Unit - 3

Probabality and distributions

Review of probability:

Introduction:

The theory of probability is one of the most useful and intresting branches of modern mathematics. It is becomming prominent by its application in many fields of learning such as insurance, statistics, biological sciences, physical sciences and engineering etc...

Experiment:

An experiment is any physical action or process and it is observed and the results are noted.

Example.

1. Tossing a coin.
2. rolling a die.

There are 2 types of experiments. They are

1. Predictable experiment.
2. random experiment.

Predictable experiment:

The results of experiments can be predicted with a certainity prior to the performance of ~~random~~ experiment.

Eg: The experiment is throwing a stone upwards

Random experiment:

A random experiment is an experiment which can be repeated any number of times under the same conditions and the results of

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experiment cannot be predicted with certainity.

Trail:

The single performance of a random experiment is called a trial.

Eg: 1. Tossing a coin.

2. Draw a card from a pack.

3. Rolling a die etc...

Event (or) Outcome:

The end results of trail of a random experiment is called an event (or) outcome.

Sample Space:

A set of all possible outcome of a random experiment is called a sample space. It is denoted by "S".

Eg: 1. Rolling a die of experiment, then the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

2. Tossing a coin then $S = \{H, T\}$

Types of events:

There are 6 types of events. They are

1. elimentary event.
2. compound event
3. Equally likely event.
4. Mutually exclusive event.
5. Exhaustive event.
6. Favourable event.

1. elimentary event:

An elimentary event is an event which cannot be split into further small events.

Eg: $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$

2. Compound event:

A compound event is an event which is a combination of several elementary events.

Eg: If 2 balls are drawn from a bag containing 4 green, 6 black, 7 white balls.

The event of drawing (drawn) 2 green ball (or) 2 white balls is a compound event.

3. Equally likely event:

Events are said to be equally likely event, if they have same chance of happening.

Eg: 1. In tossing a coin all possible outcomes are equally likely events.

i.e $\{H\}$ or $\{T\}$ are equally likely events.

4. Mutually Exclusive Event:

Events are said to be mutually exclusive event if two (or) more events cannot be happened simultaneously in the same trail.

(or)

If A and B are mutually exclusive events then

$A \cap B = \phi$

Eg: If $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ Then

$$A \cap B = \phi$$

5. Exhaustive Event:

The total number of possible outcomes in the random experiment is called the exhaustive event.

Eg: In tossing two coins of experiment, then the exhaustive events are

{HH, HT, TH, TT}

6. **Favourable event:**

The event which are favourable to a particular event of an experiment is called a favourable event.

Eg: In rolling a die my getting event is an odd numbers is {1, 3, 5} is a favourable event.

probability:

The probability of an event 'E' is defined as "ratio of number of favourable events of E and the total number of possible outcomes in a random experiment" i.e $P(E) = \frac{n(E)}{n(S)} = \frac{\text{no. of favourable events}}{\text{no. of exhaustive events}}$

Note:

1. Suppose we say that occurance of an event 'E' as success, and non-occurance '$\bar{E}$' as failure. The Probability $P(E)$ of the happening of the event 'E' is known as the probability of success and the probability $P(\bar{E})$ of non-happening of the event is known as probability of failure.

2. If $P(E) = 1$, the event 'E' is called certain event and if $P(E) = 0$, the event 'E' is called an impossible event.

3. Suppose 'A' is an event with $P(\bar{A}) < P(A)$

$$1 - P(A) < P(A)$$
$$1 < P(A) + P(A)$$
$$1 < 2P(A)$$
$$1/2 < P(A)$$

$P(A) \geq \frac{1}{2}$

Eg:

1. What is the probability for a leap year to have 52 Mondays and 53 Sundays?

Sol: Saturday, Sunday before Monday, we cannot take 2 Sundays.

A leap year has 366 days.

i.e 52 weeks and 2 days.

These 2 days can any one of the following 7 days
i.e
1. Monday and Tuesday
2. Tue and wed
3. wed and Thur
4. Thur and fri
5. fri and Sat
6. Sat and sun
7. Sun and mon

let 'E' be an event of having 52 mondays and 53 Sundays in the year.

Total number of possible cases $n(S) = 7$

number of favourable case to 'E' $n(E) = 1$ (Saturday and Sunday)

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{7} = 0.1428$$

Axioms of Probability

If $P(E)$ is called "Probability of E" and it satisfies the following conditions.

Axiom 1: $P(E) \geq 0$, it means the "probability of an event may be zero (or) positive number".

Axiom 2: $P(S) = 1$, it means the "probability of an event ~~me~~ the probability of sample space is always $= 0$".

Axiom 3: $P(A \cup B) = P(A) + P(B)$ the probability of union of any number of mutually exclusive events is equal to the sum of the probability of individual events.

Eg: $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$.

Addition theoram on probability:

Statement:

If 'S' is a sample space and E_1, E_2 are any events in S. Then $P(E_1 (or) E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

Conditional event.

If 'E_1', E_2 are any events in a sample space 'S' and if E_2 occurs after the occurance of 'E_1' then the event of occurance of E_2 after the event E_1 is called "conditional event" of E_2 given E_1 it is denoted by $\frac{E_2}{E_1}$. Similarly we define E_1/E_2.

Eg: Two coins are tossed. The event of getting two tails given that there is atleast one tail is a Conditional event.

Conditional probability.

If E_1 and E_2 are 2 events, in a sample space and $P(E_1) \neq 0$, then the pty of E_2 after the event E_1 has occured is called the "conditional pty" of the event of E_2 even E_1 and it is denoted by $P(E_2/E_1)$ and defined as

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

Similarly, we define

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}, \quad P(E_2) \neq 0.$$

We have $P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$

$$= \frac{\frac{n(E_1 \cap E_2)}{n(S)}}{\frac{n(E_1)}{n(S)}} = \frac{n(E_1 \cap E_2)}{\cancel{n(S)}} \times \frac{\cancel{n(S)}}{n(E_1)}$$

$$= \frac{n(E_1 \cap E_2)}{n(E_1)}$$

$$= \frac{\text{No. of elements in } (E_1 \cap E_2)}{\text{No. of elements in } (E_1)}, \text{ Here } E_1 \subset S, \ P(E_1) > 0.$$

Multiplication theoram on probability:

Statement:

In a random experiment, if E_1 and E_2 are 2 events such that $P(E_1) \neq 0$ and $P(E_2) \neq 0$ then

(i) $P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$

(ii) $P(E_2 \cap E_1) = P(E_2) \cdot P\left(\frac{E_1}{E_2}\right)$

Problems:

1. Two marbles are drawn from a box containing 10 red ~~balls~~, 30 white, 20 blue and 15 orange marbels, with replacement being made after each draw. Find the pty that

(i) Both are white.

(ii) First one is red and second one is white.

Sol^{n}: The total no. of marbels in a box

$= 10+30+20+15$

$= 75$

let E_1 be the event of first dracon marble is cohite

then $P(E_1) = \frac{30}{75}$

let E_2 be the event of 2nd drocon marble is also cohite then $P(\frac{E_2}{E_1}) = \frac{30}{75}$

∴ The probability of both marbles are cohite with replacement is

$$P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$$

$$= \frac{30}{75} \times \frac{30}{75} = \left(\frac{30}{75}\right)^2$$

$\therefore P(E_1 \cap E_2) = 0.16$

(ii) let E_1 be the event of First dracon marble is red $P(E_1) = \frac{10}{75}$

let E_2 be the second dracon marble is cohite is

$$P\left(\frac{E_2}{E_1}\right) = \frac{30}{75}.$$

∴ The pty that first dracon marble is red and second dracon marble is cohite is

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$$

$$= \frac{10}{75} \times \frac{30}{75} = \frac{300}{(75)^2} = \frac{\overset{4}{\cancel{20}}\cancel{300}}{\underset{1}{\cancel{75}} \times 75} = \frac{4}{75}$$

$P(E_1 \cap E_2) = 0.05$

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2. 2 dice are rolled i.e thrown let 'A' be the event that the sum of the pts on the faces is 9.

Let 'B' be the event that the atleast one member is 6.

(i) Find $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A^c \cup B^c)$

Soln: There are 36 outcomes possible when 2 dies are thrown.

let the event $A = (\text{Sum of pts in the faces is } 9)$. occurs in the following was

$$A = \{(3,6), (4,5), (5,4), (6,3)\}$$

$$P(A) = 4/36 = \frac{1}{9}.$$

The event 'B' that atleast one member is 6 occurs in the following was

i.e $B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$

$$P(B) = \frac{11}{36}.$$

(i) $P(A \cap B)$

Now $n(A \cap B) = 2$ $\{\because (3,6)\ (6,3) \text{ in both}\}$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{9} + \frac{11}{36} - \frac{1}{18}$$

$$= \frac{4 + 11 - 2}{36} = \frac{13}{36}$$

(iii) $P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B)$

$$= 1 - \frac{\cancel{13}\ 1}{\cancel{36}\ 18} = \cancel{\frac{36-13}{36} = \frac{8}{3}} \quad 1 - \frac{1}{18} = \frac{18-1}{18}$$

$$= \frac{17}{18}$$

Independent events

Two events A and B are said to be independent events. If the probability of occurence of an event is not effected by the occurence of other event.

Its mathematically written as:

$$P(A \cap B) = P(A \cdot B) = P(A) \cdot P(B).$$

Total probability theorem:

let a sample space 'S' be partioned into $E_1, E_2, E_3 \ldots E_n$ are mutually exclusive events and exhaustive events then for any event A & associated with E_i, we have

$$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)$$

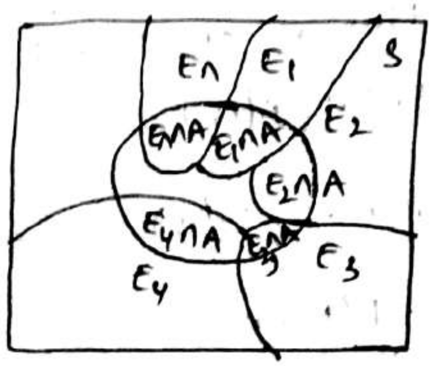
State and prove Baye's theorem:

Statement: let a sample 'S' be partioned into $E_1, E_2 \ldots E_n$ are mutually exclusive and exhaustive events with $P(E_i) > 0$ $(1 \leq i \leq n)$.

Then for any event $A(S)$, such that $P(A) > 0$ we have,

$$P(E_i/A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}$$

proof:



let S be a sample space and partioned into $E_1, E_2, E_3 \ldots E_n$ are mutually exclusive and exhaustive events

$$\because S = E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n$$
$$= \sum_{i=1}^{n} E_i$$

W.K.T, $A \subseteq S$

$$A \cap S = A$$
$$A \cap \sum_{i=1}^{n} E_i = A$$

by distributive law, $\sum_{i=1}^{n} A \cap E_i = A \quad \rightarrow (1)$

Since $E_1, E_2, E_3 \ldots E_n$ are mutually exclusive and exhaustive ents

$$(A \cap E_i) \cap (A \cap E_j) = \phi \quad for (i \neq j)$$

Taking probability on both sides in eq(1)

$$P(A) = P\left(\sum_{i=1}^{n} (A \cap E_i)\right)$$
$$= \sum_{i=1}^{n} P(A \cap E_i)$$
$$= \sum_{i=1}^{n} P(E_i \cap A)$$

by using the total probability theoram, we have

$$P(A) = \sum_{i=1}^{n} P(E_i) - P(A/E_i) \quad \rightarrow (2)$$

by Multiplication theoram on probability, we have

$$P(E_i \cap A) = P(E_i) \cdot P(A/E_i)$$
(or)
$$P(A \cap E_i) = P(A) \cdot P(E_i/A)$$

Now, $P(E_i/A) = \dfrac{P(E_i \cap A)}{P(A)}$

$$= \frac{P(E_i) \cdot P(A/E_i)}{\sum_{}^{n} P(E_i) \cdot P(A/E_i)} \quad \{\because \text{from } (2)\}$$

2/10, 18
3/5, 9
5, 3

Hence, $P(E_i/A) = \dfrac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)}$

Problems:

1. 1^{st} box contains 2 black, 3 red, 1 white balls, 2^{nd} box contain 1 black, 1 red, 2 white balls and 3^{rd} box contains 5 black, 3 red, 4 white balls. Of these a box is selected at random. From it red ball is randomly drawn. If the ball is red then find the probability that is from 2^{nd} box.

Soln: let x, y, z be the 1^{st}, 2^{nd} and 3^{rd} boxes respectively.

i.e $P(x) = \frac{1}{3}$

$P(y) = \frac{1}{3}$

$P(z) = \frac{1}{3}$.

By Baye's theoram, the required probability is

$P(R/x) = 3/6 = 1/2$

$P(R/y) = 1/4$

$P(R/z) = 3/12 = 1/4$

now $P(y/R) = \dfrac{P(y)\cdot P(R/y)}{P(x)\cdot P(R/x) + P(y)\cdot P(R/y) + P(z)\cdot P(R/z)}$

$= \dfrac{\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{3}\cdot\frac{1}{2}\right)+\left(\frac{1}{3}\cdot\frac{1}{4}\right)+\left(\frac{1}{3}\cdot\frac{1}{4}\right)}$

$= \dfrac{\frac{1}{12}}{\frac{1}{6}+\frac{1}{12}+\frac{1}{12}} = \dfrac{\frac{1}{12}}{\left(\frac{2+1+1}{12}\right)} = \frac{1}{\cancel{12}} \times \frac{\cancel{12}}{4} = \frac{1}{4}$

$\boxed{P(y/R) = 1/4}$ //.

2. A bag A contains 2 white and 3 Red balls and B contains 4 white and 5 Red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the random red ball is drawn from a bag B.

Sol: Let A, B be the 1st and 2nd bags respectively,

i.e $P(A) = 1/2$

$P(B) = 1/2$

let R_1 be the event of drawn a Red ball having selected bag A the probability to draw a Red ball is $P(R/A) = \frac{3}{5}$.

$P(R/B) = \frac{5}{9}$.

∴ One ball is drawn, at random from one of the bags and it is found to be red ball from bag B.

∴ The required probability is

$$P(B/R) = \frac{P(B)\cdot P(R/B)}{P(A)\cdot P(R/A) + P(B)\cdot P(R/B)}$$

$$= \frac{\frac{1}{2}\left(\frac{5}{9}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{2}\right)\left(\frac{5}{9}\right)}$$

$$= \frac{\left(\frac{5}{18}\right)}{\left(\frac{3}{10}\right)+\left(\frac{5}{18}\right)} = \frac{\frac{5}{18}}{\left(\frac{27+25}{90}\right)}$$

$$= \frac{5}{\cancel{18}} \times \frac{\cancel{90}\,5}{52} = \frac{25}{52}.$$

$$\therefore P(B/R) = \frac{25}{52} \; //.$$

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Random variables and Distribution Functions:

Random variable:

A random variable is a statement (or) function variable which assign a real value to the outcome of the random experiment. Generally the random variables are represented by $X, Y, Z, \ldots$ The values of a random variables are denoted by small letters like $x, y, z \ldots$

Eg: Experiment of tossing a 3 fair coins are

$S = \{HHH, HHT, HTH, HTT, TTT, THT, TTH, THH\}$

let X = no. of getting a heads

The numerical values of above expressions are

$3, 2, 2, 1, 0, 1, 1, 2$

i.e $X = \{3, 2, 1, 0\}$

$\therefore X = 3$ means the event of getting a 3 heads

i.e $P(X=3) = \frac{1}{8}$

$\therefore X = 2$ means the event of getting a 2 heads

i.e $P(X=2) = \frac{3}{8}$.

$\therefore X = 1$ means the event of getting a 1 head

i.e $P(X=1) = \frac{3}{8}$.

$\therefore X = 0$ means the event of not getting any head i.e $P(X=0) = \frac{1}{8}$.

$\therefore P(X) = P(X=3) + P(X=2) + P(X=1) + P(X=0)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$= \frac{1+3+3+1}{8} = \frac{8}{8} = 1$

$\therefore P(x) = 1$

Note:

The set $\{x \leq x\}$ be an event for any real number 'x' and the probability of this event is denoted by $P(x \leq x)$. It means the sum of probability of all the elementary events corresponding to set of $x \leq x$ i.e. $\{x \leq x\}$.

Eg: 1. $P(x \leq 2) = p(x=0) + P(x=1) + P(x=2)$

(or)

$P(x \leq 2) = 1 - P(x > 2)$

2. $P(x > 1) = P(x=2) + P(x=3) + \cdots$

$P(x > 1) = 1 - P(x \leq 1) = 1 - [P(x=0) + P(x=1)]$

Types of Random variables:

There are two types of random variables mainly. They are 1. Discrete random variables.

2. Continoues random variables.

1. Discrete Random variable:

If the random variable can take a finate number of distinct values in an intreval, then the variables are called discrete random variables.

Eg: 1. The number of deffectives in a sample electric bulbs

2. The number of telephone calls are received by telephone.

2. Continoues random variables:

If a random variables can take the values continouesly in an intreval, then the random variables are called continues random variables.

Eg: The height, weight and age of induvidual students.

Probability distribution:

The probability distribution means the total probability "1" is distributed to each possible outcome of the random experiment.

Probability Distribution Function (or) Cummulative:

Let 'X' be a relation random variable and 'x' can takes the values the values $x_1, x_2 \dots x_n$ than the function $f(x)$ (or) $F_X(x)$ is defined as $P(x) = P(X \le x)$
$$= \sum_{i=1}^{n} P(x_i) \quad \forall x_i \le x$$
is called a probability distribution function.

Properties of Distribution Function:

1. If F is the distribution function of a random variable X and if $a<b$, then

(i) $P(a < X \le b) = F(b) - F(a)$

(ii) $P(a \le X \le b) = P(X=a) + [F(b) - F(a)]$

(iii) $P(a < X < b) = [F(b) - F(a)] - P(X=b)$

(iv) $P(a \le X < b) = [F(b) - F(a)] - P(X=b) + P(X=a)$

Note:

1. If $P(X=a) = P(X=b) = 0$ then

$$P(a < X \le b) = P(a \le X \le b) = P(a < X < b) = P(a \le X < b)$$
$$= F(b) - F(a)$$

2. All distribution functions are monotonically increasing and lie between 0 and 1 i.e if F is the distribution function of the random variable X then

(i) $0 \leq P(x) \leq 1$

(ii) $F(x) < F(y)$ when $x < y$

3. (i) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0.$

(ii) $F(\infty) = \lim_{x \to \infty} F(x) = 1$

Probability density Function:

The probability density function $f_X(x)$ is defined as the derivative of the probability distribution function, $F_X(x)$ of the random variable X.

Thus $f_X(X) = \frac{d}{dx}\left[F_X(x)\right]$

Types of Probability Distributions:

There are two types of probabilities distributions They are :

1. Discrete probability distribution.
2. Continously Probability distribution.

1. Discrete Probability distribution:

Suppose a random variable 'X' is a discrete random variable which can take the values $x_1, x_2, x_3 \ldots x_n$ and the corresponding probabilities are $P_1, P_2, P_3 \ldots P_n$ respectively. If it satisifies the following conditions.

