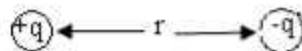


# UNIT-4

## DIELECTRICS

Dielectrics are nothing but insulators in which there are no free electrons for conduction. In dielectrics forbidden energy is greater than 6ev.

**Electric dipole:** The arrangement of equal and opposite charges separated by a distance is called electric dipole or dipole.



**Dipole moment:** Electric dipole moment is product of magnitude of charge and distance of separation between the two charges. If  $q$  is the magnitude of the charge and  $r$  is the distance of separation, the dipole moment is given by

$$\mu_e = q \times r \text{ coulomb-meter}$$

**Nonpolar dielectrics:** In an atom or molecule, if the center of gravity of positive charge coincides with the center of gravity of negative charge, the distance of separation between two charges is zero. The net dipole moment is zero ( $\mu_e = q \times 0 = 0$ ). Such a molecule is called nonpolar molecule and medium formed by these molecules is called nonpolar dielectric.

Ex.  $H_2$ ,  $N_2$ ,  $CO_2$ ,  $CH_4$ ,  $C_6H_6$  etc.

**Polar dielectrics:** when two or more atoms form a molecule and if the center of gravity of positive charge do not coincide with that of negative charge, molecule possess some permanent dipolemoment such molecule is said to be polar molecule. The medium formed by these molecules is called polar dielectric.

Example:  $H_2O$ ,  $HCl$ ,  $N_2O$ ,  $NH_3$ .....

### DIELECTRIC CONSTANT OR RELATIVE PERMITTIVITY $\epsilon_r$ :

Let us consider a parallel plate capacitor connected to voltage source  $V_0$ . Let the charges on the plates be  $+Q_0$  and  $-Q_0$ .

The capacitance of the capacitor when no medium is placed between the plates is

$$C_0 = \frac{\epsilon_0 A}{d} \text{ ----- (1) where 'd' is distance of separation between the plates and 'A' is area}$$

of the plate.

When the plates of the condenser are disconnected from the voltage source, the magnitude of charge  $Q_0$  on either plate must remain constant. When the dielectric is placed between the plates of the condenser, the potential difference decreases to a value  $V$  and capacitance of the condenser increases to ' $C$ '.

$\therefore C = \frac{\epsilon A}{d}$  where  $\epsilon$  is permittivity of the medium.

$$C = \frac{\epsilon_0 \epsilon_r A}{d} = \epsilon_r C_0$$

Dielectric Constant or relative permittivity  $\epsilon_r = \frac{C}{C_0}$

So dielectric constant is defined as the ratio of the capacity of the condenser with dielectric between the plates to the capacity of the condenser with air or vacuum in between the plates.

It is also defined as the ratio of permittivity of the medium to the permittivity of the air or free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \quad \text{where } \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m or } \text{C}^2/\text{N-m}^2 \text{ is permittivity of air or free space.}$$

### FIELD VECTORS IN DIELECTRICS:

1. Electric Field Intensity ( $\vec{E}$ ) 2. Dielectric Polarization ( $\vec{P}$ ) 3. Electric Displacement ( $\vec{D}$ )

1. Electric Field Intensity (E): Electric field intensity at any point in the electric field is defined as the force experienced by unit positive charge placed at that point. Let 'F' be the force acting on a charge 'q' then according to definition,

$$\vec{E} = \frac{\vec{F}}{q} \text{ N/coulomb}$$

The direction of 'E' is same as direction of  $\vec{F}$

2. Dielectric Polarization ( $\vec{P}$ ): The induced dipole moment per unit volume of the dielectric medium placed in the external field is called dielectric polarization  $\vec{P}$ .

$$\text{i.e. } \vec{P} = \frac{\mu}{V} \text{ ----- (1) where V is the volume of dielectric}$$

It is vector quantity whose direction is induced negative charge to induced positive charge.

But induced dipole moment is the product of induced charge and distance of separation between the charges.

If  $q^1$  and  $l$  are the induced charge and length of the dielectric (distance of separation),

$$\vec{P} = \frac{q^1 \times l}{A \times l} = \frac{q^1}{A} \text{ C/m}^2 \quad (\because V = \text{area} \times \text{length})$$

*Thus electric polarization is also equal to the induced charge per unit area*

**3. Electric Displacement ( $\vec{D}$ ):** Electric displacement is nothing but electric flux density. i.e. electric lines of force per unit area. Mathematically it can be written as real charge per unit area of the conducting surface. Let a charge 'q' be uniformly distributed on a conducting surface of area 'A',

$$D = \frac{q}{A} \text{ C/m}^2 \text{ or } q = D A.$$

In integral form  $q = \oint \vec{D} \cdot d\vec{s}$

The electric displacement is also equal to the product of absolute permittivity of the medium ( $\epsilon$ ) and resultant electric field intensity E

$$\text{i.e., } \vec{D} = \epsilon \vec{E}$$

### **Electric Susceptibility ( $\chi_e$ ):**

The electric polarization P is proportional to the electric field intensity E

$\therefore P \propto E$  or  $P = \epsilon_0 \chi_e E$  where  $\chi_e$  is proportionality constant called electric susceptibility

$$\chi_e = \frac{P}{\epsilon_0 E}$$

### **RELATION BETWEEN $\chi_e$ AND $\epsilon_r$ :**

Consider a parallel plate condenser of plate area 'A', let it be completely filled with a dielectric. Let the magnitude of the real charge on either plate is 'q' coulombs. Let a charge of magnitude 'q<sup>1</sup>' is induced on the dielectric faces as shown in figure.

Let 'E' be the electric field intensity. According to Gauss law in electrostatics,

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{q - q^1}{\epsilon_0} \text{ or}$$

$$E \cdot A = \frac{q - q^1}{\epsilon_0}$$

$$\Rightarrow \epsilon_0 E = \frac{q}{A} - \frac{q^1}{A} \text{ or } \epsilon_0 E = D - P$$

$$\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}} \text{ ----- (1) This is the relation between D, E and P}$$

Dividing the above equation by  $\epsilon_0 E$ , we get

$$\frac{\bar{D}}{\epsilon_0 \bar{E}} = 1 + \frac{\bar{P}}{\epsilon_0 \bar{E}}$$

$$\frac{\epsilon}{\epsilon_0} = 1 + \chi \quad \text{or} \quad (\because D = \epsilon E)$$

$$\epsilon_r = 1 + \chi \quad \text{or}$$

$\chi = \epsilon_r - 1$

----- (2)

**Polarizability ( $\alpha$ ):** Dipole moment is directly proportional to the electric field intensity.

$$\mu \propto E \quad \text{or} \quad \mu = \alpha E \quad \text{where '}\alpha\text{' is polarizability of the medium}$$

If medium possess 'N' molecules per unit volume, polarization

$$P = N\alpha E$$

#### INTERNAL FIELD OR LOCAL FIELD OF THE CUBIC DIELECTRICS:

Def: The electric field experienced by a dipole inside the dielectric medium is called local field or internal field  $E_{in}$ . It is different from the externally applied field.

