UNIT-II

Regular Expressions:

* Introduction
* examples
* Components
* operations.

Introduction:- Regular expressions are mathematical exp describing a language which is accepted by F.A.

* R.E describing a language called Regular language.

Definition:-

let Σ be an alphabet then RE over Σ is defined as follows.

* ϕ is a R.E then that describes an Empty set. R.E
* ϵ is a R.E then that describes NULL string set. R.E
* a is a R.E over Σ then that describes the set with a. R.E
* Let r and s are two R.E and L_1 and L_2 are two languages which are described by r and s then,
  i) $r+s$ is Equivalent to $L_1 \cup L_2$
  ii) rs is Equivalent to L_1L_2 (or) $L_1 \cdot L_2$
  iii) r^* is Equivalent to L_1^*

Components:-

union, concatenation, kleene closure

Examples:-

1) write the R.E for the language accepting all combinations of a's over $\Sigma = \{a\}$

sol:- $\Sigma = \{a\}$

$L = \{\epsilon, a, aa, aaa, aaaa, ---\}$

$= a^*$

$\boxed{\therefore RE = a^*}$

2) write a R.E for the language accepting all combinations of a's except empty string over $\Sigma = \{a\}$

Sol:- $\Sigma = \{a\}$ $\{a, aa, aaa, aaaa, ---\}$

$= a^+$

$\boxed{\therefore R.E = a^+}$

3) Write a R.E for the language accepting any no of a's and b's over, $\Sigma = \{a,b\}$

Sol:- $\Sigma = \{a,b\}$

$L = \{\epsilon, a^1, b^1, a^2, ab, ba, b^2, \ldots\}$

$= \{\epsilon, a, b, aa, ab, ba, bb, \ldots\}$

$= (a+b)^*$

$\boxed{\therefore R.E = (a+b)^*}$

4) Write a R.E for the language containing all strings which are Ending with 00 over $\Sigma = \{0,1\}$

Sol:- $\Sigma = \{0,1\}$

$L = \{ \underline{(0+1)^*} 00$

$\boxed{\therefore R.E = (0+1)^* 00}$

5) Write a R.E for the language Accepting set of all string which are starting with '1' and Ending with '0' over $\Sigma = \{0,1\}$

Sol:- $\Sigma = \{0,1\}$

$1 \underline{(0+1)}^* 0$

$\boxed{\therefore R.E = 1 \underline{(0+1)}^* 0}$

6) Write a R.E for the language accepting any no of a's followed by any no of b's, followed by any no of c's over $\Sigma = \{a,b,c\}$

Sol: $\Sigma = \{a,b,c\}$

$\boxed{\therefore R.E = a^* b^* c^*}$

7) Write a R.E for the language accepting set of all strings which contains the third character from the right end of the string is always 'a' over $\Sigma = \{a,b\}$ $\boxed{\therefore R.E = (a+b)^* a (a+b)(a+b)}$

Sol: $\Sigma = \{a,b\}$

$(a+b)^* \underline{a} \, \underline{(a+b)} \, \underline{(a+b)}$

language Associated with R.E :-

* A language which is described by r.e is called Regular language.
* Let R be a R.E then the language accepted by R.E is denoted by L(R).

eg :- If the R.E $R = (ba)^*$ then $L(R) = ?$

sol:- $R = (ba)^*$

$L(R) = \{ (ba)^0, (ba)^1, (ba)^2, (ba)^3 --- \}$

$= \{ \epsilon, ba, baba, bababa, --- \}$

Properties of R.E (Identity rules):

Let R, S be R.E then the following properties are true

1) $(R+S)+T = R+(S+T)$
2) $R+R = R$
3) $R+\phi = \phi+R = R$
4) $R\phi = \phi R = \phi$
5) $R+S = S+R$
6) $R\epsilon = \epsilon R = R$
7) $R(ST) = (RS)T$
8) $R(S+T) = RS+RT$
9) $(S+T)R = SR+TR$
10) $\phi^* = \epsilon^* = \epsilon$
11) $R^* R^* = R^* = (R^*)^*$
12) $RR^* = R^* R = R^+$
13) $(R+S)^* = (R^* S^*)^* = (R^* + S^*)^*$
14) $(R+S)^* = (R^* + S^*)^* = (R^* S^*)^*$

manipulation of R.E (Basic operations)

i) union ii) concatenation iii) kleene closure.

UNION :-

Let R & S be two R.E then union of R, S is defined as

$R \cup S = \{ x \mid x \in R \text{ or } x \in S \}$

eg :- If $R = \{ab, c\}$ and $S = \{d, ef\}$ then $R \cup S = ?$

$R \cup S = \{ ab, c, def \}$

concatenation :-

Let R & S be two R.E then concatenation of R & S is defined as $RS = \{ xy \mid x \in R \text{ and } y \in S \}$

Ex:- If $R = \{ab, c\}$ and $S = \{d, ef\}$ then $RS = ?$

Sol: $RS = \{ab, c\}\{d, ef\}$

$= \{abd, abef, cd, cef\}$

Kleene closure :-

Let R be a RE then the Kleene closure of R is denoted as R^* which contains set of all strings including Null string.

