UNIT - III ITERATIVE STATEMENTS

Algebraic equation:

A polynomial equation $f(x)=0$ is called an algebraic equation.

Ex: 1. An algebraic equation of degree one is

$ax+b=0$

2. An algebraic equation of degree two is

$ax^2+bx+c=0$

Transcedental equation:

If the polynomial equation of $f(x)=0$ involving trigonometric, hyperbolic, logarithmic and exponential function, then these equations are called transcedental equation.

Ex: 1. $x\log x-1=0$

2. $xe^x+1=0$

3. $\sin x+x\cos x-1=0$

Note: If the roots of given equations are not integers it may be difficult to solve this equation by algebraic methods. such type of equations can be solved by numerical methods. there are different types of numerical methods. they are

1. Bisection method
2. Method of successive false position or regular - falsi method

3. Iteration method

4. Newton-Raphson's method

Bisection Method

Working rule:

1. Take the given equation, let it be $f(x)$ i.e $f(x)=0$
2. Find a & b and also find $f(a)$ and $f(b)$. These are opposite signs i.e $f(a)<0$, $f(b)>0$ or $f(a)>0$, $f(b)<0$
3. Write the formula for Bisection method, $c=\frac{a+b}{2}$
4. Find the first approximation i.e $c_1=\frac{a+b}{2}$ and also find $f(c_1)$
5. If $f(c_1)=0$ then c_1 is called root solution to the given equation.
6. If $f(c_1)$ is negitive and $f(a)$ is positive then the root lies between c_1 and a. Now we can find second approximation. i.e $c_2=\frac{c_1+a}{2}$
7. If $f(c_2)$ is negitive and $f(a)$ is positive, then the root lies between c_2 and a. Also we can find third approximation i.e $c_3=\frac{c_2+a}{2}$
8. We continue like this process upto two decimal places of consequetive approximation values are equal or nearly equal.

1. Find the real root of the equation $x^3-x-11=0$ correct to 2 decimal places using Bisection method.
2. Find a real root of the equation $x\log_{10}x=1.2$ which lies between 2 and 3 using bisection method
3. By using Bisection method, find an approximate root of the equation $\sin x=\frac{1}{x}$ that lies b/w $x=1$ and $x=1.5$. carryout upto seventh stage.

4. Find a real root of the equation $f(x) = x + \tan x$ $f(x) = x + \tan x - 1 = 0$ in the interval $[0, 0.5]$ using bisection method.

5. Find a square root of 25, given $x_0 = 2$, $x_1 = 7$ using bisection method.

6. Find a real root of the equation $x - \cos x = 0$ using bisection method.

7. Find a real root of the equation $x^4 - x - 10 = 0$ using bisection method.

8. Find a real root of the equation $4\sin x = e^x$ using bisection method.

1. Given that let $f(x) = x^3 - x - 1 = 0$

Now $f(0) = -1 < 0$

$f(0.5) = -1.375 < 0$

$f(1) = -1 < 0$

$f(2) = 5 > 0$

$\therefore$ The root lies b/w 1 and 2

The successive approximations are shown in the following table.

S.No	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = c^3 - c - 1 = 0$
1.	1	-1	2	5	$c_1 = 1.5$	$0.875 > 0$
2.	1	-1	1.5	0.875	$c_2 = 1.25$	$f(c_2) = -0.296 < 0$
3.	1.25	-0.296	1.5	0.875	$c_3 = 1.375$	$f(c_3) = 0.2246 > 0$

4.	1.25	-0.348	1.375	0.2246	$1.3125 = c_4$	$f(c_4) = -0.0515$
5	1.3125	-0.0515	1.375	0.2246	$c_5 = 1.3437$	$f(c_5) = 0.0849>0$
6.	1.3125	-0.0515	1.3437	0.0424	$c_6 = 1.3281$	$f(c_6) = 0.0144>0$
7.	1.3125	-0.0515	1.3281	0.0144	$c_7 = 1.3203$	$f(c_7) = 0.0187<0$

∴ The root lies between 1.32

Hence the root of the given equation upto two decimals are 1.32

2. Given let $f(x) = x\log x - 1.2 = 0$

Given that the roots stands lies b/w given function. Now $f(2) = 2\log_{10} 2 - 1.2 = -0.5979<0$

$f(3) = 0.2314>0$

The successive approximations are shown in the following table

S.No	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = c\log_{10} c - 1.2$
1.	2	-0.5979	3	0.2314	$c_1 = 2.5$	$f(c_1) = -0.205<0$
2	2.5	-0.2051	3	0.2314	$c_2 = 2.75$	$f(c_2) = 0.0082>0$
3.	2.5	-0.2051	2.75	0.0082	$c_3 = 2.625$	$f(c_3) = -0.0997<0$
4	2.625	-0.0997	2.75	0.0082	$c_4 = 2.6875$	$f(c_4) = -0.0461<0$
5	2.6875	-0.0461	2.75	0.0082	$c_5 = 2.7187$	$f(c_5) = -0.0191<0$
6.	2.7187	-0.0191	2.75	0.0082	$c_6 = 2.7343$	$f(c_6) = -0.0053<0$
7.	2.7343	-0.0053	2.75	0.0082	$c_7 = 2.7421$	$f(c_7) = 0.0012>0$
8	2.7343	-0.0053	2.7421	0.0012	$c_8 = 2.7382$	$f(c_8) = -0.0021<0$
9.	2.7382	-0.0021	2.7421	0.0012	$c_9 = 2.7401$	$f(c_9) = -0.0004<0$
10.	2.7401	-0.0004	2.7421	0.0012	$c_{10} = 2.7411$	$f(c_{10}) = 0.0005>0$

∴ 9^{th} and 10^{th} approximation values are nearly equal upto two decimal places.

Hence, the root of given equation is 2.74

3. Given that $\sin x = \frac{1}{x}$

i.e $x \sin x = 1$

$$x \sin x - 1 = 0$$

let $f(x) = x \sin x - 1 = 0$

clearly the roots of given equations are 1 and 1.5

Now, $f(1) = -0.1585 < 0$ $= -0.1585 < 0$

$f(1.5) = +0.4962 > 0$ $= 0.4962 > 0$

The successive approximations are shown in following table.

S.N.	$a (<0)$	$f(a)$	$b (>0)$	$f(b)$	$f(c) = \frac{a+b}{2}$	$f(c) = c \sin c - 1$
1.	1	−0.1585	1.5	0.4962	$c_1 = 1.25$	$f(c_1) = 0.1862 > 0$
2.	1	−0.1585	1.25	0.1862	$c_2 = 1.125$	$f(c_2) = 0.0150 > 0$
3.	1	−0.1585	1.125	0.0150	$c_3 = 1.0625$	$f(c_3) = -0.07184 < 0$
4.	1.0625	−0.0718	1.125	0.0150	$c_4 = 1.0937$	$f(c_4) = -0.02840$
5.	1.0937	−0.0284	1.125	0.0150	$c_5 = 1.1093$	$f(c_5) = -0.0067 < 0$
6.	1.1093	−0.0067	1.125	0.0150	$c_6 = 1.1171$	$f(c_6) = 0.0040 > 0$
7.	1.1093	−0.0067	1.1171	0.0040	$c_7 = 1.1132$	$f(c_7) = -0.0013 < 0$

clearly 6^{th} and 7^{th} iterations are equal upto two decimal places.

Hence the root of given equation is 1.11.

let $x = \sqrt{25}$

Squaring on b.s, we have

$x^2 = 25$

$x^2 - 25 = 0$

let $f(x) = x^2 - 25 = 0$

clearly the roots lies b/w 2 and 7

Now $f(2) = -21$

$f(7) = 24$

the successive approximations are shown in following table

S.NO.	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = c^2 - 25$
1.	2	-21	7	24	$c_1 = 4.5$	$f(c_1) = -4.75$
2.	4.5	-4.75	7	24	$c_2 = 5.75$	$f(c_2) = 8.0625$
3.	4.5	-4.75	5.75	8.0625	$c_3 = 5.125$	$f(c_3) = 1.2656$
4.	4.5	-4.75	5.125	1.2656	$c_4 = 4.8125$	$f(c_4) = -1.8398$
5.	4.8125	-1.8398	5.125	1.2656	$c_5 = 4.9687$	$f(c_5) = -0.3120$
6.	4.9687	-0.3120	5.125	1.2656	$c_6 = 5.0468$	$f(c_6) = 0.4701$
7.	4.9687	-0.3120	5.0468	0.4701	$c_7 = 5.0077$	$f(c_7) = 0.7705$
8.	4.9687	-0.3120	5.0077	0.7705	$c_8 = 4.9882$	$f(c_8) = -0.1178$
9.	4.9882	-0.11786	5.0077	0.7705	$c_9 = 4.9979$	$f(c_9) = -0.0209$
10.	4.9979	-0.0209	5.0077	0.7705	$c_{10} = 5.0028$	$f(c_{10}) = 0.0280$
11.	4.9979	-0.0209	5.0028	0.0280	$c_{11} = 5.0005$	$f(c_{11}) = 0.0050$

Clearly 10^{th} and 11^{th} iterations are nearly equal upto two decimal places.

