

PUSH DOWN AUTOMATA

- Introduction
  - Basic model of PDA defining
  - Graphical notation
  - Informal description (a)
  - Acceptance of PDA.
- Introduction:-  
 PDA is a way to implement a CFG in a similar way we can design FA for Regular Grammar.

- PDA is more powerful than finite state machine.
- FSM has a very limited memory but a PDA has more memory.

$$PDA = FSM + stack$$

- A stack is a way we arrange elements one on the top of stack.

- A stack does two basic operations.

i) push :- A new element is added at the top of the stack.

ii) pop :- the top element of the stack is read and remove.

Ex:-  
 push(a)  
 push(b)  
 push(c)  
 push(d)

pop()  
 pop()  
 pop()



stack



stack

- Basic model of PDA:-

PDA has three components

- i) input tape
- ii) finite control unit
- iii) stack

- A stack with infinite size

- It has unlimited amount of storage space

- Used to store data and remove the data.



↑  
 Read head

finite control unit

→ output-accept or reject



push or pop

stack

Formal definition:-

- Mathematically a pta is defined with a tuple like

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F) \text{ where}$$

$Q \rightarrow$  finite and non-empty set of states

$\Sigma \rightarrow$  finite and non-empty set of input symbols

$\Gamma \rightarrow$  finite and non-empty set of stack symbols

$\delta \rightarrow$  It is a transition function which is defined as

$\delta:$

$$Q \times \{\emptyset \cup \Sigma\} \times \Gamma^* \rightarrow Q \times \Gamma^*$$

$$Q \times \Sigma^* \times \Gamma^* \rightarrow Q \times \Gamma^*$$

where  $\delta$  takes three tuples as input like  $\delta(q, a, x)$

where i)  $q$  is a state in  $Q$

ii)  $a$  is either an input symbol in  $\Sigma$  or  $a$  is also belongs  $\epsilon$

iii)  $x$  is a stack symbol i.e. member of  $\Gamma$

iv) the output of  $\delta$  is finite set of pairs like  $(p, f)$

where,  $p$ : It is a new state.

$f$ : It is a set of stack symbols that replace

$x$  at the top of the stack.

Ex:- i) If  $f = \epsilon$  then the stack is pop

ii) If  $f = x$  then the stack is unchanged (once bypass operation)

iii) If  $f = yz$  then  $x$  is replaced by  $z$  and  $y$  is pushed on to the stack.

Ex:- i)  $\delta(q_0, a, z) = (q_1, yz)$

ii) It indicates that from state  $q_0$ , reading input symbol 'a'

where, top of the stack is  $z$  then the finite control goes to  $q_1$  state and adding the element  $y$  to the top of the stack.

1)  $\delta(q_1, a, z) = (q_2, \epsilon)$

→ It indicates that  $z$  is removed from the stack and state is changed from  $q_1$  to  $q_2$

2)  $\delta(q_1, a, z) = (q_2, z)$

→ It indicates that on reading symbol  $a$  state is changing from  $q_1$  to  $q_2$  and there is exchange in the stack (bypass operation)

$q_0$  → It is the initial state.

$q_0 \in Q$

$z_0$  → It is the start stack symbol.

