

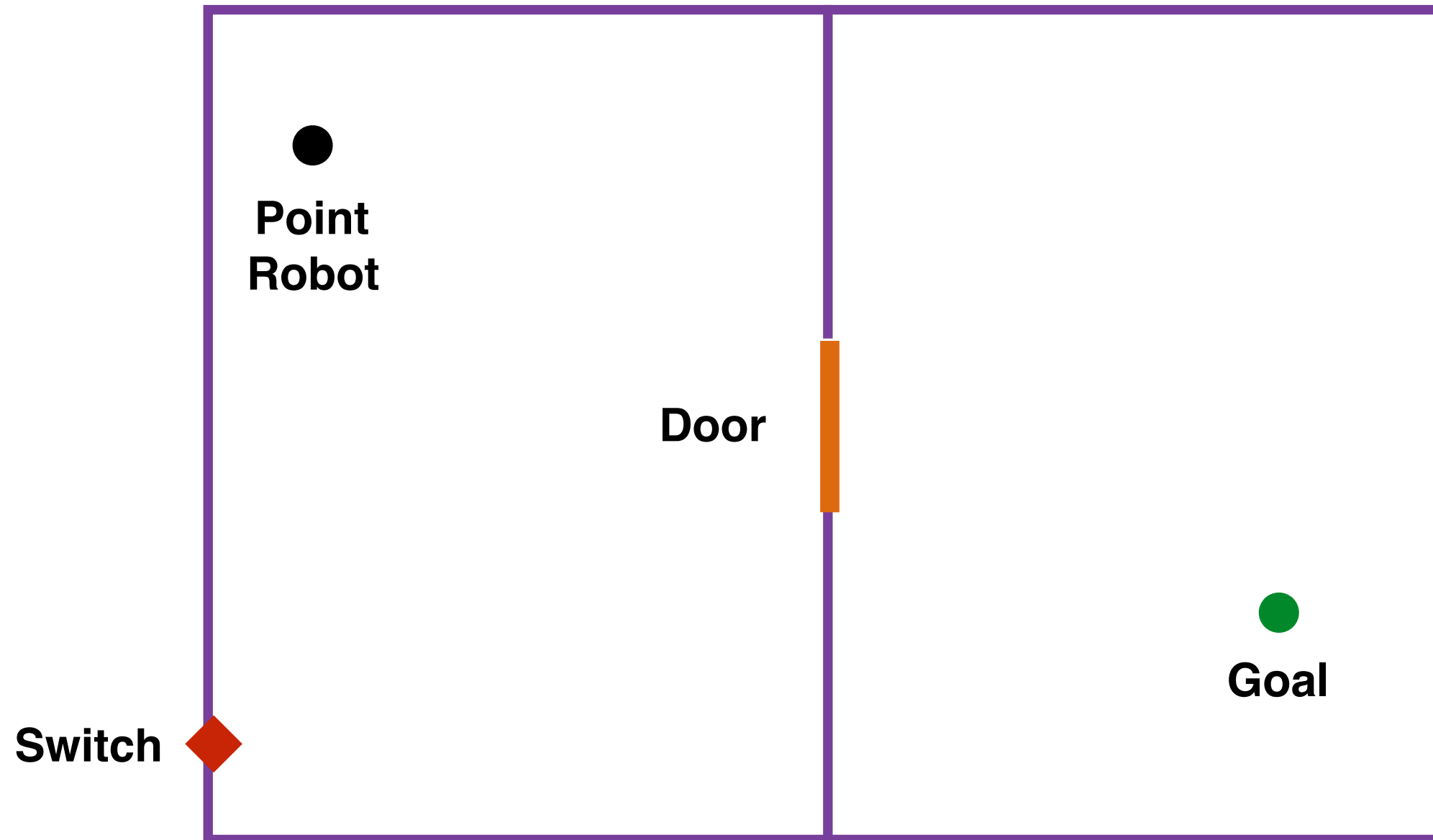
Robot Autonomy

Lecture 15: Hybrid Systems

Oliver Kroemer