Hence the root of given equation is 5.00

4. Given $f(x) = x + \tan x - 1 = 0$

clearly the roots of the quadratic equation are 0 and 0.5

Now $f(0) = -1 < 0$

$f(0.5) = 0.0463 > 0$

The following approximations are shown in the following table.

S.No	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = c + \tan c - 1$
1.	0	-1	0.5	0.0463	$c_1 = 0.25$	$f(c_1) = -0.4946$
2.	0.25	-0.4946	0.5	0.0463	$c_2 = 0.375$	$f(c_2) = -0.2313$
3.	0.375	-0.2313	0.5	0.0463	$c_3 = 0.4375$	$f(c_3) = -0.0947$
4.	0.4375	-0.0947	0.5	0.0463	$c_4 = 0.4687$	$f(c_4) = -0.7626$
5.	0.4687	-0.7626	0.5	0.0463	$c_5 = 0.4843$	$f(c_5) = 0.0103$
6.	0.4687	-0.7606	0.4843	0.0103	$c_6 = 0.4765$	$f(c_6) = -0.0073$
7.	0.4765	-0.0073	0.4843	0.0103	$c_7 = 0.4804$	$f(c_7) = 0.0015$
8.	0.4765	-0.0073	0.4804	0.0015	$c_8 = 0.4784$	$f(c_8) = -0.0030$
9.	0.4784	-0.0030	0.4804	0.0015	$c_9 = 0.4794$	$f(c_9) = -0.00075$

Clearly 8^{th} and 9^{th} iterations are equal upto two decimal places.

Hence the root of given equation is '0.47'.

6. Given that $x - \cos x = 0$

let $f(x) = x - \cos x$

clearly the roots of given equations are

Now $f(0) = -1 < 0$

$f(1) = 0.4596 > 0$

The root lies between 0 and 1

The successive approximation are shown in the following table.

no	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = c - \cos c$
1	0	−1	1	0.4596	$c_1 = 0.5$	$f(c_1) = -0.3775$
2	0.5	−0.3775	1	0.4596	$c_2 = 0.75$	$f(c_2) = 0.0183$
3	0.5	−0.3775	0.75	0.0183	$c_3 = 0.625$	$f(c_3) = -0.1859$
4	0.625	−0.1859	0.75	0.0183	$c_4 = 0.3016$	$f(c_4) = -0.6271$
5	0.3016	−0.6271	0.75	0.0183	$c_5 = 0.5358$	$f(c_5) = -0.3240$
6	0.5358	−0.3240	0.75	0.0183	$c_6 = 0.6429$	$f(c_6) = -0.1579$
7	0.6429	−0.1579	0.75	0.0183	$c_7 = 0.6964$	$f(c_7) = -0.0707$
8	0.6964	−0.0707	0.75	0.0183	$c_8 = 0.7232$	$f(c_8) = -0.0264$
9	0.7232	−0.0264	0.75	0.0183	$c_9 = 1.4732$	$f(c_9) = 1.3757$
10	0.7232	−0.0264	1.4732	1.3757	$c_{10} = 1.0982$	$f(c_{10}) = 0.6450$
11	0.7232	−0.0264	1.0982	0.6450	$c_{11} = 0.9107$	$f(c_{11}) = 0.2975$
12	0.7232	−0.0264	0.9107	0.2975	$c_{12} = 0.8169$	$f(c_{12}) = 0.1324$
13	0.7232	−0.0264	0.8169	0.1324	$c_{13} = 0.7700$	$f(c_{13}) = 0.0520$
14	0.7232	−0.0264	0.7700	0.0520	$c_{14} = 0.7466$	$f(c_{14}) = 0.0125$
15	0.7232	−0.0264	0.7466	0.0125	$c_{15} = 0.7349$	$f(c_{15}) = -0.0069$
16	0.7349	−0.0069	0.7466	0.0125	$c_{16} = 0.7407$	$f(c_{16}) = 0.0027$
17	0.7349	−0.0069	0.7407	0.0027	$c_{17} = 0.7378$	$f(c_{17}) = -0.0021$
18	0.7378	−0.0021	0.7407	0.0027	$c_{18} = 0.7392$	$f(c_{18}) = 0.0003$

Clearly 17^{th} and 18^{th} iterations are equal up to two decimal places.

Hence the root of the given equations is 0.73

7. Given Let $x^4 - x - 10 = 0$

$$f(x) = x^4 - x - 10$$

$f(0) = -10 < 0$

$f(1) = -10 < 0$

$f(2) = 4 > 0$

The successive approximations are shown in the below table.

S.No	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = c^{3}$ [illegible]
1.	1	-10	2	4	$c_1 = 1.5$	$f(c_1) = -6.4375$
2.	1.5	-6.4375	2	4	$c_2 = 1.75$	$f(c_2) = -2.3710$
3.	1.75	-2.3710	2	4	$c_3 = 1.875$	$f(c_3) = 0.4846$
4.	1.75	-2.3710	1.875	0.4846	$c_4 = 1.8125$	$f(c_4) = -1.0202$
5.	1.8125	-1.0202	1.875	0.4846	$c_5 = 1.8437$	$f(c_5) = -0.2889$
6	1.8437	-0.2889	1.875	0.4846	$c_6 = 1.8593$	$f(c_6) = 0.0091$
7.	1.8437	-0.2889	1.8593	0.0091	$c_7 = 1.8515$	$f(c_7) = -0.0$[illegible]

Clearly 6^{th} and 7^{th} iterations are equal upto two decimal places.

Hence the root of given equation is 1.85

8. Let $4\sin x = e^{x}$

$4\sin x - e^{x} = 0$

Let $f(x) = 4\sin x - e^{x}$

$f(0) = -1 < 0$

$f(1) = 0.6476 > 0$

$\therefore$ the roots lies b/w 0 and 1

The successive approximations as shown in the following table

S.No	a	f(a)	b	f(b)	$c = \frac{a+b}{2}$	$f(c) = 4\sin c - e^{c}$
1.	0	-1	1	0.6476	$c_1 = 0.5$	$f(c_1) = 0.2689$
2.	0	-1	0.5	0.2689	$c_2 = 0.25$	$f(c_2) = -0.2944$
3.	0.25	-0.2944	0.5	0.2689	$c_3 = 0.375$	$f(c_3) = 0.0100$
4.	0.25	-0.2944	0.375	0.0100	$c_4 = 0.3125$	$f(c_4) = -0.1370$
5.	0.3125	-0.1370	0.375	0.0100	$c_5 = 0.3437$	$f(c_5) = -0.0$[illegible]

6.	0.3437	-0.0622	0.375	0.0100	$c_6 = 0.3593$	$f(c_6) = -0.0258$
7.	0.3593	-0.0258	0.375	0.0100	$c_7 = 0.3671$	$f(c_7) = -0.0079$
8.	0.3671	-0.0079	0.375	0.0100	$c_8 = 0.3710$	$f(c_8) = 0.0010$
9.	0.3671	-0.0079	0.3710	0.0010	$c_9 = 0.3690$	$f(c_9) = -0.0035$
10.	0.3690	-0.0035	0.3710	0.0010	$c_{10} = 0.37$[illegible]	$f(c_{10}) = -0.0012$
11.	0.37	-0.0012	0.3710	0.0010	$c_{11} = 0.3705$	$f(c_{11}) = -0.0004$

clearly 10^{th} and 11^{th} iterations are equal upto two decimal places.

Hence the root of given equation is '0.37'.

UNIT-III : ITERATIVE METHODS

problems on Bisection method:

1. Find a real root of eq'n $x^3-x-11=0$ by bisection method.

Sol: Let $f(x) = x^3-x-11=0$

$f(0) = -11<0$

$f(1) = -11<0$

$f(2) = -5<0$

$f(3) = 13>0$

clearly $f(2)$ & $f(3)$ have opposite signs

$\therefore$ root lies between 2 & 3

$f(a)=f(2)<0$, $f(b)=f(3)>0$

$$x = \frac{a+b}{2}$$

The values of a, b, $\frac{a+b}{2}$, the sign of $f(a)$, $f(b)$, $f\left(\frac{a+b}{2}\right)$ is given in the below table.

