Homework #4

CSE 446/546: Machine Learning
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Due: **Friday** 06/12/2020 11:59 PM

Please review all homework guidance posted on the website before submitting to Gradescope. Reminders:

- Please provide succinct answers along with succinct reasoning for all your answers. Points may be deducted if long answers demonstrate a lack of clarity. Similarly, when discussing the experimental results, concisely create tables and/or figures when appropriate to organize the experimental results. In other words, all your explanations, tables, and figures for any particular part of a question must be grouped together.
- When submitting to gradescope, please link each question from the homework in gradescope to the location of its answer in your homework PDF. Failure to do so may result in point deductions. For instructions, see `https://www.gradescope.com/get_started#student-submission`.
- Please recall that B problems, indicated in boxed text are only graded for 546 students, and that they will be weighted at most 0.2 of your final GPA (see website for details). In Gradescope there is a place to submit solutions to A and B problems separately. You are welcome to create just a single PDF that contains answers to both, submit the same PDF twice, but associate the answers with the individual questions in gradescope.
- If you collaborate on this homework with others, you must indicate who you worked with on your homework. Failure to do so may result in accusations of plagiarism.

In the 'A' problems on this assignment, you are provided some choice about which subset of problems to complete. A subset of the problems are color-coded. You may choose **one RED problem from {A3, A4 } and one BLUE problem from {A5, A6 }** to complete. You do not need to complete both problems of the same color. Problems of the same color have the same total point value. { A1, A2 } are not optional and should be completed by all students.

Conceptual Questions

A1. The answers to these questions should be answerable without referring to external materials. Briefly justify your answers with a few words.

a. *[2 points]* True or False: Training deep neural networks requires minimizing a non-convex loss function, and therefore gradient descent might not reach the globally-optimal solution.

b. *[2 points]* True or False: It is a good practice to initialize all weights to zero when training a deep neural network.

c. *[2 points]* True or False: We use non-linear activation functions in a neural network's hidden layers so that the network learns non-linear decision boundaries.

d. *[2 points]* True or False: Given a neural network, the time complexity of the backward pass step in the backpropagation algorithm can be prohibitively larger compared to the relatively low time complexity of the forward pass step.

e. *[2 points]* True or False: Autoencoders, where the encoder and decoder functions are both neural networks with nonlinear activations, can capture more variance of the data in its encoded representation than PCA using the same number of dimensions.

Think before you train

A2. In class, we discussed some of the ways in which many datasets describing crime have various shortcomings in describing the entire landscape of illegal behavior in a city, and that these shortcomings often fall disproportionately on minority communities. Some examples include that crimes are reported at different rates in different neighborhoods, that police respond differently to the same crime reported or observed in different neighborhoods, and that police spend more time patrolling in some neighborhoods than others.

a. *[5 points]* Please describe two statistical problems arising from one or more of these issues (or others mentioned in class, or others from some other source that you cite relating to datasets compiled related to crime in the US), and what real-world implications might follow from ignoring these issues. A short paragraph for each statistical problem suffices.

b. *[5 points]* For each of your previously identified statistical concerns above, propose a technical "fix" (e.g. a new sampling strategy, some training method for which few or no distributional assumptions are needed on the training set).

c. *[5 points]* The solutions you described in (b) might only take you so far towards ensuring a model has similarly positive impact on different communities. Describe two reasons why a machine learning model trained to predict $f(x)$ from x might have different accuracy on two different populations, even if the training data is drawn i.i.d. from the true distribution over the event of interest. [These assumptions exclude the possibility that the two populations are sampled from at rates different from their latent frequency, though their latent frequencies may be different.]

Unsupervised Learning with Autoencoders

A3. Last homework we used PCA to project and reconstruct data from the MNIST dataset. In this exercise, we will train two simple autoencoders to perform dimensionality reduction on MNIST. As discussed in lecture, autoencoders are a long-studied neural network architecture comprised of an encoder component to summarize the latent features of input data and a decoder component to try and reconstruct the original data from the latent features.