Two Rooms Example

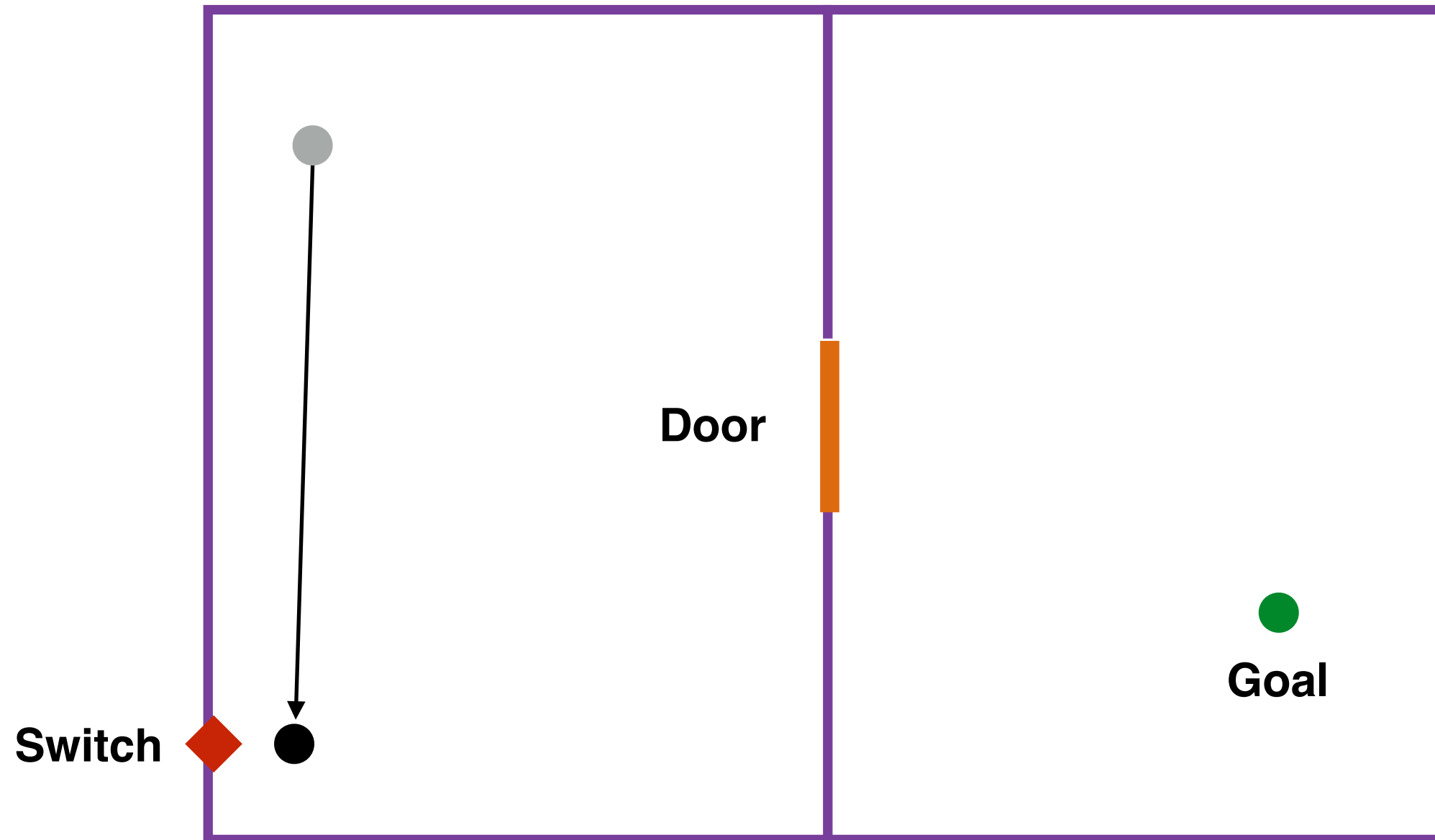
- Consider two rooms separated by switch-activated door