Ex:- If $R = \{ab\}$ then $R^* = ?$

Sol:- $R^* = \{(ab)^0, (ab)^1, (ab)^2, (ab)^3 \ldots\}$

$= \{\epsilon, ab, abab, ababab, \ldots\}$

Examples:

Construct a string set for the RE given below.

i) ab^*a

Sol: $\{ab^0a, ab^1a, ab^2a, \ldots\}$

$\{aa, aba, abba, \ldots\}$

ii) 1^*0

$\{1^0, 1^1, 1^2, 1^3, 1^4, \ldots\}0$

$= \{\epsilon, 1, 11, 111, 1111, \ldots\}0$

$= \{0, 10, 110, 1110, 11110, \ldots\}$

iii) 00^*

$= \{00^0, 00^1, 00^2, 00^3, 00^4, \ldots\}$

$= \{0, 00, 000, 0000, 00000, \ldots\}$

$= 0^+$

iv) $(100^+)^*$

$= (10\{0^1, 0^2, 0^3, \ldots\})^*$

$= (100, 1000, 10000, \ldots)^*$

$= ((100)^*, (1000)^*, (10000)^*)$

$= (\{\epsilon, 100, 100100, 100100100, \ldots\}, \{\epsilon, 1000, 10001000, 100010001000, \ldots\}, \{\epsilon, 10000, 1000010000, \ldots\})$

$\therefore \{\epsilon, 100, 100100, \ldots 1000, 10001000, \ldots 10000, 1000010000, \ldots\}$

v) $(0+1)^* = 0^* + 1^*$

$= \{\epsilon, 0, 00, 000, \ldots\} \cup \{\epsilon, 1, 11, 111, 1111, \ldots\}$

$= \{\epsilon, 0, 00, 000, \ldots 1, 11, 111, 1111 \ldots\}$

vi) $(0+1)^* 011$

$= (0^* + 1^*) 011$

$= ((\{\epsilon, 0, 00, 000, \ldots\} \cup \{\epsilon, 1, 11, 111, \ldots\}) 011)$

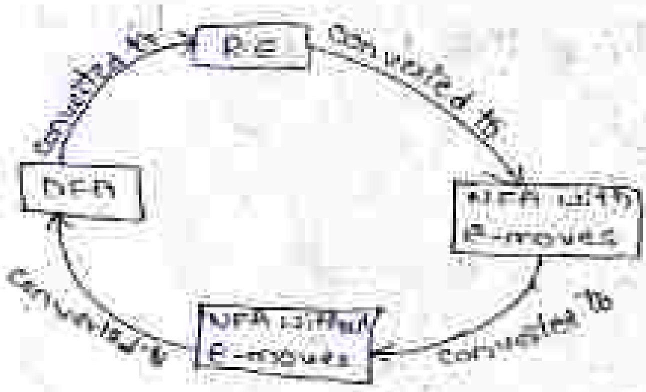
$= ((\{\epsilon, 0, 00, 000, \ldots 1, 11, 111, \ldots\}) 011)$

$= \{011, 0011, 00011, 000011, \ldots 1011, 11011, 111011, \ldots\}$

Equivalence of RE and FA:-

1. Conversion of R.E to FA
2. Conversion of FA to R.E

Relationship b/w R.E and FA:



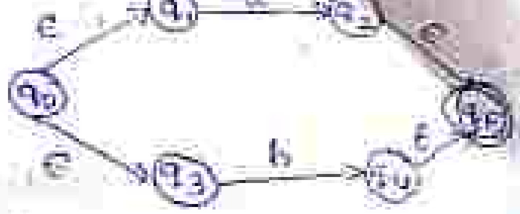
1. **Conversion of R.E to FA:**

Basic notations used.

