



# ***Resonant Level Enhancement of the Thermoelectric Power of $\text{Bi}_2\text{Te}_3$ with Tin***

Joseph P. Heremans and Christopher M. Jaworski

The Ohio State University

Vladimir A. Kulbachinskii

Moscow State University

Ackn. Vladimir Jovovic, Yibin Gao, Hyungyu Jin, Michele Nielsen, Yun Zhang

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# Outline

The physics of resonant levels: mechanisms by which they enhance ZT

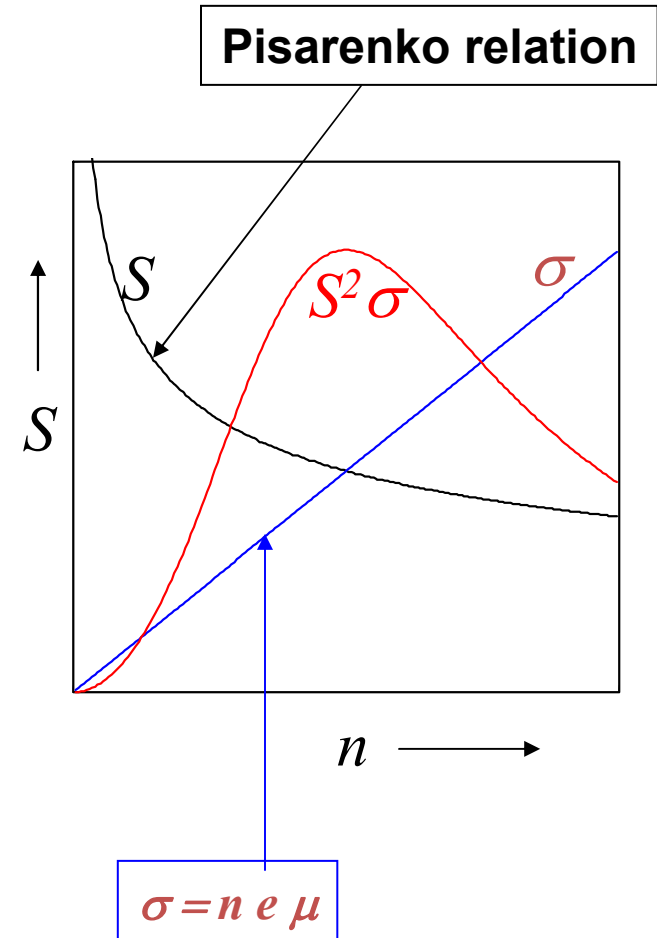
The Pisarenko (thermopower versus carrier density) relation in  $\text{Bi}_2\text{Te}_3$

Tin is a resonant level in the valence band of  $\text{Bi}_2\text{Te}_3$

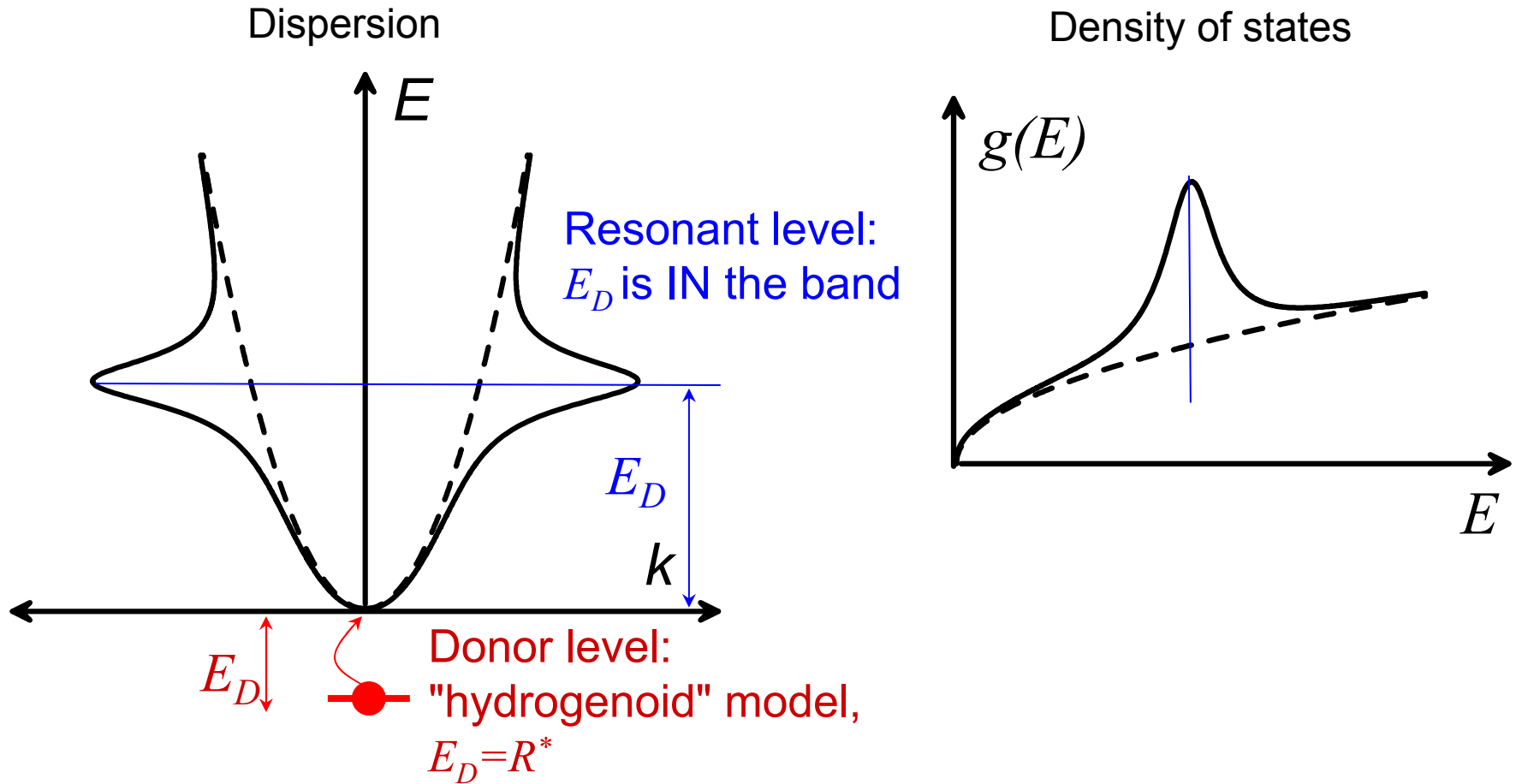
1. Band structure
2. Resistivity, Seebeck, Hall and Nernst effects

Enhancement in thermopower in single-crystal  $\text{Bi}_2\text{Te}_3$

Application to practical p-type thermoelectric  $(\text{Bi}_{30}\text{Sb}_{70})_2\text{Te}_3$  alloys for heat pumps



## Resonant energy levels: definition

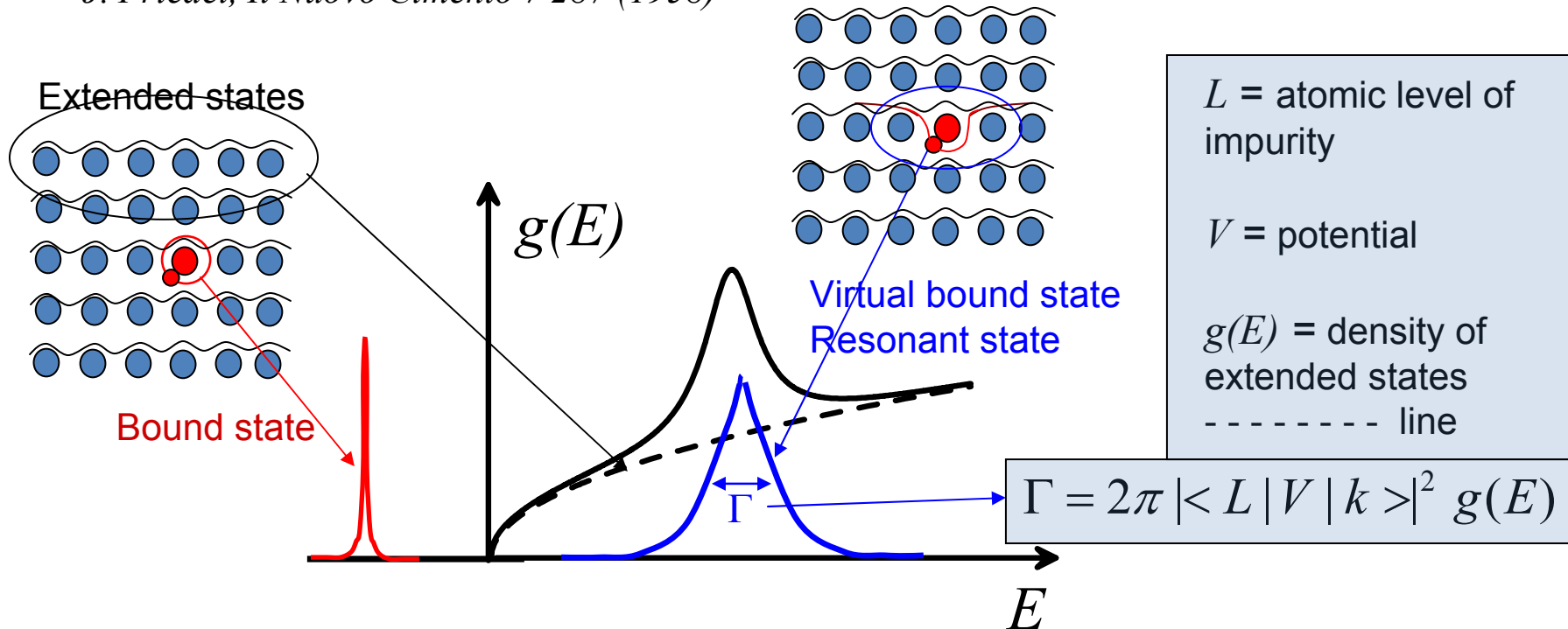


# Resonant levels in metals and semiconductors

- Concept comes from atomic physics
- First in metals: "Friedel States" or "Virtual bound states"