Body of the dielectric between the condenser plates is shown in figure. We have to calculate the electric field experienced by a dipole at the center of the dielectric. Let us consider a spherical cavity with in the dielectric as shown in the figure. Polarized charges also appear on the surface of the sphere.

Consider a molecule of dielectric at the center 'C' of the dielectric. The dipole experiences the following fields in addition to the applied field.

$$E_{in} = E_1 + E_2 + E_3 + E_4$$

Where  $E_1$  is electric field at center 'C' due to the charges on the surface of condenser plates.

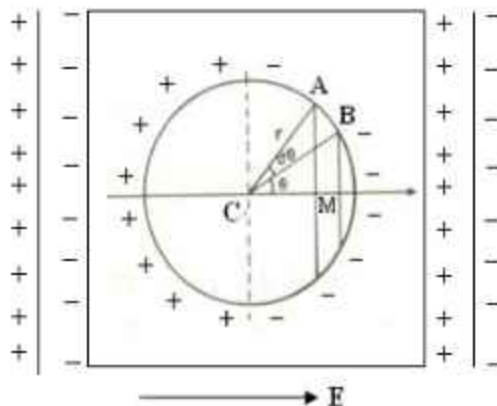
$E_2$  is electric field at center 'C' due to the charges on the surface of the dielectric

$E_3$  is electric field at center 'C' due to the charges on the surface of the sphere

$E_4$  is electric field at center 'C' due to the permanent dipoles inside the sphere.

But in our present case, nonpolar, isotropic dielectric, it is zero. i.e.  $E_4 = 0$

$$\therefore E_{in} = E + E_3 \quad \text{----- (1)} \quad (\because E_1 + E_2 = E, \text{ externally applied field})$$



Consider a small element of area 'ds' on the surface of the sphere making an angle 'dθ' with the center and 'θ' with the field direction. The polarization will be parallel to the electric field E.

The charge on the surface element is  $q^1 = P \cos \theta \, ds$  ( $\because p = \frac{q^1}{A}$ ,  $p \cos \theta$  is parallel component of polarization)

$$\begin{aligned} \text{Electric field intensity at center 'C' due to this charge is } dE_3 &= \frac{1}{4\pi\epsilon_0} \frac{q^1}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta \, ds}{r^2} \text{ ----- (2)} \end{aligned}$$

Where 'r' is radius of the sphere. This field intensity is along 'r'. Therefore parallel component of the electric field E will be  $dE_3 \cos \theta$

$$dE_3 = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta \, ds}{r^2} \cos \theta \text{ ----- (3)}$$

The area of the small surface element is  $ds = 2\pi(AB \times BM)$

$$\text{From fig. } \sin \theta = \frac{BM}{r} \text{ or } BM = r \sin \theta, \sin d\theta = \frac{AB}{r} \text{ or } d\theta \approx \frac{AB}{r} \text{ or } AB = r \, d\theta$$

$$\therefore ds = 2\pi r^2 \sin \theta \, d\theta$$

Substitute this value in equation (3), we get

$$dE_3 = \frac{P}{2\epsilon_0} \cos^2 \theta \sin \theta \, d\theta \text{ ----- (4)}$$

Electric field intensity at 'C' due to the charges on whole sphere will be

$$\begin{aligned} E_3 &= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta \, d\theta \\ &= \frac{P}{2\epsilon_0} \cdot \frac{2}{3} \quad (\because \text{put } \cos \theta = t, dz = -\sin \theta \, d\theta) \\ &= \frac{P}{3\epsilon_0} \end{aligned}$$

$$\therefore \text{Total internal field } \boxed{E_{in} = E + \frac{P}{3\epsilon_0}} \text{ ----- (5)}$$

### CLAUSIUS-MOSSOTTI EQUATION:

This equation gives the relation between the dielectric constant  $\epsilon_r$  and polarizability  $\alpha$

We know  $\chi = \epsilon_r - 1$  or

$$\frac{P}{\epsilon_0 E} = \epsilon_r - 1$$

$$P = \epsilon_0(\epsilon_r - 1) E \text{ ----- (1)}$$

$$\text{Internal field } E_{in} = E + \frac{P}{3\epsilon_0}$$

Substitute the 'P' value from equ.(1) in equ.(2)

$$\begin{aligned} E_{in} &= E + \frac{\epsilon_0(\epsilon_r - 1)}{3\epsilon_0} E \\ &= E + \left[ 1 + \frac{\epsilon_r - 1}{3} \right] E \\ &= E \frac{\epsilon_r + 2}{3} \text{ ----- (2)} \end{aligned}$$

Polarization is also proportional to the internal field  $E_{in}$ .

$$\therefore P = N\alpha E_{in} \text{ ----- (3)}$$

Where N is total no. of molecules in the dielectric,  $\alpha$  is polarizability

$$\epsilon_0(\epsilon_r - 1) E = N\alpha \frac{\epsilon_r + 2}{3} E \quad (\text{from equ. (1) and (2)})$$

$$\boxed{\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}} \text{ ----- (4) is called Clausius-Mossotti equation.}$$

Multiplying the above equation by  $\frac{M}{\rho}$ , where M is molecular weight and  $\rho$  is density.

$$\begin{aligned} \frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{M}{\rho} &= \frac{N\alpha}{3\epsilon_0} \frac{M}{\rho} \\ \frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{M}{\rho} &= \frac{\alpha N_A}{3\epsilon_0} \text{ ----- (5)} \quad \left( \because \text{number of molecules} = \frac{N_A \times \text{density}}{\text{Molecular Weight}} \right) \end{aligned}$$

Where  $N_A$  is Avogadro's number =  $6.023 \times 10^{26}/\text{kmol}$

The quantity  $\frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{M}{\rho}$  is called molar polarization of a dielectric.



