Proposition:

A proposition is a statement or declarative statement which is either true or False

Ex: $2+3=4$

The above sentence has a truth value of false. so, we called it as proposition or statement

Ex: $2\times4=8$

In the above sentence has a truth value of true. thus can be considered as statement or proposition

Ex: Robo can think like human

In the above there is an assumption but not declarative sentence either true or false. thus the above sentence can't be considered as statement.

Compound proposition

A compound propositions when more than one propositions are connected through various connectives is called compound propositions

Ex: $A \quad B$

$A \wedge B$

$A \vee B$

$A \rightarrow B$

$A \leftrightarrow B$

In the above A, B are propositions and all the rest are compound propositions. Here the connectives are $\wedge, \vee, \rightarrow, \leftrightarrow$.

Tautology:

A compound proposition that is always true no matter what the truth values of the proposition or variable that occurs in it is called Tautology

Ex:

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

Contradiction:

A compound proposition that is always false irrespective of the variable values occur in it called as contradiction

Ex:

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

Contigency:

A compound proposition that is neither a tautology nor a contradiction is called contigency.

Ex:

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional calculus

propositional calculus is the process where we use a set of rules to combine simple propositions to form compound propositions with the help of certain logic operators. The logic operators often called as connectives. Examples are $\vee, \wedge, \rightarrow, \leftrightarrow$

Note:

compound proposition is also known as formula or well-formed formula

Ex: $A \vee B$

$A \wedge B$

$A \rightarrow B$

$A \leftrightarrow B$

The above are 4 well-formed-formulas derived from simple propositions A, B using connectives such as $\vee, \wedge, \rightarrow, \leftrightarrow$.

Truth tables

It collaborates all the possible truth values of a formula

1. Negation

P	¬P
T	F
F	T

2. conjunction

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

4. Implies

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

5. Double Implies

p	q	p ↔ q
T	T	T
T	F	F
T	T	F
F	F	T

Equivalence laws

1. Identity laws

$p \wedge T = p$

$p \vee F = p$

2. Domination law

$p \vee T = T$

$p \wedge F = F$

3. Idempotent law

$p \vee p = p$

$p \wedge p = p$

4. Double negation law

$\neg(\neg p) = p$

$\neg(\neg q) = q$

5. Commutative law

$p \vee q = q \vee p$

$p \wedge q = q \wedge p$

6. Associative law

$(p \vee q) \vee r = p \vee (q \vee r)$

$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

7. Distributive law

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

8. Demorgans law

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

9. Absorption law

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

$q \vee (q \wedge p) \equiv q$

$q \wedge (q \vee p) \equiv q$

10. Negation law

$p \vee \neg p \equiv T$

$p \wedge \neg p \equiv F$

Logical Equivalences involving conditions as follows

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \sim q \to \sim p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

Proofs:

1. Identity law

p	T	F	$p \wedge T$	$p \vee F$
T	T	F	T	T
F	T	F	F	F

$\therefore p \wedge T = p$

$p \vee F = p$

2. Domination law

p	T	F	p∨T	p∧F
T	T	F	T	F
F	T	F	T	F

∴ $p \vee T = T$

$p \wedge F = F$

3. Idempotent law

p	p∨p	p∧p
T	T	T
F	F	F

∴ $p \vee p = p$

$p \wedge p = p$

4. Double negation law

p	q	¬p	¬q	¬(¬p)	¬(¬q)
T	T	F	F	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	F	F

∴ $\neg(\neg p) = p$

$\neg(\neg q) = q$

5. Commutative law

p	q	p∨q	p∧q	q∨p	q∧p
T	T	T	T	T	T
T	F	T	F	T	F
F	T	T	F	T	F
F	F	F	F	F	F

∴ $p \vee q = q \vee p$

$p \wedge q = q \wedge p$

4. Associative law

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F
T	F	T	T	T	T	F	F
T	F	F	T	T	T	F	F
F	T	T	T	T	T	F	F
F	T	F	T	T	T	F	F
F	F	T	F	T	T	F	F
F	F	F	F	F	F	F	F