*J. Friedel, Can. J. Physics 34 1190 (1956)*

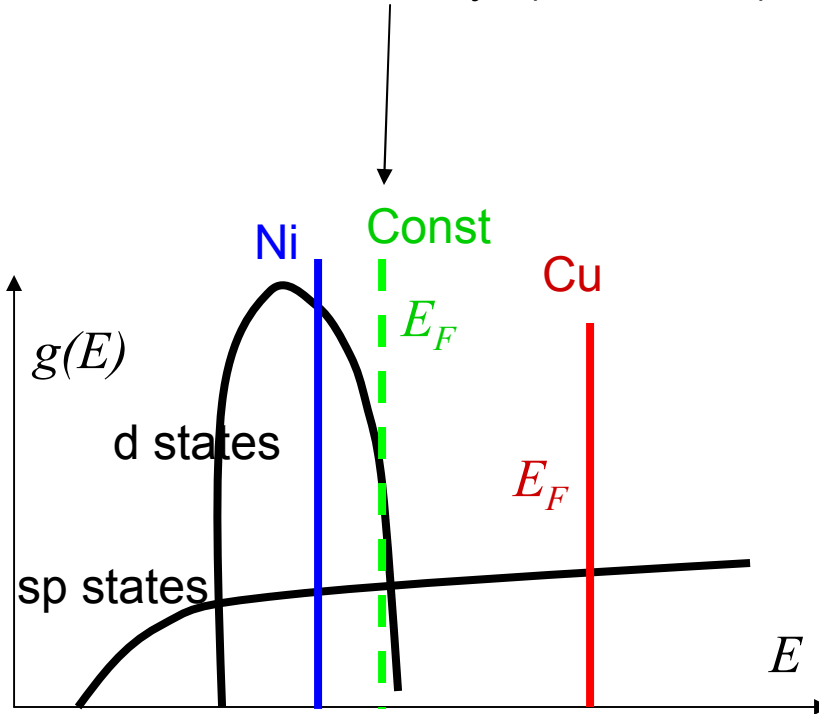
*J. Friedel, Il Nuovo Cimento 7 287 (1958)*



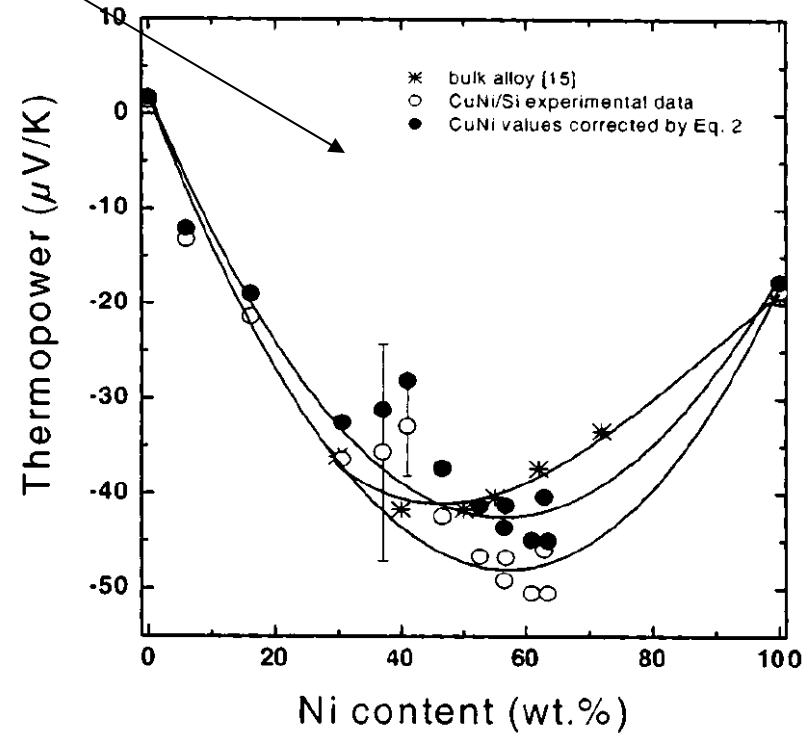
Friedel: "It is useful to think of the bound state as still existing, with a positive energy. But as it has now the same energy as an extended state, it will resonate with the  $l^{\text{th}}$  spherical component, to build up two extended states of slightly different energies; these in turn will have the same energies as the extended states with whom they will resonate, etc..."

# Similar to Kondo and thermocouple alloys

1. Isolated atoms, Friedel state, dilute limit
- 1 bis. with magnetic moment: Kondo effect (Au+0.02% Fe) *Prog. Theo. Phys.* 34 372, 1965
2. Resonant levels ( $\text{Pb}_{98}\text{Ti}_2\text{Te}$ ): semi-dilute alloys: states can interact
3. Concentrated alloys (Constantan)



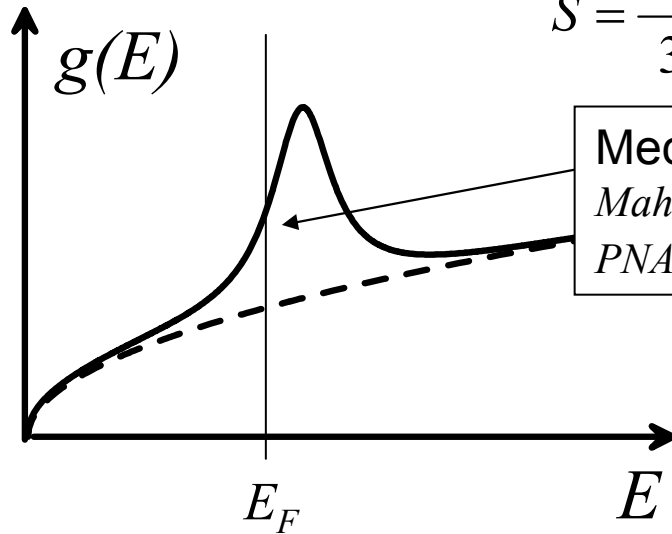
Constantan: main effect from  $g(E)$   
Thermocouple material up to 750°C



# Resonant levels increase thermopower

Mott relation for degenerate statistics

$$S = \frac{\pi^2}{3} \cdot \frac{k_B}{e} \cdot k_B T \cdot \left[ \frac{1}{g(E)} \cdot \frac{\partial g(E)}{\partial E} + \frac{1}{\mu(E)} \cdot \frac{\partial \mu(E)}{\partial E} \right]_{E=E_F}$$



**Mechanism 1**  
*Mahan and Sofo,*  
*PNAS* **93** 7436, 1996

## Mechanism 2: Resonant scattering

*A. Blandin & J. Friedel, Le Journal de Physique et le Radium* **20** 160, 1959

*In PbTe: Yu. Ravich, CRC Handbook on Thermoelectrics, D. M. Rowe, Ed. 1995*

*In Bi<sub>2</sub>Te<sub>3</sub>: M. K. Zhitinskaya, S. A. Nemov and T. E. Svechnikova, Phys. Solid State* **40** 1297, 1998

- Works great at cryogenic temperatures
- Will NOT give high  $zT$  at operating temperatures where acoustic/optic phonon scattering dominates

Which dominates? Can be proven experimentally by measuring Nernst effect

# Nernst coefficient can determine mechanism

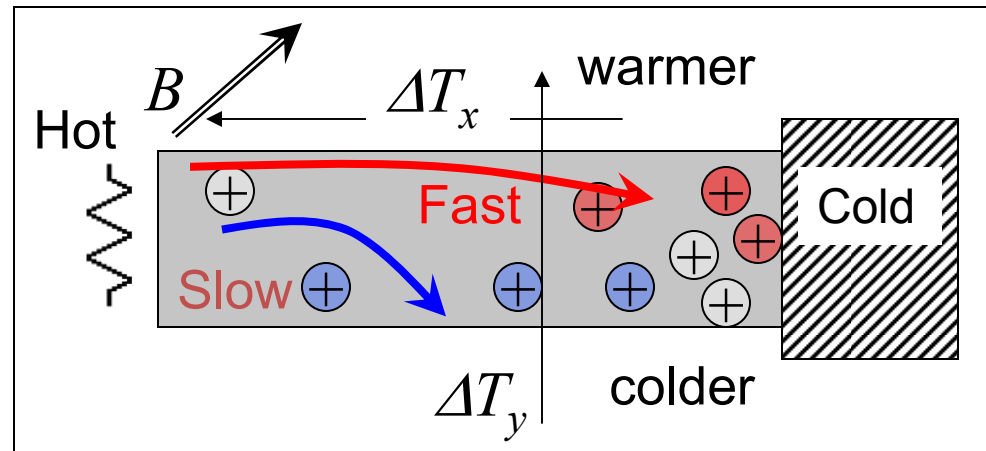
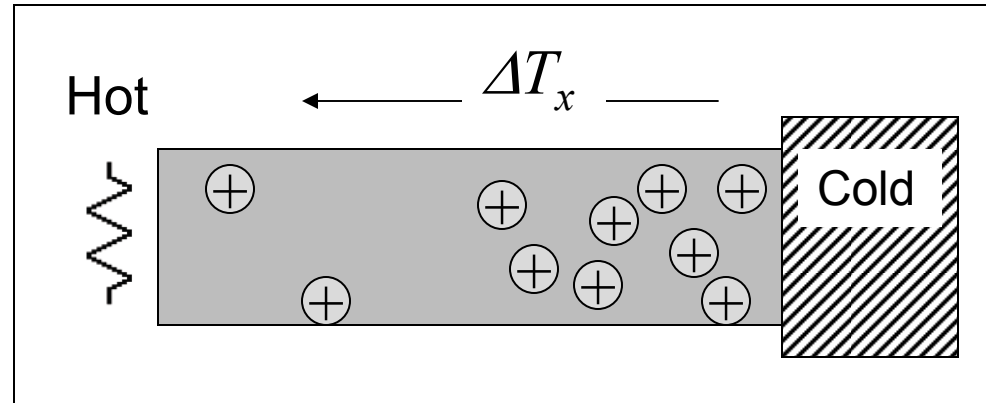
Seebeck coefficient:  
Charge carriers diffuse under  $\Delta T_x \Rightarrow$  Condense on cold side