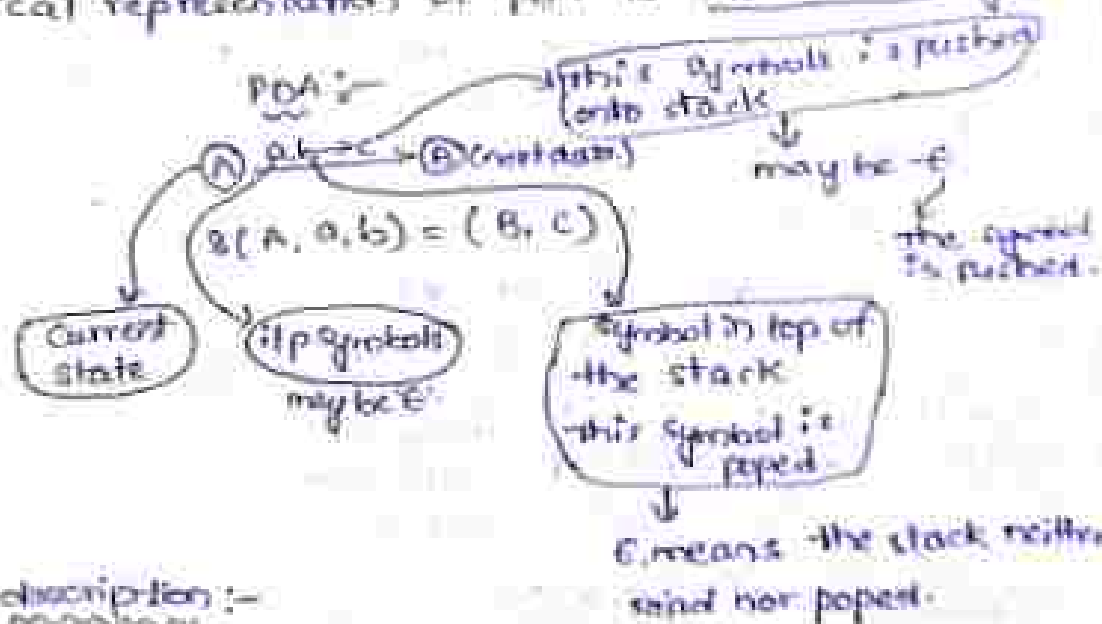
$z_0 \in T$

$F$  → It is the set of final or accepting state and  $(F \subseteq Q)$

Graphical representation:-

The Graphical representation of PDA is Transition diagram

FA:-



Instantaneous description:-

It is used to describe the configuration of PDA at given instance.

It remembers the state and stack content.

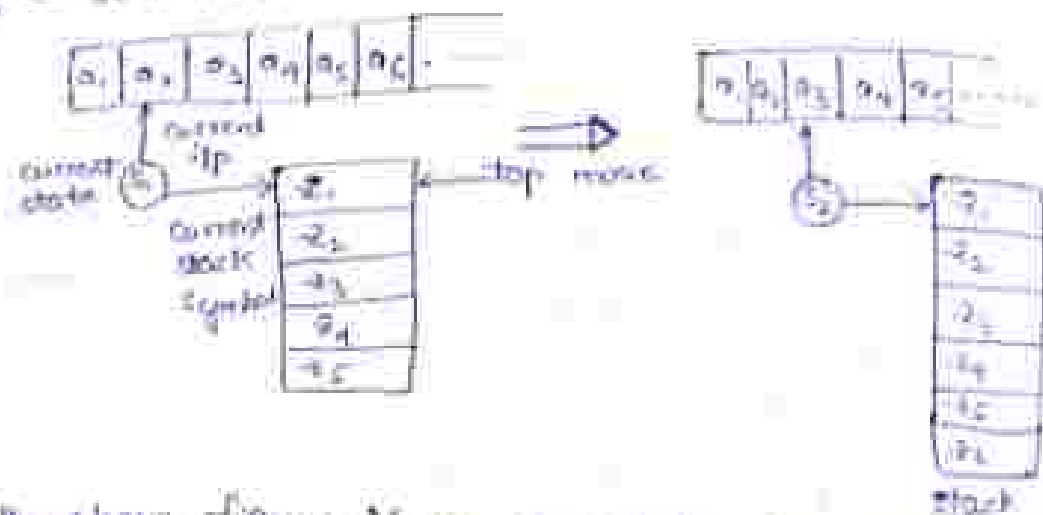
It was defined by Triple  $(q, w, Y)$  where

$q$  → is a state.

$w$  → input symbols of string

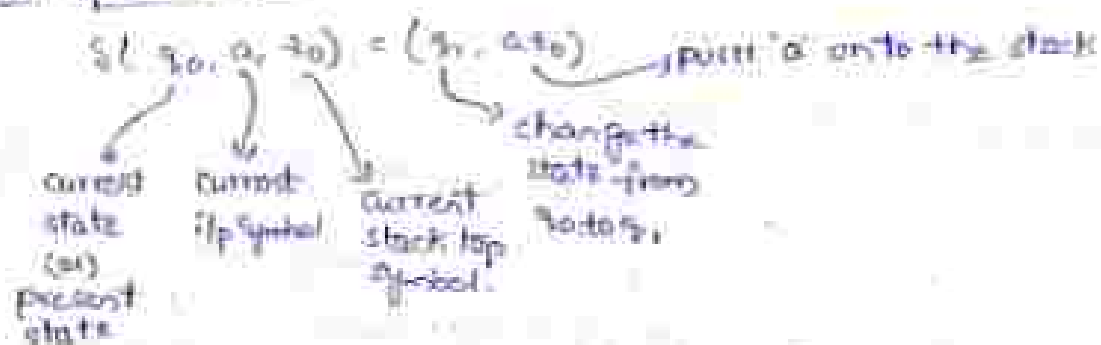
$Y$  → is a string of stack symbols.

