UNIT – 4

SAMPLING THEORY

5. Sampling Distributions

- → Sampling Distribution of mean (σ Known)
- → Sampling Distribution of proportions
- → Sampling Distribution of Differences & sums
- → Sampling Distribution of Mean (σ unknown)

6. Estimation

- → point Estimation
- → Interval Estimation
- → Bayesian Estimation

Introduction:- The outcome of a statistical experiment may be recorded either as a numerical value (or) as a descriptive representation.

When a pair of dice are tossed and the sum of the numbers on the faces is the outcome of interest, we record a numerical value. However, if the students of a certain school are given blood tests and the type of blood is of interest, then a descriptive representation might be most useful. A person's blood can be classified in 8 ways. It must be AB, A, B (or) O, with a plus (or) minus sign, depending on the presence (or) absence of the Rh antigen.

In this chapter we focus on sampling from distributions (or) populations and study which such important quantities as the sample mean and sample variance.

Population:- population is a collection of observations (or) objects.

(or)

population (or) universe is the aggregate (or) totality of statistical data forming a subject of investigation.

Ex: ① The population of the heights of Indians.

② The population of Nationalised Banks in India, etc. ---

The number of observations (or) objects in the population is called a size of the population. It may be finite (or) infinite. size of population is denoted by N.

The parameters of the population are called the mean & variance. Here, the mean of population is denoted by "μ" & the variance of population is denoted by "σ^2".

where μ (Mean of population) $= \dfrac{\text{sum of all objects in population}}{\text{size of population (N)}} = \dfrac{\sum_{i=1}^{N} x_i}{N}$

$$\sigma^2 \text{(variance of population)} = \frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$$

$$\sigma \text{(standard Deviation of population)} = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}}$$

Sample:- A sample is ~~the~~ a finite subset of a population. The no. of items in a sample is called a size of sample and it is denoted by n.

Ex:- Cars produced in India is the population and the Nano cars is the sample.

The parameters of a sample is called a Mean, variance & standard deviation. These are denoted by "$\bar{x}$", "s^2" & "s" respectively.

Here $\bar{x}$ (sample mean) $= \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\text{sum of all elements in a sample}}{\text{size of sample}(n)}$

$$s^2(\text{sample variance}) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}, \text{ if } n \geq 30 \text{ (large samples)}$$

$$= \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}, \text{ if } n<30 \text{ (small samples)}$$

$$s\text{ (Standard Deviation of sample)} = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}, \text{ if } n \geq 30$$

$$= \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}} \text{ if } n<30$$

Classification of samples:-

Samples are classified into two types. They are

1. large sample ($n \geq 30$)
2. small sample ($n<30$)

1. large sample ($n \geq 30$):- if the size of the sample $n \geq 30$, then the sample is called a "large sample".
2. small sample ($n<30$):- if the size of the sample $n<30$, then the sample is called a "small sample".

Note:-

1. Sampling with replacement (infinite population):- Each Element of the population may be choosen more than once in samples, then it is called sampling with replacement. It is also called as sampling from infinite population only.

The total number of samples of size n are drawn from a population of size N with replacement is $\boxed{N^n}$.

2. Sampling without replacement (finite population):- An element of the population cannot be choosen more than once in samples, then it is called sampling without replacement. It is also called as sampling from finite population only.

The total number of samples of size 'n' are drawn from a population of size N without replacement is $N_{C_n} = \frac{N!}{(N-n)!\,n!}$

Sampling Distribution:- The probability of distribution of a statistic is called a sampling Distribution. (or)

The Arithmetic mean of a samples is also called sampling Distribution.

(2) (2)

Sampling Distribution of a statistic:- The main characteristic of the sampling distribution of a statistic is that it approaches normal distribution even when the population distribution is not normal provided the sample size is sufficiently large (>30). Another important feature of the sampling distribution of statistic is that the mean and the standard deviation of the sampling distribution of sample mean bear a definite relation to the corresponding parameters i.e. mean and standard deviation of parent population. These characteristics of the sampling distribution helps us.

(i) To estimate the unknown population parameter from the known statistic.

(ii) To set the confidence limits of the parameter within which the parameter values are expected to lie.

(iii) To test a hypothesis and to draw a statistical inference from it.

Central limit theorem:- If $\bar{x}$ is the mean of a sample size n drawn from a population with mean μ and S.D σ. then the standardized sample mean $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ is a random variable whose distribution function approaches that of the standard normal distribution $N(z;0,1)$ as $n \to \infty$.

Standard Error (S.E) of a Statistic:-
The S.E of a statistic may be reduced by increasing sample size n, but this results in corresponding increase in cost, time & labour, etc..

Formulae for S.E:-

1. S.E. of a sample mean $\bar{x} = \frac{\sigma}{\sqrt{n}}$. It is written as $S.E(\bar{x}) = \frac{\sigma}{\sqrt{n}}$
2. S.E. of a sample proportion $p = \sqrt{\frac{PQ}{n}}$, where $Q = 1-P$
3. S.E. of a sample S.D. (s) $= \frac{\sigma}{\sqrt{2n}}$
4. S.E. of the difference of two sample means $\bar{x}_1$ & $\bar{x}_2$
   i.e, S.E of $(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ where $\bar{x}_1$ & $\bar{x}_2$ are the means of two random samples of sizes n_1 & n_2 drawn from two populations with S.D. σ_1 & σ_2 respectively.
5. S.E of $(P_1 - P_2) = \sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$, where P_1 & P_2 are the populations of two random samples of sizes n_1 & n_2 drawn from two populations with proportions P_1 & P_2 respectively.
6. S.E. of $(s_1 - s_2) = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$

For a finite population of size N, when a sample is drawn without replacement, we have (i) S.E. of sample mean $= \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$

(ii) S.E. of sample proportion $= \sqrt{\frac{PQ}{n}} \cdot \sqrt{\frac{N-n}{N-1}}$

For an infinite population when the sample is drawn without replacement, Formulae 1 & 2 remain the same.