Nernst:  
Slow-diffusing carriers are more deflected by magnetic field than fast-diffusing carriers

$\Rightarrow$  Lower energy carriers condense on one side

$\Rightarrow$  cools down  $\Rightarrow \Delta T_y$

Seebeck coefficient  $\times \Delta T_y \Rightarrow$   
Nernst coefficient  
Nernst gives energy-dependence of scattering mechanism



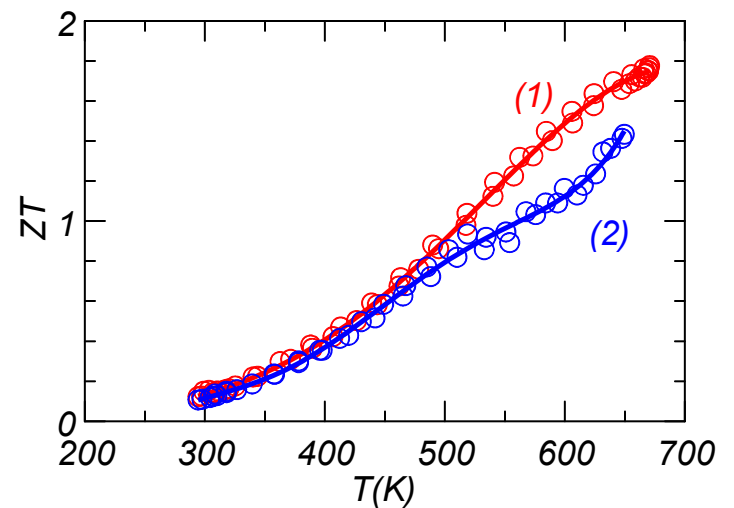
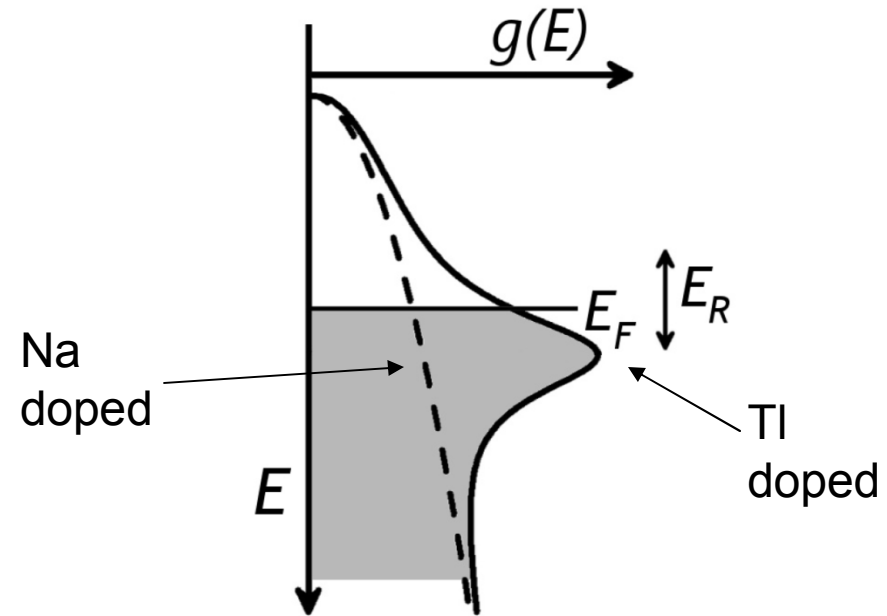
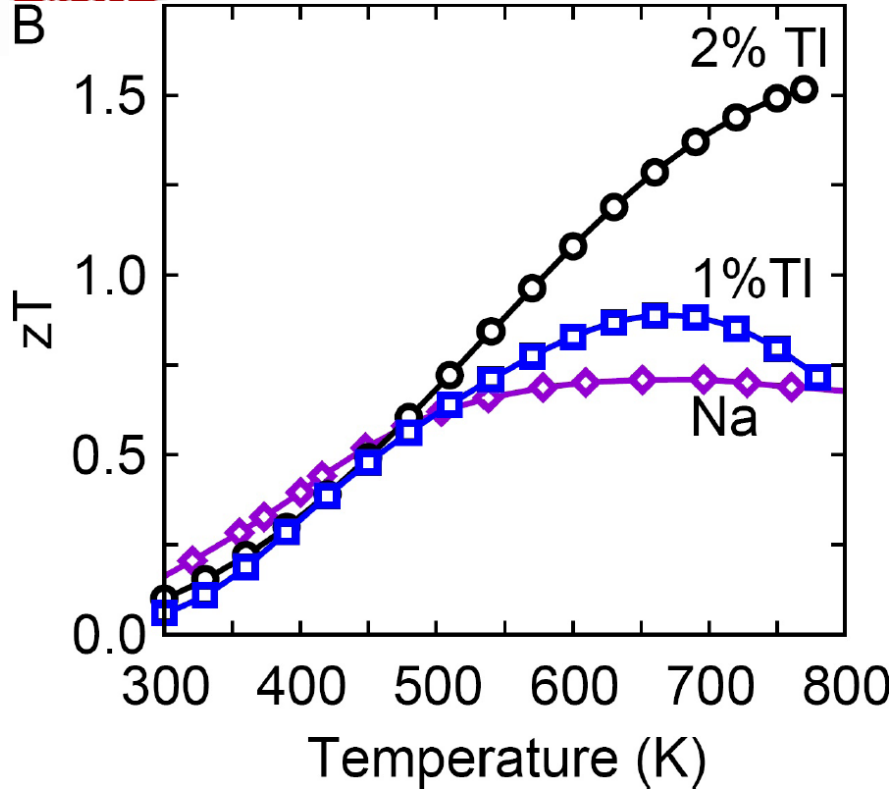
Very schematically for non-degenerate system:

$$\text{Define: } \tau = \tau_o E^\Lambda \Rightarrow N \approx \Lambda \mu \left( \frac{k_B}{|q|} \right)$$

If resonant scattering  $\Rightarrow$  Large  $\Lambda \Rightarrow$  Large  $N$

If ac. phonon scattering  $\Rightarrow \Lambda = -1/2 \Rightarrow -N/(88\mu V/K) \sim \mu/2$

# *PbTe:TI doubles $zT$*



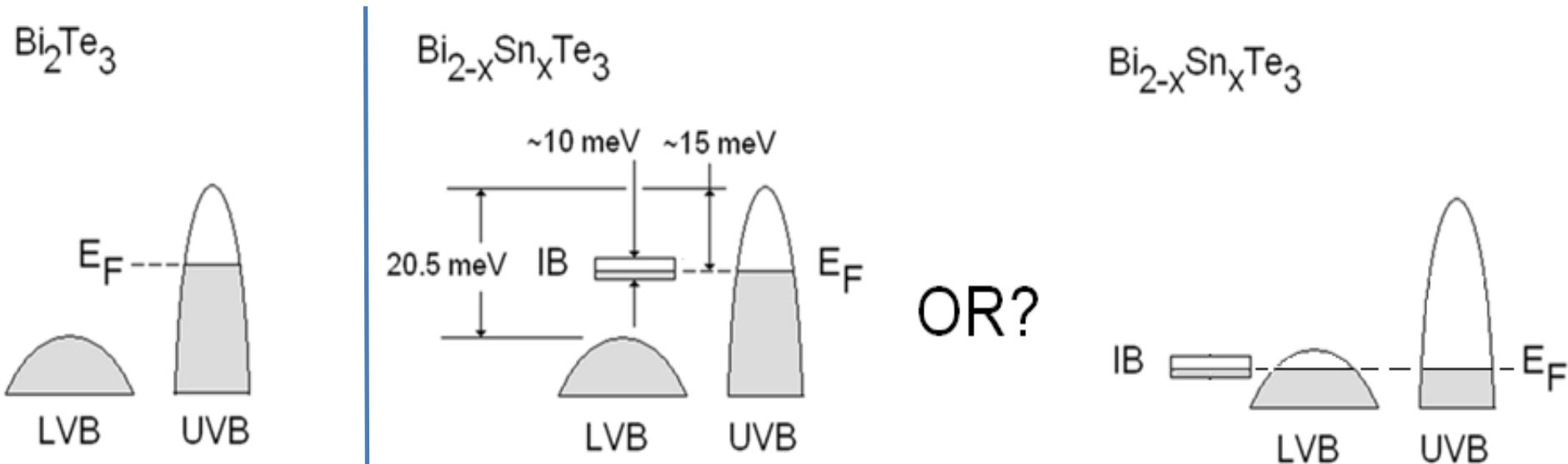


## $Bi_2Te_3:Sn$

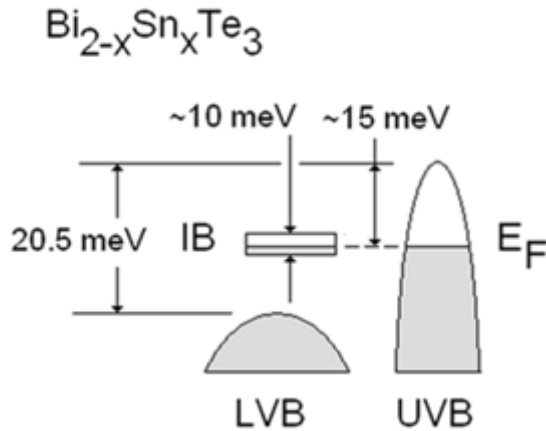
- Kulbachinskii identifies Sn as resonant level in  $Bi_2Te_3$   
*V. Kulbachinskii, N. B. Brandt et al., Phys. Stat. Sol. **150** 237 (1988)*
- Zhitinskaya suggests resonant SCATTERING boosts thermopower at 120 K (will NOT work when phonon scattering dominates, at 300K)  
*M.K. Zhitinskaya, S.A. Nemov, T.E. Svechnikova, p 72, 16th International Conference on Thermoelectrics (1997)*
- We use Kulbachinskii's Bridgeman  $Bi_{2-x}Sn_xTe_3$  single crystals with  $x=0.0025, 0.0075, 0.015$  (0.05, 0.15, 0.30 at% Sn)
- Measure four transport properties—  $S, N, R_H, \rho$  (2-400K) and use method of 4 coefficients
- Calculate Pisarenko relation (Thermopower vs. carrier concentration) for  $Bi_2Te_3$
- Measure Shubnikov-de Haas to determine area of the Fermi surface  
 $B \perp <001>$  axis, current //  $<100>$  axis for all measurements