Weight Initialization and PyTorch

Last assignment, we had you refrain from using `torch.nn` modules. For this assignment, we recommend using `nn.Linear` for your linear layers. You will not need to initialize the weights yourself; the default He/Kaiming uniform initialization in PyTorch will be sufficient for this problem. *Hint: we also recommend using the* ***`nn.Sequential`*** *module to organize your network class and simplify the process of writing the forward pass.*

Training

Use `optim.Adam` for this question. Experiment with different learning rates. Use mean squared error (`nn.MSELoss()` or `F.mse_loss()`) for the loss function and ReLU for the non-linearity in b.

a. *[10 points]* Use a network with a single linear layer. Let $W_{\text{e}} \in \mathbb{R}^{h \times d}$ and $W_{\text{d}} \in \mathbb{R}^{d \times h}$. Given some $x \in \mathbb{R}^d$, the forward pass is formulated as

$$\mathcal{F}_1(x) = W_{\text{d}} W_{\text{e}} x.$$

Run experiments for $h \in \{32, 64, 128\}$. For each of the different h values, report your final error and visualize a set of 10 reconstructed digits, side-by-side with the original image. *Note:* we omit the bias term in the formulation for notational convenience since `nn.Linear` learns bias parameters alongside weight parameters by default.

b. *[10 points]* Use a single-layer network with non-linearity. Let $W_{\text{e}} \in \mathbb{R}^{h \times d}$, $W_{\text{d}} \in \mathbb{R}^{d \times h}$, and activation $\sigma : \mathbb{R} \longmapsto \mathbb{R}$. Given some $x \in \mathbb{R}^d$, the forward pass is formulated as

$$\mathcal{F}_2(x) = \sigma(W_{\text{d}} \sigma(W_{\text{e}} x)).$$

Report the same findings as asked for in part a (for $h \in \{32, 64, 128\}$.

c. *[5 points]* Now, evaluate $\mathcal{F}_1(x)$ and $\mathcal{F}_2(x)$ (use $h = 128$ here) on the test set. Provide the test reconstruction errors in a table.

d. *[5 points]* In a few sentences, compare the quality of the reconstructions from these two autoencoders compare with those of PCA from last assignment. You may want to re-run your code for PCA using the different h values as the number of top-k eigenvalues.

Using Pretrained Networks and Transfer Learning

A4. So far we have trained very small neural networks from scratch. As mentioned in the previous problem, modern neural networks are much larger and more difficult to train and validate. In practice, it is rare to train such large networks from scratch. This is because it is difficult to obtain both the massive datasets and the computational resources required to train such networks.

Instead of training a network from scratch, in this problem, we will use a network that has already been trained on a very large dataset (ImageNet) and adjust it for the task at hand. This process of adapting weights in a model trained for another task is known as *transfer learning*.

- Begin with the pretrained AlexNet model from `torchvision.models` for both tasks below. AlexNet achieved an early breakthrough performance on `ImageNet` and was instrumental in sparking the deep learning revolution in 2012.
- Do not modify any module within AlexNet that is not the final classifier layer.
- The output of AlexNet comes from the 6th layer of the classifier. Specifically, `model.classifer[6] = nn.Linear(4096, 1000)`. To use AlexNet with CIFAR-10, we will reinitialize (replace) this layer with `nn.Linear(4096, 10)`. This re-initializes the weights, and changes the output shape to reflect the desired number of target classes in CIFAR-10.

We will explore two different ways to formulate transfer learning.

a. *[15 points]* **Use AlexNet as a fixed feature extractor:** Add a new linear layer to replace the existing classification layer, and only adjust the weights of this new layer (keeping the weights of all other layers fixed). Provide plots for training loss and validation loss over the number of epochs. Report the highest validation accuracy achieved. Finally, evaluate the model on the test data and report both the accuracy and the loss.