(i) $p(x_i) \geq 0 \quad \forall i = 1, 2, 3 \ldots, n$

(ii) $\sum_{i=1}^{n} p(x_i) = 1 \quad \forall i = 1, 2, 3 \ldots, n$

The probability distribution is followed by:

$X = x$	x_1	x_2	x_3	------	x_n
$P(X=x)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	------	$P(x_n)$

Eg: The probability distribution table as follows:

$X = x_i$:	0	1	2	3
$P(X = x_i)$:	$1/8$	$3/8$	$3/8$	$1/8$

Mean (or) Expectation, varience, Standard deviation of Discrete random variables:

Let 'X' be a discrete random variable. Then

(i) Mean (or) expectation of X is

$$\boxed{\mu = E(x) = \sum_{i=1}^{n} x_i P(x_i)}$$

(ii) variance of X is $\sigma^2 = \sum_{i=1}^{n} x_i^2 P(x_i) - \mu^2$

variance on$(X) = \sigma^2 = E(X^2) - (E(x))^2$

(iii) Standard deviation of X is

$$\boxed{\sigma = \sqrt{\sum_{i=1}^{n} x_i^2 p(x_i) - \mu^2}}$$

Properties of discrete Random variables:

1. Let X be a probability random variable and "k" is a constant then $E(X+k) = E(X)+k$

$E(kX) = k \cdot E(X)$

2. Suppose x, y are two random independent variables, then

$$E(x+y) = E(x) + E(y).$$

Problems

1. Two dies are thrown let 'x' assign to each point (a,b) in 's', the maximum of its numbers i.e $x(a,b) = max(a,b)$. Find the probability distribution and also find mean and variance of the distribution.

Solu: When 2 dies are thrown the total no. of possible outcomes $= 6^2 = 36$.

The random variable 'x' defined as maximum no. of points (a,b) in two dies are thrown.

i.e $x = \{1,2,3,4,5,6\}$.

The outcomes are

$\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3),\ (6,4)\ (6,5)\ (6,6)\}$.

For max 1:

The favourable cases for max 1 are $(1,1)$

$$n(E) = 1$$

$$n(S) = 36$$

$$P(1) = P(x=1) = \frac{n(E)}{n(S)} = \frac{1}{36}.$$

For max: 2 :

The favourable cases of max. 2 are

(1,2), (2,1) (2,2)

$\therefore n(E) = 3$

$n(S) = 36$

$P(2) = P(X=2) = \frac{3}{36}$.

For max: 3 :

The Favourable cases of max. 3 are

(1,3), (2,3), (3,1), (3,2), (3,3)

$\therefore n(E) = 5$

$n(S) = 36$

$P(3) = P(X=3) = \frac{5}{36}$,

For max: 4 :

The Favourable cases of max: 4 are

(1,4), (4,1), (4,2), (2,4), (4,3), (3,4), (4,4).

$n(E) = 7$

$n(S) = 36$

$P(4) = P(X=4) = \frac{7}{36}$.

For max: 5 :

The Favourable cases of max: 5 are

(1,5), (5,1), (2,5), (5,2), (3,5), (5,3), (4,5), (5,4)

(5,5) $n(E) = 9$

$n(S) = 36$

$P(5) = P(X=5) = \frac{9}{36}$.

For max: 6 :

The Favourable cases of max: 6 are

(1,6), (6,1), (2,6), (6,2), (3,6), (6,3), (4,6), (6,4)

(5,6), (6,5), (6,6)

$n(E) = 11$

$n(S) = 36$

$P(6) = P(X=6) = \frac{11}{36}$.

The probability distribution table is.

$X = x$:	1	2	3	4	5	6
$P(X=x)$:	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$.

To find Mean of the distribution:

$$u = \sum_{i=1}^{n} x_i p(x_i)$$

$$= \left(1\times\frac{1}{36}\right)+\left(2\times\frac{3}{36}\right)+\left(3\times\frac{5}{36}\right)+\left(4\times\frac{7}{36}\right)+\left(5\times\frac{9}{36}\right) + \left(6\times\frac{11}{36}\right)$$

$$= 0.0278 + (2\times 0.0833)+(3\times 0.1389)+(4\times 0.1944) +(5\times 0.2500)+(6\times 0.3056)$$

$$= 0.0278 + 0.1666 + 0.4167 + 0.7776 + 1.2500 + 1.8336$$

$$= 4.4723.$$

$$\therefore u = 4.4723.$$

To find variance of distribution:

It is given by $\sigma^2 = \sum_{i=1}^{n} x_i^2 p(x_i) - u^2$.

$$= \left(1\times\frac{1}{36}\right)+\left((2)^2\times\frac{3}{36}\right)+\left((3)^2\times\frac{5}{36}\right)+\left((4)^2\times\frac{7}{36}\right)+ \left((5)^2\times\frac{9}{36}\right)+\left((6)^2\times\frac{11}{36}\right)-(4.4723)^2$$

$$= \frac{1}{36}+\left(4\times\frac{3}{36}\right)+\left(9\times\frac{5}{36}\right)+\left(16\times\frac{7}{36}\right)+\left(25\times\frac{9}{36}\right) +\left(36\times\frac{11}{36}\right)-(4.4723)^2$$

$$= 0.0278 + 0.3333 + 1.2500 + 3.1111 + 6.2500 + 11 - 20.0015$$

$$= 21.9722 - 20.0015 = 1.9707$$

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$\therefore \sigma^2 = 1.9707$

To find Standard deviation of distribution:

$$\sigma = \sqrt{\sum_{i=1}^{n} x_i^2 p(x_i) - \mu^2}$$

$$= \sqrt{1.9707}$$

$\therefore \sigma = 1.4038$ //

---

2. A random variable 'x' has the following probability function.

$X=x$:	0	1	2	3	4	5	6	7
$P(X=x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$.

Determine (i) k.

(ii) evaluate $P(X\geq 6)$, $P(0<X<5)$, $P(0<x<4)$

(iii) if $P(X\leq k) > \frac{1}{2}$, then find the minimum value of k.

(iv) Determine the distribution function of x

(v) Mean and variance of x.

Sol: W.K.T

The sum of the probabalities of a random variables is always equal to '1'.

So $\sum_{i=1}^{n} p(x_i) = 1$

$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$9k+10k^2=1$$

$$10k^2+9k-1=0.$$

$$10k^2+10k-k-1=0$$

$$10k(k+1)-1(k+1)=0$$

$$(k+1)(10k-1)=0.$$

$k+1=0$ $\quad$ ~~$10-k=0$~~ $\quad$ $10k-1=0$

$k=-1$ $\quad$ ~~$k=10$~~ $\quad$ $10k=1$

$k = 1/10$.

W.K.T the probability of any event cannot be negitive. So $k = \frac{1}{10}$.

So the required probability distribution table is.

$X = x$:	0	1	2	3	4	5	6	7
$P(X=x)$:	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$.	$2\left(\frac{1}{100}\right)$	$7\left(\frac{1}{100}\right) + \frac{1}{10}$.

(i) $P(X \geq 6) = 1 - P(X < 6)$

Now $P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$+ P(X=5)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{2}{10} + \frac{1}{100}$.

$= \frac{0 + 10 + 20 + 30 + 20 + 1}{100} = \frac{81}{100}$.

Now $P(X \geq 6) = 1 - P(X<6)$

$= 1 - \frac{81}{100} = \frac{100-81}{100} = \frac{19}{100}$.

$\therefore P(X \geq 6) = \frac{19}{100}$.

(ii) $P(0<X<5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} = \frac{8}{10}$.

$\therefore P(0<X<5) = \frac{8}{10}$.

(iii) $P(0 \leq x \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} = \frac{1+2+2+3}{10} = \frac{8}{10}$

$\therefore P(0 \leq x \leq 4) = \frac{8}{10}$.

(iii) If $P(X \leq k) > \frac{1}{2}$

i.e $P(X \leq k) > 0.5$

If $k=1$ then $P(X \leq 1) = P(X=0) + P(X=1) = 0 + \frac{1}{10} = \frac{1}{10} = 0.1$

If $k=2$ then $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

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$= 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10} = 0.3.$

If $k=3$ then $P(X \le 3) = P(\overset{x=}{0}) + P(\overset{x=}{1}) + P(\overset{x=}{2}) + P(\overset{x=}{3})$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10} = 0.5$

If $k=4$ then $P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} = \frac{8}{10} = 0.8$

$\therefore$ The minimum value of $k=4$ because after taking 4 only the value is exceeding more than 0.5.

(iv) To find the probability distribution function:

The distribution function can be determine by the following $\boxed{F(x) = P(X \le x)}$

$F(0) = P(X \le 0) = 0.$

$F(1) = P(X \le 1) = P(X=0) + P(X=1) = 0 + \frac{1}{10} = \frac{1}{10}.$

$F(2) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$

$F(3) = P(X \le 3) = P(X=0) + P(X=2) + P(X=1) + P(X=3)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10} = 0.5$

$F(4) = P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} = \frac{8}{10}.$

$F(5) = P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} = \frac{10+20+20+30+1}{100}$

$= \frac{81}{100}.$

$F(6) = P(X \le 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6).$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$

$= \frac{10+20+20+30+1+2}{100} = \frac{83}{100}$

$F(7) = P(X \le 7) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$+ P(X=5) + P(X=6) + P(X=7)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100} + \frac{7}{100} + \frac{1}{10}$

$= \frac{10+20+20+30+1+2+7+10}{100} = \frac{100}{100} = 1$

(v) <u>To find the</u> ~~Variance~~ <u>Mean (u)</u>:

$$u = \sum_{i=0}^{n} x_i p(x_i)$$

$= (0 \times 0) + (1 \times \frac{1}{10}) + (2 \times \frac{2}{10}) + (3 \times \frac{2}{10}) + (4 \times \frac{3}{10}) + (5 \times \frac{1}{100})$
$+ 6(\frac{2}{100}) + (7 \times \frac{7}{100} + 7 \times \frac{1}{10})$

$= 0 + 0.1 + 0.4 + 0.6 + 1.2 + 0.0500 + 0.1200 + 0.4900 + 0.7$

$= 3.6600$

$\therefore$ Mean $(u) = 3.6600$.

(vi) <u>To find variance (σ^2)</u>:

$$\sigma^2 = \sum_{i=0}^{1} x_i^2 p(x_i) - u^2$$

$= [(0)^2 \times 0] + [(1)^2 \times \frac{1}{10}] + [(2)^2 \times \frac{2}{10}] + [(3)^2 \times \frac{2}{10}] +$
$[(4)^2 \times \frac{3}{10}] + [(5)^2 \times \frac{1}{100}] + [(6)^2 \times \frac{2}{100}] +$
$[(7)^2 \times \frac{7}{100}] + [(7)^2 \times \frac{1}{10}] - (3.6600)^2$

$= 0 + 0.1 + 0.8 + 1.8 + 4.8 + 0.2500 + 0.7200$
$+ 3.4300 + 4.9 - 13.3956$

$= 16.8000 - 13.3956$

$\sigma^2 = 3.4044.$

$\therefore \sigma^2 = 3.4044$ //

3. A sample of 4 items is selected at random from a box containing 12 items of which 5 are deffective. Find the expected number of it means "mean" deffective items.

(or)

A random variable x has the following probability determine (i) k
(ii) Mean
(iii) variance.

x:	0	1	2	3	4	5	6	7	8
P(x):	k	k	k	k	2k	6k	7k	8k	4k

Soln:
W.K.T
The sum of probabilities in a random event = 1

$$\sum_{i=1}^{n} P(x_i) = 1$$

$$k+k+k+k+2k+6k+7k+8k+4k = 1$$

$$31k = 1$$

$$k = \frac{1}{31} = 0.0323$$

$$k = 0.0323.$$

$\therefore$ The probability distribution table is

x:	0	1	2	3	4	5	6	7	8
P(x):	$\frac{1}{31}$	$\frac{1}{31}$	$\frac{1}{31}$	$\frac{1}{31}$	$\frac{2}{31}$	$\frac{6}{31}$	$\frac{7}{31}$	$\frac{8}{31}$	$\frac{4}{31}$

Mean: (μ):

$$\mu = \sum_{i=0}^{n} P(x_i) \cdot x_i$$

$$= 0 + \left(1\times\frac{1}{31}\right) + \left(2\times\frac{1}{31}\right) + \left(3\times\frac{1}{31}\right) + \left(4\times\frac{2}{31}\right) + \left(5\times\frac{6}{31}\right)$$
$$+ \left(6\times\frac{7}{31}\right) + \left(7\times\frac{8}{31}\right) + \left(8\times\frac{4}{31}\right)$$

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$= 0.0323 + 0.0645 + 0.0968 + 0.2581 + 0.9677$

$1.3548 + 1.8065 + 1.0323$

$= 5.6130.$

$\therefore$ Mean $(\mu) = 5.6130.$

Variance (σ^2):

$$\sigma^2 = \sum_{i=0}^{n} (x_i)^2 p(x_i) - \mu^2$$

$$= 0 + \left(1 \times \frac{1}{31}\right) + \left(4 \times \frac{1}{31}\right) + \left(9 \times \frac{1}{31}\right) + \left(16 \times \frac{2}{31}\right) + \left(25 \times \frac{6}{31}\right)$$

$$+ \left(36 \times \frac{7}{31}\right) + \left(49 \times \frac{8}{31}\right) + \left(64 \times \frac{4}{31}\right) - (5.6130)^2$$

$= 0 + 0.0323 + 0.1290 + 0.2903 + 1.0323 + 4.8387$

$+ 8.1290 + 12.6452 + 8.2581 - 31.5058$

$= 35.3549 - 31.5058$

$= 3.8491$

$\therefore$ variance $(\sigma^2) = 3.8491.$ //

Above problem

Solu:

Given 4 items are to be selected at random.

Total bulbs $= 12$

No. of defected bulbs $= 5$

So No. of good bulbs $= 12 - 5 = 7.$

For selection of these bulbs there are certain posibilities as follows.

No defected 4 good bulbs.

1 defected 3 good bulbs

2 defected 2 good bulbs.

3 defected 1 good bulb.

4 defected no good bulb.

(i) No defected 4 good bulbs

$$= \frac{5C_0 \times 7C_4}{12C_4} = \frac{1 \times 35}{12C_4} = \frac{35}{495}$$

(ii) 1 defected 3 good bulbs.

$$= \frac{5C_1 \times 7C_3}{12C_4} = \frac{5 \times 35}{495} = \frac{175}{495}$$

(iii) 2 defected 2 good bulbs

$$= \frac{5C_2 \times 7C_2}{12C_4} = \frac{10 * 21}{495} = \frac{210}{495}.$$

(iv) 3 defected 1 good bulb.

$$= \frac{5C_3 \times 7C_1}{12C_4} = \frac{10 \times 7}{495} = \frac{70}{495}.$$

(v) 4 defected no good bulb.

$$= \frac{5C_4 \times 7C_0}{12C_4} = \frac{5 \times 1}{495} = \frac{5}{495}$$

∴ The probability distribution table is.

$X = x$:	0	1	2	3	4
$P(X=x)$:	$\frac{35}{495}$	$\frac{175}{495}$	$\frac{210}{495}$	$\frac{70}{495}$	$\frac{5}{495}$

To find Mean (or) expected no. of defective items (μ):-

$$\text{Mean}(\mu) = \sum_{i=1}^{n} x_i P(x_i)$$

$$= 0 + \left(1 \times \frac{175}{495}\right) + \left(2 \times \frac{210}{495}\right) + \left(3 \times \frac{70}{495}\right) + \left(4 \times \frac{5}{495}\right)$$

$$= 0 + 0.3535 + \overset{0.8485}{\cancel{}} + 0.4242 + 0.0404$$

$$= \cancel{}\ 1.6666.$$

$$\therefore \mu = 1.6666. //$$

HOMEWORK

1. A random variable x has the following probability distribution

x:	1	2	3	4	5	6	7	8
$P(x)$:	k	$2k$	$3k$	$4k$	$5k$	$6k$	$7k$	$8k$

Find the value of (i) k
(ii) $p(x \leq 2)$
(iii) $p(2 \leq x \leq 5)$

Soln: W.K.T

The sum of probabilities of an event $=1$

$$\sum_{i=1}^{n} P(x_i) = 1$$

So $k + 2k + 3k + 4k + 5k + 6k + 7k + 8k = 1$

$$36k = 1$$

(i) $k = \frac{1}{36}$

Now the probability distribution table is

x:	1	2	3	4	5	6	7	8
$P(x)$:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{7}{36}$	$\frac{8}{36}$

(ii) $P(x \leq 2)$

$= p(x=0) + p(x=1) + p(x=2)$

$= \frac{1}{36} + \frac{2}{36} = \frac{3}{36} = \frac{1}{12} = 0.0833$

$\therefore P(x \leq 2) = 0.0833.$

(iii)
$p(2 \leq x \leq 5) = p(x=2) + p(x=3) + p(x=4) + p(x=5)$

$= \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} = \frac{14}{36}$

$= 0.3889.$

$\therefore P(2 \leq x \leq 5) = 0.3889$ //.

2. The probability density function of a variable x is

x:	0	1	2	3	4	5	6
$p(x)$:	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

(i) find $P(x<4)$, $P(x\geq 5)$, $P(3<x\leq 6)$

(ii) What will be the minimum value of k so that $P(x\leq k)>0.3$?

Sol: Given the probability density function.

W.K.T the sum of probabilities in a random event $=1$

So $k+3k+5k+7k+11k+9k+13k=1$

$$49k=1$$

$$k=\frac{1}{49}.$$

$\therefore$ The probability distribution table is.

x:	0	1	2	3	4	5	6
$P(x)$:	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

$P(x<4) = P(x=0)+P(x=1)+P(x=2)+P(x=3)$

$$= \frac{1}{49}+\frac{3}{49}+\frac{5}{49}+\frac{7}{49} = \frac{16}{49}$$

$P(x<4) = 0.3265$

$P(x\geq 5) = 1-P(x<5)$

$P(x<5) = p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)$

$$= \frac{1}{49}+\frac{3}{49}+\frac{5}{49}+\frac{7}{49}+\frac{9}{49}$$

$$= \frac{25}{49} = 0.5102$$

$P(x\geq 5) = 1-P(x<5)$

$= 1-0.5102$

$P(x\geq 5) = 0.4898.$

$P(3 < x \leq 6)$

$= P(x=4) + P(x=5) + P(x=6)$

$= \frac{9}{49} + \frac{11}{49} + \frac{13}{49} = 0.6735$

$= \frac{33}{49} = 0.6735$

$\therefore P(3 < x \leq 6) = 0.6735$

(ii) $P(x \leq k) > 0.3$

For $k = 1$

$P(x \leq 1) = P(x=0) + P(x=1) = \frac{1}{49} + \frac{3}{49} = \frac{4}{49} = 0.0816$

For $k = 2$

$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} = \frac{9}{49}$

$= 0.1837$

For $k = 3$

$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} = \frac{16}{49} = 0.3265$

For $k = 4$

$P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} + \frac{9}{49} = \frac{25}{49} = 0.5102$

So the minimum value of k so that $P(x \leq k) \geq 0.3$ is $k = 4$. //

---

4. Let x denote the minimum of two numbers after when a pair of fair dice are thrown once. Determine the (i) Discrete probability distribution

(ii) Expectation

(iii) Variance.

sol4: When 2 dies are thrown, No. of possible outcomes are $n(S) = 6^2 = 36$.