# $Bi_2Te_3$ :Sn Proposed Valence Structure

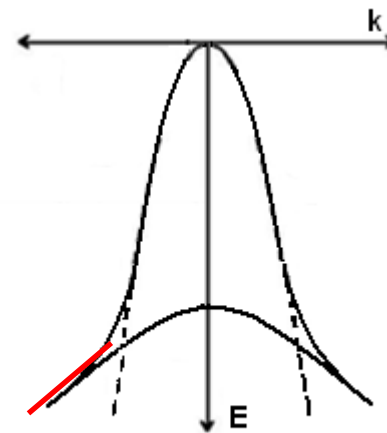
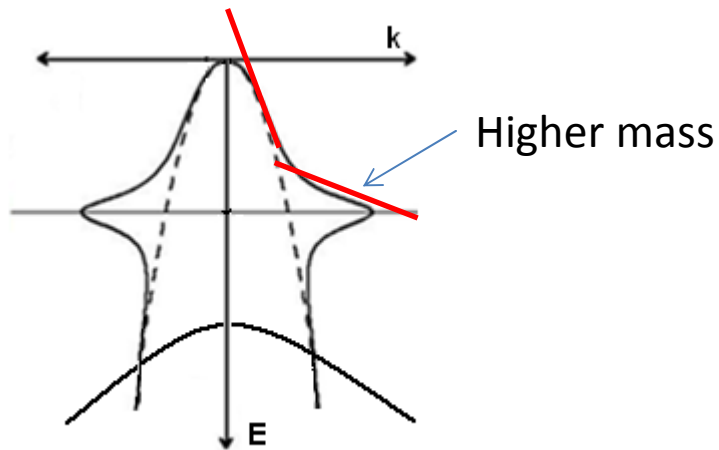
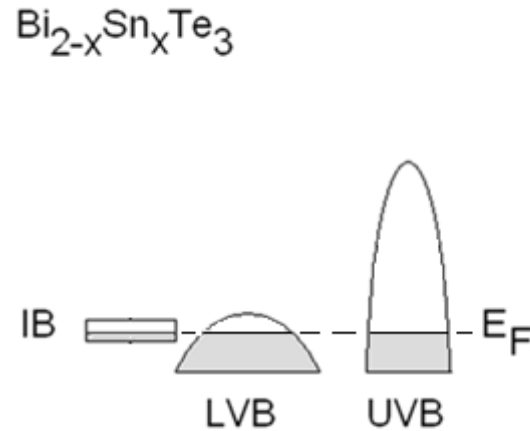
- Upper valence band with small mass  
*A. von Middendorff, G.Landwehr: Solid State Communications, 11 203 (1972)*
- Lower valence band (LVB) position: Kohler - 20.5 meV  
*H. Kohler, Physica Status Solidi (b), 74. 591 (1976)*  
In  $k$  space: LVB  $|\Gamma A|$  UVB:  $|\Gamma X|$
- Kulbachinski: Sn resonant impurity band 15meV below UVB  
*V.A.. Kulbachinskii, Physica Status Solidi (b), 199 (1997)*
- Zhitinskaya: Impurity Band (IB) is 10 meV wide  
*M. K. Zhitinskaya, Fizika Tverdogo Tela, 45, No. 7, (2003); Fizika i Tekhnika Poluprovodnikov 34 No 12 (2000)*



# *Bi<sub>2</sub>Te<sub>3</sub>:Sn Proposed Valence Structure*



OR?

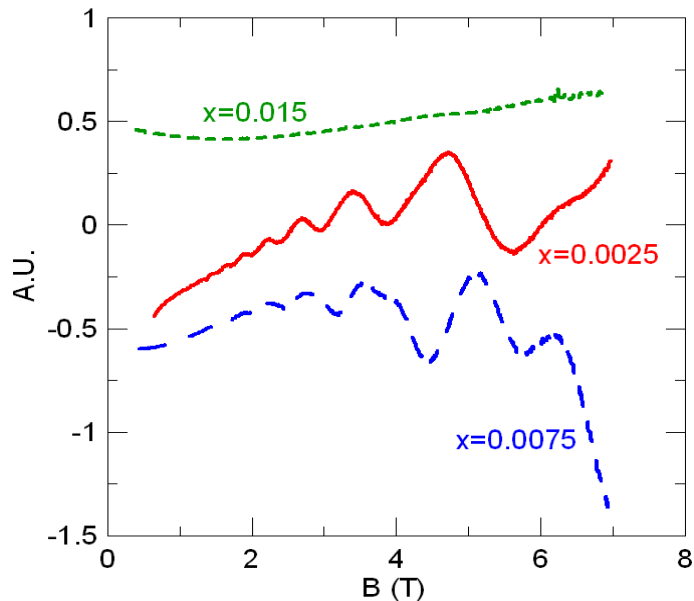


$$\frac{1}{\hbar^2} \left( \frac{\partial^2 E(k)}{\partial k^2} \right) = \frac{1}{m^*}$$

# Shubnikov-de Haas (SdH)

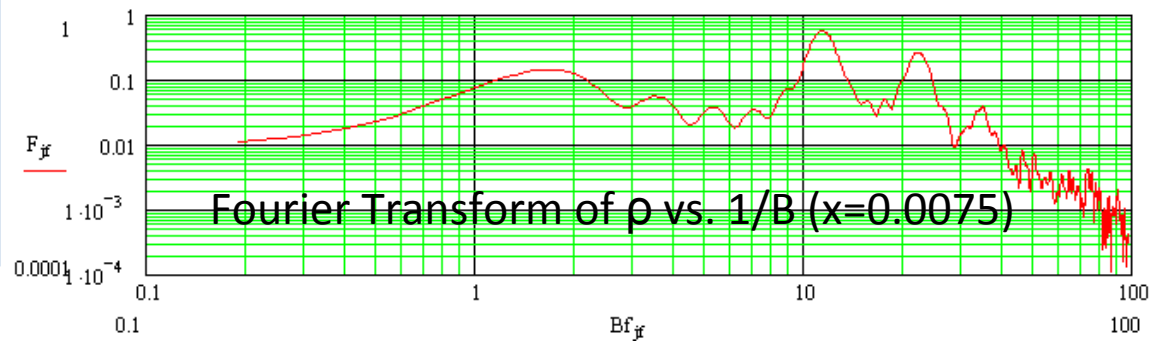
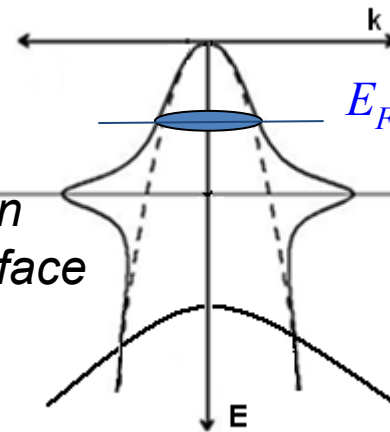
Oscillations in resistance periodic in  $1/\text{magnetic field}$

- Magnetic field quantizes allowable energy levels  $E_n = n\hbar\omega_C = n\hbar \frac{2B}{m_C^*}$
- Area of Fermi surface given by:  $\Delta \frac{1}{B} = \frac{2\pi \cdot q}{\hbar A_F}$
- Need mean free path longer than one cyclotron orbit:  $\omega_C \tau = \mu B \gg 1$



SdH oscillations in resistivity

*Period of Oscillations Measures Cross-section Of Fermi surface*

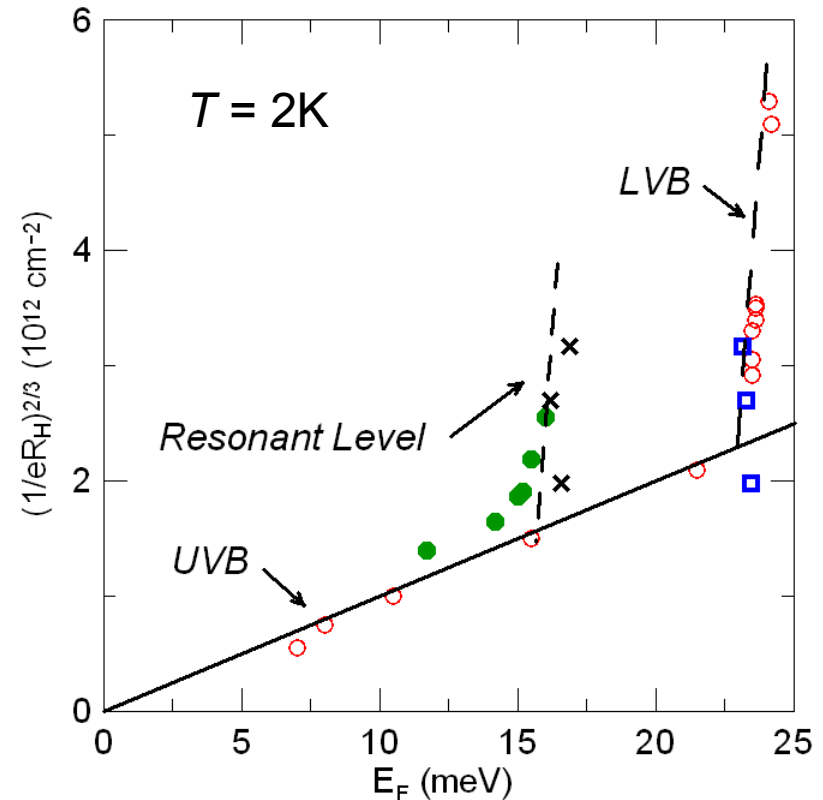


Fourier Transform of  $\rho$  vs.  $1/B$  (x=0.0075)