When using AlexNet as a fixed feature extractor, make sure to freeze all of the parameters in the network *before* adding your new linear layer:

```
model = torchvision.models.alexnet(pretrained=True)
for param in model.parameters():
    param.requires_grad = False
model.classifier[6] = nn.Linear(4096, 10)
```

b. *[15 points]* **Fine-Tuning:** The second approach to transfer learning is to fine-tune the weights of the pre-trained network, in addition to training the new classification layer. In this approach, all network weights are updated at every training iteration; we simply use the existing AlexNet weights as the "initialization" for our network (except for the weights in the new classification layer, which will be initialized using whichever method is specified in the constructor) prior to training on CIFAR-10. Following the same procedure, report all the same metrics and plots as in the previous question.

Image Classification on CIFAR-10

A5. In this problem we will explore different deep learning architectures for image classification on the CIFAR-10 dataset. If you are not comfortable with PyTorch from the previous lecture and discussion materials, the following tutorials at `http://pytorch.org/tutorials/beginner/deep_learning_60min_blitz.html` may be useful:

- *What is PyTorch?*
- *Autograd: automatic differentiation*
- *Neural Networks*
- *Training a classifier*

After completing them, you should be familiar with tensors, two-dimensional convolutions (`nn.Conv2d`) and fully-connected layers (`nn.Linear`), ReLU non-linearities (`F.relu`), pooling (`nn.MaxPool2d`), and tensor reshaping (`view`).

The final tutorial, in particular, will leave you with a network for classifying the CIFAR-10 dataset. This network is used as the starting point for this problem. You will construct a number of different network architectures and compare their performance. For all, it is highly recommended that you copy and modify the existing network code produced in the tutorial *Training a classifier*. You should not be coding this network from scratch!

A few preliminaries:

- Each network f maps an image $x^{in} \in \mathbb{R}^{32\times32\times3}$ (3 channels for RGB) to an output $f(x^{in}) = x^{out} \in \mathbb{R}^{10}$. The class label is predicted as $\arg\max_{i=0,1,\ldots,9} x_i^{out}$. An error occurs if the predicted label differs from the true label for a given image.
- The network is trained via multiclass cross-entropy loss.
- Create a validation dataset by appropriately partitioning the train dataset. *Hint*: look at the documentation for `torch.utils.data.random_split`. Make sure to tune hyperparameters like network architecture and step size on the validation dataset. Do **NOT** validate your hyperparameters on the test dataset.
- Modify the training code such that at the end of each epoch (one pass over the training data) it computes and prints the training and test classification accuracy (you may find the running calculation that the code initially uses useful to calculate the training accuracy).
- While one would usually train a network for hundreds of epochs to reach convergence and maximize accuracy, this can be prohibitively time-consuming, so feel free to train for just a dozen or so epochs.

For all of the following, apply a hyperparameter tuning method (grid search, random search, etc.) using the validation set, report the hyperparameter configurations you evaluated and the best set of hyperparameters from this set, and plot the training and validation classification accuracy as a function of iteration. Produce a separate line or plot for each hyperparameter configuration evaluated. Finally, evaluate your best set of hyperparameters on the test data and report the accuracy. **On the larger networks, you should expect to tune hyperparameters and train to at least 70% accuracy.**

Here are the network architectures you will construct and compare.

a. *[15 points]* **Fully-connected output, 0 hidden layers (logistic regression):** this network has no hidden layers and linearly maps the input layer to the output layer. This can be written as

$$x^{out} = W\text{vec}(x^{in}) + b$$

where $x^{out} \in \mathbb{R}^{10}$, $x^{in} \in \mathbb{R}^{32\times32\times3}$, $W \in \mathbb{R}^{10\times3072}$, $b \in \mathbb{R}^{10}$ since $3072 = 32 \cdot 32 \cdot 3$. For a tensor $x \in \mathbb{R}^{a\times b\times c}$, we let $\text{vec}(x) \in \mathbb{R}^{abc}$ be the reshaped form of the tensor into a vector (in an arbitrary but consistent pattern).