$S = \{1, 2, 3, 4, 5, 6\}$.

The possible outcomes are

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6).

For min: 1:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (3,1), (4,1)
(5,1), (6,1)

$n(E) = 11$
$n(S) = 36$
$P(1) = \frac{11}{36}$.

For min: 2:

(2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (4,2), (5,2), (6,2)

$n(E) = 9$
$n(S) = 36$
$P(2) = \frac{9}{36}$.

For min: 3:

(3,3), (3,4), (3,5), (3,6), (4,3), (5,3), (6,3)

$n(E) = 7$
$n(S) = 36$
$P(3) = \frac{7}{36}$.

For min: 4:

(4,4), (4,5), (4,6), (5,4), (6,4)

$n(E) = 5$
$n(S) = 36$
$P(4) = \frac{5}{36}$.

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For min 5:

(5,5), (5,6), (6,5)

$n(E) = 3$

$n(S) = 36$

$P(5) = \frac{3}{36}$.

For - min 6:

(6,6)

$n(E) = 1$ $\quad P(6) = \frac{1}{36}$.

$n(S) = 36$

(i) ∴ The Discrete probability distribution is

x :	1	2	3	4	5	6
$P(x)$:	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii) Mean(u):

$$u = \sum_{i=1}^{n} (x_i) P(x_i).$$

$$= \left(1\times\frac{11}{36}\right) + \left(2\times\frac{9}{36}\right) + \left(3\times\frac{7}{36}\right) + \left(4\times\frac{5}{36}\right) + \left(5\times\frac{3}{36}\right) + \left(6\times\frac{1}{36}\right)$$

$$= 0.3056 + 0.5 + 0.5833 + 0.5556 + 0.4167 + 0.1667$$

$$= 2.5279$$

∴ Mean (u) = 2.5279.

(iii) variance (σ^2):

$$\sigma^2 = \sum_{i=1}^{n} (x_i)^2 P(x_i) - u^2$$

$$= \left(1\times\frac{11}{36}\right) + \left(4\times\frac{9}{36}\right) + \left(9\times\frac{7}{36}\right) + \left(16\times\frac{5}{36}\right) + \left(25\times\frac{3}{36}\right) + \left(36\times\frac{1}{36}\right) - (2.5279)^2$$

$$= 0.3056 + 1 + 1.7500 + 2.2222 + 2.0833 + 1 - 6.3903$$

$$= 8.3611 - 6.3903$$

$\sigma^2 = 1.9708$ //

$\therefore$ variance $(\sigma^2) = 1.9708$ //

Standard deviation

$$\sigma = \sqrt{\sum_{i=1}^{n} x_i^2 p(x_i) - \mu^2}$$

$$= \sqrt{1.9708}$$

$$\boxed{\sigma = 1.4039}$$

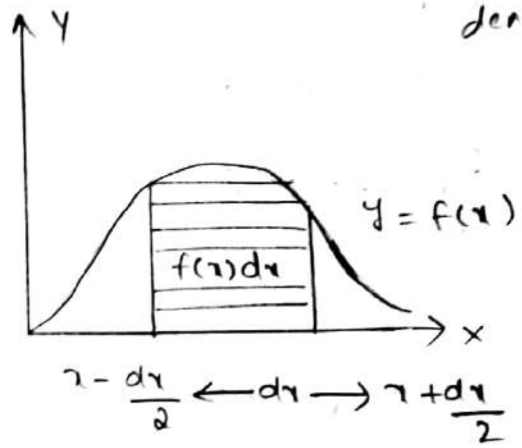
2. Continoues Probability distribution:

When a random variable 'x' takes every value in an intreval it gives to continoues distribution of 'x'. The distribution defined by the variates like as temperature, heights and weights are continoues distribution.

Integration.

probability density function:

distribution $= F(x)$
density $= f(x)$.



Let 'dx' be a length of the intreval $[x - \frac{dx}{2}, x + \frac{dx}{2}]$ around the point 'x' and $f(x)$ be any continoues function of 'x' then $f(x)\,dx$ is said to be the probability that the variable 'x' 'x' lies between $(x - \frac{dx}{2}, x + \frac{dx}{2})$ and it is denoted by $P(x - \frac{dx}{2} \le x \le x + \frac{dx}{2}) = f(x)\,dx$.

Then function $f(x)$ is said to be a probability density function of 'x' and the curve $y = f(x)$ is called probability density curve.

Properties of density Function:

1. The probability of a variable 'x' which lies in a finate interval [a,b] is given by $P(a \leq x \leq b) = \int_a^b f(x)\,dx$

2. The total probability under a density curve is unity i.e $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3. $f(x) \geq 0 \quad \forall x \in R.$

Mean, Variance and Median and Mode the continoues Probability distribution:

→ If 'x' is a continoues random variable

(i) Mean of expectation of x is

$$\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx.$$

(ii) The variance of 'x' is

$$V(x) = \sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2$$

(iii) The median of 'x' is given by

$$\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = 1/2$$

Here M = Median of distribution value.

(iv) Mode is the value 'x' for which f(x) is Maximum. It is given by

$$f'(x) = 0 \text{ and } f''(x) < 0 \text{ for } a < x < b.$$

(v) Mean deviation:

Mean deviation about the mean(μ) is given by $\sigma = \sqrt{\int_{-\infty}^{\infty} |x-\mu| f(x)\,dx} = \sqrt{\int_{-\infty}^{\infty} |x-\mu| f(x)\,dx}$

probability distribution function (or) commulative distribution function:

let 'x' be a random variable then probability distribution function of F(X) is defined as.

$$\cancel{F(x) = P(X \leq x) = \Sigma P(x_i),\ \forall x_i \leq x}$$

$$\boxed{F(x) = P(X \leq x)}$$

(i) If 'x' is a discrete random variable then the probability distribution function of x is defined as

$$F(x) = P(X \leq x) = \Sigma P(x_i),\ \forall x_i \leq x.$$

(ii) If 'x' is a continoues random variable then the probability distribution of 'x' is

$$\boxed{F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx}.$$

properties of Distribution function:

1. If $0 \leq F(x) \leq 1$, $-\infty < x < \infty$ then $F(-\infty) = 0$ and $F(\infty) = 1$
2. $P(a \leq x \leq b) = \int_{a}^{b} f(x)\,dx = F(b) - F(a)$.
3. $F(x)$ is a continoues function of x on the right.
4. The discontinuties of $F(x)$ are countable.
5. $F'(x) = f(x) \geq 0$, So that $F(x)$ is a non-decreasing function.

Relation between Density function[$f(x)$] and Distribution function[$F(x)$]:

1. If a density function $f(x)$ is given then

$$F(x) = \int_{-\infty}^{\infty} f(x)\,dx.$$

2. If a distribution function $F(x)$ is given, then

$$f(x) = \frac{d}{dx}[F(x)]$$

Problems:

1. If a random variable has the probability density function $f(x) = 2e^{-2x}$, for $x > 0$

$= 0$, for $x \leq 0$.

Find the probability that it will take a value

(i) between 1 and 3.

(ii) greater than 0.5.

Soln: Given that

$$f(x) = 2e^{-2x} \text{ for } x > 0$$
$$= 0 \text{ for } x \leq 0.$$

(i) Probability that the variate 'x' lies between 1 and 3 is $P(1 \leq x \leq 3)$

trick: Here $f(x)$ is given, so we need to find $F(x)$ {which is probability distribution function}

$$P(1 \leq x \leq 3) = \int_1^3 f(x)\,dx$$

$$= \int_1^3 2e^{-2x}\,dx = 2\int_1^3 e^{-2x}\,dx$$

$$= \cancel{2}\left(\frac{e^{-2x}}{-\cancel{2}}\right)_1^3 = -\left(e^{-2x}\right)_1^3$$

$$= -\left(e^{-6} - e^{-2}\right) = -e^{-6} + e^{-2}$$

$$= e^{-2} - e^{-6}$$

$$= 0.1353 - 0.0025$$

$$\therefore P(1 \leq x \leq 3) = 0.1328$$

(ii) probability that the variate 'x' can takes the values greater than 0.5 is

$P(x \geq 0.5)$

i.e $P(x \geq 0.5) = \int_{0.5}^{\infty} 2e^{-2x}dx$

$= 2\int_{0.5}^{\infty} e^{-2x}dx = \cancel{2}\left(\frac{e^{-2x}}{-\cancel{2}}\right)_{0.5}^{\infty} = -\left(e^{-2x}\right)_{0.5}^{\infty}$

$= -\left(e^{-\infty} - e^{-2(0.5)}\right) = -\left(e^{-\infty} - e^{-1}\right)$

$= e^{-1} - e^{-\infty}$

$= 0.3679 - 0$

$= 0.3679$

$\therefore P(x \geq 0.5) = 0.3679$ .//

---

2. Probability density of random variable 'x' given by $f(x) = k(1-x^2)$, for $0<x<1$.

$= 0$. otherwise .

Find the value of 'k' and the probability that the random variable having this probability density will taken a value

(i) b/w 0.1 and 0.2

(ii) greater than 0.5 .

Soln: We know that the total probability under a density curve is unity i.e

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx = 1$

$\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$

$\int_{-\infty}^{0} 0.dx + \int_{0}^{1} k(1-x^2)dx + \int_{1}^{\infty} 0.dx = 1$

$0 + \int_{0}^{1} k(1-x^2)dx + 0 = 1$

$$\int_0^1 k(1-x^2)dx = 1$$

$$k\int_0^1 (1-x^2)dx = 1$$

$$k\left(x-\frac{x^3}{3}\right)_0^1 = 1$$

$$k\left(1-\frac{1}{3}-0+0\right) = 1$$

$$k\left(1-\frac{1}{3}\right) = 1$$

$$k\left(\frac{3-1}{3}\right) = 1$$

$$k\left(\frac{2}{3}\right) = 1$$

$$k = 3/2$$

So $f(x) = \frac{3}{2}(1-x^2) \quad 0<x<1$

$\quad\quad\quad\; 0 \quad$ otherwise.

(i) probability that the variate 'x' between 0.1 and 0.2 is $P(0.1<x<0.2)$ i.e

$$P(0.1<x<0.2) = \int_{0.1}^{0.2} f(x)\,dx$$

$$= \int_{0.1}^{0.2} \frac{3}{2}(1-x^2)\,dx$$

$$= \frac{3}{2}\int_{0.1}^{0.2}(1-x^2)\,dx = \frac{3}{2}\left(x-\frac{x^3}{3}\right)_{0.1}^{0.2}$$

$$= \frac{3}{2}\left[(0.2)-\frac{(0.2)^3}{3}-(0.1)+\frac{(0.1)^3}{3}\right]$$

$$= \frac{3}{2}\left[0.2-\frac{0.0080}{3}-0.1+\frac{0.0010}{3}\right]$$

$$= \frac{3}{2}\left[0.2-0.0027-0.1+0.0003\right]$$

$$= 0.1464$$

doubt

$$\therefore P(0.1<x<0.2) = 0.1464$$

(ii) Probability that the variate greater than 0.5

$P(x>0.5)$

$$P(x \geq 0.5) = \int_{0.5}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx .$$

$$= \int_{0.5}^{1} \frac{3}{2}(1-x^2)\,dx + \int_{1}^{\infty} 0.\,dx .$$

$$= \frac{3}{2}\int_{0.5}^{1} (1-x^2)\,dx = \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.5}^{1}$$

$$= \frac{3}{2}\left(1 - \frac{1}{3} - (0.5) + \frac{(0.5)^3}{3}\right)$$

$$= \frac{3}{2}(1 - 0.3333 - 0.5 + 0.0417)$$

$$= \frac{3}{2}(0.2084) = 0.3126 .$$

$\therefore P(x \geq 0.5) = 0.3126$ //.

---

3. Let $f(x) = 3x^2$, when $0 \leq x \leq 1$ be the probability density function of a continoues random variable x. Determine 'a' and 'b' such that

(i) $P(x \leq a) = P(x > a)$

(ii) $P(x > b) = 0.05$.

Solu: (i) Given $P(x \leq a) = P(x > a)$

$$1 - P(x > a) = P(x > a)$$

$$1 = P(x > a) + P(x > a)$$

$$2P(x > a) = 1$$

$$P(x > a) = \frac{1}{2}$$

W.K.T $P(x > a) = \int_{a}^{\infty} f(x)\,dx .$

$$\int_{a}^{\infty} f(x)\,dx = \frac{1}{2} .$$

$$\int_{a}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = \frac{1}{2}$$

$$\int_{a}^{1} 3x^2\,dx + \int_{1}^{\infty} 0.\,dx = \frac{1}{2} .$$

$$\int_{a}^{1} 3x^2\,dx = \frac{1}{2}$$

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$$3\int_a^1 x^2 \, dx = \frac{1}{2}$$

$$\left(\frac{x^3}{3}\right)_a^1 = \frac{1}{2} \times \frac{1}{3}$$

$$\left(\frac{1}{3} - \frac{a^3}{3}\right) = \frac{1}{6}$$

$$\frac{1}{3} - \frac{1}{6} = \frac{a^3}{3}$$

$$\frac{a^3}{3} = \frac{1}{3} - \frac{1}{6}$$

$$\frac{a^3}{3} = \frac{2-1}{6}$$

$$\frac{a^3}{\cancel{3}} = \frac{1}{\cancel{6}\,2}$$

$$a^3 = \frac{1}{2}$$

$$a = \left(\frac{1}{2}\right)^{1/3} = (0.5)^{1/3} = (0.5)^{0.3} = 0.8123$$

$\therefore a = 0.8123.$

(ii)

$$P(X > b) = 0.05$$

$$\int_b^\infty f(x)\,dx = 0.05$$

$$\int_b^1 f(x)\,dx + \int_1^\infty f(x)\,dx = 0.05$$

$$\int_b^1 3x^2\,dx + \int_1^\infty 0 \cdot dx = 0.05$$

$$3\int_b^1 x^2\,dx = 0.05$$

$$\cancel{3}\left(\frac{x^3}{\cancel{3}}\right)_b^1 = 0.05$$

$$1 - b^3 = 0.05$$

$$b^3 = 1 - 0.05$$

$$b^3 = 0.9500$$

$$b = (0.9500)^{1/3} = 0.9847$$

$\therefore b = 0.9847$ //.

Is the function defined by

$$f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0 & x > 4 \end{cases}$$

a probability density function? Find the probability that a variate having $f(x)$ as density function will fall in the intreval $2 \le x \le 3$.

Sol^n: (i) A function is said to be a probability density function if $\int_{-\infty}^{\infty} f(x)dx = 1$

so lets see that

$$\int_{-\infty}^{2} f(x)dx + \int_{2}^{4} f(x)dx + \int_{4}^{\infty} f(x)dx$$

$$= \int_{-\infty}^{2} 0 \cdot dx + \int_{2}^{4} \frac{1}{18}(2x+3)dx + \int_{4}^{\infty} 0 \cdot dx.$$

$$= \frac{1}{18}\int_{2}^{4}(2x+3)dx = \frac{1}{18}\left\{\cancel{2}\left(\frac{x^2}{\cancel{2}}\right)+3x\right\}_2^4$$

$$= \frac{1}{18}\left\{x^2+3x\right\}_2^4 = \frac{1}{18}\left\{(4)^2+3(4)-(2)^2-3(2)\right\}$$

$$= \frac{1}{18}\left\{16+12-4-6\right\} = \frac{1}{\cancel{18}}(\cancel{18}) = 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)dx = 1$$

So the given function is a probability density function.

(ii) $P(2 \le x \le 3) = \int_{2}^{3} f(x)dx$.

W.K.T

$$P(a \le x \le b) = \int_{a}^{b} f(x)dx$$

$$P(2 \le x \le 3) = \int_{2}^{3} \frac{1}{18}(2x+3)dx = \frac{1}{18}\int_{2}^{3}(2x+3)dx$$

$$= \frac{1}{18}\left\{2\left(\frac{x^2}{2}\right)+3x\right\}_2^3 = \frac{1}{18}\left\{\cancel{2}\cdot\frac{(3)^2}{\cancel{2}}+3(3)-\cancel{2}\left(\frac{4}{\cancel{2}}\right)-6\right\}$$

$$= \frac{1}{18}\left\{9+9-4-6\right\} = \frac{1}{\cancel{18}_9}\left\{\cancel{8}^4\right\} = \frac{4}{9}$$

$$P(2 \le x \le 3) = \frac{4}{9}$$

$$\therefore P(2 \le x \le 3) = 0.4444 \ //.$$

If x is a continuous random variable and $y = ax+b$. P.T $E(y) = aE(x)+b$ and $v(y) = a^2 v(x)$, where v stands for variance and a, b are constants.

Solu: (i) By defination we have,

$$E[\phi(x)] = \int_{-\infty}^{\infty} \phi(x) f(x)\,dx$$

Now we have

$$E(Y) = E(ax+b) = \int_{-\infty}^{\infty} (ax+b) f(x)\,dx.$$

$$= a\int_{-\infty}^{\infty} x f(x)\,dx + b\int_{-\infty}^{\infty} f(x)\,dx$$

$$= aE(X) + b(1)$$

$$= aE(X) + b$$

$$\therefore E(Y) = aE(X)+b.$$

{Here as w.k.T it is a probablity distribution function $\int_{-\infty}^{\infty} f(x)\,dx = 1$}.

(ii) From (i), we have

$$E(Y) = aE(X)+b \longrightarrow (1)$$

where $Y = aX+b \longrightarrow (2)$

(2) − (1) ⇒ $Y - E(Y) = (aX+b) - [aE(X)+b]$

$$Y - E(Y) = aX + \cancel{b} - aE(X) - \cancel{b}$$

$$Y - E(Y) = aX - aE(X)$$

$$Y - E(Y) = a[X - E(X)]$$

Squarring on both sides

$$[Y - E(Y)]^2 = a^2[X - E(X)]^2$$

Taking Expectation on both sides, we get

doubt $E\{[Y - E(Y)]^2\} = a^2 E\{[X - E(X)]^2\}$

$$\therefore V(Y) = a^2 V(X) \quad //.$$

If x is a continoues random variable and k is a constant, then prove that

(i) $var(x+k) = var(x)$

(ii) $var(kx) = k^2 var(x)$

Sol^n: W.k.T

$$var(x) = \int_{-\infty}^{\infty} x^2 f(x)dx - u^2 \longrightarrow (1)$$

here

$$u = \int_{-\infty}^{\infty} x \cdot f(x)dx$$

Substituting in above (1) we get

$$var(x) = \int_{-\infty}^{\infty} x^2 f(x)dx - \left(\int_{-\infty}^{\infty} x f(x)dx\right)^2$$

(i)

$$var(x+k) = \int_{-\infty}^{\infty} (x+k)^2 f(x)dx - \left[\int_{-\infty}^{\infty} (x+k)f(x)dx\right]^2$$

$$= \int_{-\infty}^{\infty} (x^2+2kx+k^2)f(x)dx - \left[\int_{-\infty}^{\infty} (xf(x)+kf(x))dx\right]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)dx + 2k\int_{-\infty}^{\infty} xf(x)dx + k^2\int_{-\infty}^{\infty} f(x)dx$$

$$- \left[\int_{-\infty}^{\infty} xf(x)dx + k\int_{-\infty}^{\infty} f(x)dx\right]^2$$

According to probability distribution function

W.k.T $\int_{-\infty}^{\infty} f(x)dx = 1$

$$= \int_{-\infty}^{\infty} x^2 f(x)dx + 2k\int_{-\infty}^{\infty} xf(x)dx + k^2 - \left[\int_{-\infty}^{\infty} xf(x)dx + k\right]^2$$

$$= E(x^2) + 2kE(x) + k^2 - [E(x)+k]^2$$

$$= E(x^2) + 2kE(x) + k^2 - [(E(x))^2 + k^2 + 2kE(x)]$$

$$= E(x^2) + \cancel{2kE(x)} + \cancel{k^2} - (E(x))^2 - \cancel{k^2} - \cancel{2kE(x)}.$$

$$= E(x^2) - (E(x))^2 \qquad \left\{ E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx \right.$$

$$= var(x) \qquad \left. E(x) = \int_{-\infty}^{\infty} xf(x)dx = u \right]$$

$var(x+k) = var(x)$.

drawn from a bag

(ii) $var(kx) = \int_{-\infty}^{\infty} (kx)^2 f(x)dx - \left[\int_{-\infty}^{\infty} (kx)f(x)dx\right]^2$

$= \int_{-\infty}^{\infty} k^2 x^2 f(x)dx - \left[k\int_{-\infty}^{\infty} x f(x)dx\right]^2$

$= k^2 \int_{-\infty}^{\infty} x^2 f(x)dx - k^2\left[\int_{-\infty}^{\infty} x f(x)dx\right]^2$

$= k^2 E(x^2) - k^2[E(x)]^2$

$= k^2[E(x^2) - [E(x)]^2]$

$= k^2 \, var(x)$

$\therefore var(kx) = k^2 \, var(x)$

$var(kx) = k^2 var(x)$. //

The trouble shooting capacitor of an IC chip in a circuit is a circuit is a random variable x whose distribution function is given by.

$$F(x) = \begin{cases} 0, & \text{for } x \le 3 \\ 1-\frac{9}{x^2} & \text{for } x > 3 \end{cases}$$ where x denotes the number of years.

