* UNIT-V (R19) *

Tests of Hypothesis

Introduction:- Already earlier unit we discussed hypothesis testing, types of hypothesis, Two-tailed, one-tailed right, one-tailed left tests, Level of significance & critical values of Z.

Errors in sampling:-

we decide to accept (or) reject the population parameters after examination of a sample from the population.

There are two types of errors in hypothesis testing. They are

* Type-I Error

* Type-II Error

→ Type-I Error:- The Null-hypothesis is true but our test is reject. Then the Null-hypothesis is called Type-I Error.

→ Type-II Error:- The Null-hypothesis is false but our test is accept. then the Null-hypothesis is called Type-II Error.

procedure for testing of Hypothesis:-

(i) Null-Hypothesis (H_0):- first we setup the null-hypothesis H_0 i.e. there is no significance difference

(ii) Alternative hypothesis (H_1):- Next we setup the Alternative hypothesis this will unable to decide wheather we have to use a single tailed test (or) a two-tailed test.

(iii) Level of significance (α):- choose appropriate level of significance.

(iv) Test Statistic:-

we calculate the corresponding Z value i.e. $|Z|$.

(V). Conclusion:- compare the calculated value |Z| with the critical value Z_α at 1% (or) 5% (or) 10% level of significance.

→ If $|Z| < Z_\alpha$ then we accept the Null-hypothesis (H_0) and reject the Alternative hypothesis (H_1).

→ If $|Z| > Z_\alpha$ then we accept the Alternative hypothesis (H_1) and reject the Null hypothesis (H_0).

Critical values of Z:-

	Level of significance (α)		
	α=1% (or) 0.01	α=5% (or) 0.05	α=10% (or) 0.10
Two-tailed test (T.T.T):-	$Z_{\alpha/2} = 2.58$	$Z_{\alpha/2} = 1.96$	$Z_{\alpha/2} = 1.65$
One-tailed Right test (R.T.T):-	$Z_\alpha = 2.33$	$Z_\alpha = 1.65$	$Z_\alpha = 1.28$
One-tailed left test (L.T.T):-	$Z_\alpha = 2.33$	$Z_\alpha = 1.65$	$Z_\alpha = 1.28$

→ Test significance for large samples ($n \geq 30$):-

There are four types of test of significance for large samples. they are

1. Test of significance for single mean
2. Test of significance for differences of two means
3. Test of significance for single proportion
4. Test of significance for differences of two proportions.

① Test of significance for single mean:-

When the sample having a mean ($\bar{x}$) and standard deviation (s) of sample drawn from a population mean (μ) & standard deviation (σ) respectively.

[P.T.O]

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Then the test statistic is given by

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

If 'σ' is not known then we replace 'σ' by 's' i.e $Z = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

Problems :-

1. According to Norms established for a mechanical aptitude test persons who are 18 years old have an average height of 73.2 with a S.D. of 8.6. If Randomly 34 selected persons of that age averaged 76.7. Test the hypothesis $\mu = 73.2$ against the alternative hypothesis $\mu > 73.2$ at the 0.01 level of significance.

Sol:- Given that $\mu = 73.2$

$\sigma = 8.6$

$n = 34$

sample mean $\bar{x} = 76.7$

(i) Null-hypothesis (H_0):- The Average height of student is 73.2 i.e $\mu = 73.2$

(ii) Alternative hypothesis (H_1):- The Average height of student is greater than 73.2 i.e. $\mu > 73.2$

(iii) Level of significance :- $\alpha = 0.01$ (or) 1%

(iv) Test Statistic:-

The test statistic is $|Z| = \left|\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right|$

$$= \left|\frac{76.7-73.2}{8.6/\sqrt{34}}\right|$$

$$= \left|\frac{3.5}{1.4748}\right|$$

$$= 2.3732$$

i.e. $|Z| = 2.3732$

(v) Conclusion :- The critical value is $z_\alpha = z_{0.01} = 2.33$ for R.T.T.

i.e the calculated value $|z| > z_\alpha$, so we accept H_1 & reject H_0

Hence the average height of student greater than 73.2

2. A sample of 64 students have a mean height of 70 kgs. Can this be regarded as a sample of population with mean height 56 kgs & standard deviation 25 kgs.