- The door can be either **OPEN** or **CLOSED**

Two Rooms Example

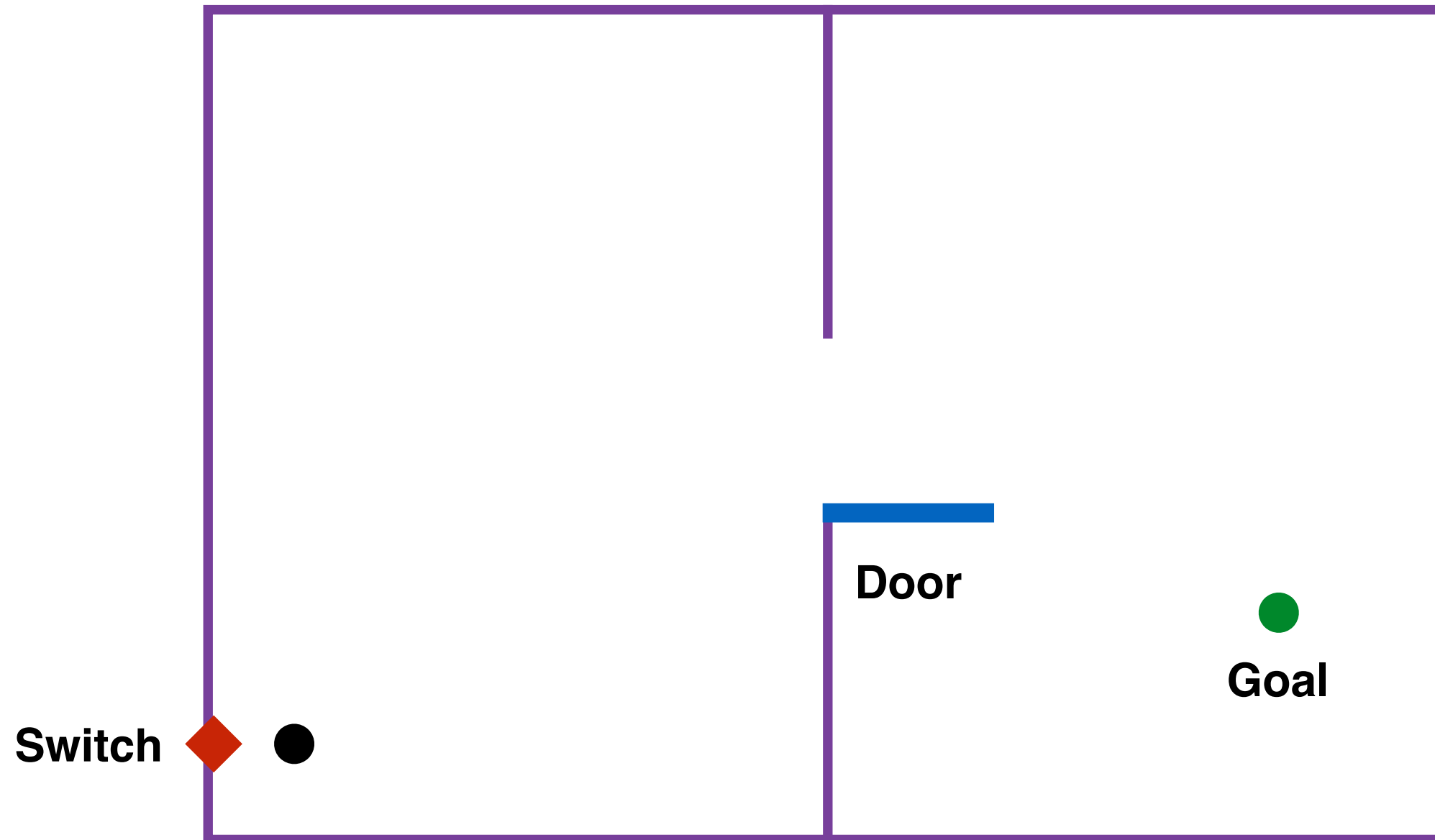
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Two Rooms Example