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
2	3	2.5	-ve	+ve	+ve
2	2.5	2.25	-ve	+ve	-ve
2.5	2.25	2.375	+ve	-ve	+ve
2.25	2.375	2.3125	-ve	+ve	-ve
2.375	2.3125	2.3438	+ve	-ve	-ve
2.375	2.3438	2.3594	+ve	-ve	-ve
2.375	2.3672	2.3711	+ve	-ve	-ve
2.375	2.3711	2.3731	+ve	-ve	+ve

2.375	2.3731	2.3741	+ve	-ve	+ve
2.3731	2.3741	2.3736	-ve	+ve	-ve

clearly $f(2.3736) = (2.3736)^3 - 2.3736 - 11$

$= -0.0008$

$\cong 0$

$\therefore x = 2.3736$ is a root of $x^3 - x - 11 = 0$

(∵ fix eq'n in casio, x – press Alpha) Right bracket $x^3 - x^{\wedge 3}$, CALC for diff values)

V.I.P

2 Using bisection method, find the negative root of $x^3 - 4x + 9 = 0$

Sol: Let $f(x) = x^3 - 4x + 9 = 0$

$f(0) = 9 > 0$, $f(2) = 12 > 0$, $f(-2) = 9 > 0$,

$f(-3) = -6 < 0$

clearly $f(-2)$ & $f(-3)$ have opposite signs

$\therefore$ root lies b/w -2 & -3

Let $a = -2$, $b = -3$, $f(a) = f(-2) = 9 > 0$, $f(b) = f(-3) = -6 < 0$

Bisection formula is $x = \frac{a+b}{2}$

The values of $a, b, \frac{a+b}{2}$ and the signs of $f(a)$, $f(b)$, $f(\frac{a+b}{2})$

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f(\frac{a+b}{2})$
-2	-3	-2.5	+ve	-ve	+ve
-3	-2.5	-2.75	-ve	+ve	-ve
-2.5	-2.75	-2.625	+ve	-ve	+ve
-2.75	-2.625	-2.6875	-ve	+ve	+ve

-2.75	-2.6875	-2.7188	+ve	+ve	-ve
-2.6875	-2.7188	-2.7032	+ve	-ve	+ve
-2.7188	-2.7032	-2.7110	-ve	+ve	-ve
-2.7032	-2.7110	-2.7071	+ve	-ve	-ve
-2.7032	-2.7071	-2.7052	+ve	-ve	+ve
-2.7071	-2.7052	-2.7062	-ve	+ve	+ve
-2.7071	-2.7062	-2.7067	-ve	+ve	-ve
-2.7062	-2.7067	-2.7065	+ve	-ve	+ve

clearly $f(-2.7065) = (-2.7065)^3 - 4(-2.7065) + 9$

$= 0.0005 \cong 0$

$\therefore x = -2.7065$ is a root of $x^3 - 4x + 9 = 0$

Here a is -ve, b is -ve

So take avg of +ve values and put -

$a = -2,\ b = -3$

$avg = \frac{2+3}{2} = 2.5$

$= -2.5$

3. Find a real root of the eqn $x \log_{10} x = 1.2$ by bisection method.

Sol: Let $f(x) = x \log_{10} x - 1.2 = 0$

$\log_{10} x = \log$

$f(1) = -1.2 < 0$

$\log_e x = \ln$

$f(2) = -0.5979 < 0$

$f(3) = 0.2314 > 0$

clearly $f(2)$ & $f(3)$ have opposite signs so root lies b/w 2 & 3 let $a = 2,\ b = 3,\ f(a) = f(2) = -0.5979 < 0$

$f(b) = f(3) = 0.2314 > 0$

Bisection formula is $x_1 = \frac{a+b}{2}$

Now, we find the values of a, b and $\frac{a+b}{2}$, the signs of $f(a), f(b)$ and $f\left(\frac{a+b}{2}\right)$ is give below

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
2	3	2.5	-ve	+ve	-ve
3	2.5	2.75	+ve	-ve	+ve
2.5	2.75	2.625	-ve	+ve	-ve
2.75	2.625	2.6875	+ve	-ve	-ve
2.75	2.6875	2.7188	+ve	-ve	-ve
2.75	2.7188	2.7344	+ve	-ve	-ve
2.75	2.7344	2.7422	+ve	-ve	+ve
2.7344	2.7422	2.7383	-ve	+ve	-ve
2.7422	2.7383	2.7403	+ve	-ve	-ve

clearly $f(2.7403) = (2.7403)\log(2.7403) - 1.2$
$= -0.0003 \simeq 0$

$\therefore x = 2.7403$ is a root of $x\log_{10}x - 1.2 = 0$

4 Using Bisection method, find a real root of the equation $\sin x = \frac{1}{x}$

Sol: let $f(x) = x\sin x - 1 = 0$

$f(0) = -1 < 0$

$f(1) = -0.1585 < 0$

$f(2) = 0.8186 > 0$

clearly $f(1)$ & $f(2)$ have opposite signs.

$\therefore$ root lies b/w 1 & 2

Let $a=1$, $b=2$, $f(a)=f(1)=-0.1585$

$f(b)=f(2)=0.8186$

The values of $a, b, \frac{a+b}{2}$, signs of $f(a), f(b)$ & $f\left(\frac{a+b}{2}\right)$

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
1	2	1.5	-ve	+ve	+ve
1	1.5	1.25	-ve	+ve	+ve
1	1.25	1.1250	-ve	+ve	+ve
1	1.125	1.0625	-ve	+ve	-ve
1.125	1.0625	1.0938	+ve	-ve	-ve
1.125	1.0938	1.1094	+ve	-ve	-ve
1.125	1.1094	1.1172	+ve	-ve	+ve
1.1094	1.1172	1.1133	-ve	+ve	-ve
1.1172	1.1133	1.1153	+ve	-ve	+ve
1.1153	1.1153	1.1143	-ve	+ve	+ve

Clearly $f(1.1143)=(1.1143)\sin(1.1143)-1$

$=0.0002 \simeq 0$

$\therefore x=1.1143$ is a root of $x\sin x-1=0$

Method 2: Iteration Method:

Let the given eq'n be $f(x)=0$

Express $f(x)=0$ as $x=g(x)$ or $\phi(x)$

where $g(x)$ satisfies the following conditions.

choose any two values a and b such that $f(a)$ & $f(b)$ have opposite signs.

$\therefore$ root lies between a & b

Let us take initial approximate solution be either a (or) b (or) $\left(\frac{a+b}{2}\right)$, it is denoted by x_0 iteration method converges only when |g'(x)|<1 and iteration formula is

$$x_{i+1} = g(x_i) \quad \forall \; i = 0, 1, 2, \cdots$$

find $x_1, x_2, x_3, \cdots$ if any two successive iterations are same then we stop the procedure which is the required root of the equation

Problems

1\. Find the root of the equation $x^3-2x-5=0$ which lies near $x=2$ by iteration method.

Sol: Let $f(x)=x^3-2x-5=0$, given $x_0=2$

Now express $f(x)=0$ as $x=\phi(x)$ we have

$$x^3-2x-5=0$$

$$x^3=2x+5$$

$$\therefore x=(2x+5)^{1/3}, \text{ clearly this is } x=\phi(x)$$

Where $\phi(x)=(2x+5)^{1/3}$

$$\phi'(x)=\frac{1}{3}(2x+5)^{\frac{1}{3}-1}\frac{d}{dx}(2x+5)$$

$$\phi'(x)=\frac{2}{3}(2x+5)^{-2/3}$$

$$|\phi'(x_0)| = \left|\frac{2}{3}(2x_0+5)^{-2/3}\right|$$

$$= \left|\frac{2}{3}(2\times 2+5)^{-2/3}\right|$$

$$= 0.1540\;2\ldots$$

$\therefore |\phi'(x_0)| < 1$, then iteration method is applicable

iteration formula is $x_{i+1} = \phi(x_i)$ $\forall i = 0, 1, 2 \cdots$

$$\therefore x_{i+1} = (2x_i + 5)^{1/3}$$

i	x_{i+1}	$(2x_i+5)^{1/3}$	$x_{i+1} = (2x_i+5)^{1/3}$
0	x_1	$(2x_0+5)^{1/3}$	$x_1 = (2x_0+5)^{1/3} = 2.0801$
1	x_2	$(2x_1+5)^{1/3}$	$x_2 = (2x_1+5)^{1/3} = 2.0924$
2	x_3	$(2x_2+5)^{1/3}$	$x_3 = (2x_2+5)^{1/3} = 2.0942$
3	x_4	$(2x_3+5)^{1/3}$	$x_4 = (2x_3+5)^{1/3} = 2.0945$
4	x_5	$(2x_4+5)^{1/3}$	$x_5 = (2x_4+5)^{1/3} = 2.0945$

Since x_4 and x_5 are identical so we stop the process and $x = 2.0945$ is the root of $x^3 - 2x - 5 = 0$.