1. If the RE is like $r = \epsilon$ then FA is

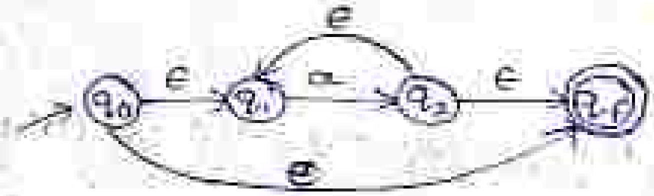
2. If the RE is like $r = a$ then FA is

3. If the RE is like $r = a+b$ then FA is



4. If the RE is like $r_1 = ab$ then FA is

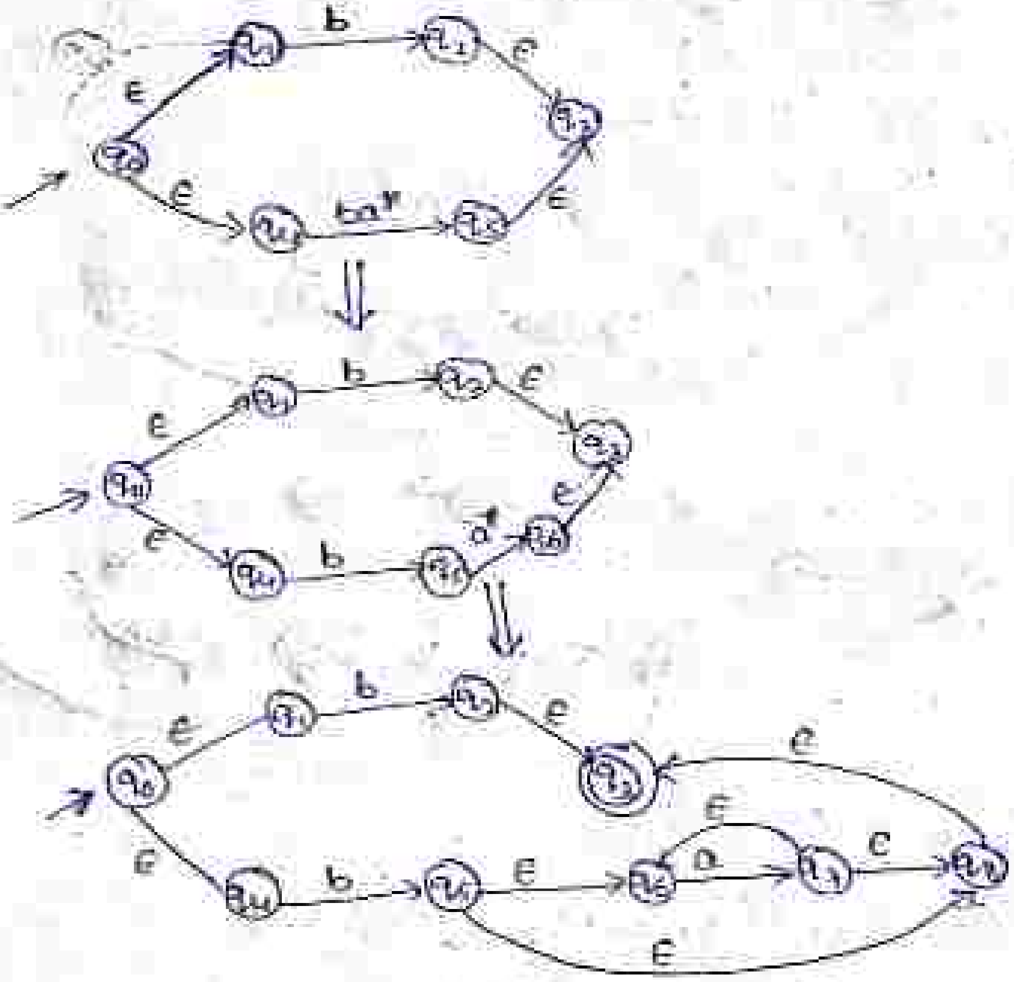
5. If the RE is like $r_1 = a^*$ then FA is



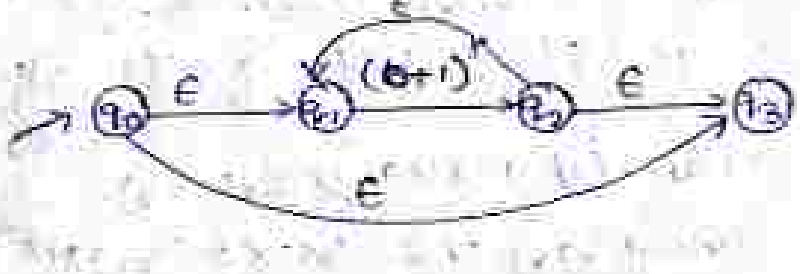
Examples:

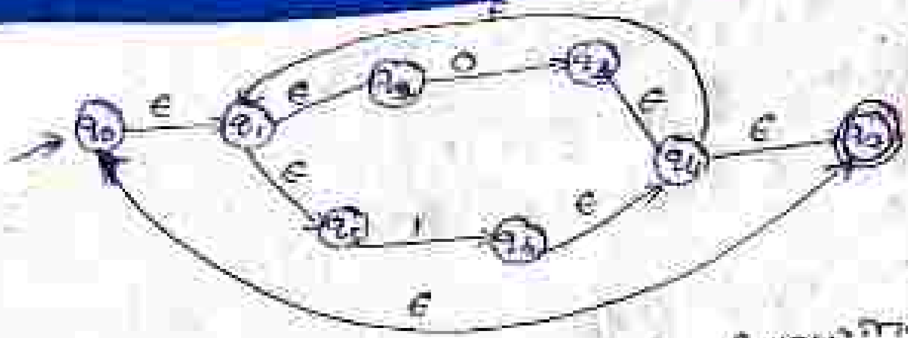
1) Construct an NFA with ϵ-moves for the RE $b+ba^*$