$q \vee r$	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T
T	F	F
T	F	F
F	F	F
T	T	F
T	F	F
T	F	F
F	F	F

$\therefore \quad (p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

7. Distributive laws

p	q	r	$q \vee r$	$p \wedge q$	$q \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	T	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$(p \wedge r)$
T	T	T	T	T
T	T	T	T	F
T	T	T	T	T
T	T	T	T	F
T	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$\therefore\ p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

8. Demorgans law

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$\neg(p \wedge q)$	$\neg(p \vee q)$
T	T	F	F	T	T	F	F
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

$\therefore$

$\neg p \wedge \neg q$	$\neg p \vee \neg q$
F	F
F	T
F	T
T	T

$\therefore \neg(p \wedge q) = \neg p \vee \neg q$

$\neg(p \vee q) = \neg p \wedge \neg q$

9. Absorption law

p	q	$p \wedge q$	$p \vee q$	$p \vee (p \wedge q)$	$p \wedge (p \vee q)$	$q \wedge p$	$q \vee p$
T	T	T	T	T	F	T	T
T	F	F	T	T	F	F	T
F	T	F	T	F	F	F	T
F	F	F	F	F	F	F	F

$q \vee (q \wedge p)$	$q \wedge (q \vee p)$
T	T
F	F
T	T
F	F

$\therefore p \vee (p \wedge q) = p$

$p \wedge (p \vee q) = p$

$q \vee (q \wedge p) = q$

$q \wedge (q \vee p) = q$

10. Negation law

p	$\neg p$	T	F	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F	T	F
F	T	T	F	T	F

$\therefore p \vee \neg p = p$

$p \wedge \neg p = p$

Logical equivalences

1.

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore p \to q \equiv \neg p \vee q$

2.

p	q	$\neg p$	$\neg q$	$\neg q \to \neg p$	$p \to q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\therefore p \to q \equiv \neg q \to \neg p$

3.

p	q	$p \vee q$	$\neg p$	$\neg p \to q$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	F

$$p \vee q \equiv \neg p \to q$$

4.

p	q	$p \wedge q$	$\neg q$	$p \to \neg q$	$\neg(p \to \neg q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	F	T	T	F

$$\therefore p \wedge q \equiv \neg(p \to \neg q)$$

5.

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

$$\neg(p \to q) \equiv p \wedge \neg q$$

6.

p	q	r	$\overset{A}{p \to q}$	$\overset{B}{p \to r}$	$A \wedge B$	$q \wedge r$	$p \to q \wedge r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$\therefore (p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

7.

p	q	r	$p \xrightarrow{A} q$	$p \xrightarrow{B} r$	$A \vee B$	$q \vee r$	$p \to q \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

$\therefore (p \to q) \vee (p \to r) \equiv p \to q \vee r$

8.

p	q	r	$p \xrightarrow{A} r$	$q \xrightarrow{B} r$	$A \vee B$	$p \wedge q$	$p \wedge q \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

9.

p	q	r	$p \xrightarrow{A} r$	$q \xrightarrow{B} r$	$A \wedge B$	$p \vee q$	$p \vee q \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Natural detection System

Description	formula	comments
Theorem	from $A \wedge B$ infer $A \wedge (B \vee C)$	To be proved
Hypothesis (given)	$A \wedge B$	1
$E{:}\wedge(1)$	A	2
$E{:}\wedge(1)$	B	3
$I{:}\vee(3)$	$B \vee C$	4
$I{:}\wedge(2,4)$	$A \wedge (B \vee C)$	proved

Ex: prove that $A \wedge (B \vee C)$ is detected from $A \wedge B$.

from the above table see comment 2 $E\wedge(1)$ i.e elimination and ($\wedge$) symbol from comment 1 done in two ways

$$A \wedge B \Rightarrow A$$
$$A \wedge B \Rightarrow B$$

which are represented in comments 2 and 3

By comment 4 we write

$$I : \vee(3)$$

Here we introduce or in comment 3 i.e B and we adde, thus we get $B \vee C$

Now $I : \wedge(2,4)$

i.e introduce '$\wedge$' symbol b/w comments 2 and 4

i.e $2 \Rightarrow A$, $4 \Rightarrow B \vee C$ thus the result is

$$A \wedge (B \vee C)$$

Hence proved using the natural detection system

The rules of Natural detection system is shown in given table.

