are small in size and does not need power and less expensive.

4. *Magnetic levitation:* maglev coaches do not slide over steel rails but float on a four inch air cushion over a strongly magnetized track. Superconducting coils produce the magnetic repulsion in order to levitate the coaches. Because of no friction between rails and wheels more speed can be easily achieved.

5. *In switching circuits:* the transistor based switches can be operated at speeds of nano seconds. In contrast logic elements based on Josephson junction can operate at a speed of a few pico seconds.

6. *SQUIDS:* several medical diagnostic equipments are now employing SQUIDS: Superconducting Quantum Interference Devices which detect very minute changes in the magnetic field of human body.

#### 7. *Cryotron as switch:*

The cryotron is a switch that operates using superconductivity. The cryotron works on the principle that magnetic fields destroy superconductivity. This simple device consists of two superconducting wires (e.g. tantalum and niobium) with

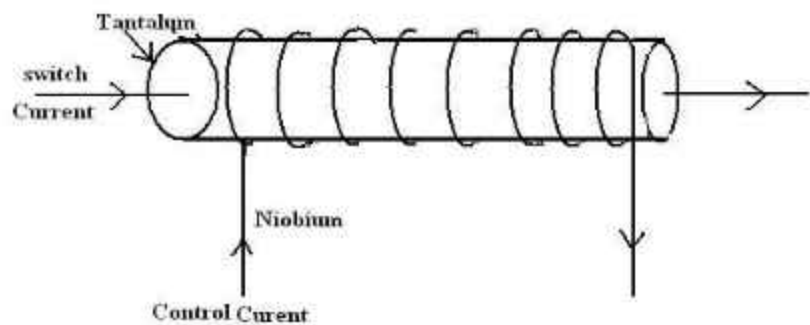


Fig. Cryotron

different critical temperature ( $T_c$ ). A straight wire of tantalum (having lower  $T_c$ ) is wrapped around with a wire of niobium in a single layer coil. Both wires are electrically isolated from each other.

When this device is immersed in a liquid helium bath both wires become superconducting and hence offer no resistance to the passage of electric current. Tantalum in superconducting state can carry large amount of current as compare to its normal state. Now when current is passed through the niobium coil (wrapped around tantalum) it produces a magnetic field, when this magnetic field reaches critical value, it kills the superconductivity of the tantalum wire and hence reduces the amount of the current that can flow through the tantalum wire. Hence one can control the amount of the current that can flow in the straight wire with the help of small current in the coiled wire. We can think tantalum straight wire as a "Gate" and coiled niobium as a "control".

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