Proposition:- The mean & S.E. of sample mean $\bar{x}$ (i.e. mean and S.D. of the sampling distribution of $\bar{x}$) when samples of the same size n are drawn from a population having mean μ and S.D. σ are given by Mean of $\bar{x} = E(X) = \mu$ and S.E. of $\bar{x} = \frac{\sigma}{\sqrt{n}}$

Sampling Distribution of Mean (σ known):-

The probability distribution of $\bar{x}$ is called the sampling distribution of means. The sampling distribution of a statistic depends on the size of the population, the size of the samples, and the method of choosing the samples.

Let $x_1, x_2, x_3, ---, x_n$ be the n random samples drawn from a population of size N with mean μ and variance σ^2 and $\bar{x}$ is the mean of samples.

(i). Infinite population (sampling with Replacement):-

→ Mean of the sampling distribution of the means is denoted by $\mu_{\bar{x}}$ and it is defined as $\mu_{\bar{x}} = \frac{\text{sum of all mean of samples}}{\text{total no. of samples}} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \cdots + \bar{x}_k}{K} = \mu$

$\therefore \boxed{\mu_{\bar{x}} = \mu}$

→ the variance of sampling distribution of the means is denoted by $\sigma_{\bar{x}}^2$ and it is defined as $\sigma_{\bar{x}}^2 = \frac{\sum_{i=1}^{k} (x_i - \mu_{\bar{x}})^2}{K}$ (or) $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$ i.e. $\text{var}(\bar{x}) = \frac{\sigma^2}{n}$

→ The S.D. of sampling distribution of means is called a standard error and it is defined as $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. (or) $S.E = \frac{\sigma}{\sqrt{n}}$

(ii). finite population (sampling without replacement):-

→ The mean of samplind distribution of means is defined as $\mu_{\bar{x}} = \mu$.

i.e. $\mu_{\bar{x}} = \frac{\text{sum of all mean of samples}}{\text{Total no. of samples}}$

→ The variance of sampling distribution of means is defined as

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \left(\frac{N-n}{N-1}\right)$$

i.e. $\text{var}(\bar{x}) = \frac{N-n}{N-1} \cdot \frac{\sigma^2}{n}$ (if the sample is drawn without replacement)

and S.D is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$

Here, the factor $\left(\frac{N-n}{N-1}\right)$, often called the finite population correction factor. We note that this term tends to become closer and closer to unity as population size becomes larger and larger.

(ii) SAMPLING DISTRIBUTION OF PROPORTIONS:-

Let 'p' be the probability of occurrence of an event (called it's success) and $q=1-p$ is the probability of non-occurrence (called it's failure). Draw all possible samples of size n from an infinite population. compute the proportion P of success for each of these samples. Then the mean μ_P & variance σ_P^2 of the sampling distribution of proportions are given by

$$\mu_P = p \text{ and } \sigma_P^2 = \frac{pq}{n} = \frac{p(1-p)}{n}$$

While population is binomially distributed, the sampling distribution of proportion is normally distributed whenever n is large. For finite population (with replacement) of size N, we have $\mu_P = p$ and $\sigma_P^2 = \frac{pq}{n}\left(\frac{N-n}{N-1}\right)$.

(iv) Sampling Distribution of Differences and Sums:-

Let μ_{S_1} and σ_{S_1} be the mean and standard deviation of sampling distribution of statistic S_1 obtained by computing S_1 for all possible samples of size n_1 drawn from population A. Also let μ_{S_2} and σ_{S_2} be the mean and standard deviation of sampling distribution of statistic S_2 obtained by computing S_2 for all possible samples of size n_2 drawn from another different population B.

Now compute the statistic S_1-S_2, the difference of the statistic from all the possible combinations of these samples from the two populations A & B.

Then the mean $\mu_{S_1-S_2}$ and the standard deviation $\sigma_{S_1-S_2}$ of the sampling distribution of differences are given by

$$\mu_{S_1-S_2} = \mu_{S_1} - \mu_{S_2} \text{ and } \sigma_{S_1-S_2} = \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2}$$

assuming that the samples are independent

Sampling distribution of sum of statistics has Mean $\mu_{S_1+S_2}$ and standard deviation $\sigma_{S_1+S_2}$ given by

$$\mu_{S_1+S_2} = \mu_{S_1} + \mu_{S_2} \text{ and } \sigma_{S_1+S_2} = \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2}$$

for example, for infinite population the sampling distribution of sums of means has mean $\mu_{\bar{x}_1+\bar{x}_2}$ and $\sigma_{\bar{x}_1+\bar{x}_2}$ given by $\mu_{\bar{x}_1+\bar{x}_2} = \mu_{\bar{x}_1} + \mu_{\bar{x}_2} = \mu_1 + \mu_2$ and

$$\sigma_{\bar{x}_1+\bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

for sampling distribution of differences of proportions, we have

$$\mu_{P_1-P_2} = \mu_{P_1} - \mu_{P_2} = p_1 - p_2 \text{ and } \sigma_{P_1-P_2} = \sqrt{\sigma_{P_1}^2 + \sigma_{P_2}^2} = \sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}}$$

Problems: what is the value of correction factor if $n=5$ and $N=200$

Sol: Given $N=$ the size of the finite population $=200$

$n=$ the size of the sample $=5$

$\therefore$ correction factor $= \frac{N-n}{N-1} = \frac{200-5}{200-1} = \frac{195}{199} = 0.98$

→ Find the value of the finite population correction factor for $n=10$ and $N=1000$

Sol: Given $N=$ The size of the finite population $=1000$

$n=$ The size of the sample $=10$

$\therefore$ correction factor $= \frac{N-n}{N-1} = \frac{1000-10}{1000-1} = \frac{990}{999} = 0.991$

→ How many different samples of size two can be chosen, from a finite population of size 25.

Sol: We can take $^{N}C_{n}$ samples of size n from the population of size N.