b. *[15 points]* **Fully-connected output, 1 fully-connected hidden layer:** this network has one hidden layer denoted as $x^{hidden} \in \mathbb{R}^M$ where M will be a hyperparameter you choose (M could be in the hundreds). The nonlinearity applied to the hidden layer will be the `relu` ($\text{relu}(x) = \max\{0, x\}$. This network can be written as

$$x^{out} = W_2 \text{relu}(W_1 \text{vec}(x^{in}) + b_1) + b_2$$

where $W_1 \in \mathbb{R}^{M \times 3072}$, $b_1 \in \mathbb{R}^M$, $W_2 \in \mathbb{R}^{10 \times M}$, $b_2 \in \mathbb{R}^{10}$.

c. *[15 points]* **Convolutional layer with max-pool and fully-connected output:** for a convolutional layer W_1 with filters of size $k \times k \times 3$, and M filters (reasonable choices are $M = 100$, $k = 5$), we have that $\text{Conv2d}(x^{in}, W_1) \in \mathbb{R}^{(33-k) \times (33-k) \times M}$.

- Each convolution will have its own offset applied to each of the output pixels of the convolution; we denote this as $\text{Conv2d}(x^{in}, W) + b_1$ where b_1 is parameterized in $\mathbb{R}^M$. Apply a `relu` activation to the result of the convolutional layer.
- Next, use a max-pool of size $N \times N$ (a reasonable choice is $N = 14$ to pool to 2×2 with $k = 5$) we have that $\text{MaxPool}(\text{relu}(\text{Conv2d}(x^{in}, W_1) + b_1)) \in \mathbb{R}^{\lfloor \frac{33-k}{N} \rfloor \times \lfloor \frac{33-k}{N} \rfloor \times M}$.
- We will then apply a fully-connected layer to the output to get a final network given as

$$x^{output} = W_2 \text{vec}(\text{MaxPool}(\text{relu}(\text{Conv2d}(x^{input}, W_1) + b_1))) + b_2$$

where $W_2 \in \mathbb{R}^{10 \times M(\lfloor \frac{33-k}{N} \rfloor)^2}$, $b_2 \in \mathbb{R}^{10}$.

The parameters M, k, N (in addition to the step size and momentum) are all hyperparameters, but you can choose a reasonable value. Tuning can be performed in the next subproblem.

d. *[5 points]* **Tuning:** Return to the original network you were left with at the end of the tutorial *Training a classifier*. Tune the different hyperparameters (number of convolutional filters, filter sizes, dimensionality of the fully-connected layers, stepsize, etc.) and train for a sufficient number of iterations to achieve a *test accuracy* of at least 70%. Provide the hyperparameter configuration used to achieve this performance.

The number of hyperparameters to tune in the last exercise, combined with the slow training times, will hopefully give you a taste of how difficult it is to construct networks with good generalization performance. State-of-the-art networks can have dozens of layers, each with their own hyperparameters to tune. Additional hyperparameters you are welcome to play with if you are so inclined, include: changing the activation function, replace max-pool with average-pool, and experimenting with batch normalization or dropout.

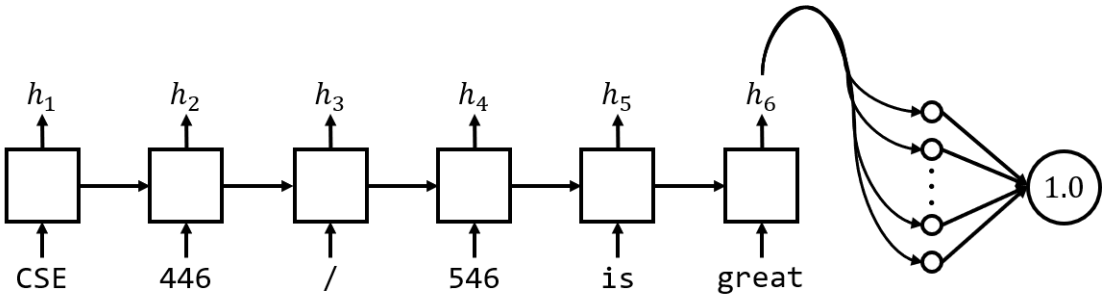
Text classification on SST-2

A6. The Stanford Sentiment Treebank (SST-2) is a dataset of movie reviews. Each review is annotated with a label indicating whether the sentiment of the review is positive or negative. Below are some examples from the dataset. Note that often times the reviews are only partial sentences or even single words.