Solu: Find the probability that the IC chip will work properly.

(i) less than 8 years.

(ii) Beyond 8 years.

(iii) Between 5 to 7 years.

(iv) Anywhere from 2 to 5 years.

Solu. We have $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$.

$\therefore F(x) = \int_{0}^{x} f(t)dt \; (\because x > 0) = \begin{cases} 0 & \text{if } x \le 3 \\ 1-\frac{9}{x^2} & \text{if } x > 3 \end{cases}$

(i) $P(X \le 8) = \int_{0}^{8} f(t)dt = F(x)$ $\quad x = 8 > 3.$

$= 1 - \frac{9}{(8)^2} = 1 - \frac{9}{64} =$

$= 1 - 0.1406$

$\therefore P(x<8) = 0.8594$ //

(ii) $P(x \geq 8) = 1 - P(x<8) = 1 - 0.8594 = 0.1406$ //

(iii) $P(5 \leq x \leq 7)$

This is in the form of $P(a \leq x \leq b)$

W.K.T $P(a \leq x \leq b) = F(b) - F(a)$

$P(5 \leq x \leq 7) = F(7) - F(5)$

$5>3$ and $7>3$ so $F(x) = 1 - \frac{9}{x^2}$

$$P(5 \leq x \leq 7) = \left(1 - \frac{9}{(7)^2}\right) - \left(1 - \frac{9}{(5)^2}\right)$$

$$= \left(1 - \frac{9}{49}\right) - \left(1 - \frac{9}{25}\right) = (1 - 0.1837) - (1 - 0.3600)$$

$= 0.8163 - 0.6400$

$= 0.1763.$

$\therefore P(5 \leq x \leq 7) = 0.1763.$ //

(iv) $P(2 \leq x \leq 5) = F(5) - F(2)$ $2<3$ so $F(2) = 0.$

$$= \left(1 - \frac{9}{(5)^2}\right) - 0$$

$$= 1 - \frac{9}{25} = 1 - 0.3600 = 0.6400.$$

$\therefore P(2 \leq x \leq 5) = 0.6400$ //.

The daily consumption of electric power (in millions 9-kw-hars) is a random variable moving and having the probability density function (p.d.f)

$$f(x) = \begin{cases} \frac{1}{9} x e^{-x/3} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

If the total production is 12 million kilohours, determine the probability that there is power cut (shortage) in any given day.

Solu. Here the total production is 12 million kilo hours

So $P(0 \le x \le 12) = \int_0^{12} f(x)\,dx$. If in a place there is a we use $P(a \le x \le b) = F(b) - F(a)$

$$= \int_0^{12} \frac{1}{9} x e^{-x/3}\,dx$$

$$P(0 \le x \le 12) = \frac{1}{9}\int_0^{12} x e^{-x/3}\,dx \longrightarrow (1)$$

$$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int\left(f'(x)\int g(x)\,dx\right)dx.$$

Evaluating $\int_0^{12} x e^{-x/3}\,dx$.

$$= \left\{x\int e^{-x/3}\,dx - \int\left((x)'\int e^{-x/3}\,dx\right)dx\right\}_0^{12}$$

$$= \left\{x \cdot \frac{e^{-x/3}}{\left(-\frac{1}{3}\right)} + \int \frac{e^{-x/3}}{\left(\frac{1}{3}\right)}\,dx\right\}_0^{12}$$

$$= \left\{-3xe^{-x/3} + 3\left(\frac{e^{-x/3}}{\left(-\frac{1}{3}\right)}\right)\right\}_0^{12}$$

$$= \left\{-3xe^{-x/3} - 9e^{-x/3}\right\}_0^{12}$$

$$= \left\{-3e^{-4}(12) - 9e^{-4} + 0 + 9e^{0}\right\}$$

$$= \left\{-36e^{-4} - 9e^{-4} + 9\right\} = -36(0.0183) - 9(0.0183) + 9$$

$$= -0.6588 - 0.1647 + 9$$

$$= 9 - 0.8235$$

$$\int_0^{12} x e^{-x/3}\,dx = 8.1765$$

Substitute this value in (1) we get

$$P(0 \le x \le 12) = \frac{1}{9}(8.1765)$$

$$= 0.9085$$

Power supply is inequate in daily consumption exceeds 12 million kW i.e

$$P(x > 12) = 1 - P(0 \le x \le 12)$$

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$= 1 - 0.9085$

$P(X > 12) = 0.0915$

Actually capacity is upto 12, if it exceeds by 12 there is power cut

$\therefore$ The Probability in a day that there will be a power cut $= 0.0915$ //

Is the function defined as follows a density function $f(x) = e^{-x}, x \geq 0$

$0, x < 0$

If so determine the probability that the variate having this density will fall in the intreval $(1,2)$? Find the Cummulative Property $F(2)$?

Soln: To prove the function is a probability density function, we have to prove $\int_{-\infty}^{\infty} f(x)dx = 1$.

So $\int_{-\infty}^{\infty} f(x)\,dx$

$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = \int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{\infty} e^{-x} dx$.

$= 0 + \int_{0}^{\infty} e^{-x} dx = \left(\frac{e^{-x}}{(-1)}\right)_0^{\infty}$

$= -(e^{-x})_0^{\infty} = -(e^{-\infty} - e^{0}) = -(0-1) = -(-1) = 1$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

So given function follows a density function.

(ii) Required probability

$= P(1 \leq x \leq 2) = \int_{1}^{2} f(x)dx$.

$= \int_{1}^{2} e^{-x} dx = \left(\frac{e^{-x}}{(-1)}\right)_1^2 = -(e^{-x})_1^2$

$= -(e^{-2} - e^{-1}) = e^{-1} - e^{-2} = 0.3679 - 0.1353$

$= 0.2326$.

$\therefore P(1 \le x \le 2) = 0.2326.$

(iii) cummulative property $F(2)$.

W.K.T

$$F(x) = P(X \le x)$$

$$F(2) = P(X \le 2).$$

$$P(X \le 2) = \int_{-\infty}^{2} f(x)\,dx.$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx = \int_{-\infty}^{0} 0.dx + \int_{0}^{2} f(x)\,dx.$$

$$= 0 + \int_{0}^{2} f(x)\,dx.$$

$$= \int_{0}^{2} e^{-x}\,dx = \left(\frac{e^{-x}}{(-1)}\right)_0^2$$

$$= -(e^{-x})_0^2$$

$$= -(e^{-2} - e^{0})$$

$$= e^{0} - e^{-2}$$

$$= 1 - 0.1353 = 0.8647$$

$$P(X \le 2) = 0.8647.$$

$\therefore$ Cumulative Property $F(2) = 0.8647$.

Show that $f(x) = \dfrac{1}{\pi(1+x^2)}$, $-\infty \le x \le \infty$ is a probability ~~distribution~~ density function.

Soln: To prove $f(x)$ is a probability density function we have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

So lets find $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\pi(1+x^2)}\,dx.$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{\pi}\left(\tan^{-1}x\right)_{-\infty}^{\infty}$$

$$= \frac{1}{\pi}\left(\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right).$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \frac{1}{\cancel{\pi}}(\cancel{\pi}) = 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\therefore f(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)$ is a probability density function.

---

5. The cumulative distributive function for a continoues random variable 'x' is

$$F(x) = 1 - e^{-2x}, \quad \text{if } x \geq 0$$
$$= 0, \quad \text{if } x < 0.$$

Find (i) density function

(ii) Mean and variance of density function.

Solu: (i) Here we want to find density function which is denoted by $f(x)$.

Given distributive function which is denoted by $F(x)$.

Now $f(x) = \frac{d}{dx}[F(x)]$ if $x \geq 0$

$$= \frac{d}{dx}\left[1 - e^{-2x}\right]$$

$$= \frac{d}{dx}(1) - \frac{d}{dx}\left(e^{-2x}\right)$$

$$= 0 - \left(-2e^{-2x}\right) = 2e^{-2x}$$

$$\therefore f(x) = 2e^{-2x}. \ //$$

If $x < 0$ then $\frac{d}{dx}(0) = 0$.

$\therefore$ The density function is

$$f(x) = 2e^{-2x}, \quad x \geq 0.$$
$$= 0, \quad x < 0.$$

drawn from a bag

Mean $u = \int_{-\infty}^{\infty} x f(x) dx$

$= \int_{-\infty}^{0} x f(x) dx + \int_{0}^{\infty} x f(x) dx$

$= \int_{-\infty}^{0} 0 \cdot x dx + \int_{0}^{\infty} x 2e^{-2x} dx$

$= 0 + \int_{0}^{\infty} x \cdot 2e^{-2x} dx$

$= 2\int_{0}^{\infty} x e^{-2x} dx$

$= 2\left\{ x \int e^{-2x} dx - \int ((x)' \int e^{-2x} dx) dx \right\}_{0}^{\infty}$

$= 2\left\{ x \left(\frac{e^{-2x}}{-2}\right) - \int (1) \cdot \frac{e^{-2x}}{(-2)} dx \right\}_{0}^{\infty}$

$= 2\left\{ -x \left(\frac{e^{-2x}}{2}\right) + \frac{1}{2}\left(\frac{e^{-2x}}{-2}\right) \right\}_{0}^{\infty}$

$= 2\left\{ -\frac{xe^{-2x}}{2} - \frac{1}{2}\left(\frac{e^{-2x}}{2}\right) \right\}_{0}^{\infty}$

$= \left\{ -xe^{-2x} - \frac{e^{-2x}}{2} \right\}_{0}^{\infty}$

$= \left\{ -(\infty)e^{-\infty} - \frac{e^{-\infty}}{2} + 0 + \frac{e^{0}}{2} \right\} = \left\{ 0 - 0 + 0 + \frac{1}{2} \right\} = \frac{1}{2}$

$\therefore$ Mean $(u) = \frac{1}{2}$ ||

Variance (σ^2):

Variance $(\sigma^2) = \int_{-\infty}^{\infty} x^2 f(x) dx - u^2$

$= \int_{-\infty}^{\infty} x^2 f(x) dx - \frac{1}{4}$

$= \int_{-\infty}^{0} x^2 f(x) dx + \int_{0}^{\infty} x^2 f(x) dx - \frac{1}{4}$

$= \int_{-\infty}^{0} 0 \cdot x^2 dx + \int_{0}^{\infty} x^2 2e^{-2x} dx - \frac{1}{4}$

$= 0 + 2\int_{0}^{\infty} x^2 e^{-2x} dx - \frac{1}{4}$

$= 2\int_{0}^{\infty} x^2 e^{-2x} dx - \frac{1}{4}$

Evaluating $\int_0^\infty x^2 e^{-2x}\,dx$:

$$\int_0^\infty x^2 e^{-2x}\,dx$$

$u = x^2 \qquad v = e^{-2x}$

$u' = 2x \qquad v_1 = -\frac{e^{-2x}}{2}$

$u'' = 2 \qquad v_2 = \frac{e^{-2x}}{4}$

$\Gamma n = (n-1)\Gamma(n-1)$

$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx$

$v_3 = -\frac{e^{-2x}}{8}$

$$= \left\{ -\frac{x^2 e^{-2x}}{2} - \frac{2x e^{-2x}}{4} - \frac{2e^{-2x}}{8} \right\}_0^\infty$$

$$= \left\{ -\frac{(\infty)^2 e^{-\infty}}{2} - \frac{2(\infty)e^{-\infty}}{4} - \frac{2e^{\infty}}{8} + 0 + 0 + \frac{2(1)}{8} \right\}$$

$$= \left\{ 0 - 0 - 0 + 0 + 0 \frac{1}{4} \right\}$$

$$= \frac{1}{4}$$

substitute this value in above we have.

$$= \cancel{8}\left(\frac{1}{\cancel{4}_2}\right) - \frac{1}{4} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$\therefore$ variance $(\sigma^2) = \frac{1}{4}$.

---

6. If 'x' is a continoues with probability density function is given by $f(x) = k x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$ ~~occurs~~.

$= 0$, otherwise.

Find the value of k and mean of x.

Sol: Given continoues probability density function.

w.k.T according to if 'x' is a continoues probability density function.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$$

$$\int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{1} k\, x^{\alpha-1}(1-x^{\beta-1})\, dx + \int_{1}^{\infty} 0 \cdot dx = 1$$

$$\int_{0}^{1} k \cdot x^{\alpha-1}(1-x^{\beta-1})\, dx = 1$$

gama

$$k\int_{0}^{1} (x^{\alpha-1} - x^{\alpha-1} \cdot x^{\beta-1})\, dx = 1$$

same base add powers.

$$k\int_{0}^{1} (x^{\alpha-1} - x^{\alpha+\beta-2})\, dx = 1$$

Relation b/w β and Γ function

$$\beta(m,n) = \frac{\Gamma m \times \Gamma n}{\Gamma(m+n)}$$

$$k\, \beta(\alpha,\beta) = 1$$

$$k \cdot \frac{\Gamma(\alpha)\cdot\Gamma(\beta)}{\Gamma(\alpha+\beta)} = 1$$

$$k = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}$$

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad \text{for } 0<x<1.$$

$= 0$ otherwise

Mean (μ): Mean of x is given by

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} x f(x)dx + \int_{0}^{1} x f(x)dx + \int_{1}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{1} x f(x)\, dx + \int_{1}^{\infty} x(0)\, dx$$

$$= \int_{0}^{1} x \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}\, dx.$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \int_{0}^{1} x \cdot x^{\alpha-1}(1-x)^{\beta-1}\, dx.$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \int_{0}^{1} x^{1+\alpha-1}(1-x)^{\beta-1}\, dx$$

{Just for formula $x^{(\alpha+1)-1}$ as $\alpha+1-1 = \alpha$ only

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \int_{0}^{1} x^{\alpha}(1-x)^{\beta-1}\, dx.$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot \beta(\alpha+1, \beta)$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\cancel{\Gamma(\beta)}} \cdot \frac{\Gamma(\alpha+1)\cdot\cancel{\Gamma(\beta)}}{\Gamma(\alpha+\beta+1)}$$

$\{ \because \Gamma(n+1) = n\Gamma(n) \}$

$$= \frac{\cancel{\Gamma(\alpha+\beta)}}{\cancel{\Gamma(\alpha)}} \cdot \frac{\alpha\cancel{\Gamma(\alpha)}}{(\alpha+\beta)\cancel{\Gamma(\alpha+\beta)}}$$

$$= \frac{\alpha}{(\alpha+\beta)}.$$

$$\because \int_0^1 x^{m-1}(1-x)^{n-1}\,dx = \beta(m,n)$$

Relation between β and Γ is

$$\boxed{\beta(m,n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}}.$$

Probability distributions:

Binomial distribution and poission distributions:

Binomial and poission distributions are depends upon on a discrete random variable

Bernoulli's trail:

1. There are 'n' independent trails.
2. There are only 2 possible outcomes of a each trail one is success and outcome is failure.
3. The probability of a success of each trail is same.

only trail which satisifies the above condition is called a "Binary trail".

Binomial distribution:

The probability of the number of successes in 'n' Bernoullie's trails of experiment is called .

"Bernouli distribution".

Defination:

A random variable 'x' has a binomial distribution if it assumes only non-negative values and it is the probability distribution is given by.

$P(x=r) = P(r) = n_{C_r} \cdot p^r \cdot q^{n-r}$ if $r = 0,1,2 \dots, n$.

where r = number of successes.

n = no. of independent trails.

P = Probability of success in single trail of experiment.

$q = 1-p$.

Mean, varience, standard deviation and mode of the binomial distribution:

Let 'x' be a binomial distribution then

(i) Mean of Binomial distribution $= np$

(ii) Variance of Binomial distribution $= npq$.

(iii) standard deviation of binomial destribution $= \sqrt{npq}$

(iv) Mode of the binomial distribution is a value of 'x' for which $P(x)$ is maximum

i.e Mode $= \begin{cases} \text{Integral part of } (n+1)p, \text{ if } (n+1)p \text{ is not to get the target data.} \\ (n+1)p \,\&\, (n+1)p-1 ; \text{ if } (n+1)p \text{ is an target.} \end{cases}$

Problems:

1. The mean and variance of a binomial distribution are 4 and 3 respectively. Then find $P(x \geq 1)$.

Solu: Given

$$\text{Mean} = np = 4$$

$$\text{variance} = npq = 3.$$

$$np = 4 \longrightarrow (1)$$

$$npq = 3 \longrightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{\cancel{np}q}{\cancel{np}} = \frac{3}{4}$$

$$q = \frac{3}{4}$$

We have $p+q=1$

$$p + \frac{3}{4} = 1$$

$$p = 1 - \frac{3}{4}$$

$$\boxed{p = \frac{1}{4}}$$

We have

$$np = 4$$

$$n\left(\frac{1}{4}\right) = 4$$

$$\boxed{n = 16}.$$

We have to find $P(x \geq 1)$

$$= 1 - P(x<1)$$

$$= 1 - P(x=0)$$

Now $P(x=0)$ here $r=0.$

$$P(x=r) = P(x=0) = nc_r \cdot p^r \cdot q^{n-r}$$

$$= 16c_0 \cdot \left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{16-0}$$

$$= 16c_0 (1) \cdot \left(\frac{3}{4}\right)^{16}$$

$$= (1)(0.75)^{16}$$

$$P(x=0) = 0.0100$$

$$\therefore P(x \geq 1) = 1 - P(x<1)$$

$$= 1 - P(x=0)$$

$$= 1 - 0.0100$$

$$= 0.9900$$

$$\therefore P(x \geq 1) = 0.9900 \;//$$

2. If the probability of a defective bolt is $1/8$.

Find (i) Mean

(ii) variance for the distribution of defective bolts of 640.