# Analysis of SdH oscillations

Evidence for resonant level from SdH alone is ambiguous: 2 harmonics or 2 periods?

| Tin Content                             | Oscillation Frequency  | Fermi Surface Area |
|---|------------------------|--------------------|
| $\text{Bi}_{2-x}\text{Sn}_x\text{Te}_3$ | $[\Delta(1/B)]^{-1} T$ | ( $\text{m}^2$ )   |
| $x=0.0025$                              | 12.7                   | 1.21E+17           |
|   | 23.5                   | 2.24E+17           |
| $x=0.0075$                              | 11.4                   | 1.09E+17           |
|   | 22.3                   | 2.13E+17           |
| $x=0.015$                               | 13.5                   | 1.29E+17           |
|   | 22.1                   | 2.11E+17           |

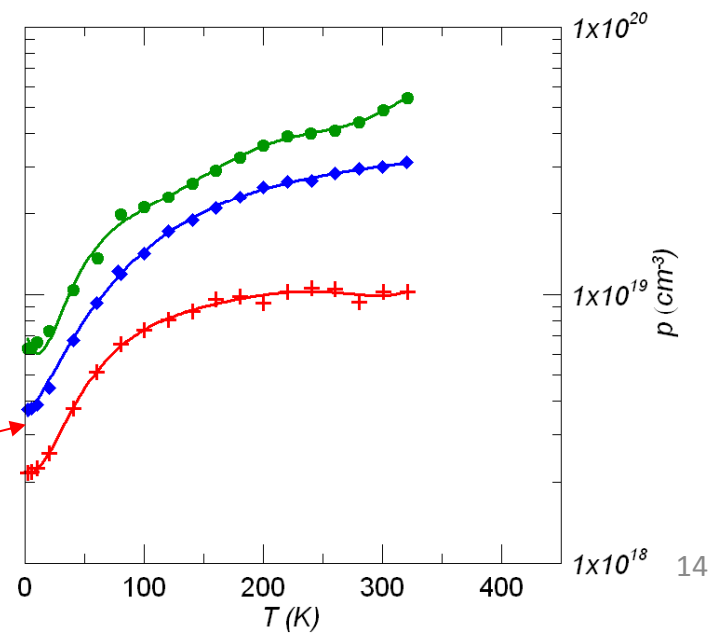
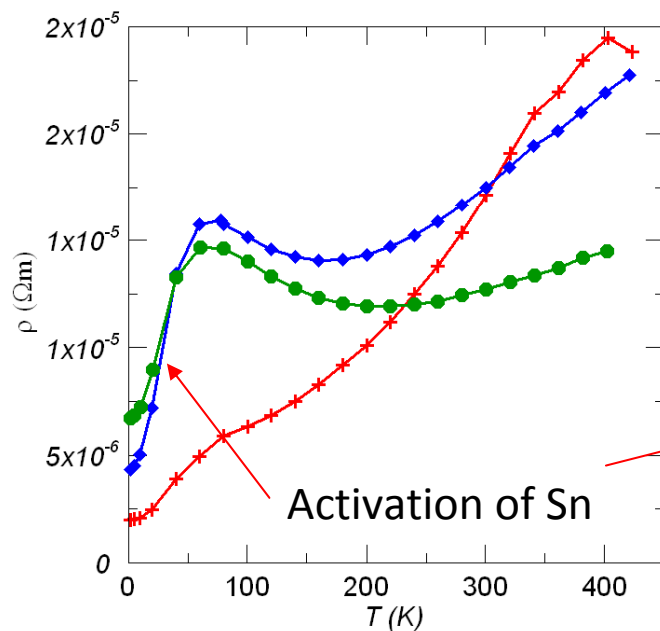
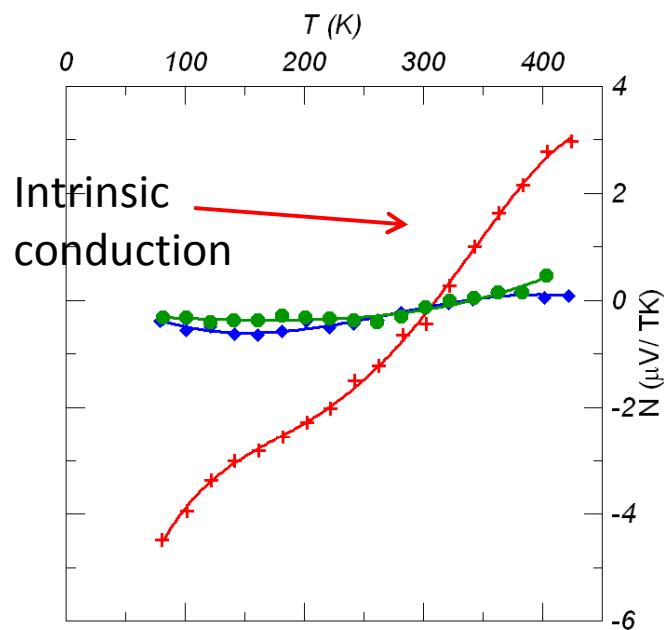
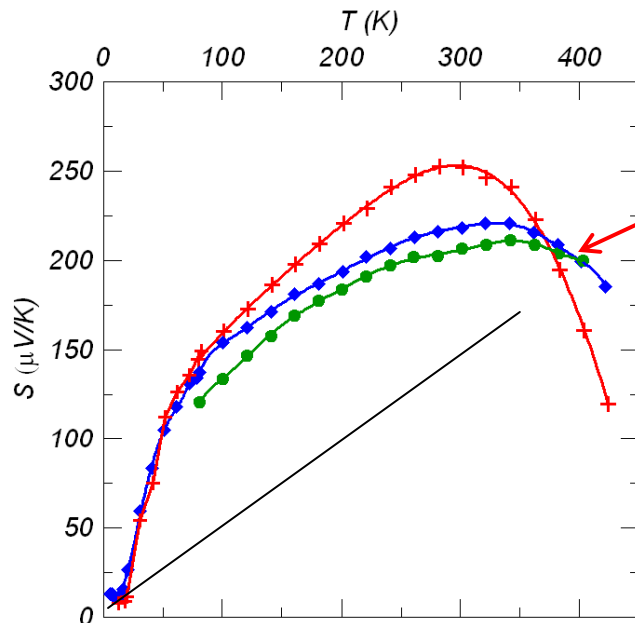
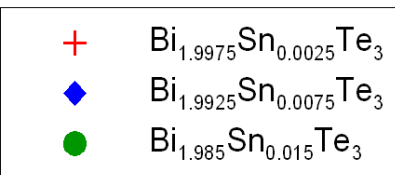


| Tin Content                             | Hall 2K carrier density      | Cyclotron mass | Fermi Level | $m_D^*$ UVB | Carriers in 1 <sup>st</sup> band | Carriers in 2 <sup>nd</sup> band | $m_D^*$ LVB |
|---|------------------------------|----------------|-------------|-------------|----------------------------------|----------------------------------|-------------|
| $\text{Bi}_{2-x}\text{Sn}_x\text{Te}_3$ | $p (\text{cm}^{-3}) 10^{18}$ | $m_c^*/m_e$    | meV         | $m_D^*/m_e$ | $p (\text{cm}^{-3}) 10^{18}$     | $p (\text{cm}^{-3}) 10^{18}$     | $m_D^*/m_e$ |
| $x=0.0025$                              | 2.78                         | 0.118          | 23.44       | 0.156       | .966                             | 1.814                            | 1.89        |
| $x=0.0075$                              | 4.44                         | 0.115          | 23.26       | 0.152       | .917                             | 3.523                            | 3.29        |
| $x=0.015$                               | 5.63                         | 0.113          | 23.14       | 0.149       | .882                             | 4.748                            | 4.19        |

(●)  $\text{Bi}_{2-x}\text{Sn}_x\text{Te}_3$  from (1),  
(○)  $\text{Bi}_2\text{Te}_3$  from (2),  
(□ and ▷)  $\text{Bi}_{2-x}\text{Sn}_x\text{Te}_3$  from this work using the method of 1(▷) and 2(□)  
The solid line is calculated from (2)

Assuming that LVB starts at 20.5 meV

# Transport Measurement Results



## Method of the four parameters

Hypotheses:

- Single-carrier system – not rigorously the case here
- Parabolic ( $\text{Bi}_2\text{Te}_3$ ) or non-parabolic bands
- Degenerate or non-degenerate statistics