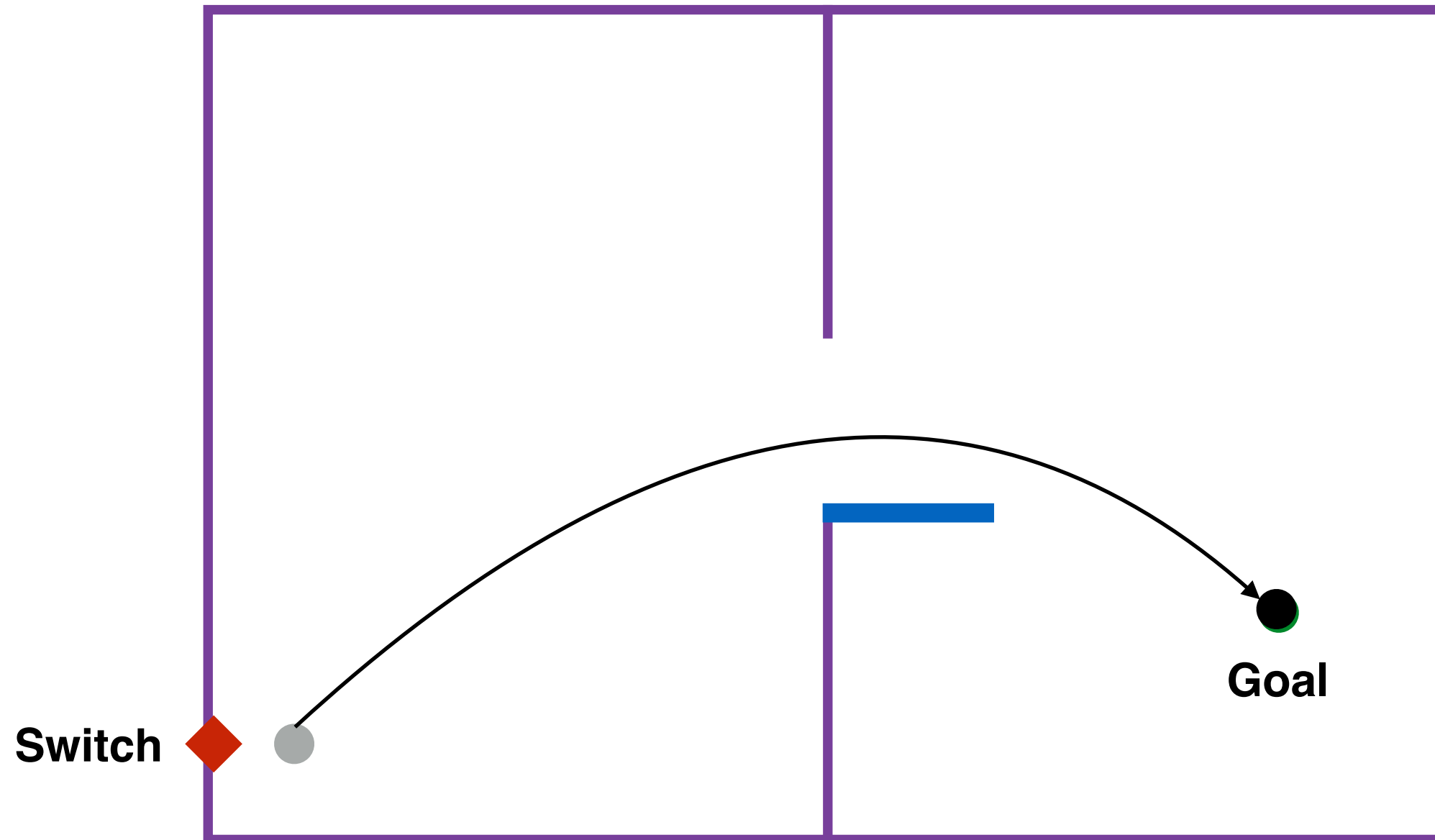
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Two Rooms Example