sol: Given that $n = 64$

$\bar{x} = 70$ kgs

$\mu = 56$ kgs

$\sigma = 25$ kgs

(i) Null hypothesis (H_0) :- The sample can be regarded as sample from the population i.e $\mu = 56$ kgs

(ii) Alternative hypothesis (H_1) :- The sample cannot be regarded as the population i.e. $\mu \neq 56$ kgs (T.T.T)

(iii) Level of significance :- $\alpha = 0.05$ (5%)

(iv) Test statistic :- The test statistic is $|z| = \left| \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \right|$

$= \left| \frac{70 - 56}{25/\sqrt{64}} \right|$

$= \frac{14}{25/8}$

$|z| = 4.48$

(v) Conclusion :- The tabulated value is $z_{\alpha/2} = z_{0.05} = 1.96$

$\therefore$ The calculated value $|z| > z_{\alpha/2}$, so we accept H_1 & reject H_0

Hence the sample cannot be regarded as the population

3. A Random sample of 100 death records in the years showed as an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years. Does this information to indicate the mean life span is greater than 70 years using 0.05 level of significance.

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sol: Given that $n=100, \mu=70, \sigma=8.9$ & $\bar{x}=71.8$

(i) Null hypothesis (H_0):- The mean life span is cannot be greaterthan 70 years ie $\mu=70$

(ii) Alternative hypothesis (H_1):- The mean life span is greaterthan 70 years ie $\mu>70$ (R.T.T)

(iii) Level of significance :- $\alpha=0.05$

(iv) Test statistic:- The test statistic is

$$|z| = \left|\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right|$$

$$= \left|\frac{71.8-70}{8.9/\sqrt{100}}\right|$$

$$|z| = 2.02$$

(v) Conclusion:- The tabulated value is $z_\alpha = z_{0.05} = 1.65$ from RTT

$\therefore$ The calculated value $|z| > z_\alpha$, so we accept H_1 & reject H_0.

Hence the Average life span is > 70 years.

4. A die is tossed 960 times and it with 5 upwards 184 items is the die unbiased at 0.01 level of significa

sol Given that $n=960$

since die is tossed so $p=1/6$ & $q=1-p=1-\frac{1}{6}=5/6$

since the value n is large approximately do the normal distribution.

ie. mean of normal distribution $\mu=np$

$=960\times\frac{1}{6}$

$\boxed{\mu=160}$

standard deviation of normal distribution $\sigma=\sqrt{npq}$

$=\sqrt{960\times\frac{1}{6}\times\frac{5}{6}}$

$=11.54$

(i) Null hypothesis (H_0):- The die is unbiased

(ii) Alternative hypothesis (H_1):- The die is biased (T.T.T)

(iii) Level of significance:- $\alpha=1\%$ (or) 0.01

(iv) Test statistic: The test statistic is

$$|Z| = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{184-160}{\frac{11.54}{\sqrt{960}}} = \frac{24}{0.3724} = 64.44$$

(v) Conclusion: The tabulated value is $Z_{\alpha/2} = Z_{0.01} = 2.58$ from T.T.T

i.e the calculated value $|Z| > Z_{\alpha/2}$ so we accept H_1 & reject H_0

Hence the die is biased.

2. Test of significance for differences of two Means:-

Let $\bar{x}_1$ & $\bar{x}_2$ be two sample means of sizes n_1 & n_2 drawn from two populations having with means μ_1 & μ_2 standard deviation σ_1 & σ_2 respectively. Then the test statistic is

$$|Z| = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right|$$ (sample means & S.D. of two populations are given)

→ If the two samples are drawn from a single large population having σ respectively. Then the test statistic is $$|Z| = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} \right|$$ (Sample mean & S.D. of population are given)

→ If the two samples having a mean $\bar{x}_1$ & $\bar{x}_2$ / S.D s_1 & s_2 of sizes n_1 & n_2 are drawn from a single large population respectively. then the test statistic is $$|Z| = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} \right|$$ (Sample mean & S.D. of samples are given & σ is not given)

where $$\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

1. The mean life of a sample of 10 electric bulbs was found to be 1456 members with S.D. of 423hrs. A Second sample of 17 bulbs choosen from a different showed a mean life of 1280 members with S.D. of 398 hrs. Is there significance difference between the means of two batches.