Sol:- The given RE $r = b+ba^*$



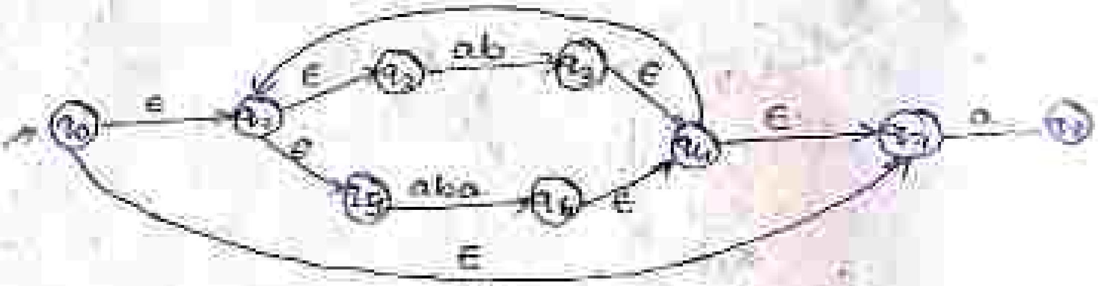
2) Construct an NFA with ϵ-moves for the RE is $(0+1)^*$

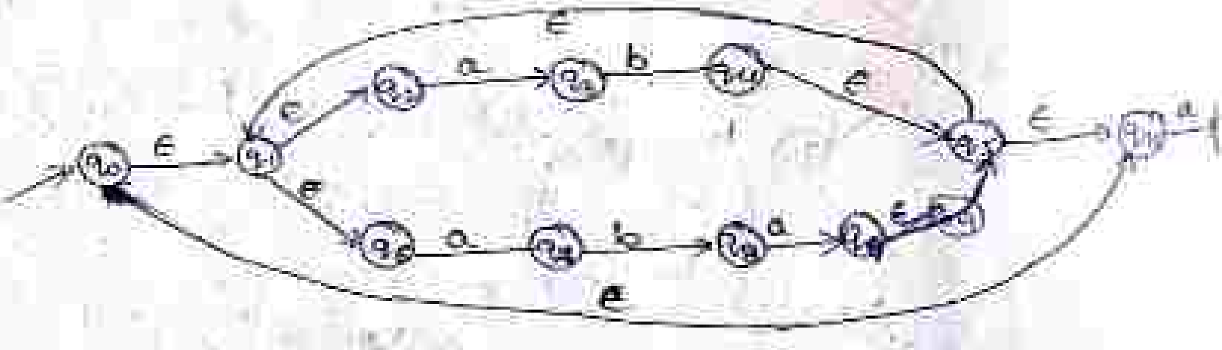




3) Construct a DFA for the following RE is (abua...

The given R.E is $(ab \cup aba)^* a$





$\Sigma = \{a, b\}$

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2, q_6, q_{10}\} = A$

$\delta'(A, a) = \epsilon\text{-closure}(\delta(A, a))$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2, q_6, q_{10}\}, a))$

$= \epsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_6, a) \cup \delta(q_{10}, a))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup q_3 \cup q_7 \cup q_{11})$

$= \epsilon\text{-closure}(q_3) \cup \epsilon\text{-closure}(q_7) \cup \epsilon\text{-closure}(q_{11})$

$= \{q_3\} \cup \{q_7\} \cup \{q_{11}\}$

$= \{q_3, q_7, q_{11}\} = B$

$\delta'(A,b) = \epsilon\text{-closure}(\delta(A,b))$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2, q_6, q_{10}\}, b))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup \phi \cup \phi \cup \phi)$

$= \phi$

$\delta'(B,a) = \epsilon\text{-closure}(\delta(B,a))$

$= \epsilon\text{-closure}(\delta(\{q_3, q_7, q_{11}\}, a))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup \phi)$

$= \phi$

$\delta'(B,b) = \epsilon\text{-closure}(\delta(B,b))$

$= \epsilon\text{-closure}(\delta(\{q_3, q_7, q_{11}\}, b))$

$= \epsilon\text{-closure}(q_4 \cup q_8 \cup \phi)$

$= \epsilon\text{-closure}(q_4) \cup \epsilon\text{-closure}(q_8)$

$= \epsilon\text{-clo}\ \{q_4, q_5, q_{10}, q_6, q_1, q_2\} \cup \{q_8\}$

$= \{q_1, q_2, q_4, q_5, q_6, q_8, q_{10}\}$

$= C$

$\delta'(C,a) = \epsilon\text{-closure}(\delta(C,a))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2, q_4, q_5, q_6, q_8, q_{10}\}, a))$