Sol$\underline{\underline{n}}$: Given $p = 1/8$

$q = 1-p = 1-\frac{1}{8} = \frac{7}{8}$.

n = no. of deffective bolts = 640.

$\therefore$ Mean of the binomial distribution $= np$

$= (\cancel{640}^{80})(\frac{1}{\cancel{8}})$

$= 80$.

$\therefore$ Mean $= 80$.

$\therefore$ Variance of the binomial distribution

$= npq$

$= \cancel{640}^{10} \times \frac{1}{\cancel{8}} \times \frac{7}{\cancel{8}}$

$= 70$

$\therefore$ Variance $= 70$.

---

3. Determine the probability of getting a sum of 9 exactly twice in three(3) throws with a pair of dice(2).

Sol$\underline{n}$: Given In rolling a die i.e 2 dies the possible outcomes are $= 36$.

Now we have $n = 3$. (throws).

No. of favourable outcomes whose sum of 9 is

$= \{(6,3), (3,6), (4,5), (5,4)\}$.

So $P = \frac{4}{36} = \frac{1}{9}$

$q = 1-p = 1-\frac{1}{9} = \frac{8}{9}$

W.K.T

$P(X=r) = {}^nC_r \cdot p^r \cdot q^{n-r}$

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Here $r=2$ (twice).

$$P(X=2) = 3C_2 \cdot \left(\frac{1}{9}\right)^2 \cdot \left(\frac{8}{9}\right)^{3-2}$$

$$= 3C_2 \cdot \left(\frac{1}{9}\right)^2 \cdot \left(\frac{8}{9}\right)^1$$

$$= (\cancel{3}) \cdot \frac{1}{81} \cdot \frac{8}{\cancel{9}\,3} = \frac{8}{81 \times 3}$$

$$= \frac{8}{243}$$

$$= 0.0329$$

$\therefore P(X=2) = 0.0329$.

<u>Homework</u>

1. If 10% of the rivets produced by a machine are defective. Find the probability that out of 5 rivets choosen at random.

(i) None will be defective

(ii) one will be defective.

(iii) Atmost 2 revets will be defective.

<u>Sol 4</u>: Given 10% of the rivets produced by a machine are defective.

$$\text{So } P = \frac{10}{100} = \frac{1}{10}$$

$$P = \frac{1}{10}$$

$$q = 1-P$$

$$= 1 - \frac{1}{10}$$

$$= \frac{9}{10}$$

Here $n=5$

(i) <u>None will be defective</u>:

$P(X=0)$, so $r=0$

W.K.T

$$P(X=r) = {}^nC_r \, p^r \cdot (q)^{n-r}$$

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$$P(X=0) = {}^5C_0 \cdot \left(\frac{1}{10}\right)^0 \cdot \left(\frac{9}{10}\right)^{5-0}$$

$$= 1 \cdot (1) \cdot \left(\frac{9}{10}\right)^5$$

$$= \left(\frac{9}{10}\right)^5$$

$$= (0.9)^5$$

$$= 0.5905$$

$$\therefore P(X=0) = 0.5905$$

$\therefore$ The Probability that none will be deffective is

$$P(X=0) = 0.5905 \,.\, //$$

(ii) <u>One Will be defective</u>

$P(X=1)$ so $r=1$

W.K.T

$$P(X=r) = {}^nC_r \, (p)^r \, (q)^{n-r}$$

$$P(X=1) = {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{5-1}$$

$$= 5\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^4$$

$$= 5(0.1)(0.9)^4$$

$$= 5(0.1)(0.6561)$$

$$\therefore P(X=1) = 0.3281$$

$\therefore$ The probability that one revet will be deffective is

$$P(X=1) = 0.3281 \quad //$$

(iii) <u>Atmost 2 revets will be defected:</u>

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2).$$

From above $P(X=0) = 0.5905$

$P(X=1) = 0.3281$

let's find $P(X=2)$

so $r=2$

W.K.T

$$P(X=r) = {}^nC_r \cdot (p)^r \cdot (q)^{n-r}$$

J. Kishani

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$P(X=2) = 5c_2 \cdot \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{5-2}$

$= (1\cancel{0})\left(\frac{1}{10\cancel{0}}\right) \cdot \left(\frac{9}{10}\right)^3$

$= (0.1)(0.9)^3$

$= (0.1)(0.7290)$

$P(X=2) = 0.0729$

$\therefore P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.3281 + 0.5905 + 0.0729$

$= 0.9915$

$\therefore P(X \le 2) = 0.9915$ //

3\. 10 coins are thrown simultaneously, find the probability of getting atleast (i) 7 heads
(ii) 6 heads.

Sol: Given $n = 10$

Generally when a coin is tossed we have two posibilities i.e head, tail

$\therefore$ $P = \frac{1}{2}$ and

$q = 1-P = 1-\frac{1}{2} = \frac{1}{2}$

Now

(i) atleast 7 heads:

$P(X \ge 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10).$

$P(X=7)$

W.K.T $P(X=r) = nc_r (P)^r (q)^{n-r}$

at $P(X=7)$, $r=7$

$P(X=7) = 10c_7 \cdot \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7}$

$= (120)(0.5)^7 \cdot (0.5)^3$

$= (120)(0.0078)(0.1250)$

$\therefore P(X=7) = 0.1170$

$$P(x=8) = {}^{10}C_8 \cdot \left(\tfrac{1}{2}\right)^8 \cdot \left(\tfrac{1}{2}\right)^{10-8}$$

$$= (45)(0.5)^8(0.5)^2$$

$$= (45)(0.0039)(0.2500)$$

$$\therefore P(x=8) = 0.0439$$

$$P(x=9) = {}^{10}C_9 \left(\tfrac{1}{2}\right)^9 \cdot \left(\tfrac{1}{2}\right)^{10-9}$$

$$= (10)(0.5)^9(0.5)^1$$

$$= (10)(0.0020)(0.5)$$

$$\therefore P(x=9) = 0.0100$$

$$P(x=10) = {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10}\left(\tfrac{1}{2}\right)^{10-10}$$

$$= (1)(0.5)^{10}(0.5)^0$$

$$= (0.5)^{10}$$

$$\therefore P(x=10) = 0.0010$$

$$\therefore P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$$

$$= 0.1170 + 0.0439 + 0.0100 + 0.0010$$

$$P(x \geq 7) = 0.1719 \;//$$

(ii) $P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$P(x=6)$

$$= {}^{10}C_6 \cdot (0.5)^6 \cdot \left(\tfrac{1}{2}\right)^{10-6}$$

$$= {}^{10}C_6 (0.5)^6 (0.5)^4$$

$$= (210)(0.0156)(0.0001)$$

$$\therefore P(x=6) = 0.0003.$$

So $P(x \geq 6) = P(x=6) + P(x \geq 7)$

$$= 0.0003 + 0.1719$$

$$= 0.1722.$$

$\therefore P(x \geq 6) = 0.1722$ //

4. The probability that John hits a target is $1/2$, he fights 6 times. Find the probability that he hits the target (i) exactly 2 times

(ii) more than 4 times

(iii) atleast once.

Soln: Given $n = 6$

The probability that John hits the target is $1/2$

$$P = 1/2$$
$$q = 1 - p$$
$$= 1 - \frac{1}{2}$$
$$q = \frac{1}{2}$$

(i) exactly 2 times: $P(x = 2)$.

W.K.T $P(x=r) = nc_r (p)^r \cdot (q)^{n-r}$; Here $r = 2$

$$P(x=2) = 6c_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2}$$
$$= 6c_2 (0.5)^2 (0.5)^4$$
$$= (15)(0.2500)(0.0625)$$
$$P(x=2) = 0.2344.$$
$$\therefore P(x=2) = 0.2344.$$

(ii) more than 4 times:

$$P(x > 4) = P(x=5) + P(x=6)$$

$P(x = 6)$

W.K.T $P(x=r) = nc_r \cdot (p)^r (q)^{n-r}$

Here $r = 6$

$$P(x=6) = 6c_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{6-6}$$
$$= 1(0.5)^6 \cdot (1)$$
$$= (0.5)^6$$
$$\therefore P(x=6) = 0.0156$$

$P(x=5) = 6c_5 \cdot (\frac{1}{2})^5 (\frac{1}{2})^{6-5}$

$= (6)(0.5)^5(0.5)^1$

$= 6(0.0313)(0.5)$

$= 0.0939$

$\therefore P(x=5) = 0.0939$

Now $P(x>4) = P(x=5) + P(x=6)$

$= 0.0939 + 0.0156$

$= 0.1095$

$\therefore P(x>4) = 0.1095$. ||

Atleast once:

$P(x \geq 1) = 1 - P(x<1) = 1 - P(x=0)$

Now

$P(x=0)$ $r=0$

$P(x=0) = 6c_0 \cdot (P)^0 (q)^{6-0}$

$= 6c_0 (\frac{1}{2})^0 (\frac{1}{2})^6$

$= 6c_0 (0.5)^0 (0.5)^6 = (1)(1)(0.5)^6$

$P(x=0) = 0.0156$

$\therefore P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(x=0)$

$= 1 - 0.0156$

$= 0.9844$

$\therefore P(x \geq 1) = 0.9844$ || .

---

5. 6 dies are thrown 729 times. How many times do expect atleast 3 dices to show a 5 (or) 6.

Sol: Here there are 6 dies

$\therefore n = 6$

Here there are 2 posibilities eith 5 (or) 6

$n(E) = 2$

Total posibilities when dice are thrown

$n(S) = 6$

$\therefore p = \frac{2}{6} = \frac{1}{3}$

$q = 1 - p = 1 - \frac{1}{3}$

atleast 3 dice:

$q = \frac{2}{3}$.

$P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$

(or)

$= 1 - P(x < 3)$

$= 1 - [P(x=1) + P(x=2)]$

Now

$P(x=1) = {}^{n}C_r \cdot (p)^r \cdot (q)^{n-r}$.

here $r = 1$

$= 6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1}$

$= \cancel{6}^{2}\left(\frac{1}{\cancel{3}}\right)\left(\frac{2}{3}\right)^5 = (2)(0.6667)^5$

$= 2 \times 0.1317$

$P(x=1) = 0.2634$.

$P(x=2) = 6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{6-2}$

$= (15)\left(\frac{1}{9}\right)\left(\frac{2}{3}\right)^4$

$= (15)(0.1111)(0.6667)^4$

$= (15)(0.1111)(0.1976)$

$= 0.3293$.

$\therefore P(x=2) = 0.3293$.

$\therefore P(x \geq 3) = 1 - [P(x=1) + P(x=2)]$

$= 1 - [0.2634 + 0.3293]$

$= 1 - 0.5927 = 0.4073$.

$\therefore P(x \geq 3) = 0.4073$ //

Now here 6 dice are thrown 729 times

so we have $P(x \geq 3) \times 729$

$= 0.4073 \times 729$

$= 296.9217$ //

---

6. The mean and variance of binomial variable 'x' with parameters 'n' and 'p' are 16 and 8. Find $P(x \geq 1)$ and $P(x \geq 2)$

Soln: Given

$\text{Mean} = np = 16$

$\text{variance} = npq = 8$

$\therefore np = 16 \rightarrow (1)$

$npq = 8 \rightarrow (2)$

$$\frac{(2)}{(1)} = \frac{\cancel{np}q}{\cancel{np}} = \frac{8}{16}$$

$q = \frac{1}{2}$

$P = 1 - q$

$= 1 - \frac{1}{2}$

$P = \frac{1}{2}$

$np = 16$

$n\left(\frac{1}{2}\right) = 16$

$n = 32$

(1)

Now

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$.

$P(x=0)$ i.e $r = 0$.

W.K.T $P(x=r) = {}^nC_r (P)^r (q)^{n-r}$

$= {}^{32}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0}$.

$= (1)(1)(0.5)^{32} = 2.328306437 \times 10^{-10}$

Now $P(X\geq1) = 1-P(X<1)$

$= 1-(0.5)^{32}$

$= 0.9999$

$\therefore P(X\geq1) = 0.9999$ //

(ii) $P(X\geq2) = 1-P(X<2)$

$= 1-[P(X=0)+P(X=1)]$

$P(X=1) = 32C_1 (\frac{1}{2})^1 (\frac{1}{2})^{32-1}$

$= (32)(0.5)(0.5)^{31}$

Now.

$P(X\geq2) = 1-[P(X=0)+P(X=1)]$

$= 1-[(0.5)^{32}+(32)(0.5)(0.5)^{31}]$

$= 1-7.683411241\times10^{-09}$

$= 0.9999.$

$\therefore P(X\geq2) = 0.9999.$ //

---

2. Out of 800 families with 5 children each, how many would you expect to have

(i) 3 boys

(ii) 5 girls

(iii) Atleast one boy

(iv) either 2 or 3 boys.

Solu: Given that $n=5$

P = probability that each children (boy (or) girl)

$P = 1/2$

Since $p+q=1$

$q = 1-p$

$q = 1-1/2$

$q = 1/2$

(i) Probability that expects 3 boys is

$$P(x=3) = \frac{e^{-5} \cdot 5^3}{3!} = \frac{(6.737946999 \times 10^{-3})(125)}{6}$$

$$= \frac{0.8422}{6}$$

$$P(x=3) = 0.1403$$

Out of 800 families would expect to 3 boys is

$$= 800 \times P(x=3)$$

$$= 800 \times 0.1403$$

$$= 112.2991$$

(ii) Probability that expect to 5 girls is

$$P(x=5) = \frac{e^{-5} \cdot 5^5}{5!} = \frac{e^{-5}(3125)}{120}$$

$$= \frac{21.0560}{120}$$

$$= 0.1754$$

$\therefore$ out of 800 families would expect to 5 girls is

$$= 800 \times 0.1754$$

$$= 140.3738.$$

(iii) probability that atleast one boy is

$$P(x \geq 1) = 1 - P(x<1)$$

$$= 1 - [P(x=0)]$$

$$= 1 - \frac{e^{-5} \cdot 5^0}{0!}$$

$$= 1 - e^{-5}$$

$$= 1 - 0.9932$$

$$P(x \geq 1) = 0.0068$$

$\therefore$ out of 800 families would expect to atleast one boy is $= 800 \times 0.0068 = 5.4400$.

(iv) probability that either two (or) three boys is

$P(X=2)+P(X=3)$

$$= \frac{e^{-5}.5^2}{2!} + \frac{e^{-5}.5^3}{3!}$$

$$= \frac{(0.0067)(25)}{2} + \frac{(0.0067)(125)}{6}$$

$$= \frac{0.1675}{2} + \frac{0.8375}{6}$$

$$= 0.0838 + 0.1396$$

$P(X=2)+P(X=3) = 0.2234$

$\therefore$ out of 800 families would expect to either two (or) three boys is $= 800 \times 0.2234$

$= 178.72$ //.

Poission Distribution:

A random variable 'x' is said to be a poision distribution if it assumes only a non-negitive values and it's probability distribution is given by

$$\boxed{\begin{aligned} P(X=r) &= \frac{e^{-\lambda}.\lambda^r}{r!}, \text{ where } r=0,1,2 \ldots n \\ &= 0, \text{ otherwise.} \end{aligned}}$$

Here $\lambda>0$ is a parameter of the distribution.

Conditions of the Poission Distribution:

The poision distribution is used under the following conditions.

(i) The no. of trails 'n' is very large.

(ii) The probability of success 'p' is very small.

(iii) $np=\lambda$ is finate.

Mean, variance and Mode of the poission distribution:

1. The mean of the poision distribution is 'λ' i.e $np = \lambda$

2. The variance of the poision distribution is also 'λ' i.e $npq = \lambda$.

3. The mode of the poision distribution means mode is the value of 'λ' for which the Corresponding Probability $P(x)$ is maximum. Hence the mode of Poision distribution is lies in between $\lambda - 1$ ~~&~~ 'λ'.

Case (i):

If 'λ' is not an integer then the mode of the poision distribution is on integral part of λ.

Case (ii):

If 'λ' is an integer and '$\lambda - 1$' is also an integer then mode of poission distribution are 'λ' and '$\lambda - 1$'.

Recurrence Relation for the poision distribution is given by:

$$P(x+1) = \left(\frac{\lambda}{x+1}\right) P(x).$$

problems:

1. A hospital switch board receives an average of 4 emergency calls in a 10 minutes intreval. What is the probability that

(i) There are atmost 2 emergency calls in a 10 minute intreval.

(ii) There are exactly 3 emergency calls in a 10 minute intreval.

Sol[4]: The avg no. of emergency calls in 10 min intrevals is $\lambda = 4$

(i) Probability that atmost 2 emergency calls in a 10 min intreval is

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-\lambda}\lambda^r}{r!} + \frac{e^{-\lambda}\cdot\lambda^r}{r!} + \frac{e^{-\lambda}\cdot\lambda^r}{r!}$$

$$= \frac{e^{-4}\cdot\lambda^0}{0!} + \frac{e^{-4}\cdot\lambda^1}{1!} + \frac{e^{-4}\lambda^2}{2!}$$

$$= \frac{e^{-4}(1)}{(1)} + \frac{e^{-4}(4)}{1!} + \frac{e^{-4}(4)^2}{2}$$

$$= e^{-4} + 4e^{-4} + \frac{e^{-4}(\cancel{16}^{8})}{\cancel{2}}$$

$$= e^{-4} + 4e^{-4} + 8e^{-4}$$

$$= e^{-4}(1+4+8)$$

$$= 13e^{-4}$$

$$\therefore P(X \le 2) = 0.2381 \;//$$

(ii) Probability that exactly three emergency calls in a 10 min intreval is.

$$P(X=3) = \frac{e^{-\lambda}\cdot\lambda^r}{r!}$$

$r=3 \quad \lambda = 4$

$$= \frac{e^{-4}\cdot(4)^3}{3!} = \frac{e^{-4}(64)}{6} = \frac{1.1722}{6} = 0.1953$$

$$\therefore P(X=3) = 0.1953 \;//$$

---

2. A car-hire firm has <u>two</u> cons which it hires out day by day. The no. of demands for a con on each day is distributed as a poission distribution with mean 1.5. Calulculate the poission of

the days.

(i) on which there is no demand.

(ii) one which demand is refused.

($\because$ refused means greater than).

Soln: Given Mean $\lambda = 1.5$

(i) Probability that on which there is no demand is $P(X=0)$

W.K.T $P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-\lambda}(1)}{(1)} = e^{-\lambda}$$

$$= e^{-1.5} = 0.2231$$

$\therefore P(X=0) = 0.2231$. //

(ii) No. of days there is no demand $= 365 \times 0.2231 = 81.442$ days.