Example:  $\delta(q_0, a_1 a_2) = (q_1, A)$

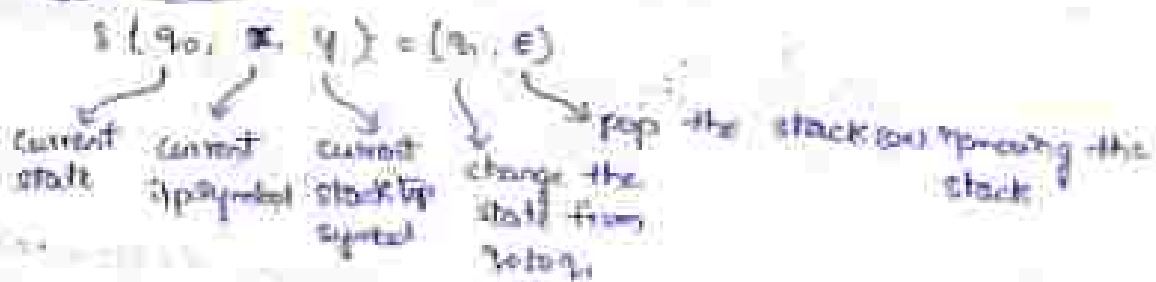


From the above figure, if we are reading the current ip symbol 'a<sub>2</sub>' at current state 'q<sub>1</sub>' and current stack symbol 'z<sub>5</sub>', then after a move we will reach to state 'q<sub>2</sub>' and there will be some new symbol on the top of the stack. This description can be represented as.

1) push operation:-



2) pop operation:-



Acceptance of PDA:-

There are two ways to accept a language by PDA, they are

i) accepted by empty stack.

ii) accepted by final state.

Accepted by empty stack:-

The given language accepted by empty stack to be defined as

$L(M) = \{w \mid \delta(q_0, w, z_0) \xrightarrow{*} (p, \epsilon, \epsilon) \text{ for some } p, n.a.\}$

that is, if stack becomes empty after scanning entire string then it is accepted by PDA otherwise, not accepted.

Accepted by final string:-

The given language accepted by final state to be defined as  
 $L(M) = \{ w \mid \delta(q_0, w, z_0) \rightarrow (p, \epsilon, f) \text{ for some } p \in F \text{ and } f \in F \}$   
 that is, even though stack is not empty, after scanning the string if the finite control reaches to the final state then it is accepted. otherwise, not accepted.

Design of PDA:-

Types of PDA:-

- i) Deterministic PDA:- if all derivations in the design are to give only single move.
- ii) Non-deterministic PDA:- if derivation generates more than one move in the designing of a particular task.

1) Design a PDA that accepts equal no. of A's and B's

Let  $\delta$ :  $\delta(q_0, a, z_0) = (q_1, a, z_0)$   
 $\delta(q_1, a, a) = (q_1, aa)$   
 $\delta(q_1, b, a) = (q_1, \epsilon)$   
 $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$   
 $\delta(q_1, a, z_0) = (q_1, a z_0)$



$\therefore$  The PDA machine for the above language is defined as

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$  where  $Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$\Gamma = \{z_0\}$

$\delta$ :

$q_0 = \{q_0\}$

$z_0 = \{z_0\}$

$F = \{q_1\}$

Read A  $\rightarrow$  push operation, Read B  $\rightarrow$  pop operation

Consider a string  $w = \{abab\}$  Read a's

$\delta(q_0, abab, z_0) = \delta(q_1, bab, a z_0)$

Q design a pda for the language  $L = \{0^n 1^n \mid n \geq 1\}$   
 sol:  $L = \{0^n 1^n \mid n \geq 1\}$