$= \epsilon\text{-closure}(\phi \cup q_3 \cup \phi \cup \phi \cup q_7 \cup q_9 \cup q_{11})$

$= \epsilon\text{-closure}(q_3) \cup \epsilon\text{-closure}(q_7) \cup \epsilon\text{-closure}(q_9) \cup \epsilon\text{-closure}(q_{11})$

$= \{q_3\} \cup \{q_7\} \cup \{q_9, q_5, q_{10}, q_1, q_2, q_6\} \cup \{q_{11}\}$

$= \{q_1, q_2, q_3, q_5, q_6, q_7, q_9, q_{10}, q_{11}\}$

$= D$

$\delta'(C,b) = \epsilon\text{-closure}(\delta(C,b))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2, q_4, q_5, q_6, q_8, q_{10}\}, a))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup \phi \cup \phi \cup \phi \cup \phi \cup \phi)$

$= \phi$

$\delta'(D,a) = \epsilon\text{-closure}(\delta(D,a))$

$= \epsilon\text{-closure}(\phi \cup q_3 \cup \phi \cup \phi \cup q_7 \cup \phi \cup \phi \cup q_{11} \cup \phi)$

$= \epsilon\text{-closure}(q_3) \cup \epsilon\text{-closure}(q_7) \cup \epsilon\text{-closure}(q_{11})$

$= \{q_3\} \cup \{q_7\} \cup \{q_{11}\}$

$= \{q_3, q_7, q_{11}\}$

$= B$

$\delta'(D,b) = \epsilon\text{-closure}(\delta(D,b))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup q_4 \cup \phi \cup \phi \cup q_8 \cup \phi \cup \phi \cup \phi)$

$= \epsilon\text{-closure}(q_4) \cup \epsilon\text{-closure}(q_8)$

$= \{q_4, q_5, q_1, q_{10}, q_2, q_6\} \cup \{q_8\}$

$= \{q_1, q_2, q_4, q_5, q_6, q_8, q_{10}\}$

$= C$

The previous NFA with ϵ-moves has the Initial state as q_0 and final state as q_{11}.

Now the DFA has initial state A because it contains q_0 as an element which can be initial state NFA.

The Final state of DFA is B, D because both states contain q_{11} as an element. The DFA 'M' is like,

$M = \{Q, \Sigma, \delta, q_0, F\}$ where

$Q = \{A, B, C, D\}$

$\Sigma = \{a, b\}$

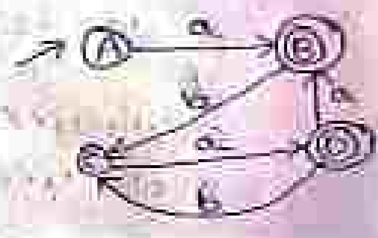
δ: is a transition function which is defined as

δ	a	b
→A	B	ϕ
(B)	ϕ	C
C	D	ϕ
(D)	B	C

$q_0 = A$

$F = \{B, D\}$

∴ transition diagram for DFA is



1st method:

conversion of FA to RE.

ARDEN'S theorem:-

Let P and Q be two RE. over the input alphabet Σ. The RE 'R' is given as $\boxed{R = Q + RP}$ which has a unique solution $\boxed{R = QP^*}$

conversion algorithm:

* Let q_1 be a initial state.
* there are $q_2, q_3, q_4, \ldots q_n$ are no. of states. The final state may be some q_j where $j \leq n$.
* Let α_{ij} be a transition from q_i to q_j state
* calculate $q_j = \alpha_{ij} q_i$. If q_j is an initial state then $q_j = \alpha_{ij} q_i + \epsilon$
* Similarly compute the final state equation which gives the RE

Q:- construct RE from given DFA.

Sol:

$q_1 = aq_1 + \epsilon \longrightarrow (1)$

$q_2 = bq_1 + bq_2 \longrightarrow (2)$

$q_3 = aq_2 \longrightarrow (3)$

$q_1 = aq_1 + \epsilon \quad \left(\begin{array}{l} R = Q + RP \\ R = QP^* \end{array} \right)$

where

$R = q_1$

$Q = \epsilon$

$P = a$

solution for above equ. is

$R = QP^*$

$q_1 = \epsilon \, a^*$

$q_1 = a^* \longrightarrow (4)$

substitute equ (4) in equ (2)