Sol: Given that 1st sample size $n_1 = 10$

1st sample mean $\bar{x}_1 = 1456$

1st S.D. $s_1 = 423$

2nd sample size $n_2 = 17$

2nd sample mean $\bar{x}_2 = 1280$

2nd S.D. $s_2 = 398$

(i) Null hypothesis (H_0):- There is no significance diff between mean life of bulbs produced by two batches.

i.e. $\mu_1 = \mu_2$

(ii) Alternative hypothesis (H_1):- There is a significance diff between mean life of bulbs produced by two batches.

i.e. $\mu_1 \neq \mu_2$ (T.T.T)

(iii) Level of significance :- $\alpha = 0.05 (5\%)$

(iv) Test statistic:-

since population S.D. is not given then

$$\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

$$= \frac{10(423)^2 + 17(398)^2}{10+17}$$

$$\sigma^2 = 166005.85$$

$$\text{i.e. } \sigma = \sqrt{166005.85} = 407.438$$

$$\text{Now } |z| = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \right|$$

$$= \left| \frac{1456 - 1280}{407.438\sqrt{\frac{1}{10} + \frac{1}{17}}} \right|$$

$$= \frac{176}{164.46}$$

$|z| = 1.067$

(v) Conclusion :- The tabulated value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.T.T.

i.e. the calculated value $|z| <$ the tabulated value $Z_{\alpha/2}$

so we accept the H_0 & reject the H_1.

Hence there is no significance diff between the mean life of bulbs produced by two batches.

2. A sample of the height of 6400 English men as a mean of 67.85 inches and standard deviation of 2.56 inches. Another sample of height of 1600 as Australians as mean of 68.55 inches and S.D of 2.52 inches. The data indicate the Australian are on the Average taller than the English men. (use $\alpha = 0.01$)

Sol: Given that $n_1 = 6400$, $\bar{x}_1 = 67.85$, $s_1 = 2.56$

$n_2 = 1600$, $\bar{x}_2 = 68.55$, $s_2 = 2.52$

$\alpha = 0.01$

(i) Null-hypothesis (H_0) :- The data indicate the Australian are on the Average not taller than the English men i.e. $\mu_1 = \mu_2$

(ii) Alternative hypothesis (H_1) :- The data indicate the Australian are on the Average taller than the English men i.e. $\mu_1 > \mu_2$ (R.T.T)

(iii) Level of significance :- $\alpha = 0.01$

(iv) Test statistic :-

Since population S.D. is not given then

$$\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

$$= \frac{6400(2.56)^2 + 1600(2.52)^2}{6400 + 1600}$$

$$\sigma^2 = 6.51296$$

i.e. $\sigma = 2.5520$

Now $|Z| = \left| \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \right|$

$= \left| \dfrac{67.85 - 68.55}{2.55\sqrt{\frac{1}{6400}+\frac{1}{1600}}} \right|$

$= \left| \dfrac{-0.7}{0.0334} \right|$

$|Z| = 20.95$

(V) Conclusion:- The tabulated value is $Z_\alpha = Z_{0.01} = 2.33$ from F.T.T.

i.e the calculated value $|Z| >$ the tabulated value Z_α, so we accept the H_1 & reject the H_0.

Hence the data indicate the Australians are on the Average are taller than the English mens.

3. The mean of two large samples of sizes 1000 & 2000 members are 67.5 inches & 68.0 inches respectively can the samples be regarded as drawn from the same population of S.D. 2.5 inches.

Sol:- Let μ_1 & μ_2 be the means of two populations.

Given that $n_1 = 1000$, $n_2 = 2000$

$\bar{x}_1 = 67.5$ inches, $\bar{x}_2 = 68$ inches

Population S.D. $\sigma = 2.5$ inches

(i) Null-hypothesis (H_0):- The samples are drawn from the same population of S.D. 2.5 inches i.e $\mu_1 = \mu_2$ & $\sigma = 2.5$

(ii) Alternative hypothesis (H_1):- The samples are not drawn from the same population of S.D. 2.5 inches i.e $\mu_1 \neq \mu_2$ & $\sigma \neq 2.5$ (T.T.T)

(iii) Level of significance:- $\alpha = 0.05$ (5% Assumed)

(iv) Test Statistic:-

since population S.D are given then

$|Z| = \left| \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \right|$

$$= \left| \frac{67.5-68}{2.5\sqrt{\frac{1}{1000}+\frac{1}{2000}}} \right|$$

$$= \left| \frac{-0.5}{0.0968} \right|$$

$$= |-5.161|$$

$$|Z| = 5.16$$

(V) <u>Conclusion</u>:- The tabulated value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.T.T

i.e. the calculated value $|Z| >$ the tabulated value $Z_{\alpha/2}$, so we accept the H_1 & reject the H_0.