Sequence	Sentiment
is one big , dumb action movie .	Negative
perfectly executed and wonderfully sympathetic characters ,	Positive
until then there 's always these rehashes to feed to the younger generations	Negative
is like nothing we westerners have seen before .	Positive

In this problem you will use a Recurrent Neural Network (RNN) for classifying reviews as either Positive (1) or Negative (0).

Using an RNN for binary classification



Above is a simplified visualization of the RNN you will be building. Each token of the input sequence (*CSE*, *446*, ...) is fed into the network sequentially. Note that in reality, we convert each token to some integer index. But training with discrete values does not work well, so we also "embed" the words in a high-dimensional continuous space. We already provided an `nn.Embedding` layer to you to do this. Each RNN cell (squares above) generates a hidden state h_i. We then feed the last hidden state into a simple fully-connected layer, which then produces a single prediction between 0 and 1.

Setup

1. Download the GloVe embeddings from `http://nlp.stanford.edu/data/glove.6B.zip`. Extract the zip file and move the file `glove.6B.50d.txt` into the same folder as the starter code (`util.py, train.py, hw4_a6.py`).
2. Download the SST-2 dataset from here `https://gluebenchmark.com/tasks` (Click on download next to `The Stanford Sentiment Treebank`). Extract the zip file into the same folder as above.

You only need to modify `hw4_a6.py`, however you are free to also modify the other two files. You will only submit `hw4_a6.py`, but if you make changes to the other files you should also include them in your submission.

Problems

a. *[10 points]* In Natural Language Processing (NLP), we usually have to deal with variable length data. In SST-2, each data point corresponds to a review, and reviews often have different lengths. The issue is that to efficiently train a model with textual data, we need to create batches of data that are fixed size matrices. In order to store a batch of sequences in a single matrix, we add padding to shorter sequences so that each sequence has the same length. Given a list of N sequences, we:

   1. Find the length of the longest sequence in the batch, call it `max_sequence_length`

2. Append padding tokens to the end of each sequence so that each sequence has length `max_sequence_length`
3. Stack the sequences into a matrix of size `(N, max_sequence_length)`

In this process, words are mapped to integer ids, so in the above process we actually store integers rather than strings. For the padding token, we simply use id 0. In the file `hw4_a6.py`, fill out `collate_fn` to perform the above batching process. Details are provided in the comment of the function.

b. *[15 points]* Implement the constructor and forward method for `RNNBinaryClassificationModel`. You will use three different types of recurrent neural networks: the vanilla RNN (`nn.RNN`), Long Short-Term Memory (`nn.LSTM`) and Gated Recurrent Units (`nn.GRU`). For the hidden size, use 64 (Usually this is a hyperparameter you need to tune). We have already provided you with the embedding layer to turn token indices into continuous vectors of size 50, but you will need a linear layer to transform the last hidden state of the recurrent layer to a shape that can be interpreted as a label prediction. The code for each of the three networks will only differ by the recurrent layer you use. In your code submission, use any of the three layers.

c. *[5 points]* Implement the `loss` method of `RNNBinaryClassificationModel`, which should compute the binary cross-entropy loss between the predictions and the target labels. Also implemented the `accuracy` method, which given the predictions and the target labels should return the accuracy.

d. *[15 points]* We have already provided all of the data loading, training and validation code for you. Choose an appropriate batch size, learning rate and number of epochs and set the constants `TRAINING_BATCH_SIZE`, `NUM_EPOCHS`, `LEARNING_RATE` accordingly. With a good learning rate, you shouldn't have to train for more than 16 epochs. Report your best validation loss and corresponding validation accuracy, corresponding training loss and training accuracy for each of the three types of recurrent neural networks.

e. *[5 points]* Currently we are only using the final hidden state of the RNN to classify the entire sequence as being either positive or negative sentiment. But can we make use of the other hidden states? Suppose you wanted to use the same architecture for a related task called tagging. For tagging, the goal is to predict a tag for each token of the sequence. For instance, we might want predict the part of speech tag (verb, noun, adjective etc.) of each token. In a few sentences, describe how you would modify the current architecture to predict a tag for each token.

f. (Extra Credit: *[1 points]*) When you run training, the model will print out 8 random reviews. What is the funniest review you have encountered after running the code a few times?