Read one 0  $\rightarrow$  push

Read two 1's  $\rightarrow$  pop

$$\delta(q_0, \epsilon, \epsilon) = (q_1, z_0)$$

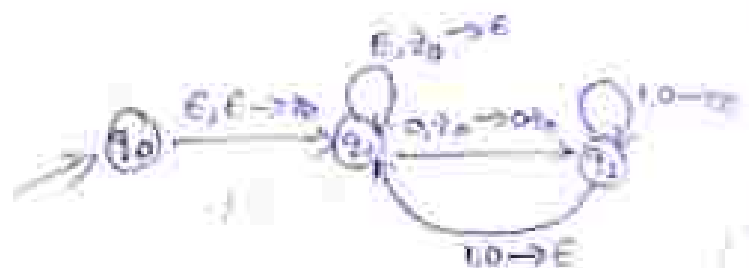
$$\delta(q_1, 0, z_0) = (q_2, 0z_0)$$

$$\delta(q_2, 0, 0) = (q_2, 00)$$

$$\delta(q_2, 1, 0) = (q_2, 00)$$

$$\delta(q_2, 1, 0) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon, \epsilon)$$



$$\delta(q_1, 1, 0) = (q_1, \epsilon)$$

$$\delta(q_1, 1, 0) = (q_1, \epsilon)$$

Q consider the string  $w = \{001111\}$

$$\delta(q_1, 001111, z_0) = \delta(q_2, 0111, 0z_0)$$

$$= \delta(q_2, 111, 00)$$

$$= \delta(q_2, 11, 00)$$

$$= \delta(q_1, 1, 0z_0)$$

$$= \delta(q_1, 1, 0z_0)$$



$$= S(q_1, \epsilon, \bar{q}_0)$$

$$= S(q_1, \epsilon, \epsilon)$$

Design a PDA for the language  $L = \{0^n 1^n \mid n \geq 1\}$

Read 0's  $\rightarrow$  push

Read 1's  $\rightarrow$  pop



$$S(q_0, \epsilon, \epsilon) = (q_1, \bar{q}_0)$$

$$S(q_1, 0, \bar{q}_0) = (q_2, 0\bar{q}_0)$$

$$S(q_2, 0, 0) = (q_2, \emptyset)$$

$$S(q_2, 1, 0) = (q_3, \epsilon)$$

$$S(q_3, 1, 0) = (q_3, \epsilon)$$

$$S(q_3, \epsilon, \bar{q}_0) = (q_3, \epsilon, \epsilon)$$

\* Design a PDA for the language  $L = \{w w^R \mid w \in (a+b)^*\}$

$$w \in \{w w^R \mid w \in (a+b)^*\}$$

$$id) L = \{w w^R \mid w \in (a+b)^*\}$$

In this language contains palindrome string. i.e.,

if  $w = ab$ ,  $w^R = ba$  then  $ww^R = abba$  is a palindrome

\* We can read no. of a's and b's and pushed them into stack until we can reach the mid position of  $ww^R$  string.

\* In the mid position we can't read any  $ww^R$  and can't push onto stack.

\* After mid position when we read a or b then pop them from the stack. This process is repeated until stack is empty.