Here $N=25, n=2$

$\therefore$ we can take $^{25}C_{2} = 300$ samples of size 2 from finite population of size 25.

→ Samples of size 2 are taken from the population 3, 6, 9, 15, 27 with replacement. Find (a) the mean of the population

(b) The standard deviation of the population

(c) Mean of the sampling distribution of means

(d) ~~the~~ standard deviation of the sampling distribution of means

Sol: (a) Mean of the population, $\mu = \frac{3+6+9+15+27}{5} = \frac{60}{5} = 12$

(b) Standard deviation of the population, $\sigma = \sqrt{\frac{1}{N}\sum_i (x_i-\mu)^2}$

$= \sqrt{\frac{1}{5}\left[(3-12)^2+(6-12)^2+(9-12)^2+(15-12)^2+(27-12)^2\right]}$

$= \sqrt{\frac{1}{5}(81+36+9+9+225)} = \sqrt{\frac{360}{5}} = 8.4853$

(c) The sampling distribution with replacement is $N^n = 5^2 = 25$

$$\left\{\begin{matrix} (3,3), & (3,6), & (3,9), & (3,15), & (3,27) \\ (6,3), & (6,6), & (6,9), & (6,15), & (6,27) \\ (9,3), & (9,6), & (9,9), & (9,15), & (9,27) \\ (15,3), & (15,6), & (15,9), & (15,15), & (15,27) \\ (27,3), & (27,6), & (27,9), & (27,15), & (27,27) \end{matrix}\right\}$$

The means are
$$\left\{\begin{matrix} 3 & 4.5 & 6 & 9 & 15 \\ 4.5 & 6 & 7.5 & 10.5 & 16.5 \\ 6 & 7.5 & 9 & 12 & 18 \\ 9 & 10.5 & 12 & 15 & 21 \\ 15 & 16.5 & 18 & 21 & 27 \end{matrix}\right\}$$

The mean of the sampling distribution of means,

$$\bar{x} = \frac{3+4.5+6+9+15+4.5+\dots+18+21+27}{25} = \frac{300}{25} = 12$$

(d) The standard deviation of the sampling distribution of means $= \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{25}}$

$$= \sqrt{\frac{1}{25}\left[(3-12)^2+(4.5-12)^2+(6-12)^2+(9-12)^2+(15-12)^2+\dots+(21-12)^2+(27-12)^2\right]}$$

$$= \sqrt{\frac{1}{25}\left[81+56.25+36+9+9+56.25+36+20.25+2.25+20.25+9+20.25+9+0+36+9+2.25+0+9+81+9+20.25+36+81+225\right]}$$

$$= \sqrt{\frac{895}{25}} = 5.983$$

→ Let $S = \{1,5,6,8\}$, find the probability distribution of the sample mean for random sample size 2 drawn without replacement.

(or)

A population consists of the four numbers 1, 5, 6, 8. Consider all possible samples of size two that can be drawn without replacement from this population. Find (i) The population mean (ii) The population standard deviation (iii) The mean of the sampling distribution of means, (iv) The standard deviation of the sampling distribution of means.

Sol Let $S = \{1,5,6,8\}$

(a) Mean of the population $\mu = \frac{1+5+6+8}{4} = \frac{20}{4} = 5$

(b) S.D. of the population, $\sigma = \sqrt{\frac{1}{n}\sum_i (x_i-\mu)^2}$

$$= \sqrt{\frac{1}{4}\left((1-5)^2+(5-5)^2+(6-5)^2+(8-5)^2\right)}$$

$$= \sqrt{\frac{1}{4}(16+0+1+9)} = \sqrt{\frac{26}{4}} = \sqrt{\frac{13}{2}} = \sqrt{6.5}$$

(c) Mean of the sampling distribution of means is $N_{C_n} = {}^4C_2 = \frac{4\times 3}{2\times 1} = 6$ samples

$\{(1,5), (1,6), (1,8),$
$(5,6), (5,8),$
$(6,8)\}$

The sample mean are $\{3, 3.5, 4.5, 5.5, 6.5, 7\}$

∴ Mean of sampling distribution, $\bar{x} = \frac{3+3.5+4.5+5.5+6.5+7}{6} = \frac{30}{6} = 5$

(d) S.D of sampling distribution of means,

$$\sigma = \sqrt{\frac{1}{6}\left((3-5)^2+(3.5-5)^2+(4.5-5)^2+(5.5-5)^2+(6.5-5)^2+(7-5)^2\right)}$$

$$= \sqrt{\frac{4+2.25+0.25+0.25+2.25+4}{6}}$$

$$= \sqrt{\frac{13}{6}}$$

$$= 1.4696$$

→ A population consists of six numbers 4, 8, 12, 16, 20, 24. Consider all samples of size two which can be drawn without replacement from this population.
Find (a) The population Mean
(b) The population standard deviation
(c) The Mean of the sampling distribution of means
(d) The standard deviation of the sampling distribution of means.

→ samples of size 2 are taken from the population 3, 6, 9, 15, 27 without replacement. Find (i) The Mean of the population
(ii) The standard deviation of the population
(iii) The standard deviation of the sampling distribution of means.
(iv) Mean of the sampling distribution of means
(v) The standard deviation of the sampling distribution of means.

→ If the population is 3, 6, 9, 15, 27
(a) List all possible samples of size 3 that can be taken without replacement from the finite population.
(b) calculate the mean of each of the sampling distribution of means
(c) find the standard deviation of sampling distribution of means.

→ samples of size 2 are taken from the population 1, 2, 3, 4, 5, 6 with replacement. Find (a) the mean of the population
(b) The standard deviation of population
(c) The mean of the sampling distribution of means
(d) The standard deviation of the sampling distribution of means.

→ A population consists of six numbers 4, 8, 12, 16, 20, 24. Consider all samples of size two which can be drawn with replacement from this population. find (a) The mean of the population
(b) standard deviation of the population.
(c) The mean of the sampling distribution of means
(d) The standard deviation of the sampling distribution of means.

