Fuzzy Logic — UNIT : V

It is an approach based on degree of truth rather than true or false.

In Traditional logic i.e predicate & proposition logic every stmt has a Truth value i.e True or False

Traditional Logic

Fuzzy logic

Nani Honest
- → (1.0) extremely honest
- → (0.8) honest highly
- → (0.4) slight honest
- → (0.0) extremely dishonest

Traditional logic

Nani Honest
- → True
- → False

Fuzzy set.

A regular set is nothing but collection of related items.

Ex: fruits = {apple, banana, Orange}

Here apple, banana, Orange are the elements of set fruits

Fuzzy set is a collection of membership and its degree of truth.

Ex: $A = \left\{ \frac{0.4}{train} + \frac{0.6}{car} + \frac{0.7}{cycle} \right\}$

membership nothing but elements in a fuzzy set

Here membership-train slightly belongs to set A

membership car highly belongs to set A
membership cycle extremely belongs to set A

Linguistic variable

Linguistic variable is a variable which store words are called linguistic variables.

Ex: Age = {child, Young, Old}

The formal definition of a linguistic variable is

$(x, T(x), U, G, M)$

x : Variable Name

$T(x)$: set of values x takes

U : All values a variable can take

G : Syntactic rules

M : Semantic rules

Ex: Age = {Child, Young, Old}

~~x-child, Young~~

x : Age

$T(x)$: Child, Young, Old

U: Set of Ages of all people

M: Give meaning to each term

$M_{child}(x) : (x \leq 15)$

$M_{young}(x) : (15 \leq x \leq 55)$

$M_{old}(x) : (50 \leq x \leq 100)$

Fuzzy propositions

Stmt: Ram is a boy

$T(stmt) = 0.8$

Stmt = Ram is not a boy

$T(stmt) = 1 - 0.8$
$= 0.2$

Fuzzy proposition is nothing but a proposition with truth value. We can modify the given proposition and we can combine the given proposition

Ex: Q: Ram is intelligent

$T(Q) = 0.6$

and new proposition is Stmt and Q

i.e Stmt and Q $(\text{stmt} \wedge Q)$

Ram is boy and Ram is intelligence

$$T(\text{stmt} \wedge Q) = \min(T(P), T(Q))$$
$$= \min(0.8, 0.6)$$
$$= 0.6$$

Fuzzy set operations

There are two sets B_1 & B_2

The operations performed on fuzzy sets are Union, intersection, difference, complement

Union:

$$\underset{\sim}{B_1} \cup \underset{\sim}{B_2} = \text{Max}\left[M_A(x), M_B(x)\right]$$

Ex: $$B_1 : \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$B_2 : \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B_1} \cup \underset{\sim}{B_2} : \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

Intersection:

$$\underset{\sim}{B_1} \cap \underset{\sim}{B_2} : \min\left[M_A(x), M_B(x)\right]$$

$$\therefore \underset{\sim}{B_1} \cap \underset{\sim}{B_2} = \left\{ \frac{1}{1\cdot 0} + \frac{0\cdot 6}{1\cdot 5} + \frac{0\cdot 2}{2\cdot 0} + \frac{0\cdot 1}{2\cdot 5} + \frac{0}{3\cdot 0} \right\}$$

<u>complement</u>,

$$\overline{\underset{\sim}{B_1}} = (1 - \mu_{B_1}(x)) \qquad \overline{\underset{\sim}{B_2}} = (1 - \mu_{B_2}(x))$$

$$\overline{\underset{\sim}{B_1}} = \left\{ \frac{0}{1\cdot 0} + \frac{0\cdot 25}{1\cdot 5} + \frac{0\cdot 7}{2\cdot 0} + \frac{0\cdot 85}{2\cdot 5} + \frac{1}{3\cdot 0} \right\}$$