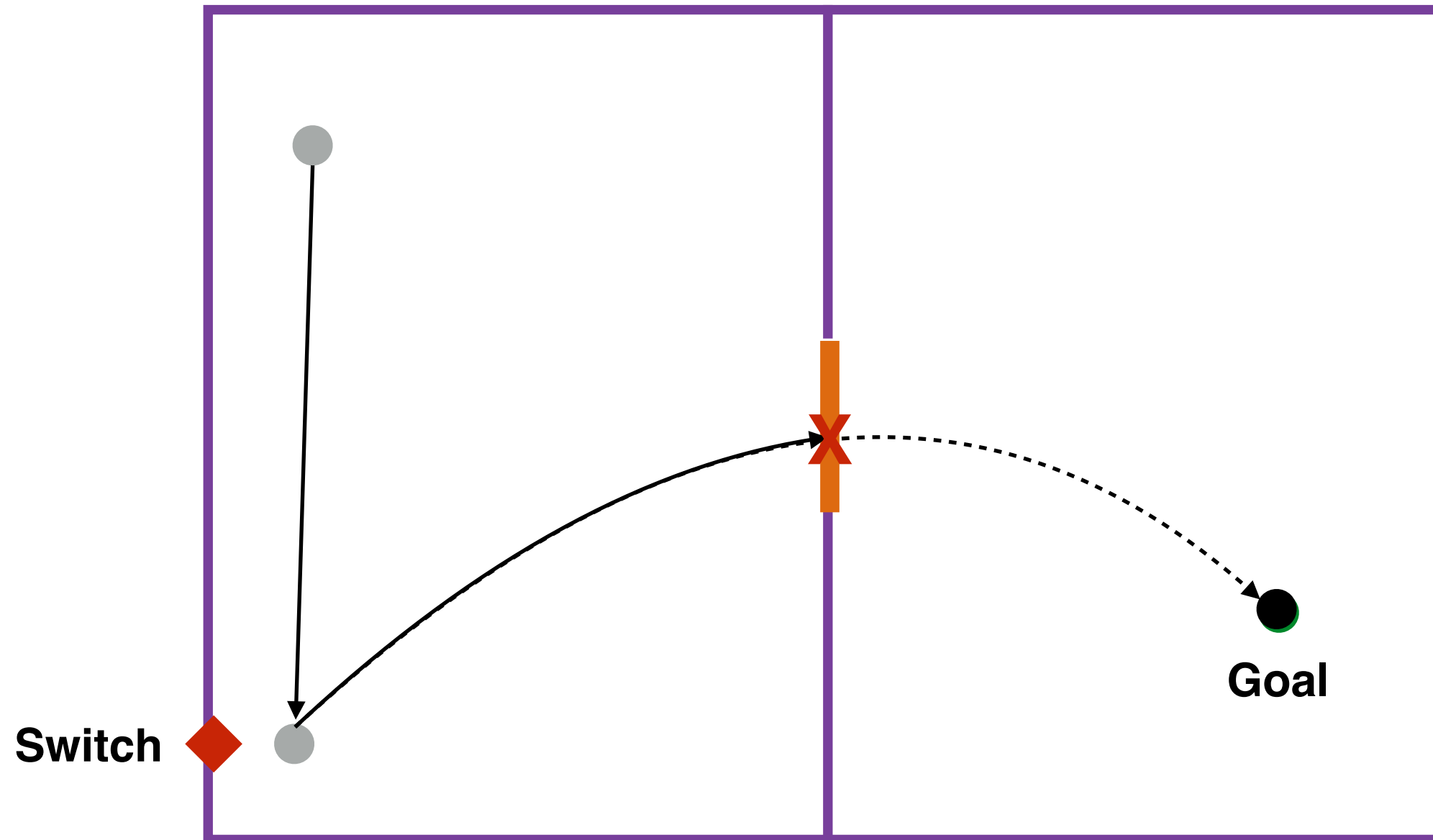
- Consider two rooms separated by switch-activated door