$$S(q_0, \epsilon, \epsilon) = (q_1, \bar{q}_0)$$

$$S(q_1, a, \bar{q}_0) = (q_1, a\bar{q}_0)$$

$$S(q_1, b, \bar{q}_0) = (q_1, b\bar{q}_0)$$

$$S(q_1, \epsilon, \epsilon) = (q_2, \bar{q}_0)$$

$$S(q_2, a, a) = (q_2, \epsilon)$$

$$S(q_2, b, b) = (q_3, \epsilon)$$

$$S(q_3, \epsilon, \bar{q}_0) = (q_3, \epsilon, \epsilon)$$



(i)  $L = \{ w \in \{a,b\}^* \mid w \text{ contains } a^2b \}$

$w = ab$

$w^R = ba$

$w \in L$  because  $ba$

$$\delta(q_0, \epsilon, \epsilon) = (q_1, \epsilon, \epsilon)$$

$$\delta(q_1, a, \epsilon) = (q_1, a, \epsilon)$$

$$\delta(q_1, b, \epsilon) = (q_1, b, \epsilon)$$

$$\delta(q_1, a, a) = (q_1, aa)$$

$$\delta(q_1, a, b) = (q_1, ab)$$

$$\delta(q_1, b, a) = (q_1, ba)$$

$$\delta(q_1, b, b) = (q_1, bb)$$

$$\delta(q_1, \epsilon, \epsilon) = (q_2, \epsilon, \epsilon)$$

$$\delta(q_2, \epsilon, \epsilon) = (q_2, \epsilon, \epsilon)$$

$$\delta(q_2, \epsilon, b) = (q_2, b)$$

$$\delta(q_2, a, \epsilon) = (q_3, \epsilon)$$

$$\delta(q_3, b, \epsilon) = (q_3, b)$$

$$\delta(q_3, \epsilon, \epsilon) = (q_4, \epsilon, \epsilon)$$

Deterministic pushdown Automata:-

→ DPA is a tuple like  $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

where  $Q$  is finite and non-empty set of states

$\Sigma$  is finite and non-empty set of IP Alphabet

$\Gamma$  is finite set of stack symbols

$\delta$  is a mapping function used for mapping (or) moving from current state to next state, i.e.,

$$\delta(q_0, x, z_0) = (q_1, xp) \text{ where}$$

$q_0$  is current state

$x$  is current IP symbol

$z_0$  is current stack symbol

$q_1$  is next state

$xp$  shows top of the stack

if  $\delta$  denotes a unique transition for each IP then PDA is said to be deterministic PDA

$$L = \{ a^n b^n \mid n \geq 1 \}$$

$$L = \{ w \in \{a,b\}^* \mid w = (a^n b^n)^k \}$$

Non-deterministic PDA:-

It is a tuple like  $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$  where

$Q$  is finite and non-empty set of states

$\Sigma$  is finite and non-empty set of IP Alphabet

$\Gamma$  is finite set of stack symbols.



$\delta$  is a mapping function used for moving from current state to next state and is defined as  $\delta(q_0, x, z) = \{q_1, p\}$

- $q_0$  is current state
- $x$  is current ip symbol
- $z$  is stack symbol
- $q_1$  is next state
- $p$  is top of the stack

If  $\delta$  denotes more than one transition for a particular ip symbol then the PDA is said to be non-deterministic (NPD).

$$L = \{w \mid (w \in L)^n\}$$

Context-free grammars and push down automata:-

conversion of CFG to PDA

conversion of PDA to CFG

i) Conversion of CFG to PDA:-

• for constructing a PDA from given CFG it is necessary to convert this CFG to some normal form like GNF.

• for converting given CFG to PDA, by this method the necessary condition is that the first symbol on RHS of production rule must be a terminal symbol. This rule that can be used to obtain PDA from CFG.

Algorithm:-

Rule 1:- for non-terminal symbols, add following rule

$\delta(q, A, \epsilon) = (q, A)$  where the production rule is  $A \rightarrow A$

Rule 2:- for each terminal symbol, add following rule

$\delta(q, A, a) = (q, \epsilon)$  for every terminal symbol 'a' in given CFG.

Ex:- Construct a PDA for the given CFG

$S \rightarrow AB$

$B \rightarrow OS$

$B \rightarrow IS$

$B \rightarrow \epsilon$

Sol:- The given CFG  $G = (V, T, P, S)$  where  $V$  = non-terminals  $\{S, B\}$

$$\Sigma = \{0, 1\}$$

$$P: \begin{aligned} S &\rightarrow 00S \\ S &\rightarrow 0S \\ S &\rightarrow 1S \\ S &\rightarrow 0 \end{aligned}$$

$$\Sigma = \{0, 1\}$$

$$P_{1,1} = \begin{aligned} S &\rightarrow 00S \\ \delta(q, \epsilon, S) &= (q, 00S) \end{aligned}$$