→ samples of size 2 are taken from the population 1, 2, 3, 4, 5, 6 without replacement. Find (a) The mean of the population.
(b) standard deviation of the population.
(c) The mean of the sampling distribution of means
(d) The standard deviation of the sampling distribution of means.

→ When a sample is taken from an infinite population, what happen to the standard error of the mean if the samples size is decreased from 800 to 200.

Sol: The standard error of mean $= \frac{\sigma}{\sqrt{n}}$

sample size $= n$, Let $n = n_1 = 800$

Then $S.E_1 = \frac{\sigma}{\sqrt{800}} = \frac{\sigma}{20\sqrt{2}}$

when n_1 is reduced to 200

Let $n_2 = 200$. Then $S.E_2 = \frac{\sigma}{\sqrt{200}} = \frac{\sigma}{10\sqrt{2}}$

$\therefore S.E_2 = \frac{\sigma}{10\sqrt{2}} = 2\left[\frac{\sigma}{20\sqrt{2}}\right] = 2(S.E_1)$

If sample size is reduced from 800 to 200, S.E of mean will be multiplied by 2.

→ If a 1-gallon can of paint covers on an average 513 square feet with a standard deviation of 31.5 square feet, what is the probability that the mean area covered by a sample size of 40 of these 1-gallon cans will be any where from 510 to 520 square feet?

Sol: Given $n = 40$, $\mu = 513$ & $\sigma = 31.5$ sq. feet

The test statistic is $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

When $\bar{x}_1 = 510$, $Z_1 = \frac{510 - 513}{31.5/\sqrt{40}} = -0.6$

When $\bar{x}_2 = 520$, $Z_2 = \frac{520 - 513}{31.5/\sqrt{40}} = 1.4$

$\therefore$ Required probability $= P(-0.6 < Z < 1.4)$

$= P(-0.6 < Z < 0) + P(0 < Z < 1.4)$

$= A(0.6) + A(1.4)$

$= 0.2258 + 0.4192$

$= 0.645$

Ex-18 (pg No: 257)

Ex-19 (pg No: 257 & 258)

Ex-20 (pg. No:- 258)

Ex-21 (pg. No:- 258 & 259)

Ex-24 (pg. No:- 261)

Ex-26 (pg. No:- 262)

Ex-29 (pg. No:- 263)

Estimation

Parameters:- Quantities appearing in distributions, such as p in the binomial distribution and μ and σ in the normal distribution are called parameters.

Estimate: An estimate is a statement made to find an unknown population parameter.

Estimator:- The procedure (or) rule to determine an unknown population parameter is called an estimator.

Types of Estimation:- There are two types of estimations. They are
(a) point estimation
(b) Interval Estimation

(a) **point estimation:-** If an estimate of the population parameter is given by a single value, then the estimate is called a point estimation.

Ex:- If the height of the student measured as 155 cms, then the measurement gives a point estimation.

(b) **Interval estimation:** If an estimate of a population parameter is given by two different values between which the parameter may be considered to lie, then the estimate is called an interval estimation of the parameter.

Ex:- If the height is given as (163 ± 3.5 cms), then the height lies between 159.5 cms & 166.5 cms and measurement gives an interval estimate.

Unbiased & biased Estimates:- A statistic is said to be an unbiased estimator of the corresponding parameter if the mean of the sampling distribution of the statistic is equal to the corresponding population parameter. otherwise the statistic is called a biased estimator of the corresponding parameter.

The values of statistics in the above two cases are called unbiased and biased estimates respectively.

If 't' be a statistic and 'θ' be the corresponding parameter and $E(t)=\theta$, then t is an unbiased estimator of θ. otherwise t is a biased estimator of θ and the bias is $E(t)-\theta$.

Ex:- sample mean $\bar{x}$ is an unbiased estimator of population mean μ (since $E(\bar{x})=\mu$).

Unbiased estimator:- A statistic (or) point estimator $\hat{\theta}$ is said to be an unbiased estimator of the parameter θ if $E(\hat{\theta})=\theta$.

properties of estimators:-
A good estimator is one which is as close to the true value of the parameter as possible. The important properties of a good estimator are:
(i) consistency (ii) unbiasedness (iii) Efficiency and (iv) sufficiency

(i) An estimator $\hat{\theta}_n$ of a parameter θ is consistent if it converges to θ, as $n \to \infty$.

(ii) A statistic $\hat{\theta}$ is said to be an unbiased estimate of θ if $E(\hat{\theta}_n)=\theta$ for all θ.

(iii) A statistic $\hat{\theta}_1$ is said to be a more efficient unbiased estimator of the parameter θ than the statistic $\hat{\theta}_2$ if (a) $\hat{\theta}_1$ & $\hat{\theta}_2$ are both unbiased estimators of θ. (b) $V(\hat{\theta}_1) < V(\hat{\theta}_2)$

(iv) An estimator is said to be sufficient for a parameter, if it contains all the information in the sample regarding the parameter.

Sampling Error:- Let $\bar{x}$ be the mean of sample drawn from a population 'μ' and 'σ' then the sampling error is given by
$E = |\bar{x}-\mu|$

Confidence Interval:-

It means the area is covered under the normal curve leaving $\frac{\alpha}{2}$ area and both ends are normal curve. where 'α' is level of significance.