Four unknown parameters

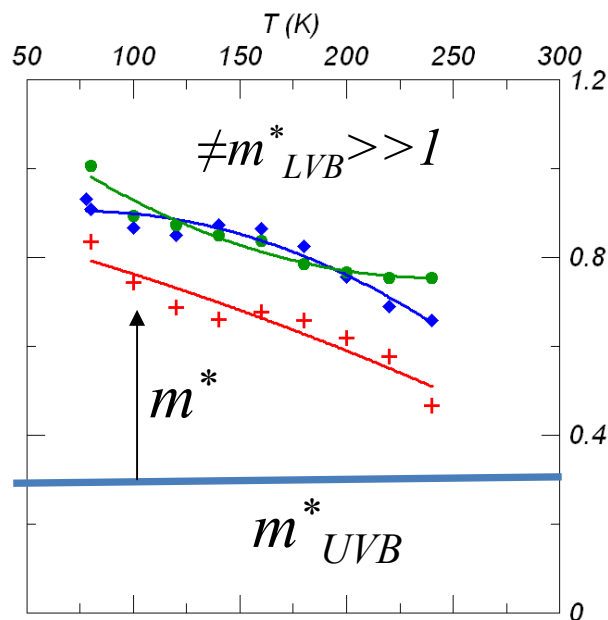
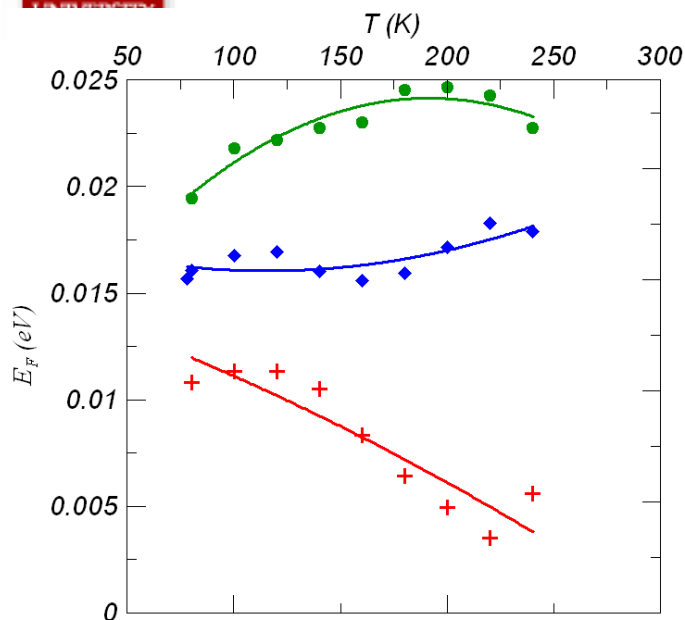
1. Density of carriers  $n$
2. Mobility of carriers  $\mu$
3. Effective mass  $m_{DOS}^*$ 
  - $(n) + (m_{DOS}^*) \Rightarrow (E_F)$
4. Energy dependence of scattering
 
$$\tau = \tau_0 E^\lambda$$

$\lambda$  = scattering exponent

Use four independent measurements at each temperature  $T$

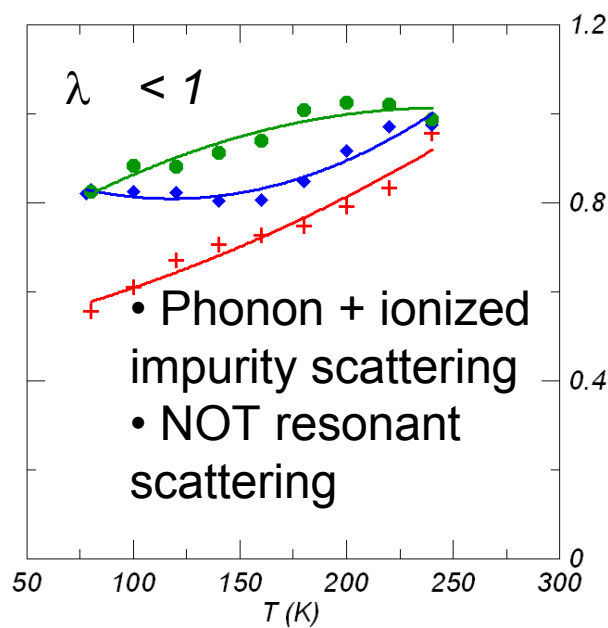
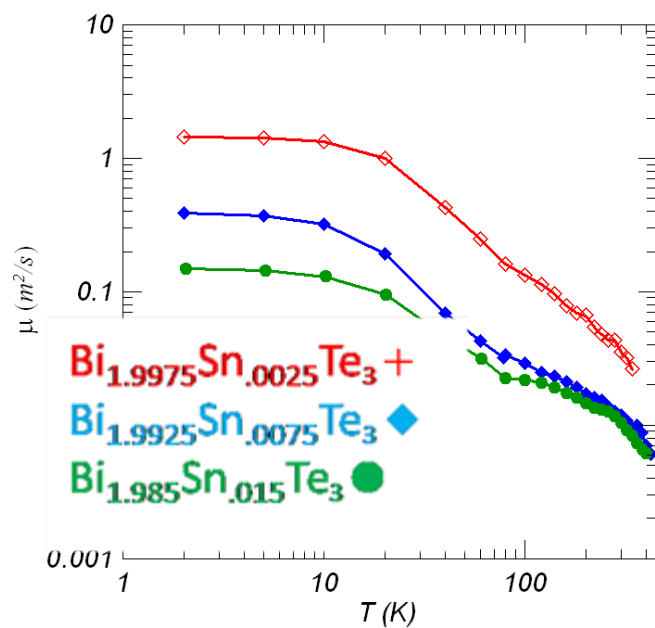
1. Resistivity  $\rho(T)$
2. Hall coefficient  $R_H(T)$
3. Thermopower  $S(T)$
4. Transverse isothermal Nernst-Ettingshausen coefficient  $N(T)$

# Results of 4-Parameter Fit



$$m_{UVB}^* < m^* < m_{LVB}^*$$

Increase in  $m^*$  for higher tin concentrations



- Phonon + ionized impurity scattering
- NOT resonant scattering

$\lambda$  Scattering exponent increases with Sn:  
Sn acts as an ionized impurity



## Calculation of Pisarenko Relation

Qualitative (here for non-degenerate statistics, for didactic purposes only):  $S$  depends on scattering mechanism  $\lambda$ , carrier concentrations, effective masses, and mobility

$$S \approx \frac{k}{q} \left( A(\Lambda) + \ln \frac{2(2\pi \cdot m_1^* k_b T)^{3/2}}{h^3 p_1} \right)$$

Quantitative: use Fermi integrals, assume parabolic model, multi-carrier conduction

$$p_{UVB} = \frac{6}{3\pi^2 \hbar^3} (2m_{UVB}^* k_B T)^{3/2} \int_0^\infty \left[ \frac{x^{3/2} e^{x-x_F}}{(1 + e^{x-x_F})^2} \right] dx \quad p_{LVB} = \frac{6}{3\pi^2 \hbar^3} (2m_{LVB}^* k_B T)^{3/2} \int_0^\infty \left[ \frac{x^{3/2} e^{x-(x_F-\Delta E_{UL})}}{(1 + e^{x-(x_F-\Delta E_{UL})})^2} \right] dx$$

$$\Delta E_{UL} = 20 \text{ meV}$$

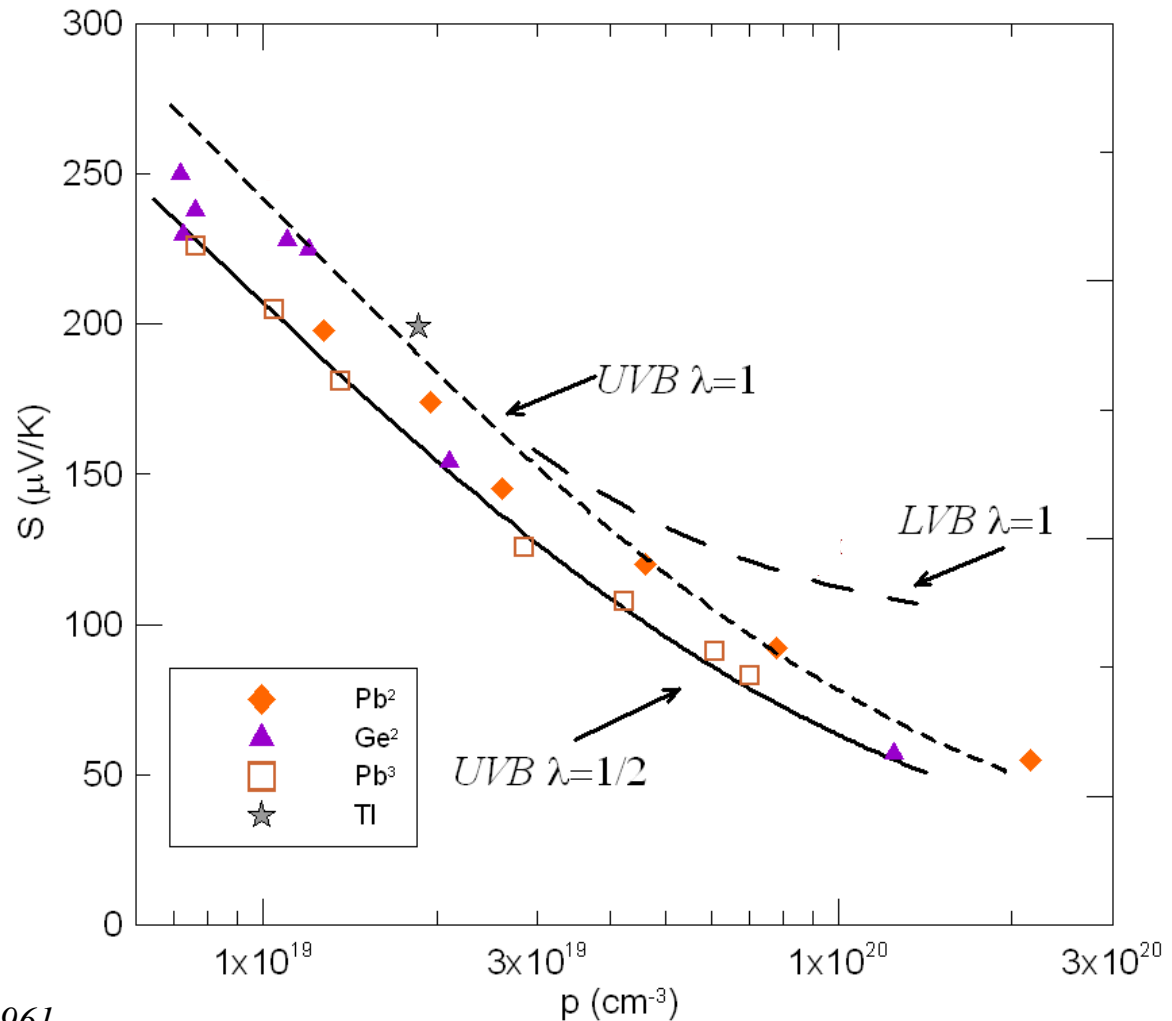
$$S_{LVB,UVB} = \frac{k_B}{q} \frac{\int_0^\infty \left[ \frac{x^{5/2+\lambda} e^{x-x_F}}{(1 + e^{x-x_F})^2} \right] dx}{\int_0^\infty \left[ \frac{x^{3/2+\lambda} e^{x-x_F}}{(1 + e^{x-x_F})^2} \right] dx} - x_F$$

$$\sigma_{ratio} = \frac{\int_0^\infty \left[ \frac{x^{3/2+\lambda} e^{x-x_F}}{(1 + e^{x-x_F})^2} \right] dx}{\int_0^\infty \left[ \frac{x^{3/2+\lambda} e^{x-(x_F-\Delta E_{UL})}}{(1 + e^{x-(x_F-\Delta E_{UL})})^2} \right] dx} \left( \frac{m_{UVB}^*}{m_{LVB}^*} \right)^{1/2}$$

$$S = \frac{S_{UVB} \sigma_{UVB} + S_{LVB} \sigma_{LVB}}{\sigma_{UVB} + \sigma_{LVB}} = \frac{S_{UVB} \cdot \sigma_{ratio} + S_{LVB}}{\sigma_{ratio} + 1}$$