$$S \rightarrow 00S$$

$$\delta(q, \epsilon, S) = (q, 00S)$$

$$S \rightarrow 0S$$

$$\delta(q, 0, S) = (q, 0S)$$

$$S \rightarrow 1S$$

$$\delta(q, 0, S) = (q, 1S)$$

$$S \rightarrow 0$$

$$\delta(q, \epsilon, S) = (q, 0)$$

$$P_{1,2}$$

Terminals

$$\delta(q, 0, 0) = (q, \epsilon)$$

$$\Sigma = \{0, 1\}$$

$$\delta(q, 0, 0) = (q, \epsilon)$$

$$\delta(q, 1, 1) = (q, \epsilon)$$

$\therefore$  the corresponding PDA for the given CFG is defined as

$$M = (Q, \Sigma, \Gamma, q, q_0, q_f, \delta)$$

$$Q = \{q\}$$

$$\Sigma = \{0, 1\}$$

$$\Gamma = \{S, 0, 1\}$$

$\delta$  = it is a transition function defined as

$$\delta(q, \epsilon, S) = (q, 00S)$$

$$\delta(q, 0, S) = (q, 0S)$$

$$\delta(q, 0, S) = (q, 1S)$$

$$\delta(q, 0, S) = (q, 0)$$

$$\delta(q, 0, 0) = (q, \epsilon)$$

$$\delta(q, 1, 1) = (q, \epsilon)$$

$$q_0 = \{q\}$$

$$q_f = \{q_0\}$$

$$\Gamma = \{ \}$$

2) construct a PDA for the following CFG  
 $S \rightarrow OSI$   
 $S \rightarrow A$   
 $A \rightarrow IAD \mid S \mid \epsilon$

3) The given CFG is  
 $S \rightarrow OSI$   
 $S \rightarrow A$   
 $A \rightarrow IAD$   
 $A \rightarrow S$   
 $A \rightarrow \epsilon$

elimination of  $\epsilon$ -production:-

$A \rightarrow \epsilon$	$A \rightarrow IAD$	$S \rightarrow OSI \mid OI$
$S \rightarrow A$	$A \rightarrow SIO$	$S \rightarrow A$
$S \rightarrow \epsilon$	$A \rightarrow IO$	$A \rightarrow IAD \mid IO$
$S \rightarrow OS$	$A \rightarrow S$	$A \rightarrow S$
$S \rightarrow OSI$	$A \rightarrow \epsilon$	
$S \rightarrow OI$		

elimination of unit production:-

$S \rightarrow A$	$A \rightarrow S$
$S \rightarrow IAD \mid IO$	$A \rightarrow OSI \mid OI$

$\therefore$  the resultant CFG is

$S \rightarrow IAD$   
 $S \rightarrow IO$   
 $A \rightarrow OSI$   
 $A \rightarrow OI$

$\therefore$  the simplified CFG is

$S \rightarrow IAD$   
 $S \rightarrow IO$   
 $A \rightarrow OSI \mid IO \mid OI$   
 $A \rightarrow OI$

Method 2

$P \rightarrow I$   
 $Q \rightarrow O$

$S \rightarrow IAD$	$S \rightarrow IO$	$A \rightarrow OSI$	$A \rightarrow IO$	$S \rightarrow OSI$
$S \rightarrow IAS$	$S \rightarrow IS$	$A \rightarrow OSP$	$A \rightarrow IS$	$S \rightarrow OSP$
$A \rightarrow IAD$	$A \rightarrow OI$	$S \rightarrow OI$		
$A \rightarrow IAS$	$A \rightarrow OSP$	$S \rightarrow OSP$		

$\therefore$  the simplified CFG in GNF is

$S \rightarrow IAS$      $A \rightarrow OSP$   
 $S \rightarrow IS$      $A \rightarrow IS$   
 $S \rightarrow OSP$      $A \rightarrow IAS$   
 $S \rightarrow OP$      $A \rightarrow OP$   
 $P \rightarrow I$   
 $Q \rightarrow O$

4) The PDA is

Ans: 1  
 $Q \rightarrow IAS$      $S \rightarrow OSP$      $A \rightarrow OSP$   
 $S(q, \epsilon, q) = (q, IAS)$      $S(q, \epsilon, s) = (q, OSP)$      $S(q, \epsilon, A) = (q, OP)$   
 $S \rightarrow IS$      $S \rightarrow OP$      $A \rightarrow IS$   
 $S(q, \epsilon, s) = (q, IS)$      $S(q, \epsilon, s) = (q, OP)$      $S(q, \epsilon, A) = (q, IS)$