Maximum error of estimate 'E' for large samples ($n \geq 30$):-

$P(-z_{\alpha/2} \leq z < z_{\alpha/2}) = 1-\alpha$

where $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

The maximum error of estimate E with $(1-\alpha)$ probability is given by $E = z_{\alpha/2}[\sigma/\sqrt{n}]$

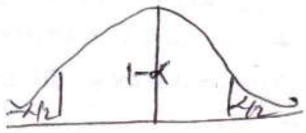
The confidence interval for mean (μ) for large values is given by
$\bar{x} = z_{\alpha/2}(\sigma/\sqrt{n})$

When σ, E, α are known, then the sample size n is given by $n=\left[\frac{z_{\alpha/2}\cdot\sigma}{E}\right]^2$

If maximum error 'E' and $z_{\alpha/2}$ given & p is unknown then $n=\frac{1}{4}\left[\frac{z_{\alpha/2}}{E}\right]^2$

Maximum error of estimate 'E' for small sample ($n<30$):-

$P(-t_{\alpha/2}\le t\le t_{\alpha/2})=1-\alpha$



The maximum error of estimate for small samples is given by

$E=t_{\alpha/2}\left[s/\sqrt{n}\right]$ s→ standard deviation

The confidence interval for 'μ' for small samples is given by $\bar{x}\pm t_{\alpha/2}(s/\sqrt{n})$.

The maximum error of estimate 'E' of population proportion error 'p' is given by $\boxed{E=z_{\alpha/2}\sqrt{\frac{PQ}{n}}}$, $Q=1-P$

If maximum error 'E' of the population proportion 'p' are known then find the sample size is given by $n=PQ\left[\frac{z_{\alpha/2}}{E}\right]^2$

The population proportion 'p' is not given $z_{\alpha/2}$, maximum error 'E' are known then sample size is given by

$$n=\frac{1}{4}\left[\frac{z_{\alpha/2}}{E}\right]^2$$

Bayesian Estimation:- The Bayesian estimation method involves sample information to be combined with prior distribution of 'μ', it is gives a "posterior" distribution of 'μ' which is approximately normal distribution

$$\mu_1=\frac{n\bar{x}\sigma_0^2+\mu_0\sigma^2}{n\sigma_0^2+\sigma^2}$$

$$\sigma_1=\sqrt{\frac{\sigma_0^2\sigma^2}{n\sigma_0^2+\sigma^2}}$$

where $\bar{x}$ = mean of sample,
n = size of sample
μ_0 = Mean of prior distribution
σ_0^2 = varience of prior distribution
μ_1 = mean of posterior distribution
σ_1^2 = varience of posterior distribution

Bayesian Limits (or) Bayesian interval for μ:-

$(1-\alpha)100\%$ Bayesian interval for μ is given by $\mu_1-z_{\alpha/2}\cdot\sigma_1<\mu<\mu_1+z_{\alpha/2}\cdot\sigma_1$

Confidence interval estimates of parameters:

The formulae for confidence limits of some well-known statistic for large random samples are given below.

I. confidence limits for population mean μ:-

(i) 95% confidence limits are $\bar{x}\pm 1.96$ (S.E of $\bar{x}$)
(ii) 99% confidence limits are $\bar{x}\pm 2.58$ (S.E of $\bar{x}$)
(iii) 99.73% confidence limits are $\bar{x}\pm 3$ (S.E of $\bar{x}$)
(iv) 90% confidence limits are $\bar{x}\pm 1.64$ (S.E of $\bar{x}$)

II. confidence limits for population proportion P:-

(i) 95% confidence limits are $P\pm 1.96$ (S.E of P)
(ii) 99% confidence limits are $P\pm 2.58$ (")
(iii) 99.73% " " $P\pm 3$ (")
(iv) 90% " " $P\pm 1.64$ (")

III. confidence limits for the difference $\mu_1-\mu_2$ of two population means μ_1 & μ_2:-

(i) 95% confidence limits are $(\bar{x}_1-\bar{x}_2)\pm 1.96$ (S.E of $(\bar{x}_1-\bar{x}_2)$)
(ii) 99% " " $(\bar{x}_1-\bar{x}_2)\pm 2.58$ (")
(iii) 99.73% " " $(\bar{x}_1-\bar{x}_2)\pm 3$ (")
(iv) 90% " " $(\bar{x}_1-\bar{x}_2)\pm 1.64$ (")

IV. confidence limits for difference (P_1-P_2) of two population proportions:-

(i) 95% confidence limits are $(P_1-P_2)\pm 1.96$ (S.E of (P_1-P_2))
(ii) 99% " " $(P_1-P_2)\pm 2.58$ (")
(iii) 99.73% " " $(P_1-P_2)\pm 3$ (")
(iv) 90% " " $(P_1-P_2)\pm 1.64$ (")

(1) If we can assert with 95% that the maximum error is 0.05 and $p=0.2$, find the size of the sample.

Sol. Given $P=0.2$, $E=0.05$

& $Q=1-P=1-0.2=0.8$ & $z_{\alpha/2}=1.96$ (for 95%)

We know that maximum error, $E=z_{\alpha/2}\sqrt{\frac{PQ}{n}}$

$\Rightarrow 0.05=(1.96)\sqrt{\frac{0.2\times 0.8}{n}}$

$\Rightarrow n=\frac{0.2\times 0.8\times(1.96)^2}{(0.05)^2}$

$=246$

②

→ It is desired to estimate the mean number of hours of continuous use until a certain computer will first require repairs. If it can be assumed that $\sigma = 48$ hours, how large a sample be needed 'so' that one will be able to assert with 90% confidence that the sample mean is off by atmost 10 hours.

Sol: It is given that maximum error, $E = 10$ hours & $\sigma = 48$ hours &

$Z_{\alpha/2} = 1.645$ (for 90%)

$\therefore n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.645 \times 48}{10}\right)^2 = 62.3 \simeq 62$

Hence sample size $= 62$

→ What is the maximum error one can expect to make with probability 0.90 when using the mean of a random sample of size $n = 64$ to estimate the mean of population with $\sigma^2 = 2.56$.

Sol: Here $n = 64$, the probability $= 0.90$

$\sigma^2 = 2.56 \Rightarrow \sigma = \sqrt{2.56} = 1.6$

confidence interval $= 90\%$

$\therefore Z_{\alpha/2} = 1.645$

Hence maximum error $E = Z_{\alpha/2} \cdot \sigma/\sqrt{n}$

$= 1.645 \times \frac{1.6}{\sqrt{64}}$

$= 0.329$

→ What is the size of the smallest sample required to estimate an unknown proportion to within a maximum error of 0.06 with atleast 95% confidence.