Matrix Completion and Recommendation System

B1. You will build a personalized movie recommender system. We will use the 100K MovieLens dataset available at `https://grouplens.org/datasets/movielens/100k/`. There are $m = 1682$ movies and $n = 943$ users. Each user rated at least 20 movies, but some watched many more. The total dataset contains $100,000$ total ratings from all users. The goal is to recommend movies the users haven't seen.
Consider a matrix $R \in \mathbb{R}^{m \times n}$ where the entry $R_{i,j} \in \{1, \ldots, 5\}$ represents the jth user's rating on movie i. A higher value represents that the user is more satisfied with the movie.

We may think of our historical data as some observed entries of this matrix while many remain unknown, and we wish to estimate the unknown ratings that each user would assign to each movie. We could use these ratings to recommend the "best" movies for each user.

The dataset contains user and movie metadata which we will ignore. We strictly use the ratings contained in the `u.data` file. Use this data file and the following python code to construct a training and test set:

```
import csv
import numpy as np
data = []
with open('u.data') as csvfile:
    spamreader = csv.reader(csvfile, delimiter='\t')
    for row in spamreader:
        data.append([int(row[0])-1, int(row[1])-1, int(row[2])])
data = np.array(data)

num_observations = len(data)   # num_observations = 100,000
num_users = max(data[:,0])+1   # num_users = 943, indexed 0,...,942
num_items = max(data[:,1])+1   # num_items = 1682 indexed 0,...,1681

np.random.seed(1)
num_train = int(0.8*num_observations)
perm = np.random.permutation(data.shape[0])
train = data[perm[0:num_train],:]
test = data[perm[num_train::],:]
```

The arrays `train` and `test` contain R_{train} and R_{test}, respectively. Each line takes the form "j, i, s", where j is the user index, i is the movie index, and s is the user's score.

Using `train`, you will train a model that can predict $\widehat{R} \in \mathbb{R}^{m \times n}$, how every user would rate every movie. You will evaluate your model based on the average squared error on `test`:

$$\mathcal{E}_{\text{test}}(\widehat{R}) = \frac{1}{|\text{test}|} \sum_{(i,j,R_{i,j}) \in \text{test}} (\widehat{R}_{i,j} - R_{i,j})^2.$$

Low-rank matrix factorization is a baseline method for personalized recommendation. It learns a vector representation $u_i \in \mathbb{R}^d$ for each movie and a vector representation $v_j \in \mathbb{R}^d$ for each user, such that the inner product $\langle u_i, v_j \rangle$ approximates the rating $R_{i,j}$. You will build a simple latent factor model.

You will implement multiple estimators and use the inner product $\langle u_i, v_j \rangle$ to predict if user j likes movie i in the test data. For simplicity, we will put aside best practices and choose hyperparameters by using those that minimize the test error. You may use fundamental operators from `numpy` or `pytorch` in this problem (`numpy.linalg.lstsq, SVD, autograd,` etc.) but not any precooked algorithm from a package

like `scikit-learn`. If there is a question whether some package is not allowed for use in this problem, it probably is not appropriate.

a. *[5 points]* Our first estimator pools all users together and, for each movie, outputs as its prediction the average user rating of that movie in `train`. That is, if $\mu \in \mathbb{R}^m$ is a vector where μ_i is the average rating of the users that rated the ith movie, write this estimator $\widehat{R}$ as a rank-one matrix. Compute the estimate $\widehat{R}$. What is $\mathcal{E}_{\text{test}}(\widehat{R})$ for this estimate?

b. *[5 points]* Allocate a matrix $\widetilde{R}_{i,j} \in \mathbb{R}^{m \times n}$ and set its entries equal to the known values in the training set, and 0 otherwise. Let $\widehat{R}^{(d)}$ be the best rank-d approximation (in terms of squared error) approximation to $\widetilde{R}$. This is equivalent to computing the singular value decomposition (SVD) and using the top d singular values. This learns a lower-dimensional vector representation for users and movies, assuming that each user would give a rating of 0 to any movie they have not reviewed.