Rule name	Symbol	Rule	Description
Eliminating $\rightarrow$	$(E:\rightarrow)$	If $A_1 \rightarrow A, A_1$ then A	If $A_1 \rightarrow A$ and A_1 are true then A is also true. This is called modes pones rule
introducing $\leftrightarrow$	$(I:\rightarrow)$	If $A_1 \rightarrow A_2$; $A_2 \rightarrow A_1$ then $A_1 \leftrightarrow A_2$	If $A_1 \rightarrow A_2$ & $A_2 \rightarrow A_1$ are true then $A_1 \leftrightarrow A_2$ is also true
Eliminating $\leftrightarrow$	$(E:\leftrightarrow)$	If $A_1 \leftrightarrow A_2$ then $A_1 \rightarrow A_2$, $A_2 \rightarrow A_1$	If $A_1 \leftrightarrow A_2$ is true then $A_1 \rightarrow A_2$ & $A_2 \rightarrow A_1$ are true
Introducing $\sim$	$(I:\sim)$	If from A infer A_1 $A_1 \sim A$ is proved then $\sim A$ is proved	If from A (which is true) a contradiction proved then truth of $\sim A$ is also proved.
Eliminating $\sim$	$(E:\sim)$	If from $\sim A$ infer A_1 $A_1 \sim A_1$ is proved then A is proved	If from $\sim A$, a contradiction is proved then truth of A is also proved

Axiomatic System

As Without truth tables we can say two equations are same for this we use the method called Axiomatic System. Axiom means principle.

Axiomatic system uses three principles/Axioms stated below

Axiom 1: $\alpha \rightarrow (\beta \rightarrow \alpha)$

Axiom 2: $[\alpha \rightarrow (\beta \rightarrow \gamma) \rightarrow [(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)]]$

Axiom 3: $(\sim\alpha \rightarrow \sim\beta) \rightarrow (\beta \rightarrow \alpha)$

Description	Formula	comments
Theorem	$\{A \to B, B \to C\} \vdash (A \to C)$	proved
Hypothesis 1	$A \to B$	1
Hypothesis 2	$B \to C$	2
Instance of Axiom1	$(B \to C) \to [A \to (B \to C)]$	3
MP(2,3)	$[A \to (B \to C)]$	4
Instance of Axiom2	$[A \to (B \to C)] \to [(A \to B) \to (A \to C)]$	5
MP(4,5)	$(A \to B) \to (A \to C)$	6
MP(1,6)	$A \to C$	proved

problem Statement

From $\{A \to B; B \to C\}$ infer $(A \to C)$

The solution is shown in above table as explained in the below steps.

Step-1: write the problem statement

Step-2: write the equation to be derived from as components based on our intuition

Step-3: write the equation to be derived from as components based on our intuition

Step-4: Instance of Axiom1

Here Axiom1 is follows

$\alpha \to (\beta \to \alpha)$ assume

$\alpha = (B \to C)$, $\beta = A$

sub α, β in Axiom1, we have

$(B \to C) \to [A \to (B \to C)]$

Step-5: Apply Modus ponens rule on eq (2,3)

$$[A \rightarrow (B \rightarrow C)]$$

Step-6: Instance of Axiom 2

Here Axiom 2 is as follows

$$[\alpha \rightarrow (\beta \rightarrow \gamma) \longrightarrow [(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)]]$$

we know $\alpha = B \rightarrow C$, $\beta = A$, assume $\gamma = C$

$$[(B \rightarrow C) \longrightarrow (A \rightarrow C) \longrightarrow [(B \rightarrow C) \longrightarrow (B \rightarrow C) \rightarrow C]$$

By applying implies law the above equation is simplified to

$$[A \rightarrow (B \rightarrow C)] \longrightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)]$$

Step-7: Apply Modus ponens rule on eq (4,5)

we have $(A \rightarrow B) \longrightarrow (A \rightarrow C)$.