- The door can be either **OPEN** or **CLOSED**

Two Rooms Example

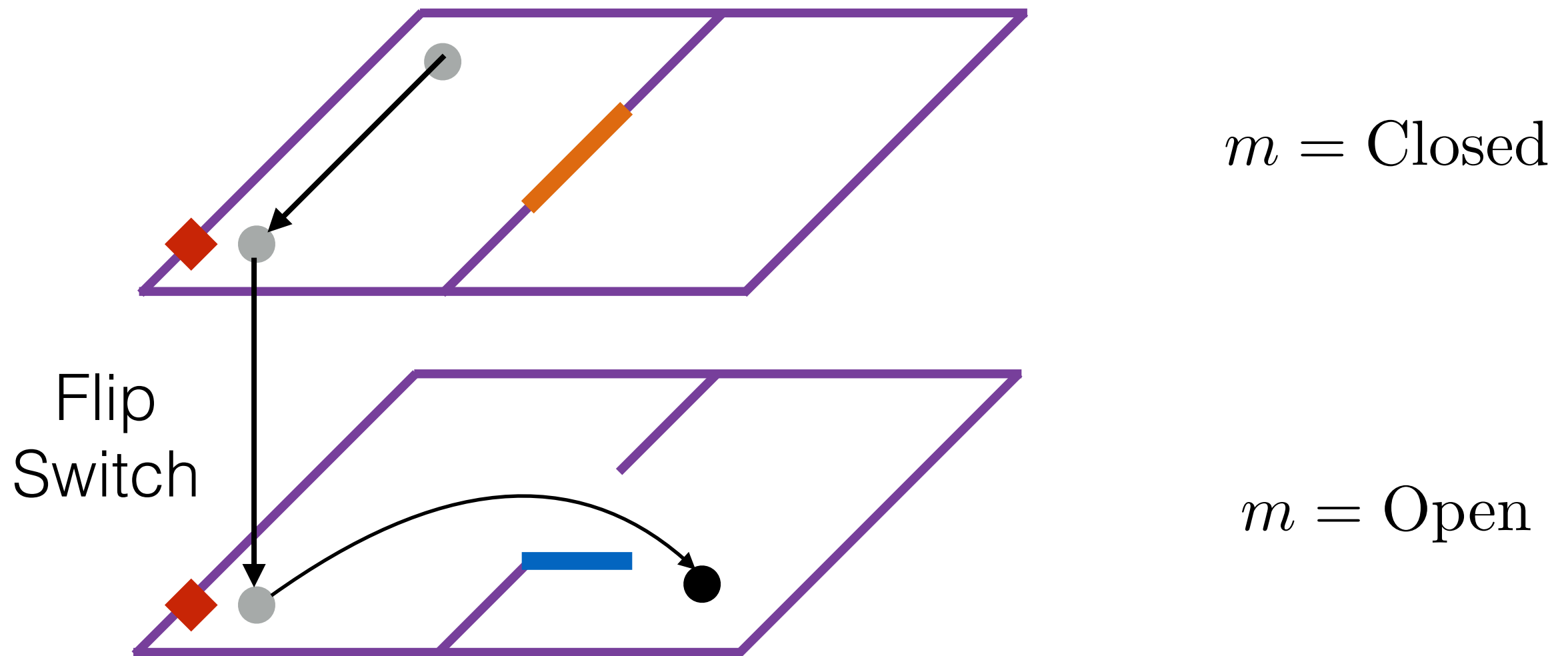
- Consider two rooms separated by switch-activated door