- For each $d = 1, 2, 5, 10, 20, 50$, compute the estimator $\widehat{R}^{(d)}$. We recommend using an efficient solver such as `scipy.sparse.linalg.svds`.
- Plot the average squared error of predictions on the training set and test set on a single plot, as a function of d.

Note that, in most applications, we would not actually allocate a full $m \times n$ matrix. We do so here only because our data is relatively small and it is instructive.

c. *[10 points]* Replacing all missing values by a constant may impose strong and potentially incorrect assumptions on the unobserved entries of R. A more reasonable choice is to minimize the MSE (mean squared error) only on rated movies. Define a loss function:

$$L\Big(\{u_i\}_{i=1}^m, \{v_j\}_{j=1}^n\Big) := \sum_{(i,j,R_{i,j}) \in \text{train}} (\langle u_i, v_j \rangle - R_{i,j})^2 + \lambda \sum_{i=1}^m \|u_i\|_2^2 + \lambda \sum_{j=1}^n \|v_j\|_2^2 \tag{1}$$

where $\lambda > 0$ is the regularization coefficient. We will implement algorithms to learn vector representations by minimizing (1).

Since this is a non-convex optimization problem, the initial starting point and hyperparameters may affect the quality of $\widehat{R}$. You may need to tune λ and σ to optimize the loss you see.

- *Alternating minimization*: First, randomly initialize $\{u_i\}$ and $\{v_j\}$. Then, alternatate between (1) minimizing the loss function with respect to $\{u_i\}$ by treating $\{v_j\}$ as fixed; and (2) minimizing the loss function with respect to $\{v_j\}$ by treating $\{u_i\}$ as fixed. Repeat (1) and (2) until both $\{u_i\}$ and $\{v_j\}$ converge. Note that when one of $\{u_i\}$ or $\{v_j\}$ is given, minimizing the loss function with respect to the other part has a closed-form solution.
- Try $d \in \{1, 2, 5, 10, 20, 50\}$ and plot the mean squared error of train and test as a function of d.

Some hints: Common choices for initializing the vectors $\{u_i\}_{i=1}^m$, $\{v_j\}_{j=1}^n$ include: entries drawn from `np.random.rand()` scaled by some scale factor $\sigma > 0$ (σ is an additional hyperparameter), or using one of the solutions from part b or c. You should never be allocating an $m \times n$ matrix for this problem.

d. *[10 points]* Repeat part c, using batched SGD rather than alternating minimization.

Stochastic Gradient Descent: First, randomly initialize $\{u_i\}$ and $\{v_j\}$. Then take a batch of random samples (of size b from your training set and compute a gradient step, and repeat until convergence.

Note that batch size b, regularization constant λ, scaling parameter σ and learning rate η are all hyperparameters. Since this is a non-convex optimization, the results may be quite sensitive to these hyperparameters and to the initialization.

One strategy for choosing η is to select the largest constant value such that the loss L still tends to decrease. Another strategy is to pick a relatively large value of η and then scale it by some factor $\beta \in (0, 1)$ so that $\eta \mapsto \beta\eta$ every time a number of examples are seen that exceeds the size of the training set.

Feel free to modify the loss function to, say, different regularizers if it helps reduce the test error. See `http://www.optimization-online.org/DB_FILE/2011/04/3012.pdf` for some ideas.

e. *[5 points]* Briefly, in words, compare the results of the prior two parts. This is an example where the loss functions are identical, but the *algorithm* used has drastic impact on how much the model fits, overfits, or generalizes to new, unseen data.

f. (Extra credit: *[5 points]*). Implement any algorithm you'd like (you must implement it yourself; do not use an off-the-shelf algorithm from e.g. `scikit-learn`) to find an estimator that achieves a test error of no more than 0.9. Please include your code.