$A \rightarrow \epsilon, 0/0$

$\delta(q, \epsilon, A) = (q, 0/0)$

$A \rightarrow op$

$\delta(q, \epsilon, A) = (q, op)$

$p \rightarrow 1$

$\delta(q, \epsilon, p) = (q, 1)$

$A \rightarrow 0$

$\delta(q, \epsilon, A) = (q, 0)$

method 2

the given CFG is  $S \rightarrow 0S1$

$S \rightarrow A$

$A \rightarrow 1A0$

$A \rightarrow S$

$A \rightarrow \epsilon$

the resultant PDA is  $S \rightarrow 0S1$

$\delta(q, \epsilon, S) = (q, 0S1)$

$S \rightarrow A$

$\delta(q, \epsilon, S) = (q, A)$

$A \rightarrow S$

$\delta(q, \epsilon, A) = \delta(q, S)$

$A \rightarrow 1A0$

$A \rightarrow \epsilon$

$\delta(q, \epsilon, A) = (q, 1A0)$

$\delta(q, \epsilon, A) = (q, \epsilon)$

Construct PDA for the following CFG  $S \rightarrow aABB/aAA$

$A \rightarrow aBB/a$

$B \rightarrow bBB/b$

sol: the given CFG is  $S \rightarrow aABB$

$S \rightarrow aAA$

$A \rightarrow aBB$

$A \rightarrow a$

$B \rightarrow bBB$

$B \rightarrow b$

elimination of unit production:-

$B \rightarrow Aa$

$B \rightarrow aBB$

$B \rightarrow a$

after eliminating unit production  $B \rightarrow Aa$ , the resultant

CFG in CNF is  $S \rightarrow aABB$

$S \rightarrow aAA$

$A \rightarrow aBB$

$A \rightarrow a$

$B \rightarrow bBB$

$B \rightarrow aBB$

$B \rightarrow a$

$$E \rightarrow aBB$$

$$s(q, E, S) = (q, aBB)$$

$$E \rightarrow aAA$$

$$s(q, E, S) = (q, aAA)$$

$$A \rightarrow aBB$$

$$s(q, A, S) = (q, aBB)$$

$$A \rightarrow a$$

$$s(q, A, S) = (q, a)$$

$$B \rightarrow bBB$$

$$s(q, B, S) = (q, bBB)$$

$$B \rightarrow aBB$$

$$s(q, B, S) = (q, aBB)$$

$$B \rightarrow a$$

$$s(q, B, S) = (q, a)$$

conversion of PDA to CFG :-

If  $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$  is a PDA then there exists CFG  $G$  which is accepted by PDA ( $M$ )

Let  $G$  be a CFG which is generated by PDA. The  $G$  can be defined as  $G = (V, T, P, S)$  where  $S$  is the start symbol and the set of non-terminals  $V = \{q, q', z_0\}$  where  $q, q' \in Q$  and  $z_0 \in \Gamma$ .

Now, we get set of production rules using the following algorithm.

Algorithm :-

Rule 1 :- The start symbol production rule can be  $S \rightarrow [q, z_0, q']$  where  $q$  indicates present state,  $q'$  indicates next state,  $z_0$  is the stack symbol.

Rule 2 :- If there exists a move of PDA as then the production rule can be return as  $s(q, a, z_0) = (q', c)$

$[q, z_0, q'] \rightarrow a$

if there exists a move of push as  $\delta(q, a, z_0) = (q', z_1 z_2)$ , then the production rules can be written as

$$[q, z_0, \epsilon] \Rightarrow \alpha [q', z_1, z_2] [q', z_1, z_2] = [q', z_1 z_2, \epsilon]$$

ex: construct a CFA from the following non  $\epsilon$ -NFA  $M = (Q, \Sigma, \delta, q_0, \epsilon)$  and

- $\delta$ :
- $\delta(q_0, 1, \epsilon) = (q_0, AA)$
  - $\delta(q_0, 0, \epsilon) = (q_0, 0)$
  - $\delta(q_0, 1, A) = (q_0, AA)$
  - $\delta(q_0, 0, A) = (q_1, A)$
  - $\delta(q_1, 1, A) = (q_1, \epsilon)$
  - $\delta(q_1, 0, \epsilon) = (q_1, \epsilon)$

sol: let us will construct a CFA  $G = (V, T, P, S)$  where  $T = \{0, 1\}$

$$V = \{ \text{all } [q_0, s, q_0], [q_0, s, q_1], [q_1, \epsilon, q_0], [q_1, \epsilon, q_1], [q_0, A, q_0], [q_0, A, q_1], [q_1, A, q_0], [q_1, A, q_1] \}$$

now let us build the production rules as  
using rule ① the production rules for start symbol  $S$