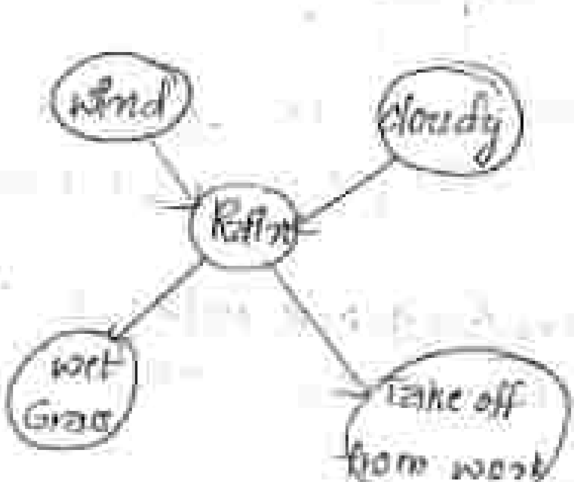
<u>Difference</u>

$$\underset{\sim}{B_1} / \underset{\sim}{B_2} = \underset{\sim}{B_1} \cap \overline{\underset{\sim}{B_2}}$$

$$\overline{\underset{\sim}{B_2}} = \left\{ \frac{0}{1\cdot 0} + \frac{0\cdot 4}{1\cdot 5} + \frac{0\cdot 8}{2\cdot 0} + \frac{0\cdot 9}{2\cdot 5} + \frac{1}{3\cdot 0} \right\}$$

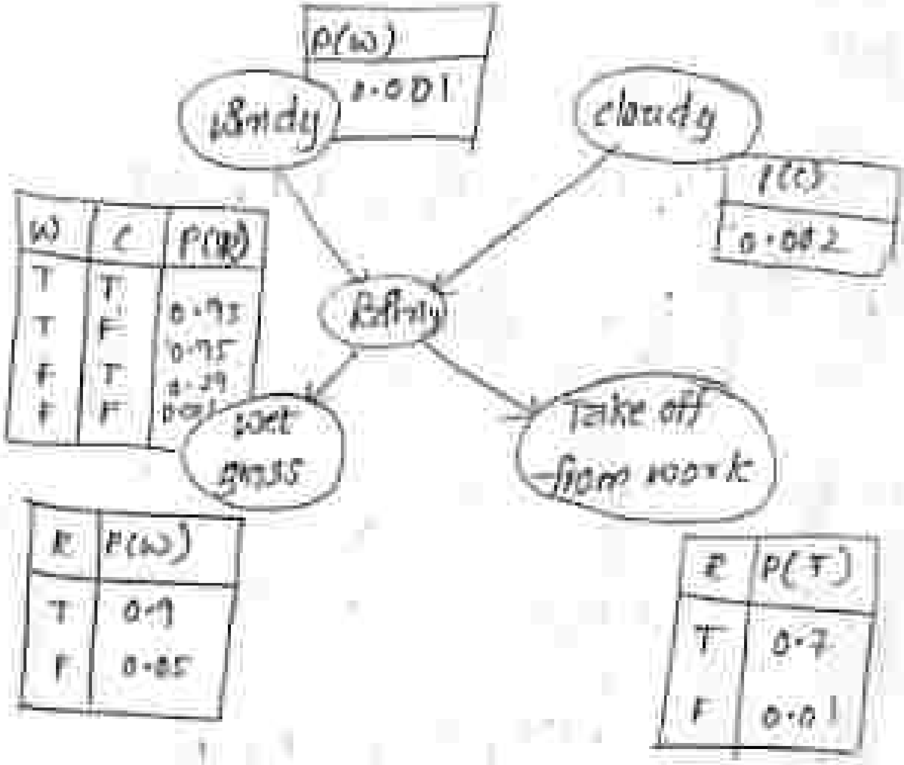
$$\underset{\sim}{B_1} \cap \overline{\underset{\sim}{B_2}} = \left\{ \frac{0}{1\cdot 0} + \frac{0\cdot 4}{1\cdot 5} + \frac{0\cdot 3}{2\cdot 0} + \frac{0\cdot 75}{2\cdot 5} + \frac{0}{3\cdot 0} \right\}$$

~~Bayesion~~ <u>Bayesian</u> <u>Belief</u> <u>Network</u>



It is a probabilistic graphical model depicts the conditional dependencies of a variable to a directed acyclic graph

Ex: From the given graph find out the propability of grass getting wet.



probability of getting wet $P(W) = P(W/R) * P(R) + P(W/\bar{R}) * P(\bar{R})$

$= 0.9 * P(R) + 0.05 * P(\bar{R})$

$P(R) = P(R/W, C) * P(W \wedge C) + P(R/\bar{W}C) * P(\neg W \wedge C) +$
$P(R/W\bar{C}) * P(W \wedge \neg C) + (R/\bar{W}\bar{C}) * P(\neg W \wedge \neg C)$