- Simply following the continuous trajectory is not enough

Hybrid System

- Configuration space C and a finite set of modes $m \in M$



State Space
 $X = C \times M$

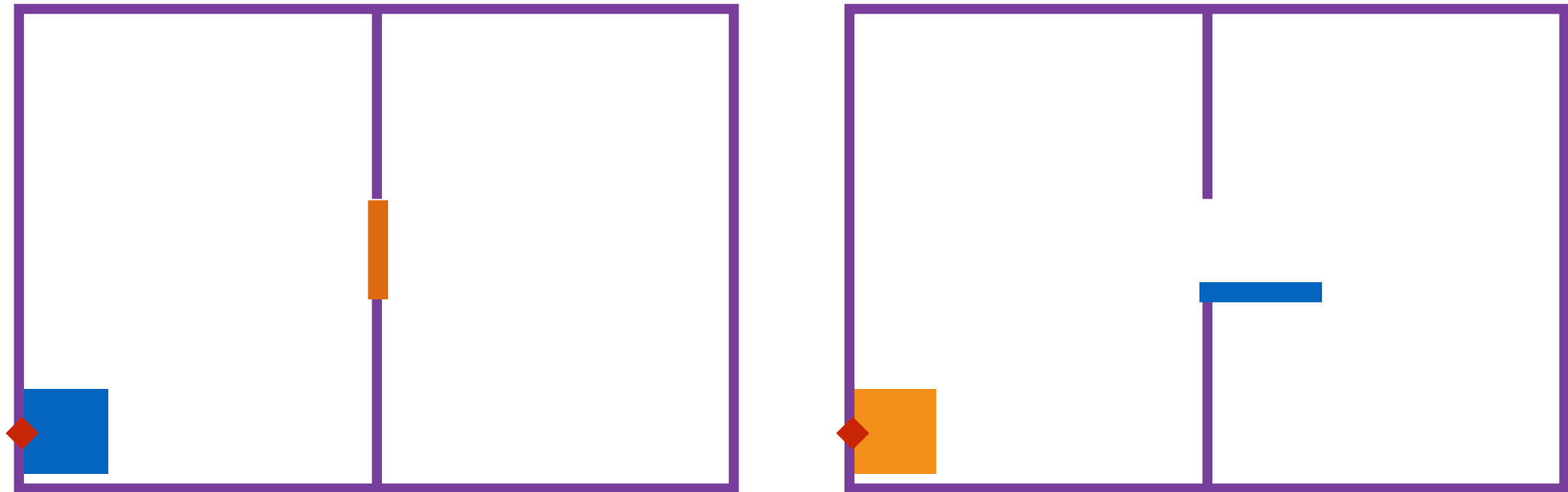
State
 $x = (q, m)$

Hybrid System

- Finite set of **actions** for transitioning between modes

$$U(x) \subset \{\text{NoAction}, \text{OpenDoor}, \text{CloseDoor}\}$$

- Available set of actions depends on state



- May have a forced action, e.g., an automatic door

$$U(x) = \text{OpenDoor}$$

- ▶ environment triggers a mode transition

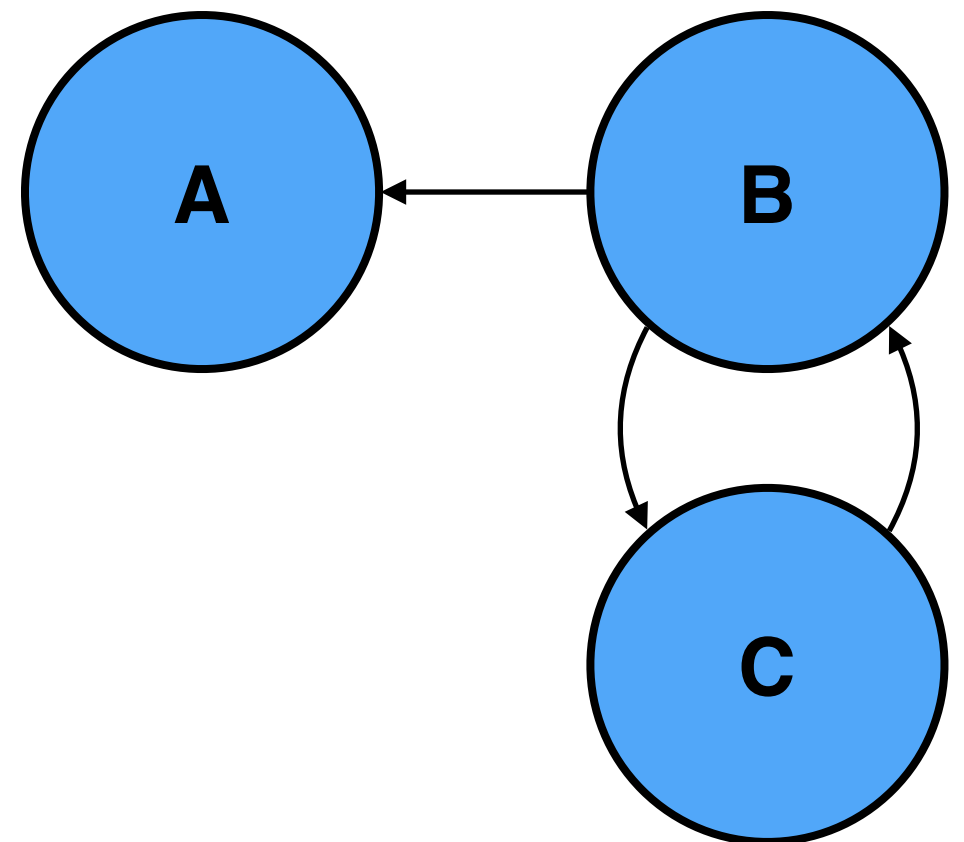
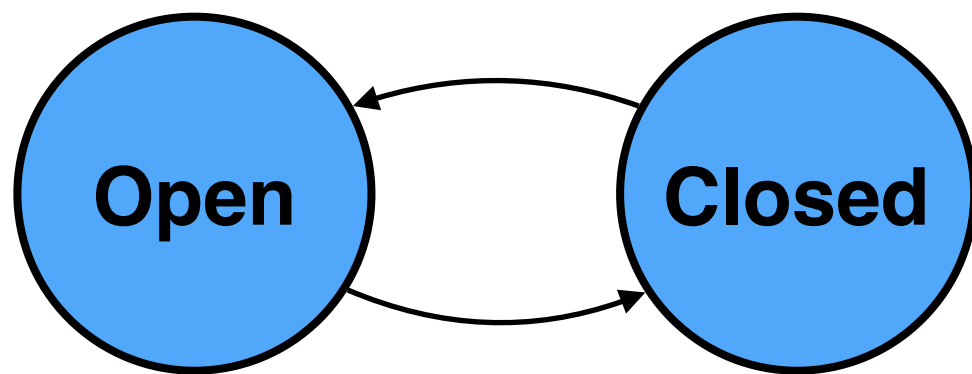
Hybrid System