Hence the sample are not drawn from the same population of S.D 2.5 inches.

H.W.

4. The two types of new cars produce in U.S.A are tested for petrol mileage. One sample is consisting of 42 cars averaged 15 kmpl while the other sample consisting of 80 cars averaged 11.5 kmpl with population variances as $\sigma_1^2 = 2.0$ & $\sigma_2^2 = 1.5$ respectively. Test wheather there is any significance difference in the petrol consumption of these two types of cars (use $\alpha = 0.01$).

5. The mean yield of wheat from a district 'A' was 210 pounds with S.D. 10 pounds per acre from a sample of 100 plots. In another district 'B' was 220 pounds per acre from a sample with S.D. of 12 pounds of 150 plots. test wheather there is any significance difference between the mean yield of crops in the two districts.

3. Test of significance for single proportion:-

If p is a sample proportion of size 'n' drawn from a Population proportion 'P' respectively. Then the test statistic is $|Z| = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$

Here $p = \frac{x}{n}$ = sample proportion = $\frac{\text{number of favourable cases}}{\text{sample size}}$

P = Probability of success (or) proportion

$Q = 1 - P$

1. Some manufacture claims that only 4% of the products are defect of the random sample of 500 were taken among which 100 were defective. Test the hypothesis at 0.05 level of significance.

Sol: Given that $P = 4\% = 0.04$

$Q = 1 - P = 1 - 0.04 = 0.96$

sample size $n = 500$

$p = \frac{\text{number of defective items}}{\text{size of sample}} = \frac{100}{500} = \frac{1}{5} = 0.2$

(i) Null hypothesis (Ho):- we can support the claim i.e. $P = 0.04$

(ii) Alternative hypothesis (H_1):- We cannot support the claim i.e $P \neq 0.04$ (T.T.T)

(iii) Level of significance:- $\alpha = 0.05$

(iv) Test statistic:- The test statistic is $|Z| = \left| \frac{p - P}{\frac{PQ}{n}} \right|$

$= \left| \frac{0.2 - 0.04}{\sqrt{\frac{(0.04)(0.96)}{500}}} \right|$

i.e. $|Z| = 18.25$

(V) conclusion:- The critical value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.T.T i.e the calculated value $|Z| >$ the critical value $Z_{\alpha/2}$, so we accept H_1 & Reject Ho. Hence we cannot support the claim.

2. 40 people were attacked by disease and only 18 are survived. Will you reject the hypothesis that the survival rate attacked by this disease is 85%. Is favour of the hypothesis that is more at 5% level of significance.

Sol: Given that $n = 40$

$$p = \frac{\text{number of survival candidates}}{\text{total people}} = \frac{18}{40} = 0.45$$

$$P = 85\% = \frac{85}{100} = 0.85$$

$$Q = 1 - P = 1 - 0.85 = 0.15$$

(i) Null hypothesis (H_0):- We can support the claim i.e. $P = 0.85$

(ii) Alternative hypothesis (H_1):- We cannot support the claim i.e. $P > 0.85$ (R.T.T)

(iii) Level of significance:- $\alpha = 0.05$

(iv) Test statistic:- The test statistic is

$$|Z| = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$$

$$= \left| \frac{0.45 - 0.85}{\sqrt{\frac{0.85 \times 0.15}{40}}} \right|$$

$$= \left| \frac{-0.4}{6.3245} \right|$$

$$|Z| = 0.0632$$

(v) Conclusion:- The tabulated value is $Z_\alpha = Z_{0.05} = 1.96$ from R.T.T.

i.e. the calculated value $|Z| <$ the tabulated Z_α, so we accept H_0 & reject H_1

Hence the proportion at the survival rate of the people is 0.85

3. A sample of 500 members from the village in A.P. 280 are found to be rice eaters and rest of members are wheat eaters. We assume that the both articles are equally popular.

