CSE546 HW0 B

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B.1

PDF is:

$$f(x) = \begin{cases} 1, & \text{for } 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

CDF is:

$$F_x(x) = \int_{-\infty}^{x} f(x)dx = x$$

Let me calculate the PDF of: $Y = Max(X_1, X_2, ..., X_n)$, and here we need to use joint probability to compute.

$$P(Max(X_1, X_2, ..., X_n) = P(X_1 \leq X, X_2 \leq X, ..., X_{n-1} \leq X)$$

So we have:

$$f_Y(x) = n[F_x(x)]^{n-1} f(x) = n(x)^{n-1} * 1 = n(x)^{n-1}$$

$$= \begin{cases} nx^{n-1}, & \text{for } 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Now we need $E[Y]$, we use mean equation for continuous random variable.

$$E[Y] = \int_0^1 x f_M(x)dx = \int_0^1 xnx^{n-1}dx = n\int_0^1 x^n dx = \frac{n}{n+1}$$

B.2

Since, $A \in \mathbb{R}^{\mathrm{n}\times\mathrm{m}}$, and $B \in \mathbb{R}^{\mathrm{m}\times\mathrm{n}}$ So,

$$\begin{aligned} tr(AB) &= \sum_{i=1}^{m} (AB)_{ii} \\ &= \sum_{i=1}^{m} \sum_{j=q}^{n} A_{i,j} B_{j,i} \\ &= \sum_{j=1}^{n} \sum_{i=1}^{m} B_{j,i} A_{i,j} \\ &= \sum_{j=1}^{n} (BA)_{j,j} \\ &= tr(BA) \end{aligned} \tag{1}$$

B.3

a.

When we consider a matrix of d by d, so,
Max rank: d and Min rank: 1;

b.

Because V is a b by n matrix, and v_i is none zero vectors, so,
Max rank: $min(d, n)$, Min rank: 1;

c.

Since A is D by d and v_i is d by 1, so Av_i is D by 1. So, we could know that $(Av_i)^T$ is 1 by D.

So $(Av_i)(Av_i)^T$ is D by D. Then,
Max rank: D, Min rank: 0.

d.

Since V is d by n, so AV is D by n, so
For AV, Max: min(d,n), Min is 0.
If V is rank d, Max: d, Min: 0.