$q_2 = bq_1 + bq_2$

$q_2 = ba^* + bq_2$

$q_2 = ba^* b^*$

$= a^* bb^*$

$= a^* b^*$

The R.E is $\boxed{a^* b^*}$

Construct R.E from given DFA

sol: $q_1 = 0q_1 + \epsilon \longrightarrow (1)$

$q_2 = 1q_1 + 1q_2 \longrightarrow (2)$

$q_3 = 0q_1 + 0q_3 + 1q_3 \longrightarrow (3)$

$q_1 = 0q_1 + \epsilon$

$q_1 = \epsilon + 0q_1$ $(\because R = Q + RP \Rightarrow R = QP^*)$

$q_1 = \epsilon 0^*$

$q = 0^* \longrightarrow (4)$

sub equ (4) in equ (2)

$q_2 = 1q_1 + 1q_2$

$q_2 = 10^* + 1q_2$

$q_2 = 10^* 1^*$

$= 0^* 11^*$

$= 0^* 1^*$

$\therefore$ The regular expression for given DFA is

$q_1 + q_2 = 0^* + 0^* 1^*$

$= 0^* (\epsilon + 1)$

$= 0^* 1^*$

Construct R.E from given DFA.

sol: $q_1 = \epsilon \longrightarrow (1)$

$q_2 = 0q_1 + 0q_3 \longrightarrow (2)$

$q_3 = 0q_2 \longrightarrow (3)$

sub equ (1) in equ (2)

$q_2 = 0q_1 + 0q_3$

$= 0 \cdot \epsilon + 0 \cdot q_3$

$q_2 = 0 + 0q_3 \longrightarrow (4)$

sub equ ③ in equ ④

$q_2 = 0 + 0q_3$

$q_2 = 0 + 0 \cdot 0q_2$ $\quad (R = Q + RP \Rightarrow R = QP^*)$

$q_2 = 0(00)^*$

The R.E for given DFA is

$\boxed{0(00)^*}$

Applications of R.E :-

* RE are used for describing a language called Regular language.
* R.E are used to implement lexical analysis in compiler design.
* R.E are used to represent a set of strings in Unix programming

closure properties of Regular languages :-

* Regular languages are closed under Union.
* " " " " " Intersection
* " " " " " Concatenation
* " " " " " Kleene closure
* " " " " " Difference
* " " " " " Complement
* " " " " " Reverse
* " " " " " Symmetric difference
* " " " " " Homomorphism
* " " " " " Inverse Homomorphism

Regular Grammar and FA :-

* Grammar
* Regular Grammar
    1. left linear Grammar
    2. Right linear Grammar
* FA from Regular Grammar
* RG from FA
* RE from RG.

Grammar :-

Grammar is a set that contains four types like

$G = (V, T, P, S)$

Regular grammar from FA:-

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a FA. It contains 'n' no. of states like $Q = \{q_1, q_2, \ldots, q_n\}$ and $\Sigma = \{a_1, a_2, a_3, \ldots, a_n\}$ therefore the Regular grammar $G = (V, T, P, S)$ is defined as

$V = \{q_1, q_2, \ldots, q_n\}$

$T = \Sigma = \{a_1, a_2, \ldots, a_n\}$

$S = \delta = q_0$

P = Transitions of FA

Rules :-

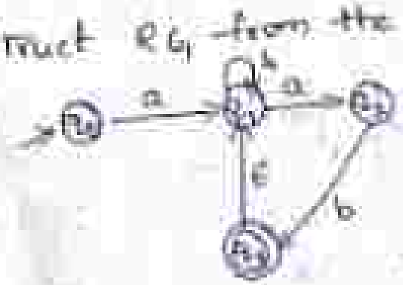
* If the transition of FA is like $q_i \xrightarrow{a} q_j$ then the production rule is $\boxed{q_i \rightarrow a q_j}$

$\Downarrow$

$q_i \rightarrow a q_j$

* If there is a final state in FA is like q_i then the production rule is $\boxed{q_i \rightarrow \epsilon}$
* If the transition of FA is like $q_i \xrightarrow{a} q_j$ then the production rules are $\boxed{\begin{matrix} q_i \rightarrow a q_j \\ q_i \rightarrow a \end{matrix}}$

Ex:- construct RG from the given FA



Sol:- The given FA is like $M = (Q, \Sigma, \delta, q_0, F)$

where $Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, \epsilon\}$

δ is a transition function is defined as

δ:

state	input symbols a	b	ϵ
$\rightarrow q_0$	q_1	ϕ	ϕ
q_1	q_2	q_1	ϕ
q_2	ϕ	q_3	ϕ
q_3	ϕ	ϕ	q_1

$q_0 = q_0$

$F = \{q_3\}$

Now, therefore the Equivalent regular Grammar G_1 of M is defined as $G_1 = (V, T, P, S)$ where