2. By the fixed point iteration process, find a root of the equation $x = \cos x$ near $x = \pi/4$

Sol: Given eqn $x = \cos x$, this is of the form $x = g(x)$ or $\phi(x)$ where $g(x) = \cos x$, given $x_0 = \frac{\pi}{4} = \frac{1}{4} \cdot \frac{22}{7}$

$$x_0 = 0.7857$$

$$g'(x) = -\sin x$$

$$|g'(x_0)| = |-\sin x_0| = \sin(0.7857) = 0.7073 < 1$$

$\therefore |g'(x_0)|$ is < 1, hence iteration method is applicable and iteration formula is

$$x_{i+1} = g(x_i) \quad \forall i = 0, 1, 2, \cdots$$

$$x_{i+1} = \cos(x_i)$$

i	x_{i+1}	$\cos x_i$	$x_{i+1} = \cos x_i$
0	x_1	$\cos x_0$	$x_1 = \cos x_0 = 0.7069$
1	x_2	$\cos x_1$	$x_2 = \cos x_1 = 0.7604$
2	x_3	$\cos x_2$	$x_3 = \cos x_2 = 0.7246$
3	x_4	$\cos x_3$	$x_4 = \cos x_3 = 0.7488$
4	x_5	$\cos x_4$	$x_5 = \cos x_4 = 0.7325$
5	x_6	$\cos x_5$	$x_6 = \cos x_5 = 0.7435$
6	x_7	$\cos x_6$	$x_7 = \cos x_6 = 0.7361$
7	x_8	$\cos x_7$	$x_8 = \cos x_7 = 0.7411$
8	x_9	$\cos x_8$	$x_9 = \cos x_8 = 0.7377$
9	x_{10}	$\cos x_9$	$x_{10} = \cos x_9 = 0.74$
10	x_{11}	$\cos x_{10}$	$x_{11} = \cos x_{10} = 0.7385$
11	x_{12}	$\cos x_{11}$	$x_{12} = \cos x_{11} = 0.7395$
12	x_{13}	$\cos x_{12}$	$x_{13} = \cos x_{12} = 0.7388$
13	x_{14}	$\cos x_{13}$	$x_{14} = \cos x_{13} = 0.7393$
14	x_{15}	$\cos x_{14}$	$x_{15} = \cos x_{14} = 0.7389$
15	x_{16}	$\cos x_{15}$	$x_{16} = \cos x_{15} = 0.7392$
16	x_{17}	$\cos x_{16}$	$x_{17} = \cos x_{16} = 0.7390$
17	x_{18}	$\cos x_{17}$	$x_{18} = \cos x_{17} = 0.7391$
18	x_{19}	$\cos x_{18}$	$x_{19} = \cos x_{18} = 0.7391$
~~19~~	~~x_{20}~~	~~$\cos x_{19}$~~	~~$x_{20} = \cos x_{19} =$~~

∴ Here x_{18} and x_{19} are same

Hence we stop the process

$x = 0.7391$ is a root of $x = \cos x$

Find a +ve root of the eqn by iteration method

$3x = \cos x + 1$

Let $f(x) = 3x - \cos x - 1 = 0$

$f(0) = -2 < 0$

$f(1) = 1.4597 > 0$

clearly $f(0)$ & $f(1)$ have opposite signs

and the roots lies b/w 0 & 1

Let us take initial approximate soln x_0 to be either 0 or 1 or b/w 0 & 1

Let us take $x_0 = 0.6$

Now from given eqn

$$3x = \cos x + 1$$

$$x = \frac{\cos x + 1}{3}$$

clearly this is of the form $x = g(x)$

where $g(x) = \frac{\cos x + 1}{3}$

Now $g'(x) = \frac{1}{3}(-\sin x + 0)$

$$|g'(x_0)| = \left|\frac{-1}{3}(\sin x_0)\right|$$

$$= \left|\frac{-1}{3}(\sin(0.6))\right|$$

Hence iteration method is applicable.

Iteration formula is

$$x_{i+1} = g(x_i) \; \forall \; i = 0, 1, 2, \cdots$$

$$x_{i+1} = \frac{1}{3}[\cos x_i + 1]$$

i	x_{i+1}	$\frac{1}{3}[\cos x_i + 1]$	$x_{i+1} = \frac{1}{3}[\cos x_i + 1]$
0	x_1	$\frac{1}{3}[\cos x_0 + 1]$	$x_1 = \frac{1}{3}[\cos x_0 + 1] = 0.6044$
1	x_2	$\frac{1}{3}[\cos x_1 + 1]$	$x_2 = \frac{1}{3}[\cos x_1 + 1] = 0.6069$
2	x_3	$\frac{1}{3}[\cos x_2 + 1]$	$x_3 = \frac{1}{3}[\cos x_2 + 1] = 0.6071$
3	x_4	$\frac{1}{3}[\cos x_3 + 1]$	$x_4 = \frac{1}{3}[\cos x_3 + 1] = 0.6071$

$\therefore$ x_3 & x_4 are identical, hence we stop process

$\therefore$ $x = 0.6071$ is a root of $3x = \cos x + 1$

4. Solve $x = 1 + \tan^{-1} x$, by iteration method.

Sol: Let $f(x) = 1 + \tan^{-1} x - x = 0$

$f(0) = 1 > 0$

$f(1) = 0.78540$

$f(2) = 0.107170$

$f(3) = -0.751040$

Clearly $f(2)$ & $f(3)$ have opposite signs

$\therefore$ root lies between 2 & 3

let $x_0 = 2.1$

from given eqn $x = 1 + \tan^{-1} x$, is of the form $x = g(x)$ where $g(x) = 1 + \tan^{-1} x$

$$g'(x) = 0 + \frac{1}{1+x^2}$$

$$|g'(x_0)| = \left|\frac{1}{1+x_0^2}\right|$$

$$|g'(x_0)| = \left|\frac{1}{1+(2.1)^2}\right|$$

$$= 0.184842$$

Hence iteration method is applicable,

Iteration formula is

$x_{i+1} = g(x_i)$ $\forall$ $i = 0,1,2, \cdots$

$x_{i+1} = 1+\tan^{-1}(x_i)$ $\forall$ $i = 0,1,2, \cdots$

i	x_{i+1}	$1+\tan^{-1}(x_i)$	$x_{i+1} = 1+\tan^{-1}(x_i)$
0	x_1	$1+\tan^{-1}(x_0)$	$x_1 = 1+\tan^{-1}(x_0) = 2.1264$
1	x_2	$1+\tan^{-1}(x_1)$	$x_2 = 1+\tan^{-1}(x_1) = 2.1312$
2	x_3	$1+\tan^{-1}(x_2)$	$x_3 = 1+\tan^{-1}(x_2) = 2.1321$
3	x_4	$1+\tan^{-1}(x_3)$	$x_4 = 1+\tan^{-1}(x_3) = 2.1322$
4	x_5	$1+\tan^{-1}(x_4)$	$x_5 = 1+\tan^{-1}(x_4) = 2.1323$
5	x_6	$1+\tan^{-1}(x_5)$	$x_6 = 1+\tan^{-1}(x_5) = 2.1323$

$\therefore$ x_5 and x_6 are identical

$\therefore$ $x = 2.1323$ is a root of $x = 1+\tan^{-1}(x)$

5. Find the positive root of $x^4 - x - 10 = 0$ by iteration

Sol: Let $f(x) = x^4 - x - 10 = 0$

$f(0) = -10 < 0$, $f(1) = -10 < 0$, $f(2) = 4 > 0$

Clearly $f(1)$, $f(2)$ have opposite signs

$\therefore$ root lies b/w 1 & 2

Let us take initial approximation soln be either 1 or 2 or b/w 1 & 2 let $x_0 = 1.8$

Given $x^4 - x - 10 = 0$

$x^4 = x + 10$

$x = (x+10)^{1/4}$

Clearly this is of the form $x = g(x)$

where $g(x) = (x+10)^{1/4}$

$$g'(x) = \frac{1}{4}(x+10)^{\frac{1}{4}-1} \frac{d}{dx}(x+10)$$

$$g'(x) = \frac{1}{4}(x+10)^{-3/4}$$

$$|g'(x_0)| = \left|\frac{1}{4}(x_0+10)^{-3/4}\right|$$

$$|g'(x_0)| = 0.0393 < 1$$

$\therefore$ Iteration method is applicable

Iteration formula is

$$x_{i+1} = g(x_i) \quad \forall\ i = 0, 1, 2, \ldots$$

$$x_{i+1} = (x_i+10)^{1/4}$$

i	x_{i+1}	$(x_i+10)^{1/4}$	$x_{i+1} = (x_i+10)^{1/4}$
0	x_1	$(x_0+10)^{1/4}$	$x_1 = (x_0+10)^{1/4} = 1.8554$
1	x_2	$(x_1+10)^{1/4}$	$x_2 = (x_1+10)^{1/4} = 1.8555$
2	x_3	$(x_2+10)^{1/4}$	$x_3 = (x_2+10)^{1/4} = 1.8556$
3	x_4	$(x_3+10)^{1/4}$	$x_4 = (x_3+10)^{1/4} = 1.8556$

$\therefore$ x_3 and x_4 are identical, hence we stop the process. $x = 1.8556$ is a root of $x^4 - x - 10 = 0$

6. Find a real root of the eqn $x + \log_{10} x = 3.375$ by iteration method.

Sol: Let $f(x) = x + \log_{10} x - 3.375 = 0$ ($\because \log_{10} x$ is not defined for $x = 0$, so take x values from 1, 2 in casio $\log_{10} x = \log x$)

$f(1) = -2.375 < 0$

$f(2) = -1.0740 < 0$

$f(2.5) = -0.4771 < 0$

$f(3) = 0.1021 > 0$

clearly $f(2.5)$ & $f(3)$ have opposite signs

$\therefore$ root lies b/w 2.5 & 3

Let us take initial approximate soln $x_0 = 2.8$

Here $f(3) = 0.1021$ and $f(2.5) = -0.4771$

i.e $0.1, -0.4, 0.1$ is near to zero. so take x_0 nearer to 3.