## Pisarenko Relation for $Bi_2Te_3$ at 300K

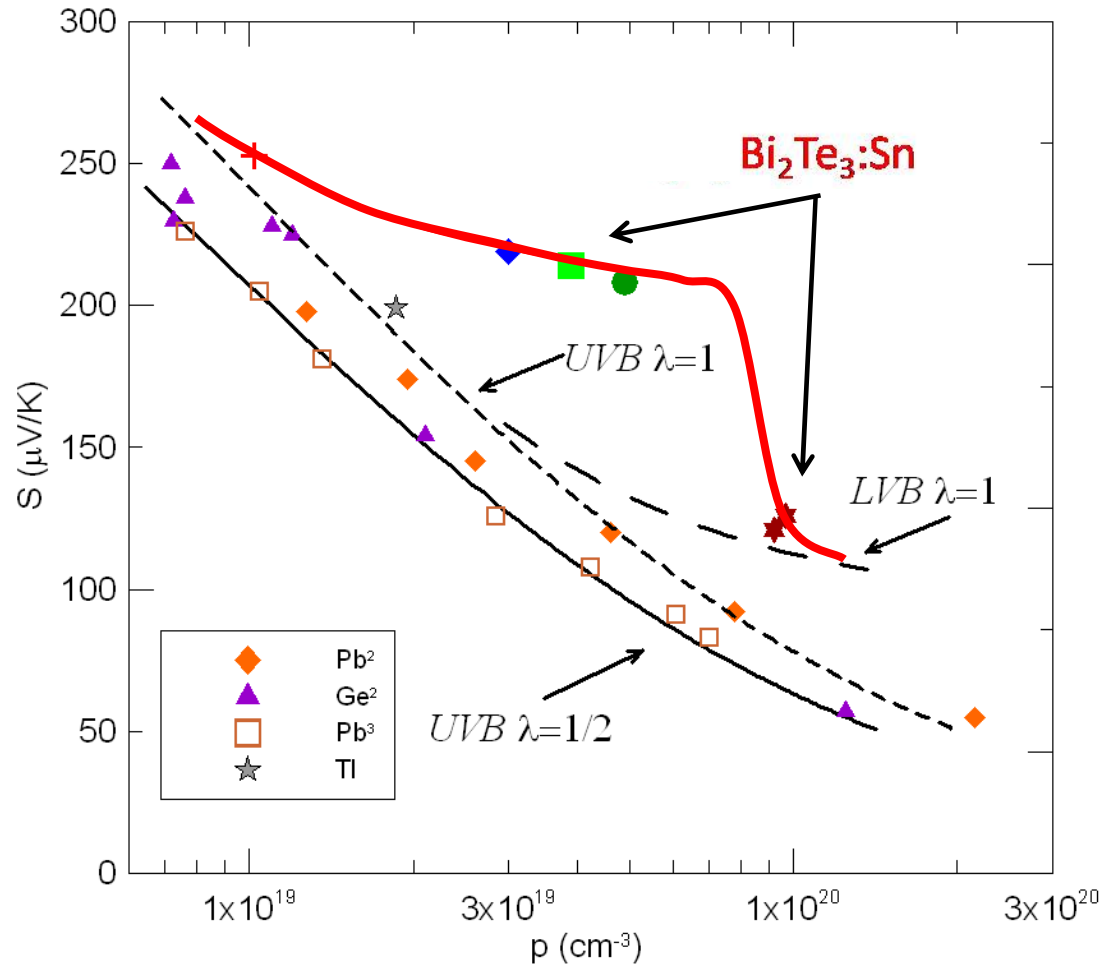
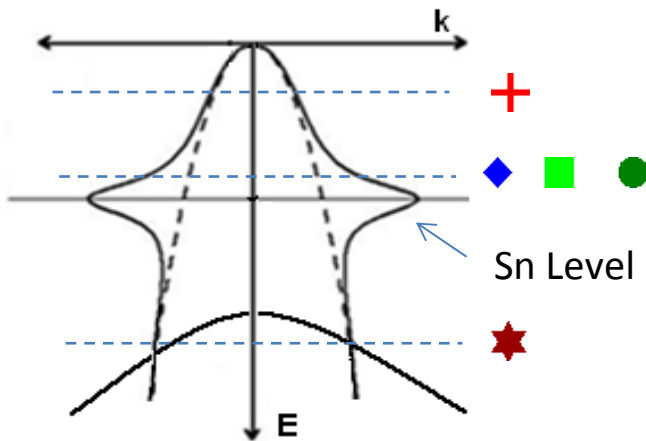
- Calculation of thermopower as function of carrier density
- UVB:  $\lambda = 1/2, 1$
- LVB:  $\lambda = 1$
- LVB starts at 20 meV below UVB



1. Ioffe, *Physics of Semiconductors*, 1961
2. Bergmann. 1169, s.l. : *Z Natuforsch*, 1963, Vol. 18a.
3. *Philosophical Magazine*, Volume 84, Issue 21 July 2004 , pages 2217 - 2228

# Pisarenko Relation for $\text{Bi}_2\text{Te}_3$ at 300K

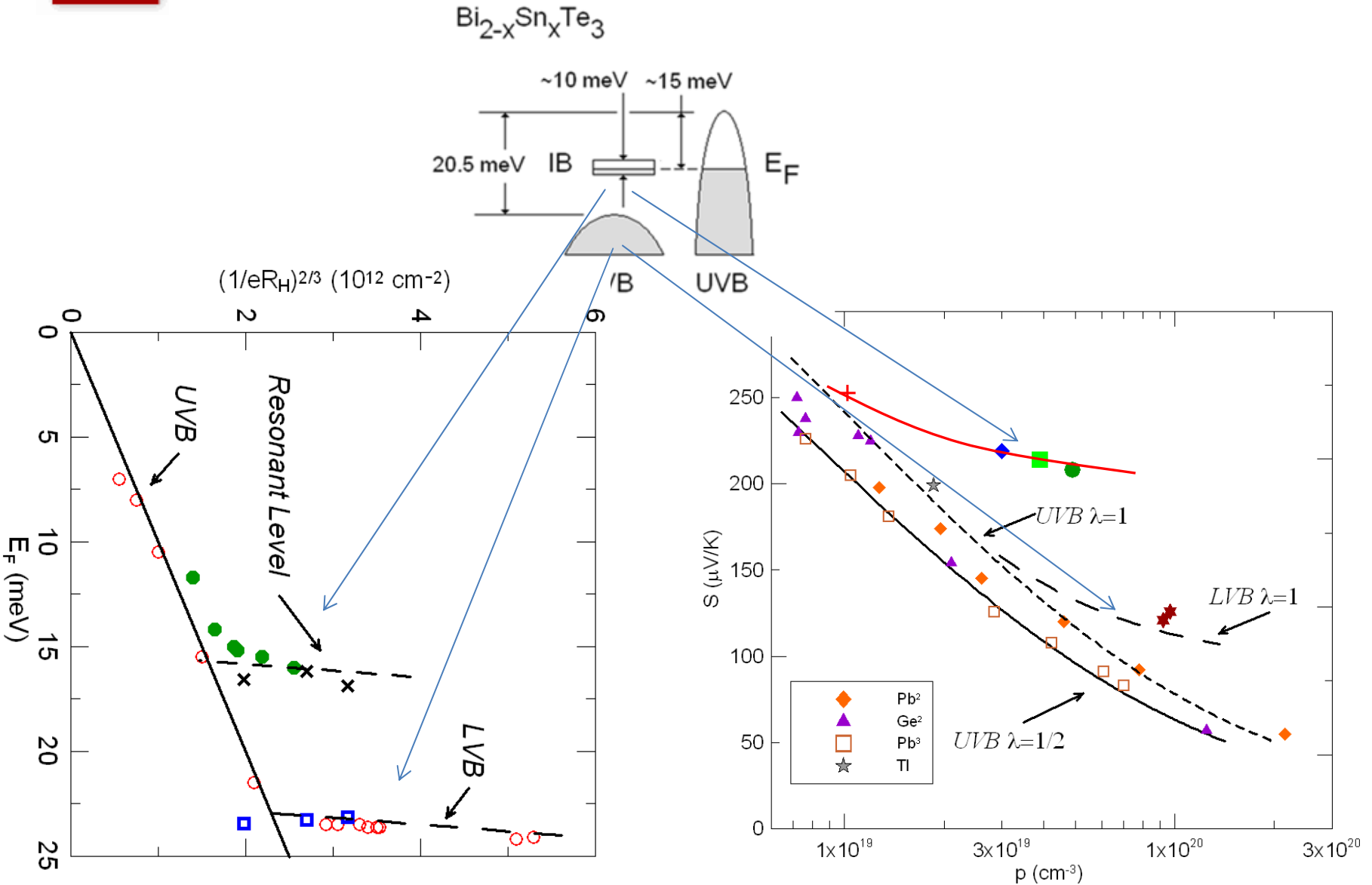
- Middle Sn concentrations have increased Seebeck over Ge and Pb doped  $\text{Bi}_2\text{Te}_3$
- Resonant level
- Highest Sn concentrations fall with 2<sup>nd</sup> valence band