## TYPES OF POLARIZATION:

1. Electronic polarization    2. Ionic polarization    3. Dipolar or orientation polarization

### Electronic polarization:

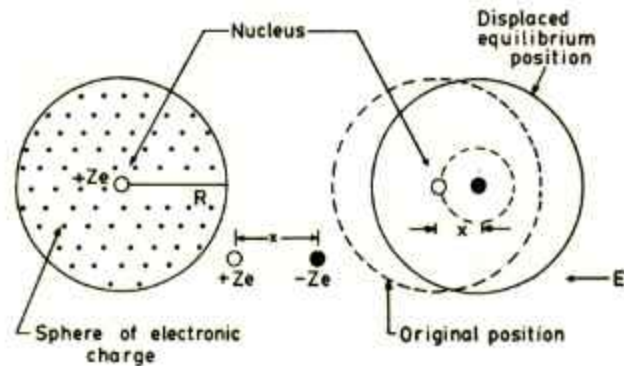
When an electric field is applied to the atom, electrons in the atom are displaced relative to the nucleus and produce dipole moment.

Polarization arises due to the displacement of electron cloud relative to the nucleus, with in the same atom is called electronic polarization.

### Electronic polarizability ( $\alpha_e$ ):

As shown in the figure  $+Ze$  is charge of nucleus is surrounded by an electronic cloud of charge  $-ze$  distributed in a sphere of radius ' $R$ '. Thus charge density of the electrons is *charge/volume*,

$$\rho = -\frac{ze}{\frac{4}{3}\pi R^3} = \left(-\frac{3}{4}\right) \frac{ze}{\pi R^3} \text{ ----- (1)}$$



When this system is subjected to an external field of intensity  $E$ , the nucleus and electron experiences a Lorentz force of magnitude  $ZeE$ . Therefore electron and nucleus pulled apart opposite direction and a Coulomb attraction force is developed between them. Let the displacement of electrons be ' $x$ '.

Thus Lorentz force =  $-ZeE$  and

$$\text{Coulomb force} = Ze \times \frac{1}{4\pi\epsilon_0} \left[ \frac{\text{charge enclosed in the sphere of radius } x}{x^2} \right]$$

The charge enclosed in the sphere of radius ' $x$ ' = charge density  $\times$  volume (with radius  $x$ )

$$\begin{aligned} &= \left(-\frac{3}{4}\right) \frac{ze}{\pi R^3} \times \frac{4}{3}\pi x^3 \\ &= -\frac{Zex^3}{R^3} \end{aligned}$$

$$\text{Hence coulomb force} = Ze \times \frac{1}{4\pi\epsilon_0 x^2} \times \left(-\frac{Zex^3}{R^3}\right) = -\frac{(ze)(ze)x}{4\pi\epsilon_0 R^3}$$

At thermal equilibrium the two forces are equal

$$\text{i.e. } -ze E = -\frac{(ze)(ze)x}{4\pi\epsilon_0 R^3} \quad \text{or} \quad E = \frac{Zex}{4\pi\epsilon_0 R^3}$$

$$x = \frac{4\pi\epsilon_0 R^3 E}{ze} \quad \text{This is expression for distance of separation between the two charges when electric field is applied.}$$

∴ Dipole moment  $\mu_e = \text{charge} \times \text{displacement}$

$$= ze \times \frac{4\pi\epsilon_0 R^3 E}{ze}$$

$$= 4\pi\epsilon_0 R^3 E$$

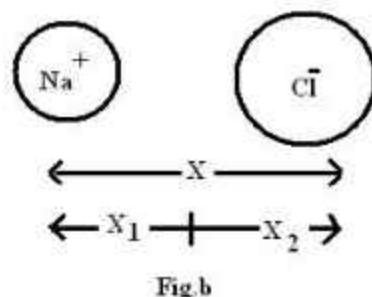
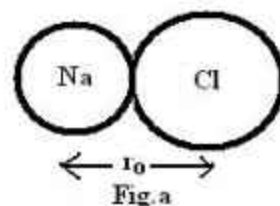
$$= \alpha_e E$$

Where  $\alpha_e = 4\pi\epsilon_0 R^3$  is called electronic polarizability

### Ionic polarization:

This polarization occurs in ionic bonding compounds. When an electric field is applied positive and negative ions displace in opposite directions causing a change in length of ionic bond. This effect of change in length causes to dipole moment. So polarization arises due to relative displacement of ions is called ionic polarization.

Let us consider  $m$  and  $M$  are the masses of the positive and negative ions respectively. When electric field  $E$  is applied on an ionic dielectric then positive ions displace in the direction of the applied field through  $x_1$  units of distance and negative ions displaced in opposite direction to the field through  $x_2$  units of distance as shown fig.b



$$\text{Hence net distance between two opposite ions } x = x_1 + x_2 \quad \text{--- (1)}$$

$$\text{Lorentz force acting on the positive ion} = eE \quad \text{--- (2)}$$

$$\text{Lorentz force acting on the negative ion} = eE \quad \text{--- (3)}$$

When ions are displaced in their respective directions from the mean positions, then the restoring force appears on the ions which tend to move the ions back to the mean positions.



∴ Restoring force acting on the positive ion =  $k_1 x_1$  ---- (4) where  $k_1$  force constant =  $m\omega_0^2$

Restoring force acting on the negative ion =  $k_2 x_2$  --- (5) where  $k_2$  force constant =  $M\omega_0^2$

At equilibrium position Lorentz force is equal and opposite to restoring force

$$\text{Hence } eE = k_1 x_1 \text{ or } x_1 = \frac{Ee}{m\omega_0^2} \text{ and } x_2 = \frac{Ee}{M\omega_0^2}$$

From equation (1)

$$x = x_1 + x_2$$

$$\begin{aligned} \therefore x &= \frac{Ee}{m\omega_0^2} + \frac{Ee}{M\omega_0^2} \\ &= \frac{Ee}{\omega_0^2} \left[ \frac{1}{m} + \frac{1}{M} \right] \end{aligned}$$

But dipole moment  $\mu$  = charge x displacement

$$\begin{aligned} &= e \frac{Ee}{\omega_0^2} \left[ \frac{1}{m} + \frac{1}{M} \right] \\ &= \frac{Ee^2}{\omega_0^2} \left[ \frac{1}{m} + \frac{1}{M} \right] \end{aligned}$$

We know that ionic polarizability  $\alpha_i = \frac{\mu}{E}$

$$= \frac{e^2}{\omega_0^2} \left[ \frac{1}{m} + \frac{1}{M} \right]$$

Hence we can say that ionic polarizability is inversely proportional to square of the angular

velocity  $\omega_0$  and reduced mass  $\left( \frac{mM}{m+M} \right)$ .