Sol: we are given the maximum error $E = 0.06$

confidence limit $= 95\%$

i.e. $1 - \alpha = \frac{95}{100}$

$\Rightarrow 1 - \alpha = 0.95$

$\Rightarrow \alpha = 0.05$

$\Rightarrow \alpha/2 = 0.025$

i.e. $Z_{\alpha/2} = 1.96$

When p is unknown, sample size

$n = \frac{1}{4}\left[\frac{Z_{\alpha/2}}{E}\right]^2$

$= \frac{1}{4}\left[\frac{1.96}{0.06}\right]^2$

$= 266.78$

$\simeq 267$

②

→ In a study of an automobile insurance random sample of 80 body repair costs had a mean of Rs. 472.36 and the S.D. of Rs. 62.35. If $\bar{x}$ is used as a point estimate to the average repair costs, with what confidence we can assert that the maximum error doesn't exceed Rs. 10?

Sol: size of random sample, $n = 80$

The mean of random sample $\bar{x} = 472.36$

$\sigma =$ Rs 62.35

Maximum error estimate $E_{max} =$ Rs. 10

we have $E_{max} = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$\Rightarrow Z_{\alpha/2} = \frac{E_{max} \cdot \sqrt{n}}{\sigma} = \frac{10\sqrt{80}}{62.35}$

$Z_{\alpha/2} = 0.9236$ (from normal distribution table)

$1 - \alpha/2 = Z_{\alpha/2} = 0.9236 \Rightarrow \alpha = 0.1528$

$\therefore$ confidence interval $= (1-\alpha)\,100\% = 84.72\%$

→ A sample of 10 cam shafts intended for use in gasoline engines has an average eccentricity of 1.02 and a standard deviation of 0.044 inch. Assuming the data may be treated a random sample from a normal population, determine a 95% confidence interval for the actual mean eccentricity of the cam shaft?

Sol: we know that confidence interval is

$\bar{x} \pm E$

where $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

given $n = 10$, $Z_{\alpha/2} = 1.96$, $\sigma = 0.044$, $\bar{x} = 1.02$

confidence interval is $1.02 \pm (1.96)\left(\frac{0.044}{\sqrt{10}}\right)$

$= 1.02 \pm 0.027$

$\therefore$ confidence interval $= (0.0993, 1.047)$

[P.T.O]

→ A sample size 10 was taken from a population s.d. of sample is 0.03. Find the maximum error with 99% confidence.

Sol: Given s = standard deviation = 0.03

n = sample size = 10

$t_{\alpha/2}$ for $\nu = 9$, 99% = ~~3.25~~ 2.58

$\therefore E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = \frac{2.58}{\cancel{3.25}} \times \frac{0.03}{\sqrt{10}} = ~~0.0325~~ \; 0.0244$

→ Find 95% confidence limits for the mean of a normality distributed population from which the following sample was taken 15, 17, 10, 18, 16, 9, 7, 11, 13, 14.

Sol: $\bar{x} = \frac{15+17+10+18+16+9+7+11+13+14}{10}$

$= 13$

$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

$= \frac{1}{9}\left[(15-13)^2 + (17-13)^2 + (10-13)^2 + (18-13)^2 + (16-13)^2 + (9-13)^2 + (7-13)^2 + (11-13)^2 + (13-13)^2 + (14-13)^2\right]$

$= \frac{40}{3} = 13.3$

Since $t_{\alpha/2} = 1.96$, we have

$t_{\alpha/2} \cdot \frac{\sqrt{s^2}}{n} = \frac{1.96}{\cancel{2.26}} \frac{\sqrt{40}}{\sqrt{10} \cdot \sqrt{3}} = 2.26$

$\therefore$ confidence limits are $\bar{x} \pm t_{\alpha/2} \cdot \frac{\sqrt{s^2}}{n}$

$= 13 \pm 2.26$

$= (10.74, 15.26)$

→ A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs. 487 with a standard deviation Rs 48. With what degree of confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between 472 to 502?

Sol: Given $\mu = 487$, $\sigma = 48$, $n = 100$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 487}{\frac{48}{\sqrt{100}}} = \frac{\bar{x} - 487}{4.8}$

standard variable corresponding to Rs. 472 is $z_1 = \frac{472 - 487}{4.8} = -3.125$

standard variable corresponding to Rs. 502 is $z_2 = \frac{502 - 487}{4.8} = 3.125$

Let 'x' be the mean of salary of teachers

Then $P(472 < x < 502) = P(-3.125 < z < 3.125)$

$= 2P(0 < z < 3.125)$

$= 2\int_0^{3.125} \phi(z)\,dz$

$= 2(0.4991)$

$= 0.9982$

Thus we can ascertain with 99.82% confidence

(3)

Assuming that $\sigma = 20.0$, how large a random sample be taken to assert with probability 0.95 that the sample mean will not differ from the true mean by more than 3.0 points?

Sol. Given maximum error $E = 3.0$ & $\sigma = 20.0$

we have $Z_{\alpha/2} = 1.96$

we know that, $n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$

$$\Rightarrow n = \left(\frac{1.96 \times 20}{3}\right)^2 = 170.74$$

$$\therefore n \cong 171$$

→ To estimate the average time it takes to assemble a certain computer component, the industrial engineer at an electronic firm timed 40 technicians in the performance of the task, getting a mean of 12.73 min, and a s.d. of 2.06 min.

(i) what can we say with 99% confidence about the maximum error if $\bar{x} = 12.73$ is used as a point estimate of the actual average time required to do the job?

(ii) Use the given data to construct 98% confidence interval.