from given eqn $x = 3.375 - \log_{10} x$

clearly this is of the form $x = g(x)$

where $g(x) = 3.375 - \log_{10} x$

but $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\therefore \frac{d}{dx}(\log_{10} x)$ does not exist

w.k.t $\log_{10} x = \log_e x \times \log_{10} e$

$\log_{10} x = \log_e x \times \log_{10}(2.7183)$

$\log_{10} x = 0.4343 \times \log_e x$

$\therefore g(x) = 3.375 - 0.4343 \times \log_e x$

$g'(x) = \frac{-0.4343}{x}$

$|g'(x_0)| = \left|\frac{-0.4343}{x_0}\right|$

$= 0.1551 < 1$

$\therefore$ Iteration method is applicable

Iteration formula is $x_{i+1} = g(x_i)$

$\therefore x_{i+1} = 3.375 - 0.4343 \times \log_e x_i$

$\forall\ i = 0, 1, 2, \cdots$

i	x_{i+1}	$3.375-0.4343\log_e x_i$	$x_{i+1}=3.375-0.4343\log_e x_i$
0	x_1	$3.375-0.4343\log_e x_0$	$x_1 = 2.9278$
1	x_2	$3.375-0.4343\log_e x_1$	$x_2 = 2.9085$
2	x_3	$3.375-0.4343\log_e x_2$	$x_3 = 2.9113$
3	x_4	$3.375-0.4343\log_e x_3$	$x_4 = 2.9109$
4	x_5	$3.375-0.4343\log_e x_4$	$x_5 = 2.9110$
5	x_6	$3.375-0.4343\log_e x_5$	$x_6 = 2.9110$

$\therefore x_5$ & x_6 are identical, hence we stop process

and $x = 2.9110$ is a root of $x+\log_{10} x = 3.375$

7. Evaluate $\sqrt{12}$ and $1/\sqrt{12}$ by the fixed point iteration method.

Sol: Let $x = \sqrt{12}$

$x^2 - 12 = 0$

Let $f(x) = x^2 - 12 = 0$

$f(3) = -3 < 0$

$f(4) = 4 > 0$

clearly $f(3)$ & $f(4)$ have opposite signs

$\therefore$ root lies b/w 3 & 4

Let $x_0 = 3.5$

from the given equation $x^2 - 12 = 0$

$\Rightarrow x^2 = 12 \Rightarrow x = \frac{12}{x}$

where $g(x) = \frac{12}{x}$

$|g'(x)| = \frac{-12}{x^2}$

$|g'(x_0)| = \left|\frac{-12}{3.5^2}\right|$

$|g'(x_0)| = 0.9796 < 1$

∴ Iteration method is applicable

Iteration formula is

$$x_{i+1} = g(x_i) \quad \forall\ i = 0,1,2,\ldots$$

$$x_{i+1} = \frac{12}{x_i} \quad \forall\ i = 0,1,2,\ldots$$

i	x_{i+1}	$\frac{12}{x_i}$	$x_{i+1} = \frac{12}{x_i}$
0	x_1	$\frac{12}{x_0}$	$x_1 = 3.4286$
1	x_2	$\frac{12}{x_1}$	$x_2 = 3.5$
2	x_3	$\frac{12}{x_2}$	$x_3 = 3.4286$
3	x_4	$\frac{12}{x_3}$	$x_4 = 3.5$

∴ The iterations oscillate b/w 3.4286 & 3.5, the average of 3.4286 & 3.5 is taken as approximate value of $\sqrt{12}$ which is 3.4643

$$\therefore \sqrt{12} = 3.4643$$

ii) To find $\frac{1}{\sqrt{12}}$ value

let $x = \frac{1}{\sqrt{12}}$

$x^2 = \frac{1}{12}$

$12x^2 - 1 = 0$

$f(x) = 12x^2 - 1 = 0$

$f(0) = -1 < 0$

$f(0.2) = -0.0857 < 0$

$f(0.3) = 0.0113 > 0$

(Here $\frac{1}{\sqrt{12}}$ value is 0.2887 so take its nearest value as next value) clearly $f(0.2)$ & $f(0.3)$ have opp signs

$\therefore$ root lies b/w 0.2 & 0.3

let $x_0 = 0.3$

from given eqn.

$$x^2 = \frac{1}{12}$$

$$x = \frac{1}{12x} \text{ is of form } x = g(x)$$

where $g(x) = \frac{1}{12x}$

$$g'(x) = \frac{-1}{12x^2}$$

$$|g'(x_0)| = 0.9259 < 1$$

$\therefore$ Iteration method is applicable,

Iteration formula is

$$x_{i+1} = g(x_i) \ \forall\ i = 0, 1, 2, \dots$$

$$x_{i+1} = \frac{1}{12x_i}$$

i	x_{i+1}	$\frac{1}{12x_i}$	$x_{i+1} = \frac{1}{12x_i}$
0	x_1	$\frac{1}{12x_0}$	$x_1 = \frac{1}{12x_0} = 0.2778$
1	x_2	$\frac{1}{12x_1}$	$x_2 = \frac{1}{12x_1} = 0.3$
2	x_3	$\frac{1}{12x_2}$	$x_3 = \frac{1}{12x_2} = 0.2778$

Here, the iteration oscillates b/w 0.3 & 0.2778,

The average of these values gives the approximate value of $\frac{1}{\sqrt{12}}$ $\therefore$ $\frac{1}{\sqrt{12}} = \left(\frac{0.3 + 0.2778}{2}\right) = 0.2889$

Method-3 : Regula - false method or false position

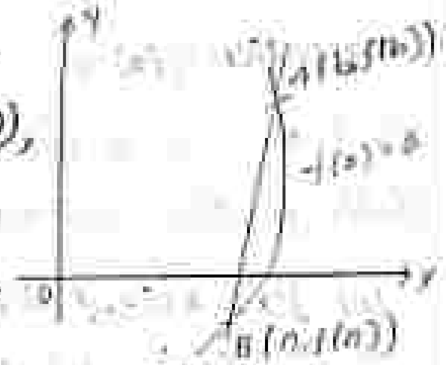
let the given eqn be $f(x)=0$.

choose any two values a & b such that $f(a)$ & $f(b)$ have opposite signs

$\therefore$ root lies b/w a & b

without loss of generality, take $f(a)<0$, $f(b)>0$

clearly $f(x)=0$ is a curve on xy-plane with $B(a,f(a))$, $A(b,f(b))$ are any two points on the curve joining of these two points is a straight line which interacts x-axis at $p(x_1,0)$



W.K.T eq'n of straight line joining the points $A(b,f(b))$, $B(a,f(a))$ is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-f(b) = \frac{f(a)-f(b)}{a-b}(x-b)$$

but this line touches x-axis at $p(x_1,0)$ we have

$$0-f(b) = \frac{f(a)-f(b)}{a-b}(x_1-b)$$

$$x_1-b = \frac{-(a-b)f(b)}{f(a)-f(b)}$$

$$x_1 = \frac{bf(a)-bf(b)-af(b)+bf(b)}{f(a)-f(b)}$$

$$x_1 = \frac{bf(a)-af(b)}{f(a)-f(b)}$$

$$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

x_1 is the first approximation by using Regula-falsi method

Find $f(x_1)$, if $f(x_1)=0$, then x_1 is the root of $f(x)=0$. If $f(x_1)\neq 0$, then check $f(x_1)<0$ or $f(x_1)>0$

suppose $f(x_1)<0$ & clearly $f(b)>0$

$\therefore$ next approximation x_2 lies b/w x_1 and b

$\therefore$ second approximation $x_2 = \dfrac{x_1 f(b) - b f(x_1)}{f(b)-f(x_1)}$

continue this procedure to get a root of the equation $f(x)=0$

1. Find the root of the eqn $x\log_{10}(x)=1.2$ using false position method.

Sol: Let $f(x) = x\log_{10}x - 1.2 = 0$

$f(1) = -1.2 < 0$

$f(1.5) = -0.9359 < 0$

$f(1.6) = -0.8734 < 0$

$f(1.7) = -0.8082 < 0$

$f(2) = -0.5979 < 0$

$f(2.5) = -0.2051 < 0$

$f(2.6) = -0.1211 < 0$

$f(2.7) = -0.0353 < 0$

$f(2.8) = 0.0520 > 0$

clearly $f(2.7)$ & $f(2.8)$ have opp signs

$\therefore$ root lies b/w 2.7 & 2.8

Let $a = x_0 = 2.7$, $b = 2.8$

$f(a) = f(2.7) = -0.0353$

$f(b) = f(2.8) = 0.0520$

By regula falsi method

Ist approximation:

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.7(0.0520) - 2.8(-0.0353)}{0.0520 - (-0.0353)}$$

$$= \frac{0.2394}{0.0873}$$

$$x_1 = 2.7423$$

$$f(x_1) = (2.7423)\log_{10}(2.7423) + 2$$

$$f(x_1) = 0.001470$$

Here $f(x_1) > 0$ & $f(a) < 0$

$\therefore$ root lies b/w a & x_1

IInd approximation.

$$x_2 = \frac{a f(x_1) - x_1 f(a)}{f(x_1) - f(a)}$$

$$x_2 = \frac{2.7(0.0014) - 2.7423(-0.0353)}{0.0014 - (-0.0353)}$$

$$x_2 = 2.7407$$

$$f(x_2) = 0$$

$\therefore x = 2.7407$ is a root of $x\log_{10}x - 1.2 = 0$

(you can check any values as a & b but these are consecutive roots i.e 2.5, 2.6 or 2,8 like this)

2 Find a real root of $xe^x = 3$ using regula falsi method.

Sol: Let $f(x) = xe^x - 3 = 0$

$f(0) = -3$

$f(1) = -0.2817 < 0$

$f(1.1) = 0.3046 > 0$

clearly $f(1)$ and $f(1.1)$ have opposite signs

$\therefore$ root lies b/w 1 & 1.1

let $a = 1$, $b = 1.1$, $f(a) = f(1) = -0.2817$

$f(b) = f(1.1) = 0.3046$

By regula falsi method

I approximation:

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{1(0.3046) - 1.1(-0.2817)}{0.3046 - (-0.2817)}$$

$x_1 = 1.0480$

$f(x_1) = -0.012 < 0$

$\therefore$ root lies b/w x_1 & b

II approximation:

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)}$$

$$= \frac{(1.0480)(0.3046) - 1.1(-0.012)}{(0.3046) - (-0.012)}$$

$= 1.0498$

$f(x_1) = -0.0006 \simeq 0$

$\therefore x = 1.0498$ is a root of $xe^x = 3$

Find a real root for $e^x \sin x = 1$ by R.F method

Let $f(x) = e^x \sin x - 1 = 0$

$f(0) = -1 < 0$

$f(0.5) = -0.2096 < 0$

$f(0.6) = 0.0288 > 0$

clearly $f(0.5)$ and $f(0.6)$ have opposite signs

$\therefore$ root lies b/w 0.5 & 0.6

Let $a = 0.5$, $b = 0.6$

$f(a) = f(0.5) = -0.2096$

$f(b) = f(0.6) = 0.0288$

by regula falsi method

I approximation.

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{0.5(0.0288) - 0.6(-0.2096)}{0.0288 - (-0.2096)}$$

$x_1 = 0.5879$

$f(x_1) = -0.0016 < 0$

clearly $f(x_1) < 0$ & $f(b) > 0$

$\therefore$ root lies b/w x_1 & b

II approximation.

$$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)}$$

$$x_2 = \frac{0.5079(0.0288) - 3.6(-0.0002)}{0.0288 - (-0.0016)}$$

$x_2 = 0.5885$

$f(x_2) = -0.0001 \cong 0$

$\therefore x = 0.5885$ is a root of $e^x \sin x = 1$

4. Find the real root of the eqn $2x - \log_{10} x = 7$ by Regula-falsi method

Sol: Let $f(x) = 2x - \log_{10} x - 7 = 0$

$f(1) = -5 < 0$

$f(2) = -3.3010 < 0$

$f(3) = -1.4771 < 0$

$f(3.5) = -0.5441 < 0$

$f(3.6) = -0.3563 < 0$

$f(3.7) = -0.1682 < 0$

$f(3.8) = 0.0202 > 0$

clearly $f(3.7)$ & $f(3.8)$ have opposite signs

$\therefore$ roots lies b/w 3.7 & 3.8

Let $a = 3.7$, $b = 3.8$

$f(a) = f(3.7) = -0.1682$

$f(b) = f(3.8) = 0.0202$

By regula-falsi method

I approximation

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x_1 = \frac{3.7(0.0202) - 3.8(-0.1682)}{0.0202 - (-0.1682)}$$

$x_1 = 3.7893$

$f(x_1) = 0$

$\therefore x = 3.7893$ is a root of $2x - \log_{10} x = 7$

Find the root of the equation $x\sin x + \cos x = 0$ or $x\tan x + 1 = 0$ by regula-falsi method.

Let $f(x) = x\sin x + \cos x = 0$

$f(0) > 0,\ f(1) > 0,\ f(2) > 0$

$f(2.5) = 0.6950 > 0$ (to solve in radians mode)

$f(2.6) = 0.4534 > 0$

$f(2.7) = 0.2499 > 0$

$f(2.8) = -0.0043 < 0$

clearly $f(2.7)$ and $f(2.8)$ have opposite signs

$\therefore$ root lies b/w $2.7 < 2.8$

Let $a = 2.7,\ b = 2.8$

$f(a) = f(2.7) = 0.2499$

$f(b) = f(2.8) = -0.0043$

By regula-falsi method

<u>I approximation.</u>

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.7(-0.0043) - 2.8(0.2499)}{-0.0043 - 0.2499}$$

$x_1 = 2.7983$

$f(x_1) = 0.0003 \cong 0$

$\therefore x = 2.7983$ is a root of $x\sin x + \cos x = 0$

Note: In examination if they given the problem is $x\tan x+1=0$ then convert this into

$$x\frac{\sin x}{\cos x}+1=0$$

$x\sin x+\cos x=0$ and do above procedure

Method-IV: Newton-Raphson Method (or) Newtons Iterative Method.

Let $f(x)=0$ be the given eq'n

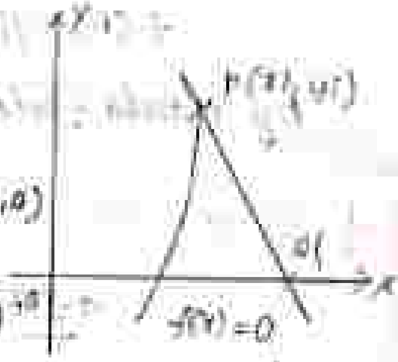
choose any two values a and b such that $f(a)$ & $f(b)$ have opposite signs.

$\therefore$ root lies between a and b

Let us take initial approximate solution to be either a or b or $\frac{a+b}{2}$ or between a & b

clearly $f(x)=0$ is a curve on X-Y plane.

Let $p(x_i,y_i)$ be any point on the curve at this point, this line touches X-axis at $Q(x_{i+1},0)$



W.K.T eqn of Tangent at $p(x_i,y_i)$ having slope m is $y-y_i=m(x-x_i)$

m = slope $=f'(x_i)$

$$y-y_i=f'(x_i)(x-x_i)$$

but this line touches x-axis at $Q(x_{i+1},0)$

$$\therefore\ 0-y_i=f'(x_i)(x_{i+1}-x_i)$$

$$x_{i+1}-x_i=\frac{-y_i}{f'(x_i)} \quad (\because y=f(x))$$

$$x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)}$$

$\therefore x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall i = 0,1,2,\cdots$

is the required Newton Raphson formula find $x_1, x_2, x_3 \cdots$ values, if any two iterations are same then stop the process.

Apply Newton Raphson formula to find an approximate root of the equation $x^3 - 3x - 5 = 0$ which lies near $x=2$.

Let $f(x) = x^3 - 3x - 5 = 0$, given $x_0 = 2$

$f'(x) = 3x^2 - 3$

By Newton Raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall i = 0,1,2,\cdots$$

$$x_{i+1} = x_i - \frac{(x_i^3 - 3x_i - 5)}{(3x_i^2 - 3)}$$

$$x_{i+1} = \frac{2x_i^3 + 5}{3x_i^2 - 3} \longrightarrow (1) \quad \forall i = 0,1,2,3\cdots$$

put $i=0$

$$x_1 = \frac{2x_0^3 + 5}{3x_0^2 - 3} = 2.3333$$

$$x_2 = \frac{2x_1^3 + 5}{3x_1^2 - 3} = 2.2806$$

$$x_3 = \frac{2x_2^3 + 5}{3x_2^2 - 3} = 2.2790$$

$$x_4 = \frac{2x_3^3 + 5}{3x_3^2 - 3} = 2.2790$$

$\therefore x_3$ and x_4 are same, stop the process

$\therefore x = 2.2790$ is a root of $x^3 - 3x - 5 = 0$

V.I.P 2. $x\sin x+\cos x=0$

Sol: Let $f(x)=x\sin x+\cos x=0$

$f(0)=1>0$, $f(1)=1.3818>0$, $f(2)=1.4024>0$

$f(2.5)=0.6950>0$

$f(3)=-0.5666<0$

clearly $f(2.5)$ and $f(3)$ have opposite signs

$\therefore$ root lies b/w 2.5 and 3

Let us take initial approximate soln be $x_0=2.8$

$$f'(x)=x\frac{d}{dx}(\sin x)+\sin x\frac{dx}{dx}+\frac{d}{dx}(\cos x)$$

$$=x\cos x+\cancel{\sin x}-\cancel{\sin x}$$

$$f'(x)=x\cos x$$

By Newton raphson Method

$$x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)}\quad \forall\ i=0,1,2,\dots$$

$$=x_i-\frac{(x_i\sin x_i+\cos x_i)}{x_i\cos x_i}$$

$$x_{i+1}=\frac{x_i^2\cos x_i-x_i\sin x_i-\cos x_i}{x_i\cos x_i}$$

$$x_1=\frac{x_0^2\cos x_0-x_0\sin x_0-\cos x_0}{x_0\cos x_0}=2.7914$$

$$x_2=\frac{x_1^2\cos x_1-x_1\sin x_1-\cos x_1}{x_1\cos x_1}=2.7984$$

$\therefore$ x_1 and x_2 are identical, hence stop the process

$\therefore$ $x=2.7984$ is the root of $x\sin x+\cos x=0$

Using Newton Raphson method find the root of the equation $x + \log_{10} x = 3.375$.