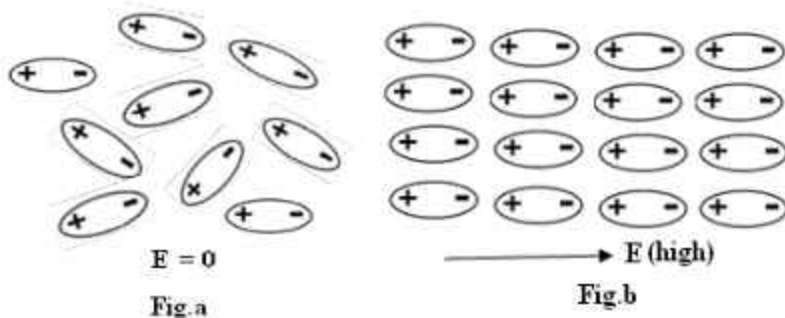
### Dipolar or Orientation Polarization:

This type of polarization only occurs in polar substances. In the absence of an external field the orientation of these dipoles is random. So that the net polarization is zero

When applied field is very strong these dipoles come to align. For ordinary fields these dipoles not come to align completely because the orientation of dipoles is continuously disturbed by temperature.

Anyway the dipole moment is induced when electric field is applied to polar molecules. This

polarization is known as dipolar polarization. This polarization is strongly temperature dependent.



$$\text{Polarizability } \alpha_0 = \frac{\mu^2}{3K_B T}$$

where  $\mu$  is the average dipole moment of all molecules.

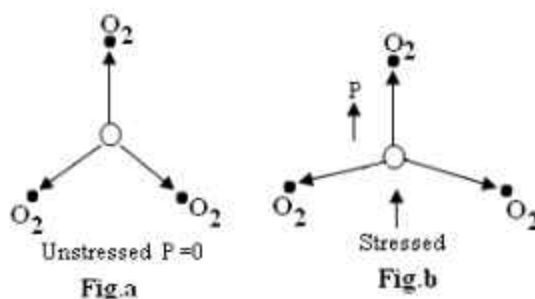
### **PIEZO ELECTRICITY:**

Definition: Certain crystals exhibit polarization when they are subjected to external stress are called piezo electrics and this property is called piezo electricity.

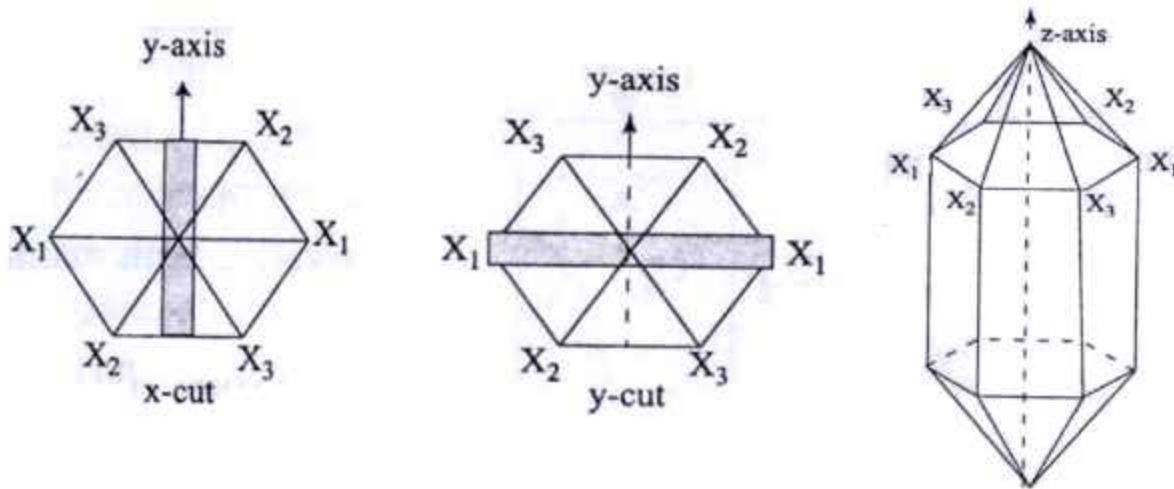
Converse is also true. i.e. when electric field is applied, strain is produced in the crystal. Generally very large electric fields are needed to produce very small strains.

Examples: Quartz.

When stress is applied to piezo electric crystals, the ions in the material are displaced the original charge distribution and causes increase in dipole moment as shown in figure (b).



### Piezo electricity in Quartz crystal:



Natural quartz crystal has the shape of hexagonal prism with a pyramid attached to each end. Fig. a shows the shape of the quartz crystal, the line joining the apex points of pyramids is called *optical axis* or *z-axis*. Optic axis has three fold symmetry. The three lines joining opposite corners are called *electrical axes* or *x-axes*. Similarly the three lines which are joining opposite sides constitute *mechanical axes* or *y-axes*.

The three electrical and mechanical axes are at right angles to the optic axis and at  $120^\circ$  to each other. Thin plates of quartz crystal cut perpendicular to one of its x-axis are known as x-cut planes. Similarly, thin plates of the crystal cut perpendicular to one of its y-axis are known as y-cut planes.

Let us consider an x-cut crystal plate. Let its thickness be 't' and length 'l'. If an alternating voltage is applied to the crystal plate, alternating stresses and strains are set up both in its thickness and length. The frequency of the thickness vibrations is given by

$$f = \frac{m}{2t} \left[ \frac{Y}{\rho} \right]^{1/2} \text{ ----- (1)}$$

and frequency of the length vibration is given by

$$f = \frac{m}{2l} \left[ \frac{Y}{\rho} \right]^{1/2} \text{ ----- (2)}$$

where  $m = 1, 2, 3, \dots$  etc. stand for fundamental, first overtone, second overtone..... respectively

$Y$  is young's modulus and  $\rho$  the density of the crystal plate.