$$P_1: S \rightarrow [q_0, \epsilon, q_0]$$

$$P_2: S \rightarrow [q_0, 1, q_0]$$

using Rule ③ of the Algorithm, for the  $\delta(q_0, 1, \epsilon) = (q_0, AA)$

$$\begin{aligned} P_3: [q_0, 1, q_0] &\rightarrow_1 [q_0, A, q_0] [q_0, 1, q_0] \\ P_4: [q_0, 1, q_0] &\rightarrow_1 [q_0, A, q_1] [q_1, \epsilon, q_0] \\ P_5: [q_0, 1, q_0] &\rightarrow_1 [q_0, A, q_1] [q_1, \epsilon, q_1] \\ P_6: [q_0, 1, q_1] &\rightarrow_1 [q_0, A, q_1] [q_1, 1, q_1] \end{aligned}$$

now, for  $\delta(q_0, 0, \epsilon) = (q_0, 0)$  using Rule ② of Algorithm we get

$$P_7: [q_0, 0, q_0] \rightarrow \epsilon$$

$$P_8: [q_0, 0, A] \rightarrow 0A$$

now for  $\delta(q_1, 1, A) = (q_1, \epsilon)$  using Rule ④ of Algorithm

$$P_9: [q_1, 1, A] \rightarrow_1 [q_1, A, q_0] [q_0, A, q_0]$$

now for  $S(q_0, 0, A) = (q_1, A)$  using Rule ② of algorithm

$$P_7: [q_0, A, q_0] \rightarrow 0 [q_0, A, q_1] [q_1, A, q_0]$$

$$P_8: [q_0, A, q_1] \rightarrow 1 [q_0, A, q_0] [q_0, A, q_1]$$

$$P_9: [q_1, A, q_1] \rightarrow 1 [q_0, A, q_1] [q_1, A, q_1]$$

now for  $S(q_0, 0, A) = (q_1, A)$  using



$$P_{10}: [q_0, A, q_0] \rightarrow 0 [q_1, A, q_0]$$

$$P_{11}: [q_0, A, q_1] \rightarrow 0 [q_1, A, q_1]$$

now for  $S(q_1, 1, A) = (q_1, \epsilon)$

$$P_{12}: [q_1, A, q_1] \rightarrow 1$$

now for  $S(q_1, 0, S) = (q_0, S)$



$$P_{13}: [q_1, S, q_0] \rightarrow 0 [q_0, S, q_0]$$

$$P_{14}: [q_1, S, q_1] \rightarrow 0 [q_0, S, q_1]$$

PDA with two stacks : —

① PDA with one stack :

$$L = \{ a^n b^n \mid n \geq 1 \}$$

consider the string  $w = aabb$

read  $a$ 's  $\rightarrow$  push

read  $b$ 's  $\rightarrow$  pop



stack

a a b b \$  
↑ ↑ ↑ ↑

when stack is empty then  
the string  $aabb$  is accepted

Q. 2.  $\{a^n b^n c^n \mid n \geq 1\}$

consider string  $w = aabbcc$

read  $a$ 's  $\rightarrow$  push

read  $b$ 's  $\rightarrow$  pop

read  $c$ 's  $\rightarrow$  exchange



$aabbcc$   
 $\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$

when read  $c$  there is no change in stack without completion of reading string i.e. the stack is empty so, string is not accepted

Q. 6. PDA with two stacks:—

$L = \{a^n b^n c^n \mid n \geq 1\}$



$w = aabbcc$   
 $\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$

read  $a$ 's  $\rightarrow$  push (in stack<sub>1</sub>)

read  $b$ 's  $\rightarrow$  push (in stack<sub>2</sub>)

read  $c$ 's  $\rightarrow$  pop (a from stack<sub>1</sub> and b from stack<sub>2</sub>)

when two stacks are empty then string  $w$  is accepted

$\therefore$  The PDA with two stacks is more powerful than a PDA with one stack.

FA + 0-stack = NFA = DFA

FA + 1-stack = PDA

FA + 2-stack = PDA with two stacks

Applications of PDA:—

• used for deriving a string from the grammar.

• used for designing top-down parser and bottom-up parser in compiler design.

• It works on regular grammar and context-free grammar.

• It accepts regular language and CFL.

• It has remembering capability by maintaining a stack.

• It is more powerful than FA.