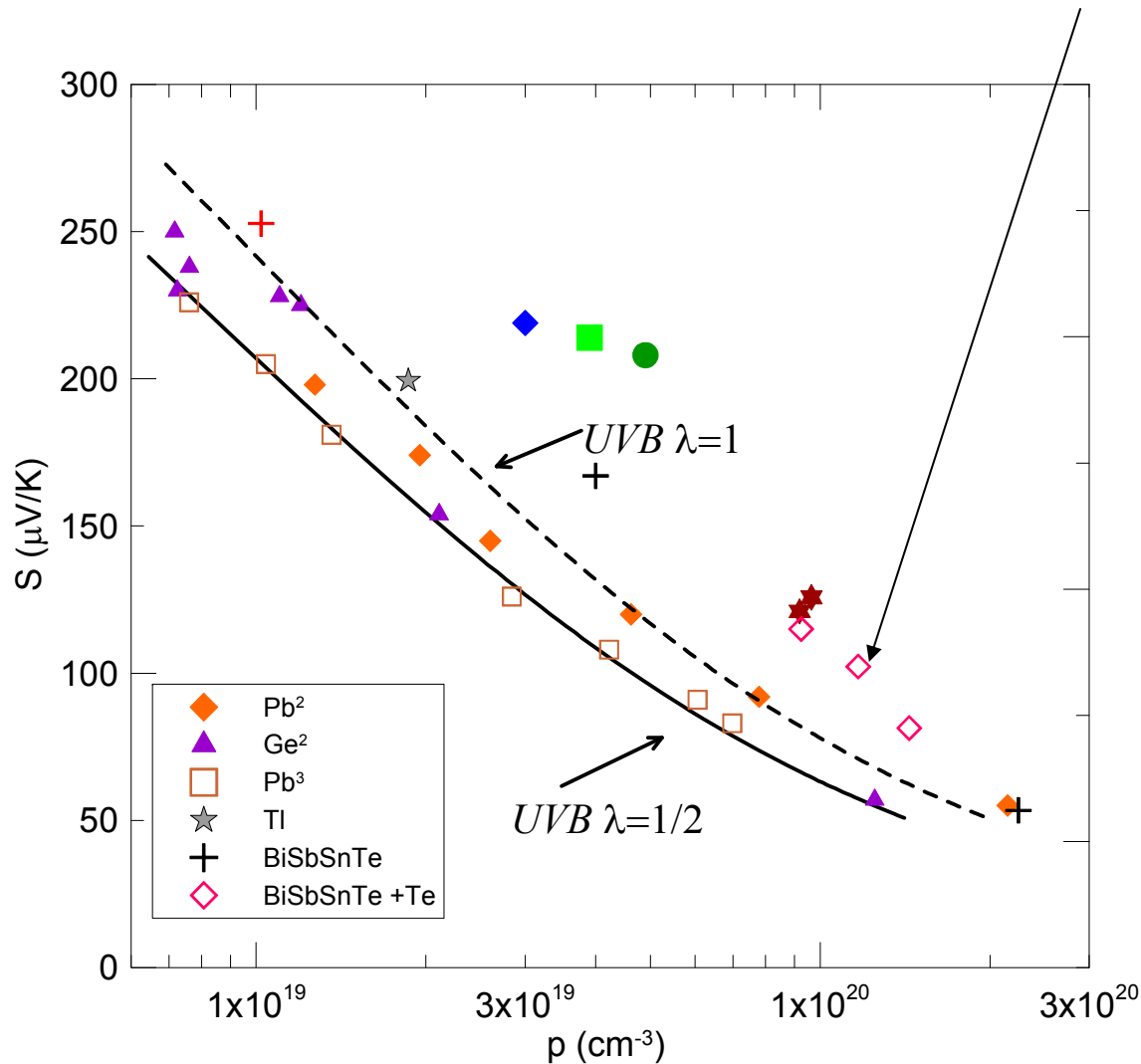
Resonant level much narrower (10 meV) than thallium in PbTe (30meV)

=> Optimization of Fermi level more delicate

# Consistent data: SdH / Pisarenko



# Extend to $(\text{Bi}_{30}\text{Sb}_{70})_2\text{Te}_3$

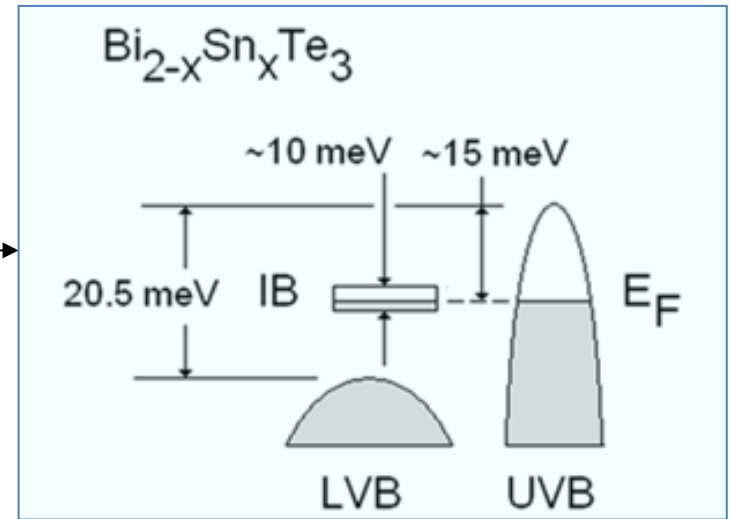


Sb more prone to  
antisite defects than  
Bi

=> Fermi level  
optimization is  
harder.

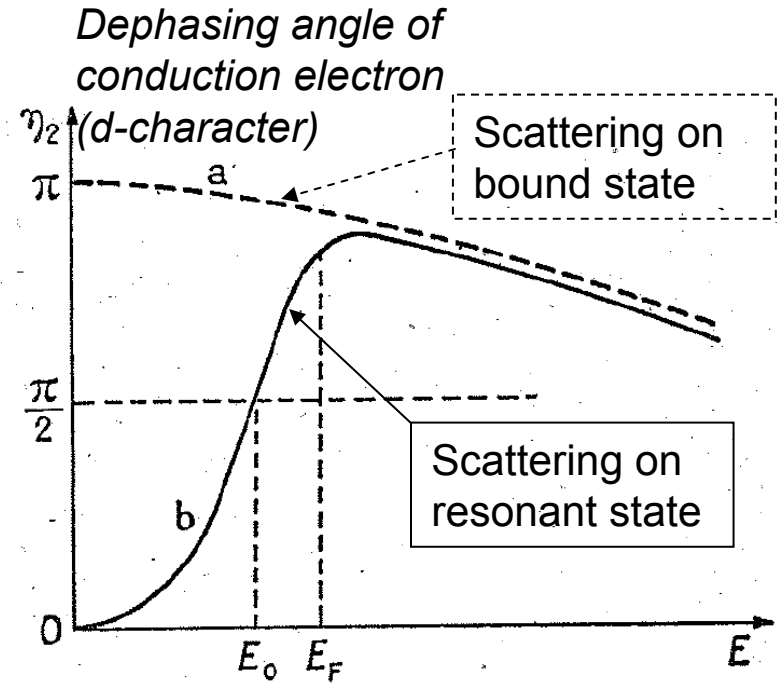
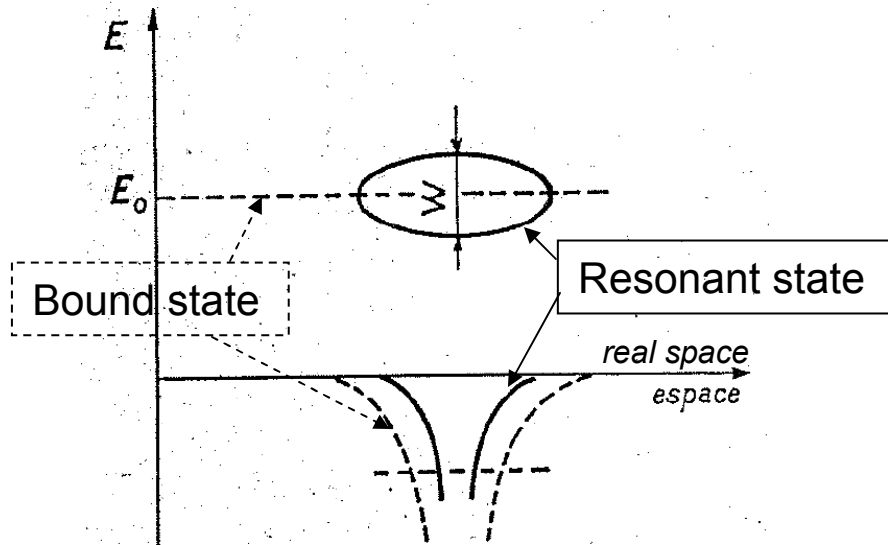
## Conclusions

1. Correct picture for Bi<sub>2</sub>Te<sub>3</sub> from SdH & method of 4 coefficients →
2. Pisarenko relation for Bi<sub>2</sub>Te<sub>3</sub> : Sn different from Bi<sub>2</sub>Te<sub>3</sub> : (Ge, Sn, Pb)
3. Effect is due to an increase in effective mass, NOT resonant scattering as suggested by Zhitinskaya
4. Sn boosts thermopower EVEN at room temperature: Sn enhances power factor  $S^2n$  at useful temperatures for Peltier coolers
5. Resonant level much narrower (10 meV) than thallium in PbTe (30meV)  
=> Optimization of Fermi level more delicate
6. Applicable to commercial (Bi<sub>0.3</sub>Sb<sub>0.7</sub>)<sub>2</sub>Te<sub>3</sub> – type alloys?  
**YES: ONGOING WORK**



# Resonant scattering

Blandin & Friedel *J. Phys. Rad.* **20**  
160 1959



If phase is 0 or  $\pi$   
Wave is reconstructed

Scattering  
strong  
exactly when  
 $E=E_0$   
Resonance  
in scattering

If phase is  $\pi/2$   
Wave is reflected