Sol) Given that $n = 500$

$$p = \frac{\text{number of rice eaters}}{\text{Total members}} = \frac{280}{500} = 0.56$$

$$P = \frac{1}{2} = \text{rice eaters}$$

$$Q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$$

(i) Null hypothesis (H_0):- The rice & wheat eaters are equally popular in A.P ie $P = 0.5$

(ii) Alternative hypothesis (H_1):- The rice & wheat eaters are not equal popular in A.P. ie $P \neq 0.5$ (T.T.T.)

(iii) Level of significance:- $\alpha = 0.05$

(iv) Test statistic:- The test statistic is $|z| = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$

$$= \left| \frac{0.56 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{500}}} \right|$$

$$= \frac{0.06}{0.022}$$

$$|z| = 2.72$$

(v) Conclusion:- The critical value is $z_{\alpha/2} = z_{0.05} = 1.96$ from T.T.T.

ie the calculated value $|z| >$ the critical value $z_{\alpha/2}$, so we accept H_1 & reject H_0.

Hence the rice & wheat eaters are not equally popular in A.P.

4. In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

Sol: Given that $n = 600$

$$p = \frac{\text{number of smokers}}{\text{Number of mens}} = \frac{325}{600} = 0.5417$$

$P = \frac{1}{2}$ = population proportion of smokers in the city

$$Q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$$

(i) Null hypothesis (H_0):- The number of smokers & Non-smokers are equal in the city i.e. $P = 0.5$

(ii) Alternative hypothesis (H_1):- The number of smokers & Non-smokers are not equal in the city $P > 0.5$ (R.T.T)

(iii) Level of significance:- $\alpha = 0.05$

(iv) Test statistic:- The test statistic is

$$|Z| = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$$

$$= \left| \frac{0.5417 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}} \right|$$

$$|Z| = 2.04$$

(v) Conclusion:- The critical value is $Z_\alpha = Z_{0.05} = 1.65$ from R.T.T.

i.e. the calculated value $|Z| >$ The critical value Z_α, So we accept the H_1 & reject the H_0

Hence the majority of men in city are smokers.

5. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test this claim that at 5% level of significance

Sol Given that $n = 200$

Number of pieces confirming to specification $= 200 - 18 = 182$

i.e. $p = \frac{182}{200} = 0.91$

$P = \frac{95}{100} = 0.95$

$Q = 1 - P = 1 - 0.95 = 0.05$

(i) Null-hypothesis (H_0):- The proportion of pieces confirming to specifications i.e. $P = 95\%$

(ii) Alternative hypothesis (H_1):- The proportion of pieces not confirming to specifications i.e. $P \neq 95\%$ (T.T.T.)

(iii) Level of significance :- $\alpha = 0.05$ (5%)

(iv) Test statistic: The test statistic is

$$|Z| = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$$

$$= \left| \frac{0.91 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}} \right|$$

$$= |-2.591|$$

i.e. $|Z| = 2.59$

(v) Conclusion:- The tabulated value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.T.T.

i.e. the calculated value $|Z| >$ the critical value $Z_{\alpha/2}$, so we accept the H_1 & reject the H_0.

Hence the proportion of pieces not confirming to specification

4. Test of significance for differences of two proportions:-

Let P_1 & P_2 be the two proportions of large sample sizes n_1 & n_2 drawn from two populations having proportions P_1 & P_2 respectively. Then the test statistic is

$$|Z| = \left| \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \right|$$

Where $P = \dfrac{n_1 P_1 + n_2 P_2}{n_1+n_2}$

$Q = 1-P$

Problems:-

1. A Random samples of 400 men and 600 women were asked wheather there would like to have near their residences. 200 men and 325 women were in favour of the proposal. Test the hypothesis that propotions of the men and women favour of the proposal are same at 5% level of significance.

Sol Given that $n_1 = 400$, $n_2 = 600$

population proportions of men $P_1 = \frac{200}{400} = \frac{1}{2}$

population proportions of women $P_2 = \frac{325}{600}$

(i) Null hypothesis (H_0):- The favour of the proposal men & women are same ie, $P_1 = P_2$

(ii) Alternative hypothesis (H_1):- The favour of the proposal men & women are not same i.e $P_1 \neq P_2$ (T.T.T.)