Probability that on which demand is refused is

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-\lambda} \cdot \lambda^r}{r!} + \frac{e^{-\lambda} \cdot \lambda^r}{r!} + \frac{e^{-\lambda} \cdot \lambda^r}{r!}\right]$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)}{1!} + \frac{e^{-1.5} \cdot (1.5)^2}{2!}\right]$$

$$= 1 - \left[e^{-1.5} + 1.5e^{-1.5} + \frac{(1.5)^2}{2} e^{-1.5}\right]$$

$$= 1 - e^{-1.5}\left[1 + 1.5 + \frac{(1.5)^2}{2}\right]$$

$$= 1 - e^{-1.5}[1 + 1.5 + 1.125] = 1 - e^{-1.5}(3.625)$$

$$= 1 - 0.8088$$

$P(X > 2) = 0.1912$

$\therefore$ No. of days that the demand was refused

$= P(X > 2) \times 365$

$= 0.1912 \times 365$

$= 69.788$

$= 70$ days (nearly).

HOMEWORK

1. If the probability that an induvidual suffers a bad reaction from a certain junction is 0.0001. Determine the probability that out of 2000 induviduals.

   (i) exactly 3

   (ii) More than 2 induviduals.

   (iii) None in more than one induvidual suffer a bad reaction.

Soln: Given $P = 0.001$

$n = 2000$.

Mean, $\lambda = np = 0.001 \times 2000$

$\therefore \lambda = 2$

(i) exactly 3:

$P(X=3)$

W.K.T $P(X=r) = \dfrac{e^{-\lambda} \cdot \lambda^r}{r!}$

$P(X=3) = \dfrac{e^{-\lambda} \cdot \lambda^3}{3!}$

$= \dfrac{e^{-2} \cdot (2)^3}{6} = \dfrac{e^{-2}(\cancel{8}^{4})}{\cancel{6}\,3}$

$= \dfrac{4e^{-2}}{3} = \dfrac{4}{3}(0.1353)$

$= \dfrac{0.5412}{2} = 0.1804$

$\therefore P(X=3) = 0.1804$ ||

(ii) More than 2 induviduals:

$$P(X>2) = 1-P(X\leq 2)$$

$$= 1-[P(X=0)+P(X=1)+P(X=2)]$$

$$= 1-\left[\frac{e^{-\lambda}.\lambda^r}{r!}+\frac{e^{-\lambda}.\lambda^r}{r!}+\frac{e^{-\lambda}.\lambda^r}{r!}\right]$$

$$= 1-\left[\frac{e^{-2}.(2)^0}{0!}+\frac{e^{-2}(2)}{1!}+\frac{e^{-2}(2)^2}{2!}\right]$$

$$= 1-\left[\frac{e^{-2}(1)}{1}+2e^{-2}+\frac{\cancel{4}^{2}e^{-2}}{\cancel{2}}\right]$$

$$= 1-[e^{-2}+2e^{-2}+2e^{-2}]$$

$$= 1-5e^{-2}$$

$$= 1-5(0.1353)$$

$$= 1-0.6766$$

$$= 0.3234$$

$\therefore P(X>2) = 0.3234$ ||

(iii) More than one induvidual:

$$P(X>1) = 1-P(X\leq 1)$$

$$= 1-[P(X=0)+P(X=1)]$$

$$= 1-\left[\frac{e^{-\lambda}.(\lambda)^r}{r!}+\frac{e^{-\lambda}.\lambda^r}{r!}\right]$$

$$= 1-\left[\frac{e^{-2}.(2)^0}{0!}+\frac{e^{-2}(2)^1}{1!}\right]$$

$$= 1-[e^{-2}+2e^{-2}]$$

$$= 1-3e^{-2} = 1-3(0.1353)$$

$$= 1-0.4059$$

$$= 0.5941$$

$\therefore P(X>1) = 0.5941.$ //

2. A Manufacturer of cotton pins knows that 5% of his product is defective pins is sold in boxes of 100. He garanties that not more than 10 pins will be defective. What is the approximate probability that a box will fail to meet the guranted quality.

Sol^n: Given $n = 100$.

$$P = 5\% = \frac{5}{100} = \frac{1}{20}.$$

Here
Mean $= \lambda = np$

$$= 100\left(\frac{1}{20}\right)$$

$$= 5$$

$$\lambda = 5.$$

More than 10 pins will be defective

So $P(X>10) = 1 - P(X \le 10)$.

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=7) + P(X=8) + P(X=9) + P(X=10)].$$

$$= 1 - \left[\frac{e^{-5}.(5)^0}{0!} + \frac{e^{-5}(5)^1}{1!} + \frac{e^{-5}(5)^2}{2!} + \frac{e^{-5}(5)^3}{3!} + \frac{e^{-5}(5)^4}{4!} + \frac{e^{-5}(5)^5}{5!} + \frac{e^{-5}(5)^6}{6!} + \frac{e^{-5}(5)^7}{7!} + \frac{e^{-5}(5)^8}{8!} + \frac{e^{-5}(5)^9}{9!} + \frac{e^{-5}(5)^{10}}{10!}\right]$$

$$= 1 - \left[e^{-5} + 5e^{-5} + \frac{25}{2}e^{-5} + \frac{125}{6}e^{-5} + \frac{625}{24}e^{-5} + \frac{3125}{120}e^{-5} + \frac{15625}{720}e^{-5} + \frac{78125e^{-5}}{5040} + \frac{390625}{40320}e^{-5} + \frac{1953125}{362880}e^{-5} + \frac{9765625}{3628800}e^{-5}\right]$$

$$= 1 - e^{-5}[1 + 5 + 12.5 + 20.8333 + 26.0416 + 26.0416 + 21.7013 + 15.5009 + 9.6881 + 5.3822 + 2.6911]$$

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$= 1 - e^{-5}(146.3801)$

$= 1 - 0.9863$

$= 0.0137$

$\therefore P(x \geq 10) = 0.0137$ //.

---

3. Suppose 2% of the people on the avg are left handed. find (i) The probability of finding 3 (or) more left handed

(ii) The Probability of finding none (or) one left handed.

Solu: Given

$$\lambda = \frac{2}{100} = 0.02$$

(i) 3 (or) more.

$$P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-0.02}(0.02)^0}{0!} + \frac{e^{-0.02}(0.02)^1}{1!} + \frac{e^{-0.02}(0.02)^2}{2!}\right]$$

$$= 1 - \left[e^{-0.02} + 0.02\,e^{-0.02} + \frac{(0.02)^2}{2}e^{-0.02}\right]$$

$$= 1 - e^{-0.02}\left[1 + 0.02 + \frac{(0.02)^2}{2}\right]$$

$$= 1 - e^{-0.02}\left[1 + 0.02 + 0.0002\right]$$

$$= 1 - e^{-0.02}\left[1.0202\right]$$

$$= 1 - 0.9999$$

$$= 0.0000013$$

$\therefore P(x \geq 3) = 0.0000013$ //.

one (or) none :

$p(x \le 1) = P(x=0) + P(x=1)$

$$= \frac{e^{-0.02}(0.02)^0}{0!} + \frac{e^{-0.02}(0.02)^1}{1!}$$

$$= e^{-0.02} + 0.02\,e^{-0.02}$$

$$= e^{-0.02}(1+0.02)$$

$$= e^{-0.02}(1.02)$$

$$= 0.9998$$

$\therefore P(x \le 1) = 0.9998$. //

---

4. If a poision distribution is such that $3/2[P(x=1)] = [P(x=3)]$

Find (i) $P(x \ge 1)$

(ii) $P(x \le 3)$

(iii) $P(2 \le x \le 5)$

Soln : Given $\frac{3}{2}P(x=1) = P(x=3)$.

$$\frac{3}{2}\left(\frac{e^{-\lambda}\cdot\lambda}{1!}\right) = \frac{e^{-\lambda}\cdot\lambda^3}{3!}$$

$$\frac{3}{2}\left(\lambda e^{-\lambda}\right) = \frac{\lambda^3\cdot e^{-\lambda}}{6}$$

$$\lambda \cancel{e^{-\lambda}} = \frac{2}{3}\cdot\frac{\lambda^3 \cancel{e^{-\lambda}}}{6}$$

$$\lambda = \frac{\cancel{2}}{3} \times \frac{\lambda^3}{\cancel{6}\,3}$$

$$\cancel{\lambda} = \frac{\cancel{\lambda^3}\,\lambda^2}{9}$$

$$\lambda^2 = 9$$

$\lambda = 3$. $\lambda = \pm 3$ λ is also Positive .

$\therefore \lambda = 3$.

(i) $P(X \geq 1)$

We have $P(X \geq 1) = 1 - P(X < 1)$

$$= 1 - P(X = 0)$$

$$= 1 - \left[\frac{e^{-\lambda} \lambda^r}{r!}\right]$$

$$= 1 - \left[\frac{e^{-3} \cdot (3)^0}{0!}\right] = 1 - \left[e^{-3}\right]$$

$$= 1 - 0.0497$$

$$= 0.9502$$

$\therefore P(X \geq 1) = 0.9502.$

(ii) $P(X \leq 3)$

$$= P(X=0) + P(X=1) + P(X=2) + P(X=3).$$

$$= \left(\frac{e^{-\lambda}\lambda^r}{r!} + \frac{e^{-\lambda}\lambda^r}{r!} + \frac{e^{-\lambda}\lambda^r}{r!} + \frac{e^{-\lambda}\lambda^r}{r!}\right).$$

$$= \left(\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!}\right)$$

$$= \left(e^{-3} + e^{-3}(3) + \frac{9}{2}e^{-3} + \frac{27}{6}e^{-3}\right)$$

$$= e^{-3}(1 + 3 + 4.5 + 4.5)$$

$$= e^{-3}(13)$$

$$= 13e^{-3}$$

$$= 13(0.0497)$$

$\therefore P(X \leq 3) = 0.6472$

(iii) $P(2 \leq X \leq 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5).$

$$= \left(\frac{e^{-3} \cdot (3)^2}{2!} + \frac{e^{-3}(3)^3}{3!} + \frac{e^{-3}(3)^4}{4!} + \frac{e^{-3}(3)^5}{5!}\right)$$

$$= \left(\frac{9e^{-3}}{2} + \frac{27}{6}e^{-3} + \frac{81}{24}e^{-3} + \frac{243}{120}e^{-3}\right)$$

$$= e^{-3}(4.5 + 4.5 + 3.375 + 2.025)$$

$= 14.4e^{-8}$

$= 14.4(0.0497)$

$= 0.7169.$

$\therefore P(2 \leq x \leq 5) = 0.7169.$ //

3. Normal distribution:

Normal distribution is depends on a continoues random variable.

Defination: continoues random variable 'x' has a normal distribution if its density function (or) Probability distribution is given by:

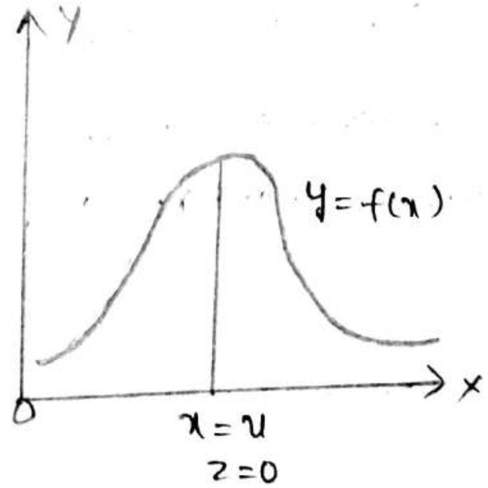
$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}, \quad -\infty < x < \infty$$

$-\infty < \mu < \infty, \sigma > 0$

Where μ and σ are two parameters of a normal distribution. The random variable 'x' is called a normal random variable.

Where $z = \frac{x-\mu}{\sigma}$, is called "standard normal variate".

Charecteristics of a Normal distribution:



(i) The Graph of a Normal distribution $y = f(x)$ in the xy-plane is called a normal curve.

(ii) The curve is bell shaped and it is symetrical about $x = u$.

(iii) The total area under the normal curve and x-axis is unity.

(iv) The points of inflexion of the curve are

$x = u \pm \sigma$

(v) probability that the normal variable 'x' lies b/w x_1 and x_2 is given by.

$$P(x_1 \leq x \leq x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-1/2\left(\frac{x-u}{\sigma}\right)^2} dx .$$

(vi) The normal distribution for $u=0$ and $\sigma=1$ is called a standard normal distribution.

Find the probability density of a normal curves:

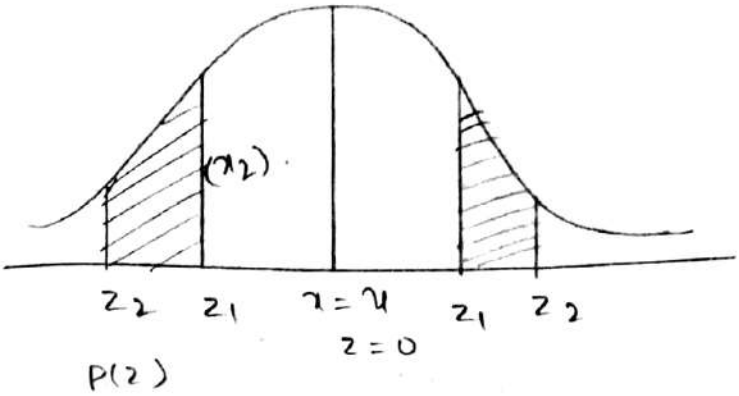
Probability that the normal variable 'x' width mean 'u', standard deviation 'σ' lies b/w the values x_1 and x_2 ($x_1 \leq x_2$) can be obtained by using area under the standard normal and distributions standard normal curve is as follows.

First we can find the value of z_1 and z_2 corresponding values of x_1 and x_2 respectively by using $z = \frac{x-u}{\sigma}$.

Type I: To find $P(x_1 \leq x \leq x_2)$

Case(i): If both z_1 and z_2 are +ve or -ve then

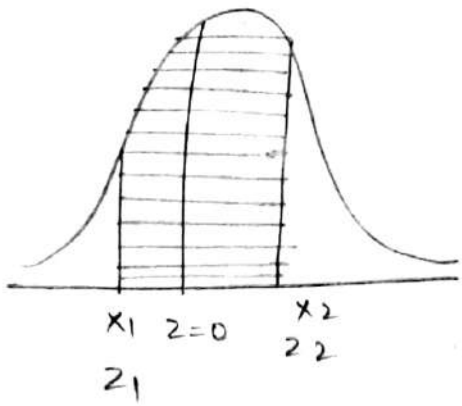
$$P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_2) - A(z_1)$$



case (ii):

If $z_1<0$ and $z_2>0$ (z_1 and z_2 are opposite signs)

then



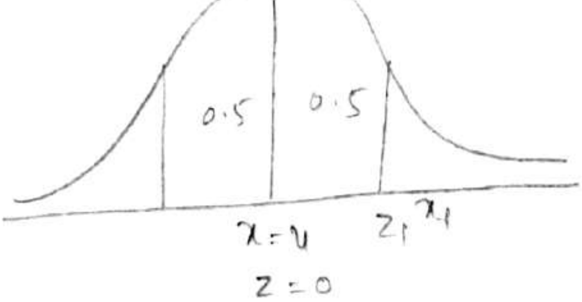
$$P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2) = A(z_2) + A(z_1).$$

Type-II:

To find $P(x \ge x_1)$ first we can find the value of z_1 to find corresponding x_1 by using

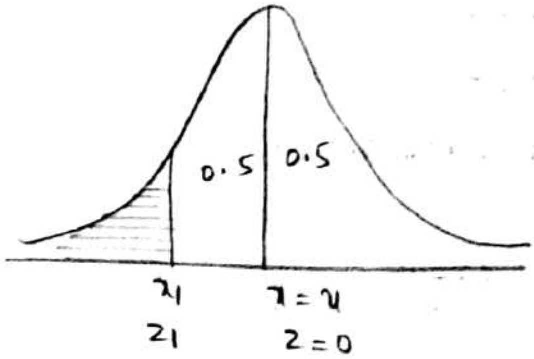
$$\boxed{z_1 = \frac{x_1 - \mu}{\sigma}}$$

case (i): If $z_1>0$ (i.e z_1 is positive) then



$$P(x \ge x_1) = P(z \ge z_1) = 0.5 - A(z_1).$$

case(ii): If $z_1 < 0$ (z_1 is Negitive (-ve)) then

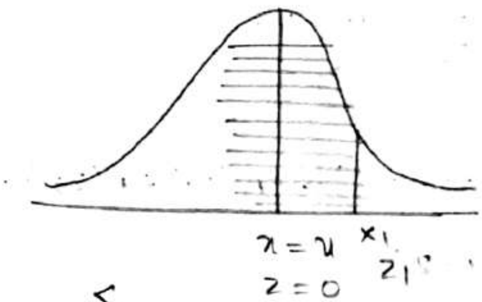


Type III: $P(x \geq x_1) = P(z \geq z_1) = 0.5 + A(z_1)$

To find $P(x \leq x_1)$:

First we find the value of z_1 corresponding to x_1 by using $z_1 = \dfrac{x_1 - \mu}{\sigma}$

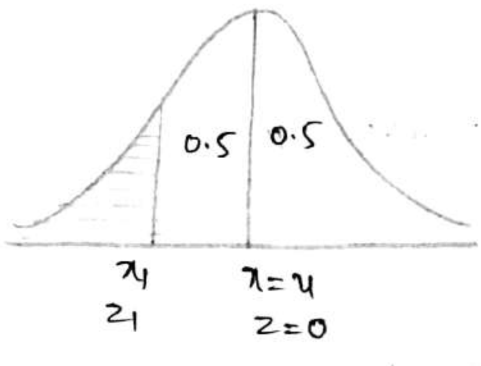
case(i): If $z_1 > 0$ (z_1 is +ve) then.



$P(x \leq x_1) = P(z \geq z_1) = 0.5 + A(z_1)$.

case(ii).

If $z_1 < 0$ (z_1 is -ve) then



$P(x \leq x_1) = P(z \leq z_1) = 0.5 - A(z_1)$.

Problems

1. If 'x' is a normal variate with mean 30 and standard deviation is 5. find the probability that

(i) $26 \leq x \leq 40$

(ii) $x > 45$.

Solu: (i) $26 \leq x \leq 40$

Let $x_1 = 26$ and $x_2 = 40$.

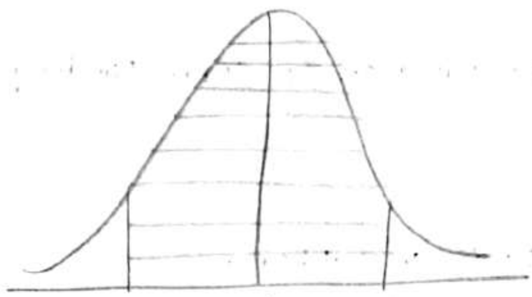
Here give Mean$(u) = 30$.

Standard deviation $(\sigma) = 5$.

We have

$$z_1 = \frac{x_1 - u}{\sigma} = \frac{26-30}{5} = \frac{-4}{5} = -0.8$$

$$z_2 = \frac{x_2 - u}{\sigma} = \frac{40-30}{5} = \frac{10}{5} = 2$$



Here z_1 and z_2 are of opposite signs so consider case (ii) in Type-1

W.K.T

$$P(x_1 < x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_1) + A(z_2).$$

$= A(-0.8) + A(2)$

$= A(-0.8) + A(2)$ Here -0.8 also becomes 0.8 only.

$= A(0.8) + A(2)$.