Applications:

1. Piezo electric crystals are used as transducers i.e.to convert electrical energy into mechanical energy and vice versa.
2. Piezo electric crystals are used in gram phone and guitar pick ups.
3. They are used to generate and detect SONAR waves.
4. They are used to generate high voltage and power sources.
5. Piezo electric sensors especially used in ultrasonic transducers for medical imaging and nondestructive testing (NDT).

## MAGNETIC PROPERTIES

The fundamental thing responsible for electric field or magnetic field is charge. The static electric field is associated with charge. But we do not have magnetic charge in nature. A moving charge gives rise to a magnetic field.

In every atom electrons are revolving around the nucleus with -ve charge in electronic orbits create current in orbit. This current creates magnetic field. So atom or molecule behaves as a magnet. So that it consist of magnetic moment.

In magnetizing substances the net magnetic moment is not zero, but in non magnetizing substance like wood the net magnetic moment is zero.

### MAGNETIC DIPOLE MOMENT ( $\mu_m$ ):

The arrangement of two equal and opposite charges separated by a distance is called magnetic dipole.

The magnetic moment of a magnet is defined as the product of length of magnet ( $2l$ ) and its pole-strength ( $m$ )

$$\text{Magnetic moment } \mu_m = 2l \times m \quad \text{amp-m}^2$$

It is vector quantity. Its direction is south to North Pole.

**Torque ( $\tau$ ):** Torque is defined as a couple acting on the bar magnet when placed at right angles to the direction of uniform magnetic field. Because of this torque magnetic dipole rotates and finally comes along the direction of field.

$$\tau = \mu_m B \sin \theta \quad \text{where } B \text{ is magnetic induction}$$

Bar magnets, current loops, current carrying coils etc., all experience a torque in magnetic field and can be regarded as magnetic dipole.

### MAGNETIC INTENSITY OR MAGNETIC FIELD STRENGTH $H$ :

Magnetic field intensity at any point in the magnetic field is defined as the magnetizing force experienced by a unit North Pole placed at that point. It is measured in **Amp/m**. It is a vector quantity. It is independent of the medium.

### MAGNETIC INDUCTION OR MAGNETIC FLUX DENSITY $B$ :

The total no. of magnetic lines of force passing through the unit cross sectional area of the substance due to both magnetizing field and induced magnetism is called magnetic flux density.

$$B = \frac{\phi}{A} \quad \text{It is vector quantity. It is measured in N/amp-m or wb/m}^2 \text{ or Tesla.}$$

### INTENSITY OF MAGNETIZATION I or M:

When a magnetizing substance is placed in an external magnetic field, the two ends of the substance acquire equal and opposite pole strength, hence substance possess some magnetic moment.

Def: The induced magnetic moment per unit volume of the substance when it placed in an external magnetic field is called intensity of magnetization. It is vector quantity and is measured in amp/m.

According to definition  $I = \frac{\text{Induced magnetic moment}}{\text{Volume}}$

$$I = \frac{2lm}{A \cdot 2l} = \frac{m}{A} \text{ amp/m}$$

So intensity of magnetization is defined as the induced pole strength per unit area of the substance when it is placed in an external magnetic field.

### MAGNETIC PERMEABILITY $\mu$ :

Permeability of a medium is measure of the conducting power of the magnetic lines of force through that medium.

Consider a magnetizing substance is placed in uniform magnetic field. The magnetic lines of force passing through unit area in a substance is directly proportional to the field strength H. Therefore

$$B \propto H$$

$$B = \mu H$$

Where  $\mu$  permeability of medium

$$\mu = \frac{B}{H}$$

Permeability is the ratio of magnetic induction to field strength.

If the material is placed in air or free space then the above equation can be written as

$\mu_0 = \frac{B_0}{H}$  where  $B_0$  is flux density in air or free space,  $\mu_0$  is permeability of air or free space,  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

$\mu_r = \frac{\mu}{\mu_0}$  is called relative permeability. It is the ratio of permeability of medium to permeability of air or free space.

For air and non magnetic materials  $\mu_r = 1$ .



### MAGNETIC SUSCEPTIBILITY $\chi$ :

The magnetic susceptibility of the specimen gives *how free the specimen can be magnetized*. The intensity of magnetization  $I$  or  $M$  of the substance is directly proportional to the magnetic intensity  $H$ .

$$\text{i.e., } M \propto H$$

$$\text{or } M = \chi H$$

where  $\chi$  is proportionality constant known as magnetic susceptibility.

$$\chi = \frac{M}{H} \text{ dimension less quantity.}$$

### RELATION BETWEEN RELATIVE PERMEABILITY $\mu_r$ AND SUSCEPTIBILITY $\chi$ :

When a material is placed in an external magnetic field, the total flux is sum of the flux in air or free space produced by external field and flux in free space produced by magnetization of the material.

$$B = B_0 + B_i$$

$B_0$  = flux in air or free space due to external field =  $\mu_0 H$

$B_i$  = flux in air or free space due to induced magnetism =  $\mu_0 I$

$$B = \mu_0 [H + M] \text{ ----- (1)}$$

This is the relation between  $B$ ,  $H$  and  $I$ .

Divide the above equation by  $H$ , we get

$$\frac{B}{H} = \mu_0 \left[ 1 + \frac{M}{H} \right]$$

$$\mu = \mu_0 (1 + \chi)$$

$$\frac{\mu}{\mu_0} = 1 + \chi$$

$$\mu_r = 1 + \chi \text{ ----- (2)}$$

### ORIGIN OF MAGNETIC MOMENT:

In general the magnetic moment of atom originate from 3 sources:

- 1) orbital motion of electrons
- 2) spin motion of electrons
- 3) nuclear spin

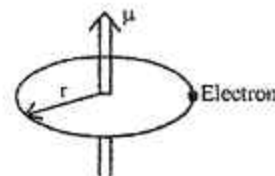
## ORBITAL MOTION OF ELECTRONS: BHOR MAGNETON:

The motion of the electron constitutes a current and the circular path of electron is identical to current loop. Such a current loop behaves as an elementary magnet having magnetic moment. The total orbital magnetic moment of magnetic moment of an atom is sum of orbital magnetic moments of individual electrons.