(iii) Level of significance:- $\alpha = 0.05$ (5%)

(iv) Test Statistic:- The test statistic is

$$|Z| = \left| \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \right|$$

Here $P = \dfrac{n_1 P_1 + n_2 P_2}{n_1+n_2}$

$$= \frac{400 \cdot \frac{200}{400} + 600 \cdot \frac{325}{600}}{400+600}$$

$$= \frac{200+325}{1000}$$

$$= \frac{525}{1000}$$

$$P = 0.525$$

$$Q = 1-P = 1-0.525 = 0.475$$

(iv) Test statistic :- The test statistic is

$$|Z| = \left| \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \right|$$

$$= \left| \frac{\frac{200}{400} - \frac{325}{600}}{\sqrt{(0.525)(0.475)\left(\frac{1}{400}+\frac{1}{800}\right)}} \right|$$

$$|Z| = 1.25$$

(v) Conclusion :- The critical value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.T.T.

i.e. the calculated value $|Z|$ < the critical value $Z_{\alpha/2}$, so we accept the H_0 & reject the H_1.

i.e the favour of the men & women are same.

2. In a random sample of 1000 persons from town 'A' are 400 found to be consumers of wheat. In a sample of 800 in town B are 500 found to be consumers of wheat. Do this data the difference between town A & town B. so far as the proportions of wheat consumers is concerned.

Sol :- Given that $n_1 = 1000$, $n_2 = 800$

$$P_1 = \frac{400}{1000} = 0.4 \,,\ P_2 = \frac{400}{800} = 0.5$$

(i) Null hypothesis (H_0) :- There is no significance difference between the proportions of wheat consumers from town A & town B. i.e $P_1 = P_2$

(ii) Alternative hypothesis (H_1) :- There is a significance difference between the proportions of wheat consumers from town A & town B. i.e $P_1 \neq P_2$ (T.T.T.)

(iii) Level of significance:- $\alpha = 0.05$ (5%)

(iv) Test statistic:- The test statistic is

$$|Z| = \left|\frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\right|$$

Here $P = \dfrac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$$= \frac{1000(0.4) + 800(0.5)}{1000}$$

$$P = 0.4$$

$$Q = 1 - P = 1 - 0.4 = 0.6$$

i.e. $|Z| = \left|\dfrac{0.4 - 0.5}{\sqrt{(0.4)(0.6)\left(\frac{1}{1000}+\frac{1}{800}\right)}}\right|$

$$= \left|\frac{-0.1}{0.0232}\right|$$

$$|Z| = 4.3103$$

(v) Conclusion:- The critical value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.O.F

i.e the calculated value $|Z| >$ the critical value $Z_{\alpha/2}$, so we accept the H_1 & reject the H_0.

Hence there is a significance difference between the proportions of wheat consumers from town A & town B.

3. In a two large populations, there are 30% & 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 & 900 respectively from two populations.

Sol Given that $n_1 = 1200$, $n_2 = 900$

P_1 = proportions of fair haired people in the 1^{st} population

$$= \frac{30}{100} = 0.3$$

P_2 = Proportion of fair haired people in the 2^{nd} population

$= \frac{25}{100}$

$P_2 = 0.25$

(i) Null hypothesis (H_0):- there is no difference in population proportions is likely to be hidden in sampling i.e. $P_1 = P_2$

(ii) Alternative hypothesis (H_1):- there is a difference in population proportions is likely to be hidden in sampling i.e. $P_1 \neq P_2$ (T.T.T.)

(iii) Level of significance:- $\alpha = 0.05$ (5%)

(iv) Test statistic:- The test statistic is

$$|Z| = \left| \frac{P_1 - P_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}} \right|$$

where $Q_1 = 1 - P_1 = 1 - 0.3 = 0.7$

$Q_2 = 1 - P_2 = 1 - 0.25 = 0.75$

Now $$|Z| = \left| \frac{0.3 - 0.25}{\sqrt{\frac{0.3 \times 0.7}{1200} + \frac{0.25 \times 0.75}{900}}} \right|$$

$$= \left| \frac{0.05}{0.0195} \right|$$

$$|Z| = 2.55$$

(v) Conclusion:- the critical value is $Z_{\alpha/2} = Z_{0.05} = 1.96$ from T.T.T.
i.e. the calculated value $|Z| >$ the critical value $Z_{\alpha/2}$, so we accept the H_1 & reject the H_0.

Hence the difference in population proportion is unlikely that the real difference will be hidden.

H.W

4. A machine puts out 10 imperfect Articles in a sample of 200 after the machine is over hauled it puts out 4 imperfect Articles in batch of 100 as the machine been improved