$= 0.2881 + 0.4772$ find 0.8 at 0.00, 2 at 0.00

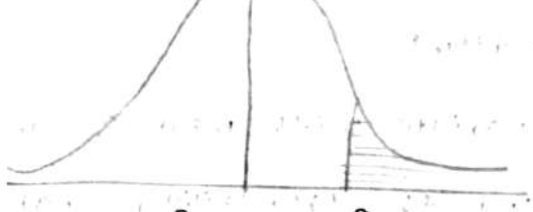
$= 0.7653$

$\therefore P(26 \leq x \leq 40) = 0.7653$ //

(ii) $P(x \geq 45)$

let $x_1 = 45$ then

$$z_1 = \frac{x_1 - u}{\sigma} = \frac{45-30}{5} = \frac{15}{5} = 3.$$



W.K.T

$P(x \geq x_1) = P(z \geq z_1) = 0.5 - A(z_1)$

$P(x \geq 45) = P(z \geq 3) = 0.5 - A(3)$

$= 0.5 - 0.4987$

$= 0.0013$

$\therefore P(x \geq 45) = 0.0013$ //.

---

2. Suppose the weights of 800 male students are normally distributed with mean 140 pounds and Standard deviation is 10 pounds. Find the no. of students whose weights are

(i) b/w 138 and 148 pounds

(ii) more than 152 pounds.

Sol4: (i) B/w 138 and 148 pounds.

$P(138 \leq x \leq 148)$

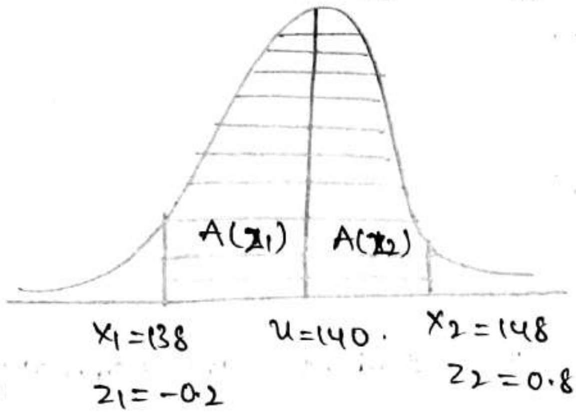
let $x_1 = 138$ and $x_2 = 148$

and Mean $(\mu) = 140$

Standard deviation $(\sigma) = 10$.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{138 - 140}{10} = \frac{-2}{10} = -0.2$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{148 - 140}{10} = \frac{8}{10} = 0.8$$



opposite signs so $A(z_1) + A(z_2)$.

W.K.T $P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_1) + A(z_2)$

$\Rightarrow P(138 \leq x \leq 148) = P(-0.2 \leq z \leq 0.8) = A(-0.2) + A(0.8)$

$= A(0.2) + A(0.8)$

$= 0.0793 + 0.2881$

$= 0.3674$.

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∴ No. of students whose weight are lies in b/w

138 to 148 pounds is $= 800 \times 0.3674$

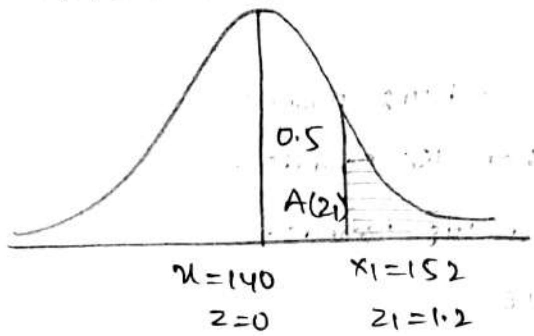
$= 293.9200$

$= 294$ (aprox)

(ii) More than 152 pounds i.e $P(x \geq 152)$

let $x_1 = 152$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{152-140}{10} = \frac{12}{10} = 1.2$$



here $1.2 > 0$
so $P(x_1 > x)$ form

W.K.T

$P(X > x_1) = P(z > z_1) = 0.5 - A(z_1)$

$P(x > 152) = P(z > 1.2) = 0.5 - A(1.2)$

$= 0.5 - 0.3849$

$= 0.1151$

∴ No. of students whose weights are more than 152

Pounds $= 800 \times 0.1151$

$= 92.0800$

$= 92$ (aprox)

3. HOMEWORK

1. If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kg. How many students have masses

(i) greater than 72 kgs

(ii) ≤ 64 kg.

(ii) B/w 65 and 71 kg

(ii) greater than 92 kgs

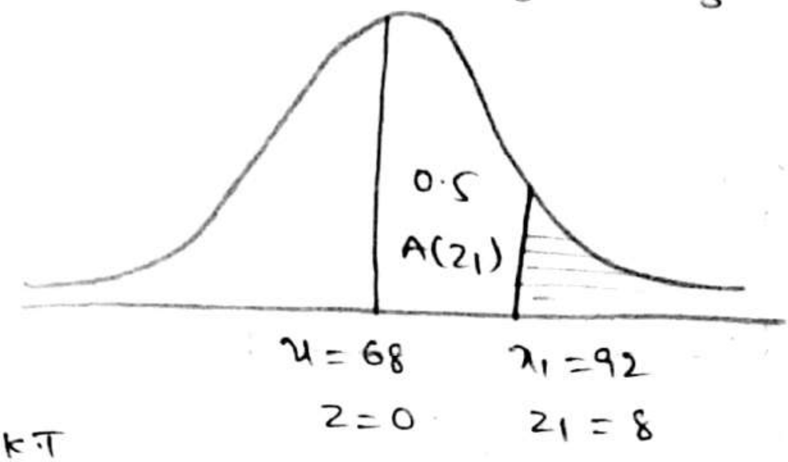
i.e $P(x > 92)$

let $x_1 = 92$

We had Mean(μ) = 68 kgs

Standard deviation (σ) = 3 kg

so $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{92 - 68}{3} = \frac{24}{3} = 8$.



W.K.T

$P(x > x_1) = P(z > z_1) = 0.5 - A(z_1)$

$P(x > 92) = P(z > 8) = 0.5 - A(8)$

$= 0.5 - 0.4994$

$= 0.0006$

$\therefore$ No. of students whose masses are greater than 92 are $= 300 \times 0.0006$

$= 0.1800$

$= 0$ students (Aprox.

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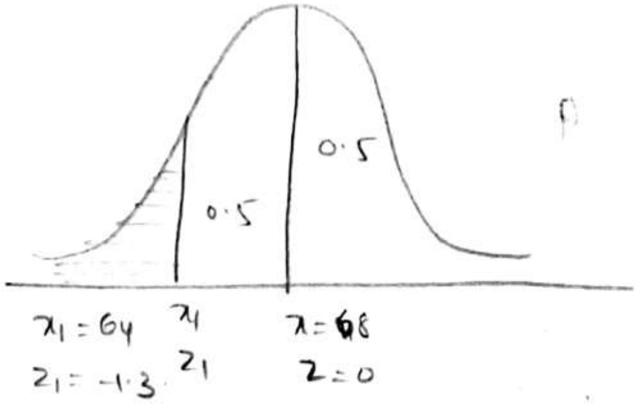
(ii) $\underline{\leq 64}$ i.e $P(x \leq 64)$

We have $x_1 = 64$.

Now $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{64-68}{3} = \frac{-4}{3} = -1.3$



W.K.T $P(x \leq x_1) = P(x \leq 64) = P(z \leq z_1) = P(z \leq z_1)$

$\Rightarrow P(z \leq -1.3) = 0.5 - A(z_1)$

$= 0.5 - A(-1.3)$

$= 0.5 - A(1.3)$

$= 0.5 - 0.4032$

$= 0.0968$.

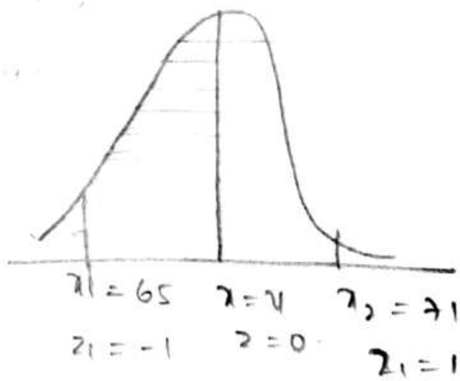
$\therefore P(x \leq 64) = 0.0968$

$\therefore$ The no. of students have mass ≤ 64 is

$= 300 \times 0.0968$

$= 29.0400$

$= 29$ (aprox).



(iii). $\underline{\text{B/w } 65 \text{ and } 71 \text{ kgs}}$:

$P(65 < x < 71)$

let $x_1 = 65$ and $x_2 = 71$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{65-68}{3} = -\frac{3}{3} = -1$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{71-68}{3} = \frac{3}{3} = 1$

W.K.T

$P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_1) + A(z_2)$.

$P(65 \leq x \leq 71) = P(-1 \leq z \leq 1) = A(-1) + A(1)$

$= A(1) + A(1)$

$= 0.3413 + 0.3413$

$= 0.6826.$

$\therefore$ The no. of students whose mass is in between 65 and 71 is $= 300 \times 0.6826$

$= 204.7800$

$= 205$ (aprox.)

---

2. A sample of 1000 cakes, the mean of certain test is 14 and σ is 2.5 assuming the distribution is normal. Find

(i) How many students score b/w 12 and 15.

(ii) How many score above 18.

(iii) How many score below 18.

Solu: b/w 12 and 15:

$P(12 \leq x \leq 15)$

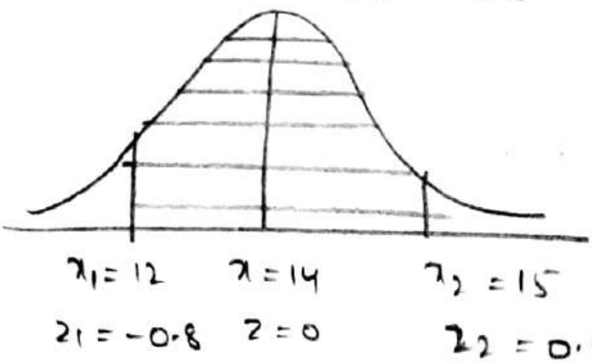
Now Given Mean $(\mu) = 14$

$\sqrt{\text{variance}}$ = standard derivation $(\sigma) = 2.5$

$x_1 = 12$ and $x_2 = 15$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{12-14}{2.5} = \frac{-2}{2.5} = -0.8.$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{15-14}{2.5} = \frac{1}{2.5} = 0.4$$



opposite signs

W.k.T

$P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_2) + A(z_1)$

$= A(-0.8) + A(0.4)$

$= A(0.8) + A(0.4) = 0.2881 + 0.1554$

$= 0.4435$

$\therefore$ No. of Students who score in b/w 12 and 15

$= 1000 \times 0.4435$

$= 443.5000$

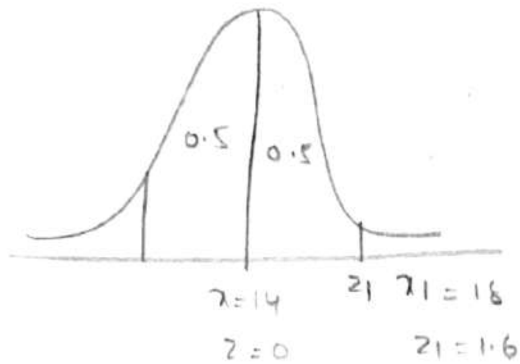
$= 444$ (aprox).

(ii) <u>above 18</u>:

$P(x \geq 18)$

$x_1 = 18$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{18-14}{2.5} = \frac{4}{2.5} = 1.6$$



W.K.T

$P(x \geq x_1) = P(z \geq z_1) = 0.5 - A(z_1)$

$\Rightarrow P(x \geq 18) = P(z \geq 1.6) = 0.5 - A(1.6)$

$= 0.5 - 0.4452$

$= 0.0548.$

$\therefore$ No of students who score above 18 are

$= 1000 \times 0.0548$

$= 54.8000$

$= 55$ (aprox).

(iii) <u>below 18</u>:

$P(x \leq 18)$.

$x_1 = 18$.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{18-14}{2.5} = \frac{4}{2.5} = 1.6.$$

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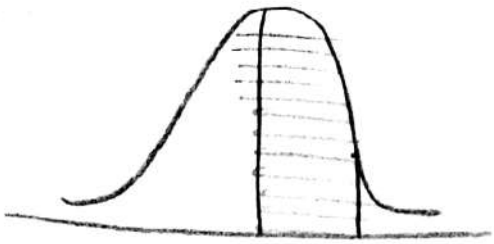


W.K.T

$$P(x \le x_1) = P(z \le z_1) = 0.5 + A(z_1)$$

$$P(x \le 18) = P(z \le 1.6) = 0.5 + A(1.6)$$

$$= 0.5 + 0.4452$$

$$= 0.9452$$

$\therefore$ No. of Students who score below 18 are

$$= 1000 \times 0.9452$$

$$= 945.2000$$

$$= 945 \text{ (aprox)}.$$

---

3. The mean and standard deviation of the marks obtained by 1000 Students in an exam are respectively 34.5 and 16.5. Assuming the normality of the distributions the approximate no. of Students expected to obtain the marks between 30 and 60.

Soln: b/w 30 and 60:

$$P(30 \le x \le 60)$$

let $x_1 = 30$ and $x_2 = 60$

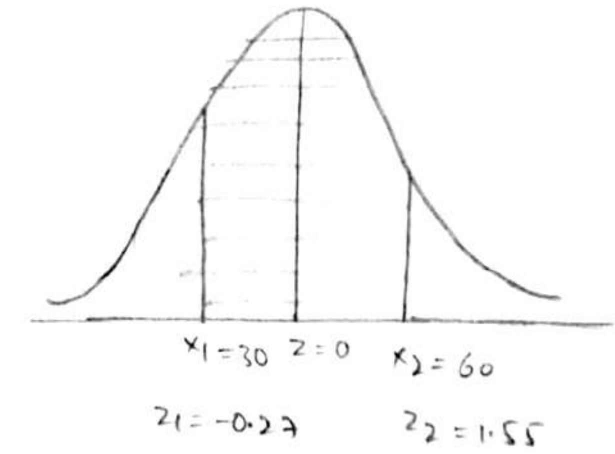
Given Mean $(\mu) = 34.5$

Standard deviation $(\sigma) = 16.5$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{30 - 34.5}{16.5} = \frac{-4.5}{16.5} = -0.27.$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{60 - 34.5}{16.5} = \frac{25.50}{16.5} = 1.55$$

Here we have opposite Signs.



W.K.T $P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_2) + A(z_1)$

$\Rightarrow P(30 \leq x \leq 60) = P(-0.27 \leq z \leq 1.55)$

$= A(1.55) + A(-0.27)$

$= A(1.55) + A(0.27)$

$= 0.4394 + 0.1084$

$= 0.5478.$

$\therefore P(30 \leq x \leq 60) = 0.5478.$

$\therefore$ No. of Students expected to get marks between 30 and 60 $= 1000 \times 0.5478$

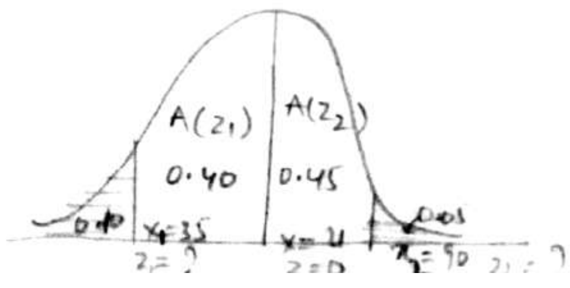
$= 547.8000$

$= 548$ Students //.

---

4. Suppose 10% of probability of a normal distribution $N(\mu, \sigma^2)$ is below 35 and 5% above 90. What are the values of μ and σ.

Sol: Given 10% below 35 means the area 0.10 is Covered under the normal curve to the left of 35 and 5% above 90 means the area 0.05 is Covered under the normal curve to the right of 90.



from the diagram $A(z_1) = 0.5 - 0.10 = 0.40$ → we have to search where this value in table and row + column gives $= z_1$ (-ve)

from the reverse tables we have $z_1 = -1.29$

$$A(z_2) = 0.5 - 0.05$$
$$= 0.45$$

from the reverse tables we have $z_2 = 1.65$

W.K.T

$$z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow -1.29 = \frac{35 - \mu}{\sigma}$$

$$-1.29\sigma = 35 - \mu$$

$$35 - \mu = -1.29\sigma$$

$$\mu - 35 = 1.29\sigma \cancel{\longrightarrow (1)}$$

$$\mu - 1.29\sigma = 35 \longrightarrow (1)$$

Now

$$z_2 = \frac{x_2 - \mu}{\sigma} \Rightarrow 1.65 = \frac{90 - \mu}{\sigma}$$

$$1.65\sigma = 90 - \mu$$

$$1.65\sigma + \mu = 90$$

$$\mu + 1.65\sigma = 90 \longrightarrow (2)$$

Solving (1) and (2) we get μ and σ values.

$$\cancel{\mu} \cancel{-} 1.29\sigma = 35$$
$$\cancel{\mu} + 1.65\sigma = 90$$
$$\cancel{(-)} \quad (-) \qquad (-)$$

$$\cancel{-}2.94\sigma = \cancel{-}55$$

$$\sigma = \frac{55}{2.94} = 18.7075$$

$$\therefore \sigma = 18.7075$$

Now substitute the value of σ in (1) we get

$$\mu - 1.29\sigma = 35$$

$$\mu - 1.29(18.7075) = 35$$

$$\mu = 35 + 1.29(18.7075)$$

$$\mu = 35 + 24.1327$$

$$\mu = 59.1327$$

$\therefore$ The values of σ and μ are.

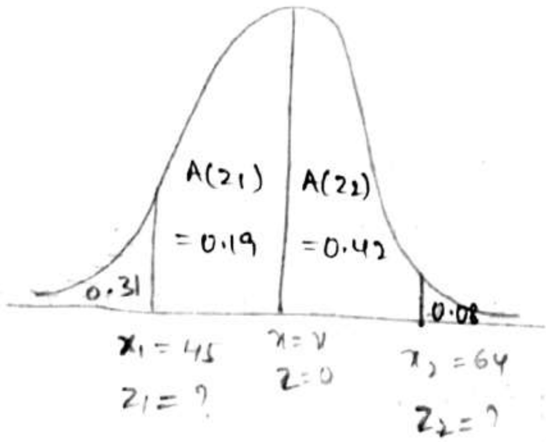
$\sigma = 18.7075$

$\mu = 59.1327$ //.

HOMEWORK.

1. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.

Solu. 31% under 45 means the area 0.31 is covered under the normal to the left of the curve and 8% over 64 means the area 0.08 is covered under the normal curve to the right of 64.