Let us consider an atom in which there is single electron revolving around the positive nucleus. The magnetic moment possessed by an electron which is revolving around the nucleus is given by

$$\mu_m = I \times A \text{ ----- (1)}$$

Where I is current in the circular loop of orbit. A, area enclosed by the circulating current.



We know  $I = \frac{ev}{2\pi r}$ , r is the radius of the circular orbit ( $I = q/t$ , t = distance/speed)

From equ(1),  $\mu_m = \frac{ev}{2\pi r} \pi r^2$

$$= \frac{evr}{2} \text{ ----- (2)}$$

The angular momentum of the electron in the circular orbit of radius r is  $L = mvr$  or  $r = L/mv$

From equ(2), we get  $\mu_m = - \frac{eL}{2m} \text{ ----- (3)}$

The negative sign indicates that the dipole moment is opposite to the vector representing the angular momentum.

The constant  $\frac{e}{2m}$  is called *gyromagnetic ratio*. It is the ratio of magnetic moment to angular momentum.

According to the modern atomic theory the angular momentum of electron in the orbit is determined by the orbital quantum number l which is restricted to a set of values  $l = 0, 1, 2, \dots (n-1)$ . Where n is principal quantum number. Which determine the energy of orbit. It can accept only integers.

The angular momentum of electron associated with a particular value of 'l' is  $\frac{lh}{2\pi}$

$$\therefore \mu_m = - \left( \frac{e}{2m} \right) \frac{lh}{2\pi}$$

$$= - \left( \frac{eh}{4\pi m} \right) l$$

$$= -\mu_B l \text{ ----- (4)}$$

The quantity  $\mu_B = \left( \frac{eh}{4\pi m} \right)$  is called Bohr magneton and has value  $9.27 \times 10^{-24} \text{ A-m}^2$

Bohr magneton is a fundamental unit of atomic magnetic moment. Electron possess the magnetic moment not less than Bohr magneton.

For filled electronic shells the total angular momentum is zero.

$\therefore$  Atoms or ions which are having only filled shells have no permanent magnetic moment, they are dia magnetic.

### Spin motion of electron:-

The spinning electron also associated with a magnetic moment, which is given by the relation  $\mu_{es} = \gamma \left( \frac{e}{2m} \right) S$ , where  $\gamma$  is spin gyromagnetic ratio depends on the structure of the spinning particle, experimental value of  $\gamma = -2.0024$ , the negative sign indicates that  $\mu_{es}$  is opposite to that of  $S$  in direction. Since  $S = \frac{h}{4\pi}$  for electron,  $\mu_{es} = 9.24 \times 10^{-24} \text{ A-m}^2$

**Nuclear Spin:** - The magnetic moment of the nucleus is given by

$$\mu_{ps} = \frac{eh}{4\pi M_p} = 5.05 \times 10^{-27} \text{ A-m}^2 \text{ where } M_p \text{ represents the mass of proton}$$

which is nearly 1/2000 as much as that of an electron.

$\therefore$  The magnetic moment due to the nuclear spin is neglected.

The two factors namely orbital motion and spin motion of electron are contributed to the permanent magnetic moment in atoms.

### **CLASSIFICATION OF MAGNETIC MATERIALS BASED ON ATOMIC MAGNETIC MOMENT:**

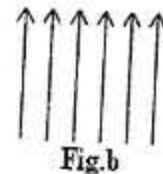
The materials may be classified on the basis of permanent magnetic moment in to five groups.

**Diamagnetic:** Materials composed of atoms or molecules having zero magnetic moment are called *diamagnetic*.

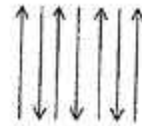
**Paramagnetic:** If the atomic magnetic dipoles are orient in random direction in the absence of external field, the material will be paramagnetic. The orientation of magnetic moments in paramagnetic substances is as shown in fig.



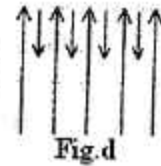
**Ferromagnetic:** If the individual dipoles of the material orient in the same direction, the material will be Ferro magnetic. The orientation of dipole moment in ferro magnetic materials is shown in fig.(b)



*Antiferro magnetic:* If the neighboring dipoles in the material orient in opposite direction to each other and with same magnitude, the materials are called antiferro magnetic. And their spin orientations as shown in fig.(c).



*Ferri magnetic:* If the neighbouring dipoles are orient anti parallel and the with unequal magnitudes, the materials are called ferri magnetic and their orientation is shown in fig.(d)



### DIAMAGNETIC MATERIALS:

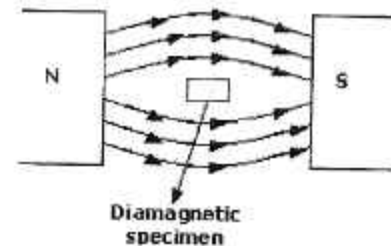
The materials which are magnetized in the direction opposite to the magnetic field are called diamagnetic materials. or

The materials which can not be magnetized in the external magnetic field are called diamagnetic materials.

Examples: All inert gases, hydrogen, air, water, gold, silver, bismuth etc.,

*The salient features are:*

1. Diamagnetic substances exhibit negative susceptibility. The value of susceptibility is small and is the order of  $10^{-6}$
2. As the diamagnetic susceptibility is negative, the relative permeability  $\mu_r$  is slightly less than unity.
3. When a small rod of diamagnetic material is placed in magnetic field it turns to a position perpendicular to the field lines. Diamagnetic materials pulled aside the field lines
4. The magnetic susceptibility of diamagnetic materials is independent of temperature.
5. The magnetization varies linearly with the applied field  $H$ , when the field is too strong.
6. when the diamagnetic substance is placed in nonuniform magnetic field, it move towards the weaker region of the field.



### LANZEVIN'S THEORY OF DIAMAGNETISM: *Susceptibility of diamagnetic materials*

When an electron is revolving in circular orbit, orbit acts as current loop. This current loop produces magnetic moment  $\mu$  which is opposite to the direction of the field.