$V = \{q_0, q_1, q_2, q_3\}$

$T = \{a, b, \epsilon\}$

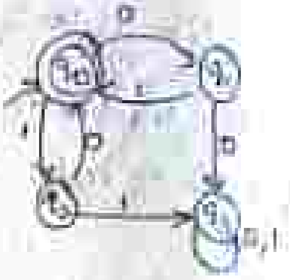
P is a set of production rules defined from transitions of M is like

P:

$\{q_0 \rightarrow aq_1$
$q_1 \rightarrow aq_2$
$q_1 \rightarrow bq_1$
$q_2 \rightarrow bq_3$
$q_2 \rightarrow bq_2$
$q_3 \rightarrow \epsilon q_1\}$

$S = \{q_0\}$

2) Construct RG from the given FA



Sol:- The Given FA is like

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

δ:

states	input symbols 0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_3	q_3

Now, therefore the Equivalent RG of M is defined as $G = (V, T, P, S)$ where

$V = \{q_0, q_1, q_2, q_3\}$

$T = \{0, 1\}$

P is a set of production rules defined from transitions of M is like

$P: \{q_0 \rightarrow 0q_1$ $q_1 \rightarrow 1$ $q_3 \rightarrow 1q_3\}$
$q_0 \rightarrow 1q_2$ $q_2 \rightarrow 0q_0$
$q_1 \rightarrow 0q_3$ $q_2 \rightarrow 0$
$q_1 \rightarrow 1q_0$ $q_2 \rightarrow 1q_3$
$q_3 \rightarrow 0q_3$

Finite automata from RG:-

Let $G=(V,T,P,S)$ be a RG. We can construct DFA 'M' whose

1. states corresponds to variables 'V'
2. starting state corresponds to starts symbol 'S'
3. Transitions in 'M' corresponding to production Rules in 'P'.
4. Inputsymbols corresponding to terminals in 'T'
5. If there is a production is of the form $q_i \to a$ then the transition is terminate at a new state called final state

Rules:-

* If the production rule is of the form $A \to \epsilon$ then 'A' is the final state (A)
* A production rule is of the form $A_i \to a$ then there is a transition from A_i to final state labelled with a

$$A_i \to a = (A_i) \xrightarrow{a} (V_f)$$

* A production rule is of the form $A_i \to aA_j$ then there is a transition from A_i to A_j labelled with 'a'.

$$(A_i) \xrightarrow{a} (A_j)$$

* If a production rule is of the form $A_i \to a_1 a_2 a_3 \cdots a_m A_j$ then there is a transition from A_i to A_j and add intermediate states labelled by $a_1, a_2, a_3 \cdots a_m$.

$$A_i \xrightarrow{a_1} (x_1) \xrightarrow{a_2} (x_2) \xrightarrow{a_3} (x_3) \cdots (x_m) \xrightarrow{a_m} (A_j)$$

Ex:- Construct a FA from the RG.

$$S \to aA/B$$
$$A \to aaB$$
$$B \to bB/a$$

Sol:- The Given $G=(V,T,P,S)$ where

$V=\{S,A,B\}$

$T=\{a,b\}$

$P=\{ S \to aA$
$S \to B$
$A \to aaB$
$B \to bB$
$B \to a \}$

$S = \{S\}$

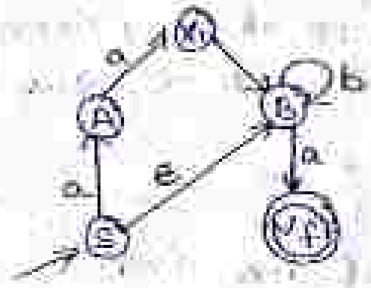
$\therefore$ The Equivalent DFA M is

$M = (Q, \Sigma, \delta, q_0, F)$ where

$Q = \{S, A, B, V_f\}$

$\Sigma = \{a, b\}$

δ:



$Q = \{S, A, B, V_f, X_1\}$

$\Sigma = \{a, b, \epsilon\}$

$q_0 = \{S\}$

$F = \{V_f\}$

3) Construct FA from the given RG:

$S \rightarrow aA \mid \epsilon$

$A \rightarrow aA \mid bB \mid \epsilon$

$B \rightarrow bB \mid \epsilon$

Sol:- The given $G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{a, b, \epsilon\}$

$P = \{ S \rightarrow aA$

$S \rightarrow \epsilon$

$A \rightarrow aA$

$A \rightarrow bB$

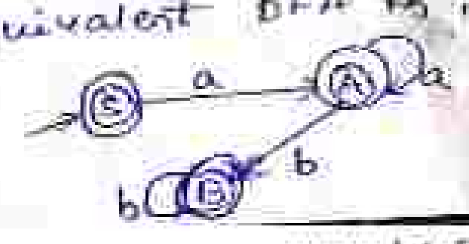
$A \rightarrow \epsilon$

$B \rightarrow bB$

$B \rightarrow \epsilon \}$

$S = \{S\}$

The Equivalent DFA M is $M = (Q, \Sigma, \delta, q_0, F)$



$Q = \{S, A, B\}$

$\Sigma = \{a, b, \epsilon\}$

$q_0 = \{S\}$

Regular expression from Regular grammar :—

Let $G = (V, T, P, S)$ be a RG. Now the RE 'R' is defined from 'G' by using the following rules