Step-8: Apply Modus ponens rule on eq (1,6)

we get $(A \rightarrow C)$

Hence proved

Semantic Tableau

Description	Formula	line number
Tableau root	$(A \wedge \sim B) \wedge (\sim B \rightarrow C)$	1
Rule 1(1)	$A \wedge \sim B$	2
	$\sim B \rightarrow C$	3
Rule 1(2)	A	4
Rule 6(2)	$\sim B$	5
	$\sim(\sim B)$ C (open)	6
	B	
	$\times$ (closed) $\{B, \sim B\}$	

Here Tableau means picture (or) image (or) representation.
Semantic Tableau means representing the given formula or equation in binary tree.

By using semantic Tableau rules, with a given formula as root

Eg: Construct a Semantic Tableau for formula $(A \wedge \sim B) \wedge (\sim B \rightarrow C)$

Ans: For solution to above table

Here we applied 3 Semantic Tableau to get the solution as follows:

Rule 1: $\alpha \wedge \beta$
$\quad\ |$
$\quad\ \alpha$
$\quad\ |$
$\quad\ \beta$

Rule 5: $\sim(\sim\alpha)$
$\quad\ |$
$\quad\ \alpha$

Rule 6: $\alpha \rightarrow \beta$
$\quad / \quad \backslash$
$\sim\alpha \quad \beta$

The Semantic Tableau rules are given below in the table.

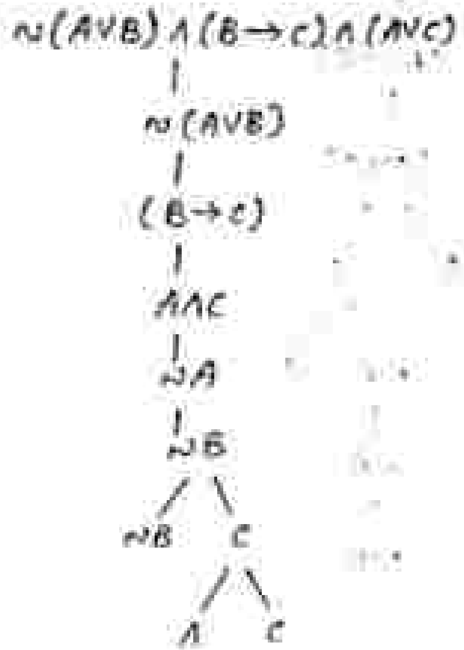
Rule no	Tableau tree	Explanation
Rule 1	$\alpha \wedge \beta$ is true if both α and β are true. $\alpha \wedge \beta$ $\vert$ α $\vert$ β	A tableau for the formula $(\alpha \wedge \beta)$ is constructed by adding both α and β to the same path
Rule 2	$\sim(\alpha \wedge \beta)$ is true either $\sim\alpha$ or $\sim\beta$ is true. $\sim(\alpha \wedge \beta)$ $/ \quad \backslash$ $\sim\alpha \quad \sim\beta$	A tableau for the formula $\sim(\alpha \wedge \beta)$ is constructed by adding two new path one containing $\sim\alpha$ and other containing $\sim\alpha$ and other containing $\sim\beta$