Let  $\omega_0$  be the angular velocity of the electron. The magnitude of the magnetic moment  $\mu$  is given by

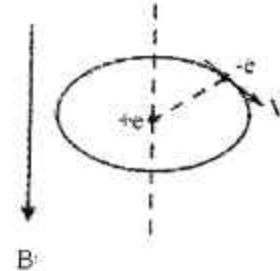
$$\mu = I \times A \text{ ----- (1)}$$

$$= \frac{ev}{2\pi r} \times \pi r^2$$

$$= -\frac{e\omega_0}{2\pi} \times \pi r^2$$

$$[\omega_0 = \frac{v}{r}]$$

$$= -\frac{1}{2} e\omega_0 r^2 \text{ ----- (2)}$$



According to Lenz's law electric field setup in the circular orbit

$$\oint E \cdot dl = -\frac{d\phi}{dt}$$

$$E \cdot 2\pi r = -\frac{d\phi}{dt}$$

$$E = -\frac{1}{2\pi r} \frac{d\phi}{dt} \text{ ----- (3)}$$

We know  $B = \frac{\phi}{A}$  or  $\phi = B \times A$

$$E = -\frac{1}{2\pi r} \frac{dB}{dt} \pi r^2$$

$$E = -\frac{r}{2} \frac{dB}{dt} \text{ ----- (4)}$$

We know  $F = ma = -eE$  or

$$ma = \frac{er}{2} \frac{dB}{dt} \text{ or}$$

$$a = \frac{er}{2m} \frac{dB}{dt}$$

$$dv = \frac{er}{2m} \frac{dB}{dt} dt \quad \text{or}$$

$$= \frac{er}{2m} dB \quad \text{----- (5)}$$

∴ The change in the angular velocity  $d\omega = \frac{dv}{r} = \frac{e}{2m} dB$

$$\int_{\omega_0}^{\omega} d\omega = \omega - \omega_0 = \frac{e}{2m} B$$

∴  $\omega = \frac{e}{2m} B$  ----- (6) This is increase in the angular velocity of the electron when magnetic field is applied to the atom.

The term  $\frac{e}{2m} B$  is called Larmour's frequency.

∴ From equ. (2)  $\mu = -\frac{1}{2} e\omega_0 r^2$

$$\text{The change in magnetic moment } d\mu = -\frac{e^2 r^2 B}{4m} \quad \text{----- (7)}$$

For spherically symmetric atom  $r_x, r_y, r_z$  be the average radii for all the electrons along the three axes then

$$r_0^2 = r_x^2 + r_y^2 + r_z^2 \quad \text{and}$$

$$r_x^2 = r_y^2 = r_z^2 = \frac{r_0^2}{3}$$

$$\text{for x and y axes } r^2 = r_x^2 + r_y^2 = \frac{r_0^2}{3} + \frac{r_0^2}{3} = \frac{2r_0^2}{3} \quad \text{----- (8)}$$

∴ from eqs (7) and (8), we get  $d\mu = -\frac{2e^2 r_0^2 \mu_0 H}{12m}$

If material contains N number of atoms per unit volume,

$$\chi_{dia} = -NZ \left[ \frac{e^2 r^2 \mu_0}{6m} \right] \quad \text{----- (9)} \quad \left( \because \chi = \frac{\text{Induced magnetic moment}}{\text{volume}} \right)$$

where z is atomic number,  $\mu_0$  is permeability of air or free space  
For diamagnetic materials the susceptibility is of the order of  $10^{-6}$ .



## PARAMAGNETIC MATERIALS:

Materials or substances which acquire weak magnetism in the direction of the field when placed in magnetic field are called paramagnetic materials.

Examples : oxygen, solutions of iron salts, copper chloride, chromium and platinum.

The properties are as follows:

1. paramagnetic materials exhibit +ve magnetic susceptibility, the susceptibility is of the order of  $10^{-6}$
2. The relative permeability  $\mu_r$  is slightly more than unity.
3. A paramagnetic material magnetized in the direction of field. Field lines are pulled towards the materials and penetrate through the material when it is placed in a magnetic field
4. The paramagnetic susceptibility is strong dependent on temperature. The susceptibility is inversely proportional to temperature. Thus

$$\chi_{\text{para}} = \frac{c}{T} \quad \text{where } c \text{ is curie's constant and relation is called curie's law.}$$

5. The magnetization M varies linearly with the applied field when the field is not too strong.
6. In no uniform field the paramagnetic substances are attracted towards stronger region of magnetic field.

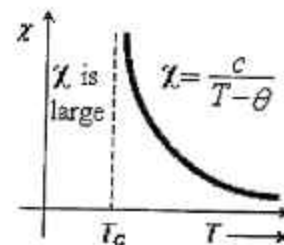
## FERRO MAGNETIC MATERIALS:

*Materials which are strongly magnetized in the direction of the field in external magnetic field are called ferromagnetic.*

Example: Iron, Nickel, Cobalt and some steels are examples.

The properties are:

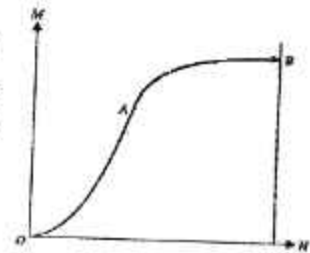
1. Ferro magnetic materials exhibit very high values of magnetic susceptibility and relative permeability. Susceptibilities are of the order of  $10^6$  and relative permeabilities are of the order of a few thousands.
2. When ferro magnetic material is kept in magnetic field, the field lines crowd in to the material.
3. As the temperature increases susceptibility decreases, above a certain temperature ferro magnetic material become ordinary paramagnetic and this temperature is called curie temperature.



Susceptibility follows the Curie's law.  $\chi = \frac{C}{T - \theta}$ , where  $\theta$  is paramagnetic Curie's temperature.

For  $T > \theta$  material transforms into paramagnetic state,  $T < \theta$  material is in ferromagnetic state.

4. Magnetization  $M$  varies non linearly with applied field  $H$ . As  $M$  varies non linearly with the applied field,  $\mu_r$  also increases with increase of field, beyond the saturation point permeability decreases rapidly as shown in fig.