(iii) with what confidence we can assert that the sample mean does not differ from the true mean by more than 30 sec

Sol: Here $\bar{x} = 12.73$, $s = 2.06$, $n = 40$.

for 99%, $Z_{\alpha/2} = 2.575$

(a) Maximum error of estimate $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$$\therefore E = (2.575) \cdot \frac{(2.06)}{\sqrt{40}} = 0.8387$$

(b) for 98% confidence, $E = (2.33) \cdot \frac{(2.06)}{\sqrt{40}}$

$$= 0.758915$$

98% confidence interval limits are

$\bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = \bar{x} \pm E = 12.73 \pm 0.7589$

i.e. confidence interval is $(11.97, 13.4889)$

(c) $\frac{30}{60}$ minutes $= \frac{1}{2}$ minutes $\Rightarrow E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$$= Z_{\alpha/2} \cdot \frac{2.06}{\sqrt{40}}$$

$\therefore Z_{\alpha/2} = 1.5350$

from normal distribution table, the area corresponding to $Z_{\alpha/2} = 1.5350$ is 0.4370

Then the area between $Z_{\alpha/2}$ to $Z_{\alpha/2}$ is

$2(0.4370) = 0.8740$

Thus we have 87.4% confidence

→ The mean & the standard deviation of a population are ~~1~~ 11795 & 14054 respectively. If $n = 50$. find 95% confidence interval for the mean.

(3)

Sol: Here mean of population $\mu = 11795$

S.D. of population, $\sigma = 14054$

$\bar{x} = 11795$

n = sample size $= 50$, Maximum error $= Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$Z_{\alpha/2}$ for 95% confidence $= 1.96$

Max. error $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{(14054)}{\sqrt{50}} = 3899$

$\therefore$ confidence interval $= \left(\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$

$$= (11795 - 3899, 11795 + 3899)$$
$$= (7896, 15694)$$

→ Determine a 95% confidence interval for the mean of a normal distribution with variance 0.25, using a sample of $n = 100$ values with mean 212.3

Sol: we have $n = 100$, $\bar{x} = 212.3$, $S.D = \sigma = \sqrt{0.25}$ and $Z_{\alpha/2} = 1.96$ (for 95%)

we know that 95% confidence interval is

$$\left(\bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

Now $Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = \frac{1.96 \times \sqrt{0.25}}{\sqrt{100}} = \frac{1.96 \times 0.5}{10} = \frac{0.98}{10}$
$= 0.098$

$\therefore$ confidence interval $= (212.3 - 0.098, 212.3 + 0.098)$

$$= (212.202, 212.398)$$

→ A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs. 487 with a standard deviation Rs 48. with what degree of confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between 472 to 502?

Sol: Given $\mu = 487$, $\sigma = 48$, $n = 100$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 487}{\frac{48}{\sqrt{100}}} = \frac{\bar{x} - 487}{4.8}$$

standard variable corresponding to Rs. 472 is

$$Z_1 = \frac{472 - 487}{4.8} = -3.125$$

standard variable corresponding to Rs. 502 is

$$Z_2 = \frac{502 - 487}{4.8} = 3.125$$

Let $\bar{x}$ be the mean salary of teacher, Then

$P(472 < \bar{x} < 502) = P(-3.125 < Z < 3.125)$
$= 2(0 < Z < 3.125)$
$= 2\int_0^{3.125} \phi(z)\,dz$
$= 2(0.4991) = 0.9982$

The mean of random samples of size is an unbiased estimate of the mean of the population 3, 6, 9, 15, 27

(i) List of all possible samples of size 3 that can be taken without replacement from the finite population.

(ii) calculate the mean of each of the samples listed in (a) & assigning each sample a probability of 1/10. verify that the mean of these $\bar{x}$ is equal to 12. which is equal to the mean of the population θ i.e. $E(\bar{x}) = \theta$ i.e. prove that $\bar{x}$ is an unbiased estimate of θ.

Sol (i) The possible samples of size 3 taken from 3, 6, 9, 15, 27 without replacement, are $5_{C_3} = 10$ samples i.e (3,6,9), (3,6,15), (3,6,27), (6,9,15), (6,9,27), (3,9,15), (3,9,27), (9,15,27), (6,15,27), (3,15,27)

(ii) Mean of the population $\theta = \frac{3+6+9+15+27}{5} = 12$

Means of the samples are 6, 8, 12, 10, 14, 9, 13, 17, 16, 15.

probability assigned to each one is $\frac{1}{10}$ each

$\bar{x}$	6	8	12	10	14	9	13	17	16	15
$p(\bar{x})$	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

$E(\bar{x}) = 6 \cdot \frac{1}{10} + 8 \cdot \frac{1}{10} + 12 \cdot \frac{1}{10} + 10 \cdot \frac{1}{10} + 14 \cdot \frac{1}{10} + 9 \cdot \frac{1}{10}$
$+ 13 \cdot \frac{1}{10} + 17 \cdot \frac{1}{10} + 16 \cdot \frac{1}{10} + 15 \cdot \frac{1}{10}$

$= \frac{1}{10} \times 120 = 12 = \theta$

$\therefore E(\bar{x}) = \theta$

$\therefore \bar{x}$ is an unbiased estimate of θ.

i.e. the mean of a random sample is an unbiased estimator of the mean of the population.

→ A random sample of size 100 is taken from an infinite population having the mean $\mu = 76$ & the variance $\sigma^2 = 256$. What is the probability that $\bar{x}$ will be between 75 & 78.

Sol: n = size of the sample = 100

μ = mean of the population = 76

σ^2 = variance of the population = 256

$\therefore \sigma = 16$

since n is large, the sample mean $\bar{x} \sim N(\mu, \sigma^2/\sqrt{n})$

$\therefore z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

when $\bar{x}_1 = 75$,

$z_1 = \frac{\bar{x}_1-\mu}{\sigma/\sqrt{n}} = \frac{75-76}{16/\sqrt{100}} = -0.625$

and when $\bar{x}_2 = 78$, $z_2 = \frac{\bar{x}_2-\mu}{\sigma/\sqrt{n}} = \frac{78-76}{16/\sqrt{100}} = 1.25$

$\therefore P(75 \le \bar{x} \le 78) = P(z_1 \le z \le z_2)$

$= P(-0.625 \le z \le 1.25)$

$= P(-0.625 \le z \le 0) + P(0 \le z \le 1.25)$

$= 0.2334 + 0.3944 = 0.628$

→ A normal population has a mean of 0.1 & standard deviation of 2.1. Find the probability that mean of a sample of size 900 will be negative.