Let $f(x) = x + \log_{10} x - 3.375 = 0$

$f(1) = -2.375 < 0$

$f(2) = -1.0740 < 0$

$f(3) = 0.1021 > 0$

clearly $f(2)$ and $f(3)$ have opposite signs

$\therefore$ root lies b/w 2 & 3

Let initial approximate soln be $x_0 = 2.8$

Given $f(x) = x + \log_{10} x - 3.375$

$$f(x) = x + (0.4343) \log_e x - 3.375$$

$$f'(x) = 1 + \left(\frac{0.4343}{x} \right)$$

By Newton Raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= x_i - \frac{(x_i + \log_{10} x_i - 3.375)}{1 + \left(\frac{0.4343}{x_i} \right)}$$

$$= x_i - \frac{(x_i + \log_{10} x_i - 3.375)}{(x_i + 0.4343)/x_i}$$

$$= x_i - \frac{x_i (x_i + \log_{10} x_i - 3.375)}{x_i + 0.4343}$$

$$= \frac{x_i^2 + x_i (0.4343) - x_i^2 - \log_{10} x_i + 3.375 x_i}{x_i + 0.4343}$$

$$x_{i+1} = \frac{x_i (0.4343) - x_i \log_{10} x_i + 3.375 x_i}{x_i + 0.4343}$$

$$x_1 = \frac{(0.43429)x_0 - x_0\log_{10}x_0 + 3.375x_0)}{(x_0 + 0.43429)}$$

$x_1 = 2.9107$

(∵ Fit this formula in calci and calculate to x_0, x_1)

$x_2 = 2.9110$

$x_3 = 2.9110$

$\therefore x_2$ and x_3 are same

$\therefore x = 2.9110$ is a root of $x\log_{10}x = 3.375$

4. Find a real root of the eqn $xe^x - \cos x = 0$ using Newton Raphson method.

Sol: Let $f(x) = xe^x - \cos x = 0$

$f(0) = -1 < 0$

$f(0.5) = -0.0532 < 0$

$f(1) = 2.1780 > 0$

clearly $f(0.5)$ and $f(1)$ have opposite signs

$\therefore$ root lies b/w 0.5 & 1

Let us take initial approximate soln be $x_0 = 0.5$

[$\because f(0.5) = -0.0532$ is nearer to zero compared

$f(1) = 2.1780$ so take 0.5 as x_0]

$f'(x) = xe^x + e^x + \sin x$

By Newton raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall\ i = 0, 1, 2, \ldots$$

$$x_{i+1} = x_i - \frac{(x_i e^{x_i} - \cos x_i)}{x_i e^{x_i} + e^{x_i} + \sin x_i}$$

$$x_{i+1} = \frac{x_i^2 e^{x_i} + x_i e^{x_i} + x_i \sin x_i - x_i e^{x_i} + \cos x_i}{x_i e^{x_i} + e^{x_i} + \sin x_i}$$

$$x_{i+1} = \frac{(x_i^2 e^{x_i} + x_i \sin x_i + \cos x_i)}{(x_i e^{x_i} + e^{x_i} + \sin x_i)}$$

$$x_1 = \frac{x_0^2 e^{x_0} + x_0 \sin x_0 + \cos x_0}{x_0 e^{x_0} + e^{x_0} + \sin x_0} = 0.5180$$

$x_2 = 0.5178$

$x_3 = 0.5178$

$\therefore$ x_2 and x_3 are same, hence $x = 0.5178$ is a root of $xe^x - \cos x = 0$

Q: Find a root of $e^x \sin x = 1$ using Newton Raphson method.

Sol: Let $f(x) = e^x \sin x - 1 = 0$

$f(0) = -1 < 0$

$f(0.5) = -0.20956 < 0$

$f(1) = 1.28735 > 0$

clearly $f(0.5)$ & $f(1)$ have opposite signs

$\therefore$ root lies b/w 0.5 & 1

Let us take an approximate initial soln be

$x_0 = 0.6$

$f'(x) = e^x \cos x + \sin x e^x$ $\left(\because \frac{d}{dx}(e^x \sin x) = u\frac{dv}{dx} + v\frac{du}{dx} \right)$

By Newton Raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall\ i = 0, 1, 2, \ldots$$

$$x_{i+1} = x_i - \frac{(e^{x_i}\sin x_i - 1)}{(e^{x_i}\cos x_i + e^{x_i}\sin x_i)}$$

$$x_{i+1} = \frac{(x_i e^{x_i}\cos x_i + x_i e^{x_i}\sin x_i - e^{x_i}\sin x_i + 1)}{(e^{x_i}\cos x_i + e^{x_i}\sin x_i)}$$

$$x_1 = \frac{(x_0 e^{x_0}\cos x_0 + x_0 e^{x_0}\sin x_0 - e^{x_0}\sin x_0 + 1)}{(e^{x_0}\cos x_0 + e^{x_0}\sin x_0)}$$

$x_1 = 0.5886$

$x_2 = 0.5885$

$x_3 = 0.5885$

$\therefore x_2$ and x_3 are identical, hence stop process.

$\therefore x = 0.5885$ is a root of $e^x \sin x = 1$

VIP 6. Using Newton Raphson method find square root of N and hence obtain square root of 10

Sol: let $x = \sqrt{N}$

$x^2 = N$

let $f(x) = x^2 - N = 0$

$f'(x) = 2x$

$\therefore$ by N-R method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall\, i = 0,1,2,\ldots$$

$$x_{i+1} = x_i - \frac{(x_i^2 - N)}{2x_i}$$

$$= \frac{2x_i^2 - x_i^2 + N}{2x_i}$$

$x_{i+1} = \frac{x_i^2 + N}{2x_i}$

$\Rightarrow x_{i+1} = \frac{1}{2}\left[x_i + \frac{N}{x_i}\right] \quad \forall\, i = 0, 1, 2, \ldots \rightarrow (1)$

is the iteration formula to find the square root of any no N

Ex- find square root of 10.

Here $N = 10$

$f(x) = x^2 - 10 = 0$.

$f(3) = -1 < 0$

$f(3.5) = 2.25 > 0$

clearly $f(3)$ & $f(3.5)$ have opposite signs

$\therefore$ root lies b/w 3 & 3.5

let $x_0 = 3$

by using (1)

$x_{i+1} = \frac{1}{2}\left[x_i + \frac{N}{x_i}\right] \quad \forall\, i = 0, 1, \ldots$

$x_1 = \frac{1}{2}\left[x_0 + \frac{10}{x_0}\right]$

$x_1 = 3.1667$

$x_2 = 3.1623$

$x_3 = 3.1623$

$\therefore x = 3.1623$ is a root of $x^2 - 10 = 0$ i.e

$\sqrt{10} = 3.1623$.

7. Using Newton Raphson method find cube root of N and hence obtain cube root of 12

Sol: let $x = \sqrt[3]{N}$

$x^3 = N$

let $f(x) = x^3 - N = 0$

$f'(x) = 3x^2$

By N-R method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall i = 0,1,2\cdots$$

$$x_{i+1} = x_i - \left(\frac{x_i^3 - N}{3x_i^2}\right)$$

$$= \frac{3x_i^3 - x_i^3 + N}{3x_i^2}$$

$$x_{i+1} = \frac{1}{3}\left[2x_i + \frac{N}{x_i^2}\right]$$

$$\therefore x_{i+1} = \frac{1}{3}\left[2x_i + \frac{N}{x_i^2}\right] \forall i = 0,1,2,\cdots \text{ is}$$

the required iteration formula to find cube root of any no N

To find cube root of 12

i.e here $N = 12$

$f(x) = x^3 - 12 = 0$

$f(2) = -4 < 0$

$f(2.5) = 3.625 > 0$

clearly $f(2)$ and $f(2.5)$ have oppo signs

$\therefore$ root lies b/w 2 & 2.5

Let us take $x_0 = 2.3$

From the above formula $x_{i+1} = \frac{1}{3}\left[2x_i + \frac{N}{x_i^2}\right]$

$$x_1 = \frac{1}{3}\left[2x_0 + \frac{12}{x_0^2}\right] = 2.2895$$

$$x_2 = \frac{1}{3}\left[2x_1 + \frac{12}{x_1^2}\right] = 2.2894$$

$$x_3 = \frac{1}{3}\left[2x_2 + \frac{12}{x_2^2}\right] = 2.2894$$

$\therefore x = 2.2894$ is a root of $x^3 - 12 = 0$

i.e $\sqrt[3]{12} = 2.2894$

Using Newton Raphson method find reciprocal of a number N and hence obtain reciprocal of 18

let $x = \frac{1}{N}$

$N = \frac{1}{x}$

let $f(x) = \frac{1}{x} - N = 0$

$f'(x) = \frac{-1}{x^2}$

By N-R method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \forall\, i = 0,1,2,3 \ldots$$

$$x_{i+1} = x_i - \frac{\left(\frac{1}{x_i} - N\right)}{\left(-1/x_i^2\right)}$$

$$x_{i+1} = x_i + x_i^2\left(\frac{1}{x_i} - N\right)$$

$$x_{i+1} = x_i + x_i - N x_i^2$$

$$x_{i+1} = 2x_i - N x_i^2$$

$$x_{i+1} = x_i\left[2 - N x_i\right] \quad \forall\, i = 0,1,2 \ldots$$

is the required iteration formula to find reciprocal of any no N by Newton raphson method.