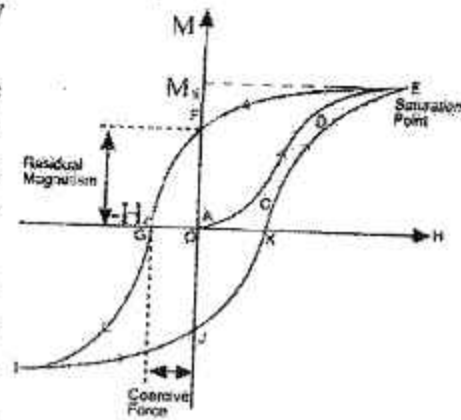
5. Ferromagnetic materials exhibit hysteresis.

When the ferromagnetic material is placed in magnetic field, magnetization increases first slowly, then more rapidly and finally attains a saturation value. Above the saturation value  $B$  or  $M$  does not vary with increase of  $H$ .

When the field  $H$  is switched off the curve does not retrace in its original path. At zero field there exists a residual field called remanent flux or retentivity. To bring back the magnetization to zero the magnetic field is applied in reverse direction which is called coercive field  $H_c$ .

As the field is increased in negative direction saturation is ultimately reached in reverse direction. Now the field is increased a closed loop is obtained called hysteresis loop.

When the magnetic field is applied in reverse direction, some magnetic energy is lost in the form of heat. This is known as hysteresis loss which is equal to the area of the hysteresis loop.



### DOMAIN HYPOTHESIS:

In order to explain why virgin sample of ferromagnetic material has no magnetic moment, Weiss postulated that *entire ferromagnetic material splits into a large no. of small regions of spontaneous magnetization. These regions are called domains.*

Every domain having a definite value and direction of the magnetic moment. In the absence of an external field the magnetic moment vectors are randomly oriented and net magnetic moment is zero. When magnetic field is applied the domains rotate and make to align their magnetic moments with the field direction. So specimen exhibits a net magnetization.

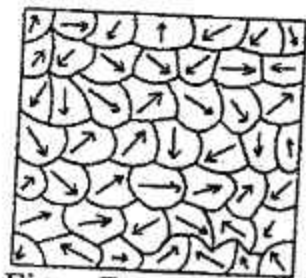
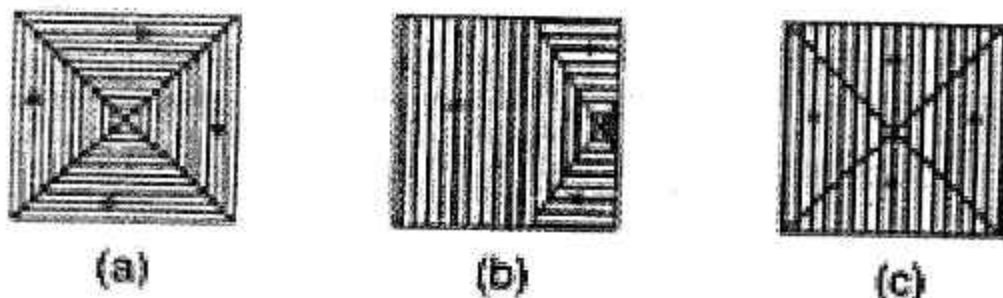


Fig. Ferromagnetic domains

How ever in the absence of external field the domains are randomly distributed. Hence the resultant magnetic moment in any direction is Zero. When a ferromagnetic material is placed in an external magnetic field, the magnetic moment (or) the magnetism of the substance increases in two different ways.

**By motion of domain walls:** Consider an unmagnetized specimen in which each domain is



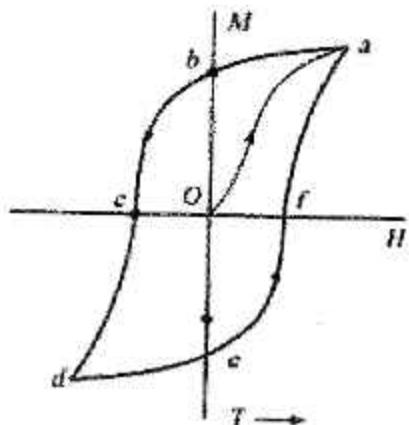
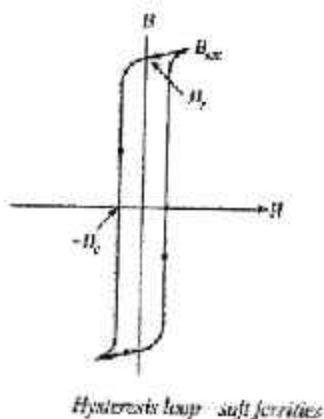
spontaneously magnetized and the direction of magnetism as shown in fig (a). The substance is placed in a magnetic field the domains increases in size while those are oriented opposite to the external field as shown in fig (b).

**By the rotation of domains:** When external magnetic field is weak, the substance is magnetized mostly by the motion of domain walls, on the other hand in strong magnetic field the magnetization takes place mostly by the rotation of domain. The domain rotates until their magnetic moments are oriented in the direction of external magnetic field. When the domain vector is oriented parallel to the direction of the field is called as saturation limit as shown in fig(c).

**Soft magnetic materials:** The materials which are easily magnetized & demagnetized are said to be soft magnetic materials

### **Properties:**

- ❖ Soft magnetic materials have low hysteresis loss due to small hysteresis loop area.
  - ❖ These materials have a large value of susceptibility & permeability.
  - ❖ The coercivity & retentivity are small.
  - ❖ The retentivity of these materials is very high and hence they have low eddy current loss.
  - ❖ These materials are free from irregularities.
- Ex: Iron - silicon alloy, Nickel – Iron alloys, Iron – cobalt alloys.



**Soft magnetic materials**

**Hard magnetic materials**

### Applications:

- soft magnetic materials are used in electrical equipment, magnetic cores of transformers.
- soft magnetic materials are used in high frequency rotating mechanics.
- soft magnetic materials are used in high frequency devices such as inductors & wide band transformers.

**Hard magnetic materials:** The materials which are very difficult to magnetize and demagnetize are said to be hard magnetic materials.

### Properties:

- Hard magnetic materials have large hysteresis loop due to large hysteresis loop area.
- These materials have low value of susceptibility & permeability.
- The coercivity & retentivity are large.
- The eddy current loss is very high.
- These materials have large amount of impurities.

### Applications:

- Hard magnetic materials are used as magnets for toys, compass needles.
- Hard magnetic materials are used in d.c meter magnets.

Ex: Carbon steel, tungsten steel, chromium steel alnico etc.