* Replace the '$\rightarrow$' in the grammar productions with equal symbol (=) to get the set of equations.
* Convert the equation of the form $A \rightarrow aA \mid ab$

$$\Downarrow$$

$$A = a^* ab$$

* Repeat the step 2 until we get the RE for the starting symbol. This gives the final R.E of given grammar 'G'.

Eg:- obtain the Regular expression from the grammar given below

$$S \rightarrow 01B \mid 0$$
$$B \rightarrow 1B \mid 1$$

Sol:- The given Regular Grammar $G = (V, T, P, S)$

where $V = \{S, B\}$

$T = \{0, 1\}$

$$P = \{ S \rightarrow 0B,\ S \rightarrow 0,\ B \rightarrow 1B,\ B \rightarrow 1 \}$$

$S = \{S\}$

Replace arrow from set of productions with equal (=)

$$S = 01B \mid 0$$
$$B = 1B \mid 1$$
$$\Downarrow$$
$$B = 1^* 1$$
$$B = 1^+$$

Sub $B = 1^+$ in $S = 01B \mid 0$

$$S = 011^+ \mid 0$$

The final RE is

$$S = 011^+ + 0$$
$$= 0(11^+ + \epsilon)$$
$$= 01^*$$

2)

$S \to baS \mid aA$
$A \to bbA \mid bb$

Sol:- The given RG $G = (V, T, P, S)$

where $V = \{S, A\}$

$T = \{a, b\}$

$P = \{ S \to baS$
$S \to aA$
$A \to bbA$
$A \to bb\}$

Replace arrow from set of productions with =

$S = baS \mid aA$
$A = bbA \mid bb$

$A = bbA \mid bb$
$A = bb^* bb$

$S = baS \mid aA$
$S = ba^* aA$

Sub $A = bb^* bb$ in $S = ba^* aA$

$S = ba^* abb^* bb$

$$\boxed{\begin{aligned} RE &= ba^* abb^* bb \\ &= ba^+ b^+ bb \end{aligned}}$$

Introduction :-

* Pumping lemma is used for checking the given language Regular or not.
* Let $M = (Q, \Sigma, \delta, q_0, F)$ be a FA with 'n' states.
* Let 'L' be a Regular language accepted by 'M'
* Let 'w' belongs to L ($w \in L$) and their Exists x, y, z such that $w = xyz$ and $xy^iz \in L$ for each $i \geq 0$.

Application :-

step 1 :- Assume 'L' be regular language and 'n' is the no of states in the FA.

2 :- choose the string 'w' such that $|w| \leq n$. use pumping lemma to write $w = xyz$. with the conditions

ii) $|xy| \le n$

iii) $|y| > 0$

3:- find a suitable integer 'i' such that $xy^iz \notin L$. Hence 'L' is not regular.

<u>examples</u>:-

1) S.T the set $L = \{0^{i^2} / i \ge 1\}$ is not regular.

<u>sol</u>:- Given $L = \{0^{i^2} / i \ge 1\}$

$$L = \{0^{1^2}, 0^{2^2}, 0^{3^2}, 0^{4^2}, \dots\}$$

$$L = \{0^1, 0^4, 0^9, 0^{16}, \dots\}$$

Assume $w = 0^{i^2}$

$$\underbrace{0^{i^2-3}}_{x} \quad \underbrace{0^1}_{y} \quad \underbrace{0^2}_{z}$$

For $i=2$

$$xy^iz = 0^{i^2-3}\, 0^2\, 0^2$$
$$= 0^{i^2+1}$$
$$= 0^{2^2+1}$$
$$= 0^5 \notin L$$

The given set is not regular.

2) S.T the set $L = \{1^p \mid p \text{ is prime}\}$ is not regular

<u>sol</u>:- Given $L = \{1^p \mid p \text{ is prime}\}$

$$L = \{1^2, 1^3, 1^5, 1^7, 1^{11}, 1^{13}, \dots\}$$

Assume $w = 1^p$

$$\underbrace{1^{p-5}}_{x} \quad \underbrace{1^2}_{y} \quad \underbrace{1^3}_{z}$$

For $i=2$

$$xy^iz = 1^{p-5}\,(1^2)^2\,(1^3)$$
$$= 1^{p-5}\, 1^4\, 1^3 \Rightarrow 1^{p+2} \notin L$$

$\therefore$ L is not regular