<u>Note</u>: $P(W \wedge C) = P(W) * P(C)$

$P(R) = 0.95 * 0.001 * 0.002 + 0.29 * (1-0.001) * 0.002 + 0.95 *$
$0.001 * (1-0.002) + 0.001 * (1-0.001) * (1-0.002)$
$= 0.00252$

$P(\bar{R}) = P(\bar{R}/WC) * P(W \wedge C) + P(\bar{R}/\bar{W}C) * P(\bar{W} \wedge C) + P(\bar{R}/W\bar{C}) *$
$P(\bar{W} \wedge \bar{C}) + P(\bar{R}/\bar{W}\bar{C}) * P(\bar{W} \wedge \bar{C})$

$= 0.99744$

$\therefore P(W) = 0.9 * 0.00252 + 0.05 * 0.99744$
$= 0.0521$

Dempster Shafer Theory

In this concept we use a term called 'plausibility' indicated by 'θ' and 'probability density' indicated by 'm'

Ex: θ contains elements allergy, flu, cold represented as below

$\theta = \{$Allergy, flu, cold$\}$

to diagnose the person has fever elements needed as flu, cold represented by 'm'

m = {flu, cold}

∴ m = 0.6

Here probability of $\theta = 1$

probability of m = 0.6

Here we have not use all the elements in the room θ

Dempster Shafer Theory gives the probability of unnecessary elements

from Ex: wastage = 1-0.6 = 0.4

Ex: For the diagnosis of running nose

m = {allergy, flu, cold}

m = 0.8

∴ wastage = 1-0.8 = 0.2

Certainity Factor

It gives the probability of given statement is true.

To calculate certainity factor we have two things

1. Measure of belief
2. Measure of disbelief

1. Measure of belief

H = Hypothesis
E = evidence

$$MB[H, E] = 0$$

This states that for provided evidence the hypothesis is false

$$MB[H, E] = 1$$

This states that for provided evidence the hypothesis is True

2. Measure of disbelief

$$MD[H, E] = 0$$

This states that for provided evidence the hypothesis is True

$$MD[H, E] = 1$$

This states that for provided evidence the hypothesis is False

Multiple evidences simple hypothesis

$MB[H, E_1 \text{ and } E_2] = MB[H, E_1] + MB[H, E_2] * [1 - MB(H, E_1)]$

$MD[H, E_1 \text{ and } E_2] = MD[H, E_1] + MD[H, E_2] * (1 - MD(H, E_1))$

$CF[H, E_1 \text{ and } E_2] = MB(H, E_1 \text{ and } E_2) - MD[H, E_1 \text{ and } E_2]$

Calculate the certainty factor for having the values given

$MB[H, E_1] = 0.4$, $MD[H, E_1] = 0$

$MB[H, E_2] = 0.3$, $MD[H, E_2] = 0.1$

$MB[H, E_1 \text{ and } E_2] = (0.4 + 0.3) * (1 - 0.4)$

$= \cancel{0.7 * 0.6} = 0.4 + 0.3 \times 0.6$

$= \cancel{0.42} \quad = 0.4 + 0.18$

$= 0.22$

$MB[H, E_1 \text{ and } E_2] = 0 + 0.1 * (1 - 0)$

$= 0 + 0.1 * 1$

$= 0 + 0.1$

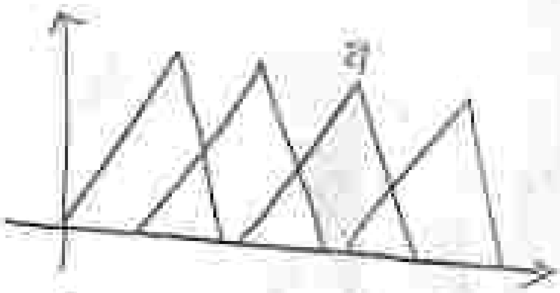
$= 0.1$

$CF = 0.22 - 0.1$

$= 0.21$

Types of Membership functions

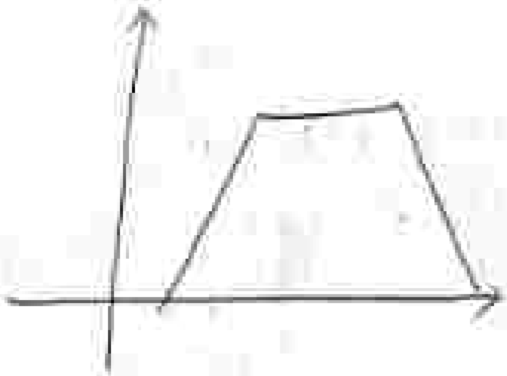
1. Triangle:



When we plot the membership

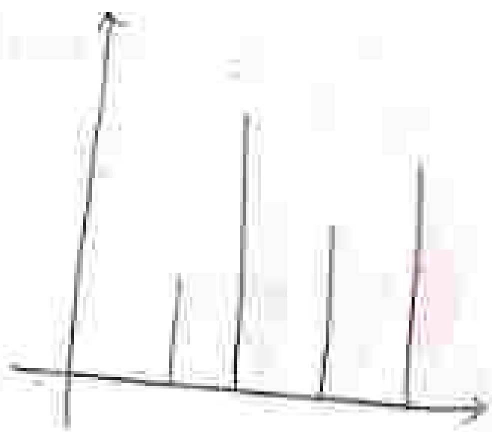
A Fuzzy set consists of membership with truth values When we plot all the truth values in a graph, If it represents as the graph drawn above then we call a fuzzy set of membership type triangle.

2. Trapizodial



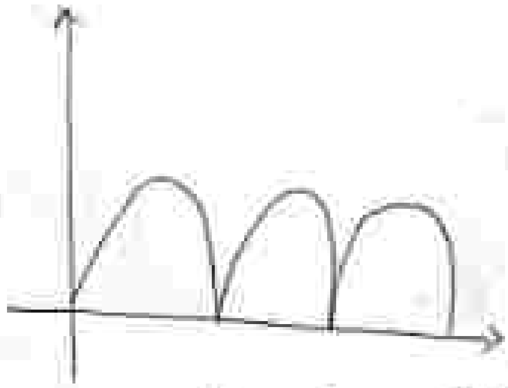
A Fuzzy set consists of membership with truth values when we plot all the truth values in a graph, if it represents as the graph shown above then we call a fuzzy set of member type trapizoidial.

3. Singleton



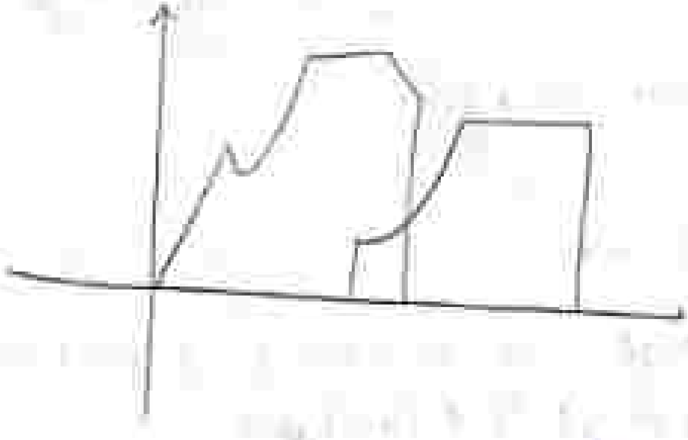
A fuzzy set consists of membership with truth values when we plot all the truth values in a graph, if it represents as the graph shown above then we call a fuzzy set of member type Singleton.