To find $\frac{1}{18}$

Here $N = 18$

$f(x) = \frac{1}{x} - 18 = 0$

Let us take $x_0 = 0.055$

($\because \frac{1}{18} = 0.0556$, we take its nearest value as x_0 from the above iteration formula.)

$$x_{i+1} = x_i\left[2 - Nx_i\right] \quad \forall\ i = 0,1,2\cdots$$

$$x_1 = x_0\left[2 - 18x_0\right] = 0.0556$$

$$x_2 = x_1\left[2 - 18x_1\right] = 0.0556$$

$\therefore x = 0.0556$ is a root of $f(x) = 0$

i.e $\frac{1}{18} = 0.0556$

Secant Method.

W.K.T by Newton Raphson method :-

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \rightarrow (1) \quad \forall\ i = 0,1,2,\cdots$$

W.K.T the slope of the curve (x_i, y_i) approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \rightarrow (2)$$

($\because$ slope $m = \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1}$)

but eq'n (2) in eq'n (1), we get

$$x_{i+1} = x_i - \frac{f(x_i)}{\frac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})}}$$

$$x_{i+1} = x_i - \frac{f(x_i)\,(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \quad \forall\ i = 0,1,2\cdots$$

which is called secant formula

$$x_{i+1} = \frac{x_i f(x_i) - x_i f(x_{i-1}) - f(x_i)x_i + f(x_i)x_{i-1}}{f(x_i) - f(x_{i-1})}$$

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})} \quad \forall\, i = 1,2,3,\ldots$$

put $i=1$ $\quad x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$ where $f(x_0)$ & $f(x_1)$ have opp signs

put $i=2$ $\quad x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$ and so on

Solve $x^3 - 5x + 1 = 0$ using secant method.

Let $f(x) = x^3 - 5x + 1 = 0$

clearly $f(0) = 1 > 0$

$f(0.5) = -1.375 < 0$

clearly $f(0)$ & $f(0.5)$ have opp signs

$\therefore$ root lies b/w 0 & 0.5

Let us take $x_0 = 0$, $x_1 = 0.5$

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.5) = -1.375$

By secant method $x_{i+1} = x_i - f(x_i)\left[\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}\right]$

on simplification this formula is

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})} \quad \forall\, i = 1,2,\ldots$$

put $i=1$ $\quad x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{0 - 0.5(1)}{-1.375 - 1} = 0.2105$

$f(x_2) = -0.0432$

put $i=2$ $\quad x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{0.5(-0.0432) - 0.2105(-1.375)}{-0.0432 - (-1.375)}$

$x_3 = 0.2011$

$f(x_3) = 0.0026$

put $i=3$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \frac{0.2105(0.0026) - 0.2011(-0.0432)}{0.0026 - (-0.0432)}$$

$x_4 = 0.2016$

$f(x_4) = 0.0002 \cong 0$

$\therefore x = 0.2016$ is a root of $x^3 - 5x + 1 = 0$

<u>Note:</u> Since $x^3 - 5x + 1 = 0$ is a cubic eqn so it has 3 roots, one root lies b/w 0 & 0.5

lly we can find another roots lies b/w another pair check another root of their eq'n lies b/w 2 & 2.5

2 Find a real root of the equation $x = e^{-x}$ using secant method.

Sol: Let $f(x)$ is $x = e^{-x}$

$f(x) = xe^{x} - 1 = 0$

clearly, $f(0) = -1 < 0$

$f(0.5) = -0.1756 < 0$

$f(1) = 1.7183 > 0$

$\therefore f(0.5)$ & $f(1)$ have opposite signs

$\therefore$ roots lies b/w 0.5 & 1

Let $x_0 = 0.5$, $x_1 = 1$,

$f(x_0) = f(0.5) = -0.1756$

$f(x_1) = f(1) = 1.7183$

By secant method, $x_{i+1} = x_i - f(x_i)\left[\dfrac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}\right]$

on simplification

$$\therefore x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}, \quad i = 1, 2, 3 \cdots$$

$i=1$: $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \dfrac{0.5(1.7183) - 1(-0.1756)}{1.7183 - (-0.1756)}$

$x_2 = 0.5464$

$f(x_2) = -0.0564$

$i=2$: $x_3 = \dfrac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \dfrac{1(-0.0564) - 0.5464(1.7183)}{-0.0564 - 1.7183}$

$x_3 = 0.5608$

$f(x_3) = -0.0174$

$i=3$: $x_4 = \dfrac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \dfrac{0.5464(-0.0174) - 0.5608(-0.0564)}{-0.0174 - (-0.0564)}$

$x_4 = 0.5672$

$f(x_4) = 0.0002 \simeq 0$

$\therefore x_4 = 0.5672$ is a root of $x = e^{-x}$ or $xe^x = 1$

3. Find a real root of the eqn. $xe^x - \cos x = 0$ using secant method.

A: Let $f(x) = xe^x - \cos x = 0$

$f(0) = -1 < 0$

$f(0.5) = -0.0532 < 0$

$f(1) = 0.1780 > 0$

Clearly $f(0.5)$ and $f(1)$ have opposite signs

$\therefore$ root lies b/w 0.5 & 1

Let us take $x_0 = 0.5,\ x_1 = 1$

$$f(x_0) = f(0.5) = -0.0532$$

$$f(x_1) = f(1) = 2.1780$$

By secant method $x_{i+1} = x_i - f(x_i)\left[\dfrac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}\right]$

on simplification, this is

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})} \quad \forall\ i = 1, 2, 3, \ldots$$

$\underline{i=1}$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{0.5(2.1780) - 1(-0.0532)}{2.1780 - (-0.0532)}$$

$$x_2 = 0.5119$$

$$f(x_2) = -0.0177$$

$\underline{i=2}$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{1(-0.0177) - 0.5119(2.1780)}{-0.0177 - 2.1780}$$

$$x_3 = 0.5158$$

$$f(x_3) = -0.0059$$

$\underline{i=3}$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$x_4 = \frac{0.5119(-0.0059) - 0.5158(-0.0177)}{-0.0059 - (-0.0177)}$$

$$x_4 = 0.5178$$

$$f(x_4) = 0.0001 \simeq 0$$

$\therefore x = 0.5178$ is a root of $xe^{x} - \cos x = 0$

Convergence of Newton Raphson Method

W.K.T Newton Raphson formula is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \rightarrow (1) \quad \forall\, i = 0,1,2,\cdots$$

we compare it with the general iteration formula

$$x_{i+1} = \phi(x_i) \rightarrow (2) \quad \forall\, i = 0,1,2,\cdots$$

$$\therefore \phi(x_i) = x_i - \frac{f(x_i)}{f'(x_i)} \rightarrow (3)$$

we have already noted that the iteration method converges is $|\phi'(x)| < 1$

In general we write eq (3) as

$$\phi(x) = x - \frac{f(x)}{f'(x)}$$

diff this w.r.t x on b.s

$$\phi'(x) = 1 - \frac{\left\{ f'(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[f'(x)] \right\}}{(f'(x))^2}$$

$$\phi'(x) = \frac{[f'(x)]^2 - [f'(x)f'(x) - f(x)f''(x)]}{[f'(x)]^2}$$

$$\phi'(x) = \frac{\cancel{[f'(x)]^2} - \cancel{[f'(x)]^2} + f(x)f''(x)}{[f'(x)]^2}$$

but given $|\phi'(x)| < 1$

$$\therefore \left| \frac{f(x)f''(x)}{[f'(x)]^2} \right| = |\phi'(x)| < 1$$

$|f(x)f''(x)| < [f'(x)]^2$ is the convergence condition for N-R method.

∴ In the considered interval, N-R-formula converges provided the initial approximation x_0 is choosen sufficiently close to the root $f(x)$ and $f''(x)$ are continuous as bounded in any small interval containing the root. as bounded