From the diagram $A(z_1) = 0.19$ i.e $0.5 - 0.31 = 0.19$

from the reverse table we have

$z_1 = 0.50.$

From the diagram $A(z_2) = 0.5 - 0.08 = 0.42$

from the reverse table we have

$z_2 = 1.41$

W.k.T

$$z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow 0.50 = \frac{45-\mu}{\sigma}$$

$$0.50\sigma = 45 - \mu$$

$$\mu + 0.50\sigma = 45 \longrightarrow (1)$$

$$z_2 = \frac{x_2 - \mu}{\sigma} \Rightarrow 1.41 = \frac{64-\mu}{\sigma}$$

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$1.41\sigma = 64 - \mu$

$\mu + 1.41\sigma = 64 \longrightarrow (2)$

Saving (1) and (2) we get

$$\mu + 0.50\sigma = 45$$
$$\mu + 1.41\sigma = 64$$
$$(-) \quad (-) \quad (-)$$
$$-0.91\sigma = -19$$
$$\cancel{-}0.91\sigma = \cancel{-}19$$
$$\sigma = \frac{19}{0.91}$$
$$\sigma = 20.8791.$$

Now substitute σ in (1)

$$\mu + 0.50\sigma = 45$$
$$\mu + 0.5(20.8791) = 45$$
$$\mu = 45 - 0.5(20.8791)$$
$$\mu = 45 - 10.4396$$
$$\mu = 34.5604.$$

$\therefore$ The values of σ and μ are

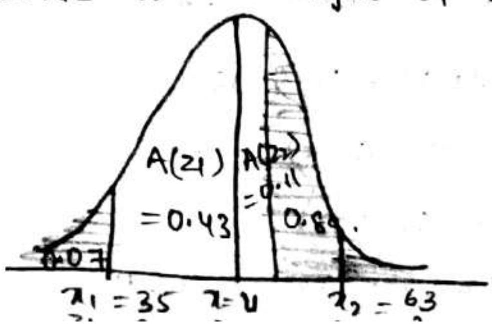
$\sigma = 20.8791$ Varience $= \sigma^2 = (20.8791)^2$

$\mu = 34.5604.$ // $= 435.9368$ //.

---

2. In a normal distribution 7% of items one under 35 and 89% of items are under 63, determine the mean variance of the distribution?

Solu: 7% under 35 means the area 0.07 covered under the normal to the left of the curve and 89% over 63 means the area 0.89 is covered under the normal curve to the right of 63.



From the diagram $A(z_1) = 0.43 \Rightarrow 0.5 - 0.07 = 0.43$

From the reverse of the table we have

$z_1 = 1.48$

From the diagram $A(z_2) = 1 - 0.89 = 0.11$

From the reverse of the table we have.

$z_2 = 0.03$.

$z_1 = \frac{x_1 - u}{\sigma} \Rightarrow 1.48 = \frac{35 - u}{\sigma}$

So $1.48\sigma = 35 - u$

$u + 1.45\sigma = 35 \rightarrow (1)$

$z_2 = \frac{x_2 - u}{\sigma} \Rightarrow 0.03 = \frac{63 - u}{\sigma}$

$0.03\sigma = 63 - u$

$u + 0.03\sigma = 63 \rightarrow (2)$

Solving (1) and (2) we get

$\cancel{u} + 1.45\sigma = 35$

$\cancel{u} + 0.03\sigma = 63$

$(-)\ (-)\ \ \ (-)$

$1.42\sigma = -28$

$\sigma = \frac{-28}{1.42}$

$\sigma = -19.7183$.

Varience $\sigma^2 = (-19.7183)^2 = 388.8114$.

Now substitute σ in (1)

$u + 1.45\sigma = 35$

$u + 1.45(-19.7183) = 35$

$u = 35 + 1.45(19.7183)$

$u = 35 + 28.5915$

$u = 63.5915$

So Mean$(u) = 63.5915$

Varience$(\sigma^2) = 388.8114$. //

3. In a test of 2000 electric bulbs. it was found that the life of a particular makes was normally distributed with an average life of 2040 hours be standard deviation of 60 hours. Find the number of bulbs likely to burn for

(i) More than 2150 hours

(ii) less than 1950 hours and

(iii) More than 1920 hours and but less than 2160 hours.

Solu: Given no. of bulbs to be tested = 2000.

Mean(μ) = 2040.

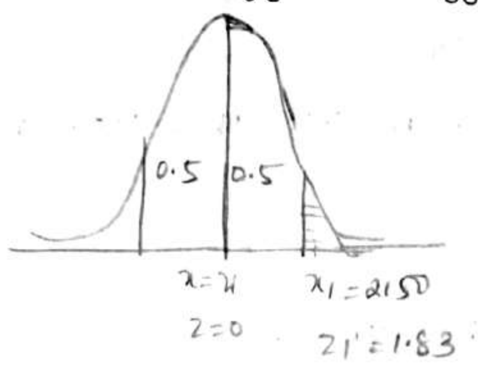
Standard deviation(σ) = 60

(i) More than 2150 hrs:

$P(X > 2150)$.

let $x_1 = 2150$.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{2150 - 2040}{60} = \frac{110}{60} = 1.83.$$



$P(x \geq x_1) = P(z \geq z_1) = 0.5 - A(z_1)$

$P(X \geq 2150) = P(z \geq 1.83) = 0.5 - A(1.83)$

$= 0.5 - 0.4664$

$= 0.0336$.

∴ No. of bulbs likely to burn more than 2150 hrs is

$= 2000 \times 0.0336$

$= 67.2000$

$= 67$ bulbs (aprox).

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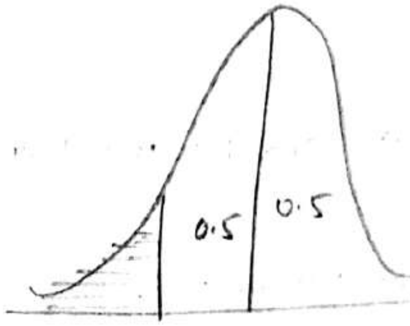
(ii) <u>less than 1950 hrs</u>.

$P(X < 1950)$

let $x_1 = 1950$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{1950 - 2040}{60} = \frac{-90}{60} = -1.5$$



$x_1 = 1950$ $\quad x = \mu$

$z_1 = -1.5$ $\quad z = 0$

W.K.T $P(X \le x_1) = P(z \le z_1) = 0.5 - A(z_1)$

$= 0.5 - A(-1.5)$

$= 0.5 - A(1.5)$

$= 0.5 - 0.4332$

$= 0.0668$

$\therefore P(X < 1950) = 0.0668$.

$\therefore$ No. of bulbs to burn less than 1950 hrs

$= 2000 \times 0.0668$

$= 133.6000$

$= 134$ bulbs // aprox.

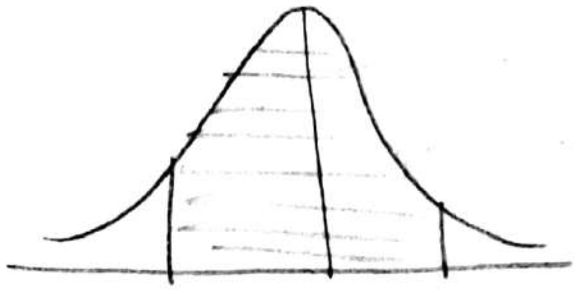
(iii) $P(1920 \le x \le 2160)$

here $x_1 = 1920$ $\quad x_2 = 2160$.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{1920 - 2040}{60} = -2.0$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{2160 - 2040}{60} = 2.0$$

Both are of opposite signs.



W.K.T

$$P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2) = A(z_2) + A(z_1)$$

$$P(1920 \leq x \leq 2160) = P(-2 \leq z \leq 2) = A(2) + A(-2)$$

$$= A(2) + A(2)$$

$$= 2 \cdot A(2)$$

$$= 2(0.4772)$$

$$= 0.9544.$$

$\therefore$ No. of bulbs burn in b/w 1920 and 2160 are

$$= 2000 \times 0.9544$$

$$= 1908.8000.$$

$= 1909$ bulbs aprox //

---

4. 1000 students have written an examination the mean of test is 35 and standard deviaton is 5. Assuming the distribution to be normal, find.

(i) how many students lie b/w 25 and 40.

(ii) how many students set more than 40.

(iii) how many students set below 20.

(iv) how many students set more than 50.

Solu: (i) $P(25 \leq x \leq 40)$:

let $x_1 = 25$ and $x_2 = 40$.

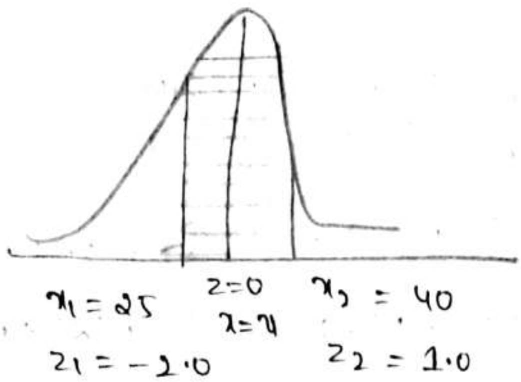
Given Mean $(\mu) = 35$

standard deviation $(\sigma) = 5$.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{25 - 35}{5} = \frac{-10}{5} = -2.0.$$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{40-35}{5} = \frac{5}{5} = 1.0$.

Both are of opposite signs.



W.K.T $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2) = A(z_2) + A(z_1)$.

$P(25 \le x \le 40) = P(-2.0 \le z \le 1.0) = A(1.0) + A(-2.0)$

$= A(1.0) + A(2.0)$

$= 0.3413 + 0.4772$

$= 0.8185$.

No. of Students lie between 25 and 40 are

$= 1000 \times 0.8185$

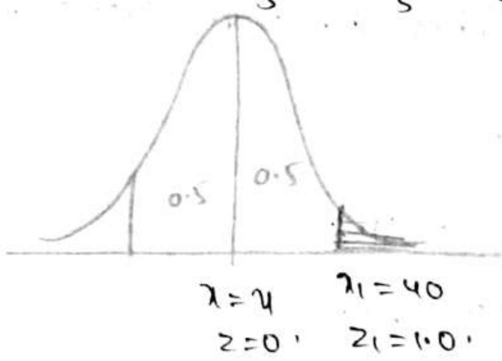
$= 818.5000$.

$= 819$ students aprox.

(ii) $P(x > 40)$:

let $x_1 = 40$.

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{40-35}{5} = \frac{5}{5} = 1.0$.



W.K.T $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2) = 0.5 - A(z_1)$.

$= 0.5 - A(1)$

$= 0.5 - 0.3413$

$= 0.1587$.

$\therefore$ No. of students lie ~~below~~ more than 40

$= 1000 \times 0.1587$

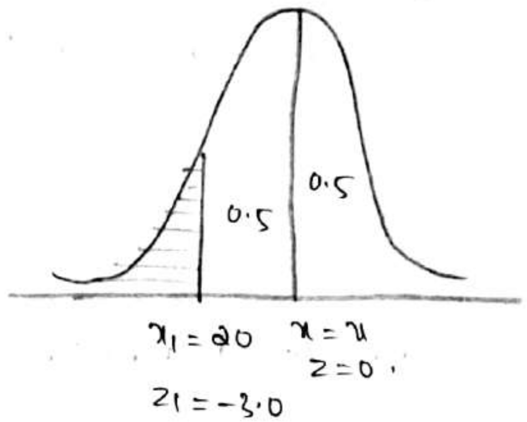
$= 158.7$

$= 159$ aprox.

(iii) $P(X < 20)$:

let $x_1 = 20$.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{20-35}{5} = \frac{-15}{5} = -3.0$$



W.K.T $P(X \le x_1) = P(z < z_1) = 0.5 - A(z_1)$

$P(X \le 20) = P(z < -3) = 0.5 - A(-3)$

$= 0.5 - A(3)$.

$= 0.5 - 0.4987$

$= 0.0013$

$\therefore$ No. of students less than 20 is

$= 1000 \times 0.0013$

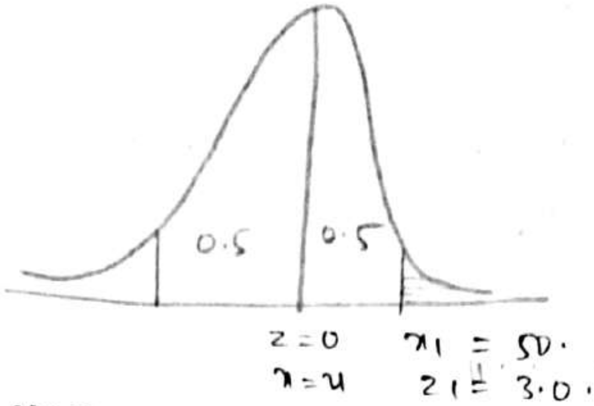
$= 1.3000$

$= 1$ aprox.

(iv) $P(X > 50)$:

let $x_1 = 50$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{50-35}{5} = \frac{15}{5} = 3.0.$$



$P(\cancel{x_1 < x < x_2}\ x_1 > 50) = P(\cancel{z_1 < z < z_2}\ z_1 > 30) = 0.5 - A(z_1)$

$P(x_1 > 50) = P(z_1 > 3.0) = 0.5 - A(3.0)$

$= 0.5 - 0.4987$

$= 0.0013.$

∴ No. of Students whose marks are more than 50 are

$= 1000 \times 0.0013$

$= 1.3000$

$= 1$ aprox. //

Normal Approximation to the binomial distribution:

Suppose that no. of success 'x' ranges from x_1 and x_2 then the probability of getting x_1 to x_2 is given by

$$\sum_{r=x_1}^{x_2} {}^{n}C_r\, p^r q^{n-r}$$

for large 'n' the caluculation of a binomial distribution is very difficult. In such cases we can replace binomial distribution by Normal distribution.

We are having a two cases in this method. They are

Case (i): When $P = q = 1/2$, then we can find values of u and σ are given by

$u = np$ and

$\sigma = \sqrt{npq}$

* Next we can find the lower and upper limits of 'z' are given by

$$z_1 = \frac{x_1 - \mu}{\sigma} \text{ and } z_2 = \frac{x_2 - \mu}{\sigma}$$

* The required probability is $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$

$$= \int_{z_1}^{z_2} \phi(z) dz.$$

Case(ii).

when $p \neq q$, then we can find the values of μ and σ are given by $\mu = np$

and

$$\sigma = \sqrt{npq}$$

* Next we can find lower and upper limits of 'z_1' is given by.

$$z_1 = \frac{(x_1 - \frac{1}{2}) - \mu}{\sigma} \text{ and } z_2 = \frac{(x_2 + \frac{1}{2}) - \mu}{\sigma}$$

* The required probability is

$$P(x_1 \le x \le x_2) = \int_{z_1}^{z_2} \phi(z) dz$$

Problems

1. Determine the probability that getting an even number of faces takes 3 to 5 times in throwing 10 dice together.

Solu: P = probability of success (getting an even number). in a single trail of throwing a die

$$= \frac{3}{6} = \frac{1}{2}.$$

$p + q = 1$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore p = q = 1/2$ at this time we use case(i)

Mean$(\mu) = np = 10(\frac{1}{2}) = 5$

$$\sigma = \sqrt{npq}$$

$$= \sqrt{10\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)}$$

$$= \sqrt{\cancel{10}\left(\tfrac{1}{\cancel{4}_2}\right)}$$

$$= \sqrt{5/2} = \sqrt{2.5}$$

$$\sigma = 1.5811$$

Probability that getting an even number of face 3 to 5 times is $P(3 \le x \le 5)$

here $x_1 = 3$ and $x_2 = 5$

then $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{3-5}{1.5811} = \dfrac{-2}{1.5811} = -1.2649.$

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{5-5}{1.5811} = 0.$

Required probability is $= \displaystyle\int_{z_1 = -1.2649}^{0} \phi(z)\,dz$

$= P(-1.27 \le z \le 0)$

$= A(1.27) + A(0)$

$= 0.3980 + 0$

$= 0.3980.$ //.

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2. Find the probability that out of 100 patients b/w 84 and 95 inclusive will survive a heart-operation given that the chances of survival is 0.9?

Soln: Here $p = 0.9$ and $n = 100.$

$p + q = 1$

$q = 1 - p$

$q = 1 - 0.9$

$q = 0.1$

Here $p \ne q$ so we go for case (ii)

where $\text{Mean}(\mu) = np$

$$= (100)(0.9)$$

$$= 1\cancel{0}\cancel{0} \times \frac{9}{1\cancel{0}}$$

$$\mu = 90$$

$$\sigma = \sqrt{npq}$$

$$= \sqrt{(100)(0.9)(0.1)}$$

$$= \sqrt{\cancel{100} \times \frac{9}{\cancel{10}} \times \frac{1}{\cancel{10}}}$$

$$= \sqrt{9} = 3$$

$$\sigma = 3.$$

Now b/w 84 and 95 $P(84 \le x \le 95)$.

$x_1 = 84, \; x_2 = 95$

$$z_1 = \frac{(x_1 - \frac{1}{2}) - \mu}{\sigma} = \frac{(84 - \frac{1}{2}) - 90}{3}$$

$$= \frac{84 - 1/2 - 90}{3} = \frac{-6 - 1/2}{3} = \frac{-12-1}{2 \times 3} = \frac{-13}{6}$$

$$= -2.1667.$$

$$\therefore z_1 = -2.1667 = -2.16$$

$$z_2 = \frac{(x_2 + \frac{1}{2}) - \mu}{\sigma} = \frac{(95 + \frac{1}{2}) - 90}{3} = \frac{95 + \frac{1}{2} - 90}{3} = \frac{5 + \frac{1}{2}}{3} = \frac{11}{2 \times 3}$$

$$= \frac{11}{6} = 1.8333 = 1.83$$

Hence the required probability is

$$= \int_{-2.16}^{1.83} \phi(z)\,dz$$

$$= P(-2.16 \le x \le 1.83)$$

$$= A(-2.16) + A(1.83)$$

$$= A(2.16) + A(1.83).$$

$$= 0.4846 + 0.4664$$

$$= 0.9510$$

$\therefore$ The required probability is $= 0.9510$ //.

3. Eight coins are tossed together. Find the probability of getting 1 to 4 heads in a single toss?

Solu: Here $p = 1/2$

$q = 1 - p = 1 - 1/2 = 1/2$

$p = q = 1/2$

So we use case(i)

Now $\text{Mean}(\mu) = np = 8\left(\frac{1}{2}\right) = 4$

$\sigma = \sqrt{npq} = \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{8 \times \frac{1}{4}}$

$\sigma = \sqrt{2}$

$\sigma = 1.4142$

$\sigma = 1.4142$

$P(1 \leq X \leq 4)$

$x_1 = 1 \quad x_2 = 4.$

Now $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{1-4}{1.4142} = \frac{-3}{1.4142} = -2.1213$

$= -2.12$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{4-4}{1.4142} = 0.$

Now, the required probability is

$= \int_{-2.12}^{0} \phi(z)dz$

$= P(-2.12 \leq x \leq 0)$

$= A(-2.12) + A(0)$

$= A(2.12) + A(0)$

$= 0.4830 + 0$

$= 0.4830.$

$\therefore$ The required probability is $= 0.4830$.//

Table : Area under standard normal curve from 0 to $\frac{x-\mu}{\sigma}$

$\frac{x-\mu}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	0159	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1084	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	1368	.1406	1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1916	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2611	.2642	.2671	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	2939	.2967	.2995	.3023	.3051	3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4430	.4441
1.6	.4452	.4463	.4474	.4485	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4654	.4564	.4573	.4582	.4591	4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	4744	.4750	.4756	.4762	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	[illegible]
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4865	.4868	.4871	.4875	.5878	.4881	.4884	.4887	[illegible]
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	4948	.4949	.4951	[illegible]
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	[illegible]
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	[illegible]
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4980	.4980	[illegible]
2.9	.4981	.4982	.4983	.4983	.4984	.4984	.4985	.4985	.4986	[illegible]
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	[illegible]
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	[illegible]