<table>
<tr><td>Rule 3</td><td>$\alpha \vee \beta$ is true if either α and β is true
$\alpha \vee \beta$
/ \
α β</td><td>A tableau for a formula $(\alpha \vee \beta)$ constructed by adding two new paths one containing α and other containg β</td></tr>
<tr><td>Rule 4</td><td>$\sim(\alpha \vee \beta)$ is true if both $\sim\alpha$ and $\sim\beta$ are the true
$\sim(\alpha \vee \beta)$
|
$\sim\alpha$
|
$\sim\beta$</td><td>A tableau for the formula $\sim(\alpha \vee \beta)$ is constructed by adding both $\sim\alpha$ and $\sim\beta$ to the same path.</td></tr>
<tr><td>Rule 5</td><td>$\sim(\sim\alpha)$ is true then α is true
$\sim(\sim\alpha)$
|
α</td><td>A tableau for $\sim(\sim\alpha)$ is constructed adding α on same path.</td></tr>
<tr><td>Rule 6</td><td>$\alpha \rightarrow \beta$ is true $\alpha \vee \beta$ is true
$\alpha \rightarrow \beta$
/ \
$\sim\alpha$ β</td><td>A tableau for a formula $\alpha \rightarrow \beta$ is constructed by adding two new paths one containing $\sim\alpha$ and other containing β.</td></tr>
<tr><td>Rule 7</td><td>$\sim(\alpha \rightarrow \beta)$ is true then $\alpha \wedge \sim\beta$ is true
$\sim(\alpha \rightarrow \beta)$
|
α
|
$\sim\beta$</td><td>A tableau for a formula $\sim(\alpha \rightarrow \beta)$ is constructed by adding both α and $\sim\beta$ to the same path.</td></tr>
<tr><td>Rule 8</td><td>$\alpha \leftrightarrow \beta$ is true $(\alpha \wedge \beta) \vee (\sim\alpha \wedge \sim\beta)$ is true
$\alpha \leftrightarrow \beta$
/ \
$\alpha \wedge \beta$ $(\sim\alpha \wedge \sim\beta)$</td><td>Tableau for $(\alpha \wedge \beta) \vee (\sim\alpha \wedge \sim\beta)$ is constructed by adding $(\alpha \wedge \beta)$, $(\sim\alpha \wedge \sim\beta)$ on two different path.</td></tr>
</table>

Satisfiability

Problem: Show that $S = \{\sim(A \vee B), (B \to C), (A \vee C)\}$ consistent

Now,



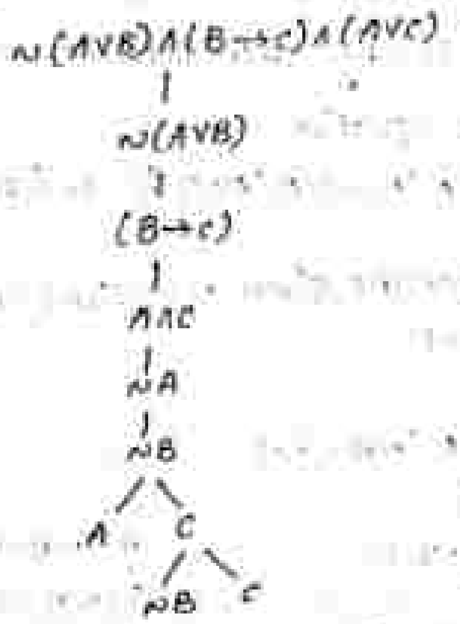
Path

∼A – ∼B – ∼B – opened

∼A – ∼B – C – A – closed

∼A – ∼B – C – C – opened

changing order:



Path

∼A – ∼B – A – closed

∼A – ∼B – C – ∼B – opened

∼A – ∼B – C – C – opened

$\sim(A \vee B) \wedge (B \rightarrow C) \wedge (A \vee C)$

$|$

$\sim(A \vee B)$

$|$

$(B \rightarrow C)$

$|$

$(A \vee C)$

$/ \quad \backslash$

$A \qquad C$

$/ \quad \backslash$

$\sim B \qquad C$

$|$

$\sim A$

$|$

$\sim B$

Path

A

$C - \sim B - \sim A - \sim B$

$C - C$

Consistency:

If the path in tableau derived from the given equation does not consists of any contradiction is called consistency

Thus the given equation

$\sim(A \vee B) \wedge (B \rightarrow C) \wedge (A \vee C)$ is consistent

Ex: check whether the given equations are consistent or inconsistent?