Sol: Given $\mu = 0.1$, $\sigma = 2.1$ & $n = 900$

The standard normal variate

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{x}-0.1}{2.1/\sqrt{900}} = \frac{\bar{x}-0.1}{2.1/30} = \frac{\bar{x}-0.1}{0.07}$$

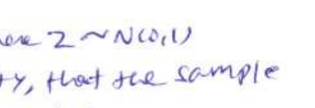
$\therefore \bar{x} = 0.1 + 0.07z$ where $z \sim N(0,1)$

$\therefore$ The required probability, that the sample mean is negative is given by

$P(\bar{x} < 0) = P(0.1 + 0.07z < 0)$

$= P(0.07z < -0.1)$

$= P(z < \frac{-0.1}{0.07})$



$= P(z < -1.43)$

$= 0.50 - P(0 < z < 1.43)$

$= 0.50 - 0.4236$

$= 0.0764$

→ The mean voltage of a battery is 15 & S.D is 0.2. Find the probability that four such batteries connected in series will have a combined voltage of 60.8 or more volts.

Sol: Let mean voltage of batteries A, B, C, D be $\bar{x}_A, \bar{x}_B, \bar{x}_C, \bar{x}_D$.

The mean of the series of the four batteries connected is

$\bar{x}_A + \bar{x}_B + \bar{x}_C + \bar{x}_D = \mu_{\bar{x}_A} + \mu_{\bar{x}_B} + \mu_{\bar{x}_C} + \mu_{\bar{x}_D}$
$= 15+15+15+15 = 60$

$\sigma_{A+B+C+D} = \sqrt{\sigma_A^2+\sigma_B^2+\sigma_C^2+\sigma_D^2} = \sqrt{4(0.2)^2} = 0.4$

Let x be the combined voltage of the series

when $x = 60.8$, $z = \frac{x-\mu}{\sigma} = \frac{60.8-60}{0.4} = 2$

Then probability that the combined voltage is more than 60.8 is given by

$P(x \ge 60.8) = P(z \ge 2) = 0.5 - 0.4772 = 0.0228$.

→ Determine the probability that the mean breaking strength of cables produced by company B will be (a) atleast 600 N more than (b) at least 450 N more than the cables produced by company A, if 100 cables of brand A and 50 cables of brand B are tested.

company	Mean Breaking strength	S.D	sample size
A	4000N	300N	100
B	4500N	200N	50

Sol: Given $\bar{x}_A = 4000$, $\bar{x}_B = 4500$

$\sigma_A = 300$, $\sigma_B = 200$

and $n_A = 100$, $n_B = 50$

$\mu_{\bar{x}_B - \bar{x}_A} = 4500 - 4000 = 500N$

$$\sigma_{\bar{x}_B - \bar{x}_A} = \sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_A^2}{n_A}} = \sqrt{\frac{(200)^2}{50} + \frac{(300)^2}{100}} = \sqrt{1700} = 41.23$$

→ A random sample of size 64 is taken from a normal population with $\mu = 51.4$ & $\sigma = 6.8$. What is the probability that the mean of the sample will (a) exceed 52.9 (b) fall between 50.5 & 52.3 (c) be less than 50.6

Sol: Given n = the size of the sample = 64

μ = the mean of the population = 51.4

σ = the S.D. of the population = 6.8

S.E, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.8}{\sqrt{64}} = \frac{6.8}{8} = 0.85$

(a) $P(\bar{x}$ exceed $52.9) = P(\bar{x} > 52.9)$

$\therefore z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{52.9-51.4}{0.85} = 1.76$

$\therefore P(\bar{x} > 52.9) = P(z > 1.76)$

$= 0.5 - P(0 < z < 1.76)$

$= 0.5 - 0.4608$

$= 0.0392$

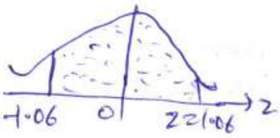
[P.T.O]

(b) $P(\bar{x}$ fall between 50.5 & $52.3)$

i.e. $P(50.5 < \bar{x} < 52.3) = P(\bar{x}_1 < \bar{x} < \bar{x}_2)$

$\therefore z_1 = \frac{\bar{x}_1 - \mu}{\sigma_{\bar{x}}} = \frac{50.5 - 51.4}{0.85} = -1.06$

$z_2 = \frac{\bar{x}_2 - \mu}{\sigma_{\bar{x}}} = \frac{52.3 - 51.4}{0.85} = 1.06$



$z = -1.06 \quad 0 \quad z = 1.06 \quad z$

$P(50.5 < \bar{x} < 52.3)$

$= P[-1.06 < Z < 1.06]$

$= P[-1.06 < Z < 0] + P[0 < Z < 1.06]$

$= P[0 < Z < 1.06] + P[0 < Z < 1.06]$

$= 2P[0 < Z < 1.06] = 2(0.3554) = 0.7108$

(c) $P(\bar{x}$ will be less than $50.6) = P(\bar{x} < 50.6)$

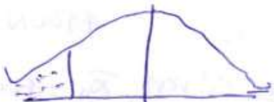
$= P(Z < -0.94) \quad [\because Z = \frac{50.6 - 51.4}{0.85} = -0.94]$

$= 0.50 - P(0.94 < Z < 0)$

$= 0.50 - P(0 < Z < 0.94)$

$= 0.50 - 0.3264$

$= 0.1736$



$z = -1.43 \quad 0$