4. Gaussian.



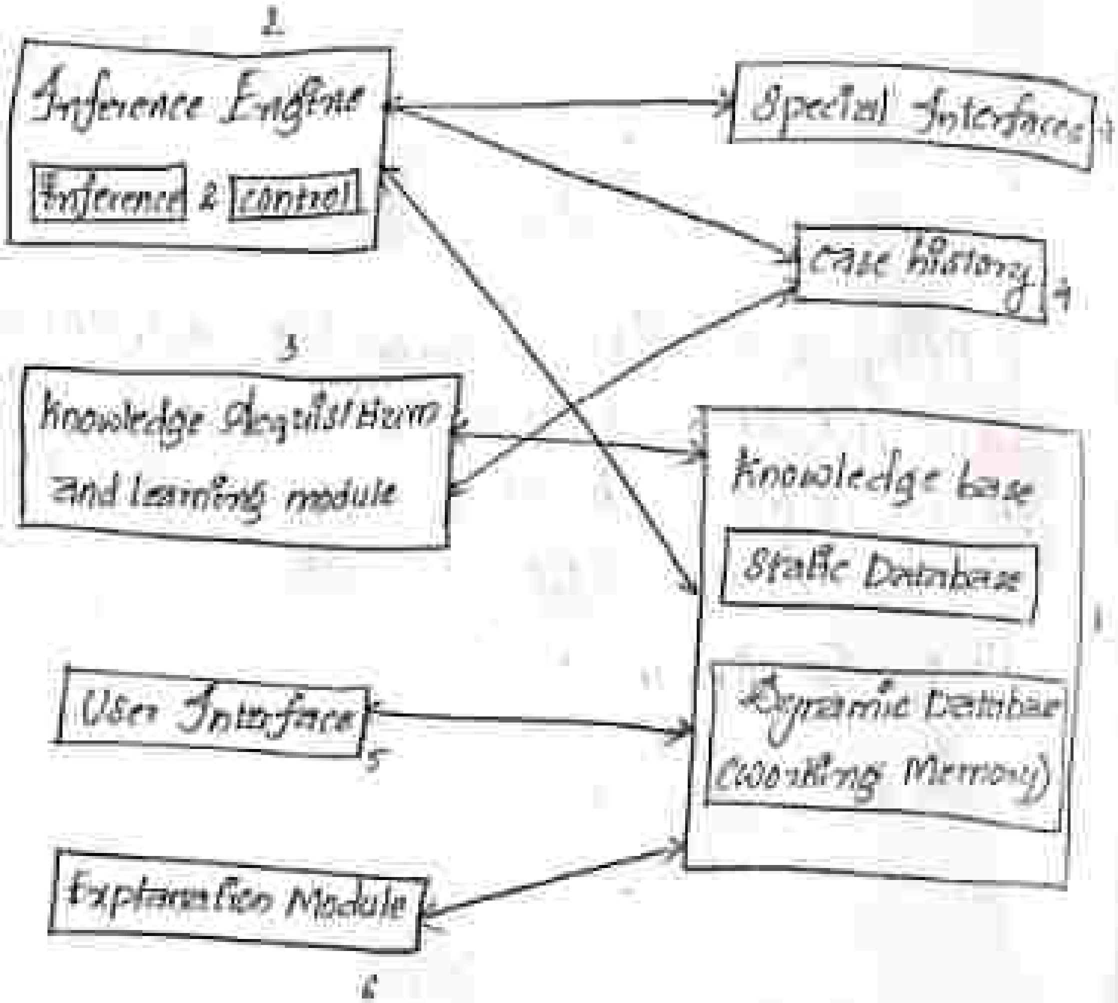
A fuzzy set consists of membership with truth values when we plot all the truth values in a graph. If it represents as the graph shown above then we call a fuzzy set of membertype Gaussian

5. Piece wise linear



A fuzzy set consists of membership with truth values when we plot all the truth values in a graph. If it represents as the graph shown above then we call a fuzzy set of membertype Piece wise linear.

Expert System Architecture



1. knowledge base

Knowledge base is nothing but database of the expert system. It is of two types.

i, Static DB

Prewritten programs or clauses are stored here

ii, Dynamic DB

While solving the problem, the intermediary results stored in dynamic DB

2. Inference Engine

It has two components

i, Inference

Bringing data from secondary to primary memory

ii, control

Responsibility of Applying the rules to solve the particular problem

3. knowledge Acquisition and learning Module:

Acquisition means addition. so add the knowledge in the system. we have to make the system learn

4. Special Interfaces

Delivery of the solution without complete Information

5. Case History

Statements converted in form of cases is stored here

6. User Interface

This module helps the communication between machine and human.

Ex: U: Hi

M: Hi

7. Explanation Module

This Module Explains the user why it comes to conclusion

M. Are you depressed

U: Why are you asking?

M: Your looks were depressed!

U: Yes

M: By seeing your looks, conclude that you are depressed.