$(A \wedge \sim B) \wedge (\sim B \rightarrow C)$

$|$

$(A \wedge \sim B)$

$|$

$(\sim B \rightarrow C)$

$|$

A

$|$

$\sim B$

$/ \quad \backslash$

$B \qquad C$

$A - \sim B - B$ - closed

$A - \sim B - C$ - opened

consistent

as one is opened here

$(A \land B) \land (B \to \sim A)$

$|$

$(A \land B)$

$|$

$(B \to \sim A)$

$|$

A

$|$

B

$\sim B \qquad \sim A$

$A - B - \sim B - \times$ closed

$A - B - \sim A - \times$ closed

as complements are there in path, so inconsistent.

Logical Consequence:

A formula 'α' is said to be logical consequence of set S if and only if 'α' is tableau proved from 'S'.

Ex: Show that B is a logical consequence of $S = \{A \to B, A\}$

Sol: Here the root should start with $\sim$(element to be found) in the tableau of given formula.

$\sim B$

$|$

$A \to B$

$|$

A

$\sim A \qquad B$ found

Rule 9:	$\sim(\alpha \leftrightarrow \beta)$ is true. $(\sim\alpha \land \beta) \lor (\alpha \land \sim\beta)$ either are true. $\sim(\alpha \leftrightarrow \beta)$ branches to $(\sim\alpha \land \beta)$ and $(\alpha \land \sim\beta)$	A tableau for a formula $\sim(\alpha \leftrightarrow \beta)$ is constructed by adding two new paths one containing $(\sim\alpha \land \beta)$ & $(\alpha \land \sim\beta)$ in different paths.

Resolution Refutation in propositional logic

This method is used to derive any equation from the given equation.

Note: Inside the parenthesis of eqn we use only two symbols $(\vee, \sim)$

Here we use some rules or steps as follows:

Rule 1: $A \longleftrightarrow B$

$(A \rightarrow B) \wedge (B \rightarrow A)$

Rule 2: $A \longrightarrow B$

$\sim A \vee B$

Rule 3: $\sim(A \vee B) = \sim A \wedge \sim B$ (Demorgans law)

$\sim(A \wedge B) = \sim A \vee \sim B$

Rule 4: Distributive law

$A \wedge (B \vee C) = AB \vee AC$

CNF (conjunctive Normal form)

If an eqn consists of only $(\vee, \sim)$ and the individual components of eqn are combined by '$\wedge$' symbol thus we called eqn is in CNF.

Ex: $(A \vee B) \wedge (A \vee \sim B)$

DNF (Disjunctive Normal form)

If an eqn consists of only $(\wedge, \sim)$ and the individual components of eqn are combined by '$\vee$' symbol. Thus we called eqn is in DNF.

Ex: $(A \vee B) \vee (A \vee \sim B)$

1. Convert the given formula $(\sim A \rightarrow B) \wedge (C \wedge \sim A)$ in to its equivalent CNF.

$(\sim A \rightarrow B) \wedge (C \wedge \sim A)$ $\quad [\because A \rightarrow B \text{ is } \neg A \vee B]$

$\Rightarrow (A \vee B) \wedge C \wedge \sim A$

2. $(A \rightarrow B) \rightarrow C$ convert it into CNF

$(A \rightarrow B) \rightarrow C$

$(\sim A \vee B) \rightarrow C$

$\sim (A \wedge \sim B) \rightarrow C$

$(A) \wedge (\sim B \vee C)$

$\Rightarrow (A \wedge \sim B) \vee C$

$\Rightarrow (A \vee C) \wedge (\sim B \vee C)$

3. Convert $\sim (A \vee \sim B) \wedge (S \rightarrow T)$ in to DNF

$\sim (A \vee \sim B) \wedge (S \rightarrow T)$

$(\sim A \wedge B) \wedge (\sim S \vee T)$

$[(\sim A \wedge B) \wedge \sim S] \vee [(\sim A \wedge B) \wedge T]$

Predicate Logic

It is also called as First order Logic (FOL). It is a powerful language that develops information about objects in more easy way and also express the relationship between those objects.

The predicate logic consists of the following things

1. Variables

   Ex: x, y

2. Symbols (or) connectives

   Ex: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

3. Quantifiers

$\forall$ for all — $\exists$ for some

Note: Predicate logic is extension to propositional logic.

Quantifiers:

1. Universal quantifier:

$\forall$ is called as Universal quantifier. During universal quantifier '$\longrightarrow$' symbol was used.

2. Existential quantifier:

$\exists$ is called as existial quantifier. During existensial quantifier '$\wedge$' symbol was used.

Examples:

1. Ram is tall

It can be represented as tall (Ram)

Inside the parenthesis we represent objects and outside the paranthesis represent characteristic (or) relation.

2. Rama love sita

It can be represented as loves (Rama, sita)

3. Siva teaches MFCS (or) AI

It can be represented as

teaches (siva, MFCS) $\vee$ teaches (siva, AI)

4. All students like football

It can be represented as

$\forall_x$ (student (x) $\rightarrow$ like (x, football))

5. Every person has a father

It can be represented as

$\forall x\,(person(x) \longrightarrow father(x))$

person(x) : x is person

father(x) : x has father

6. All dancers loves to dance.

It can be represented as

$\forall x\,(dancer(x) \longrightarrow loves(x, dance))$

Resolution Refutation in FOL

Ex: Given clauses are

1. If it is sunny and warm day. You will enjoy
2. If it is raining you will get wet
3. It is a warm day
4. It is raining
5. It is sunny

Note: Prove enjoy

Steps

Step-1: Convert the given clauses to FOL

1. $(sunny \wedge warmday) \longrightarrow enjoy$
2. $raining \longrightarrow wet$
3. warm
4. Rain
5. sunny

Step-2: convert FOL in to conjunctive Normal form

1. $\neg(sunny \wedge warm) \vee enjoy$

$= (\neg sunny \vee \neg warm \vee enjoy)$

2. ~raining v wet

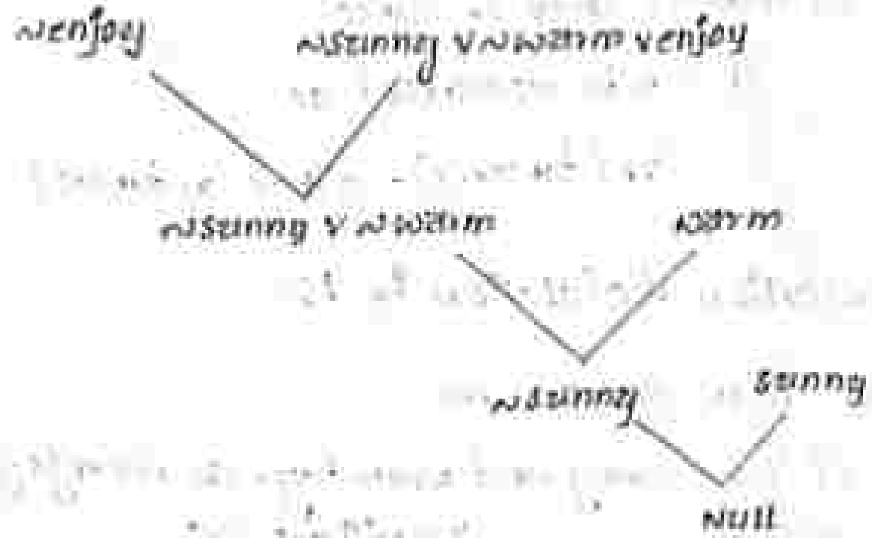
3. warm

4. rain

5. Sunny

Step-3 : negate goal i.e (~enjoy)

Step-4: Draw resolution graph



Step-5: If empty clause null or nil was produce stop and report that the theorem is proved.

De-finition:

proving the both by following same steps or rules. The rules are steps to be followed are:

1. Convert the given clauses to first order logic
2. Convert first order logic to conjunctive Normal form
3. Negate Goal
4. Draw resolution graph
5. If empty clause null is produce stop & report that the theorem is proved.