- Mode transition defined by the **mode transition function**

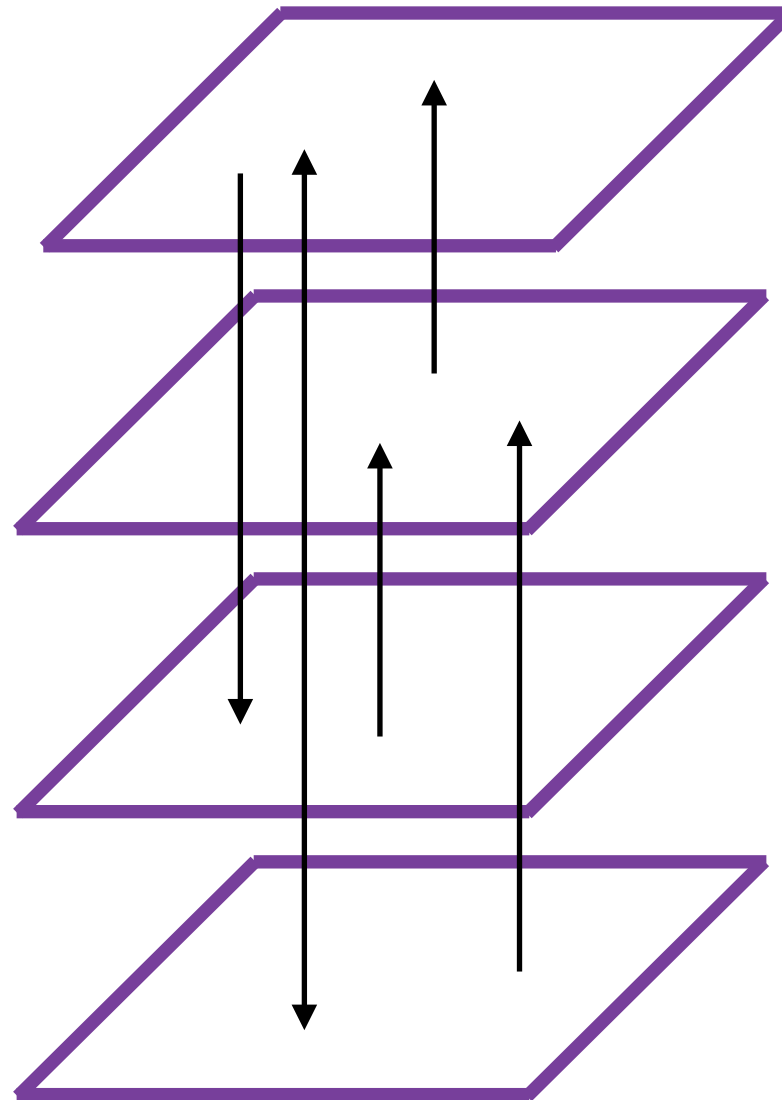
$$f_m(x, u) \in M$$

$$f_m((q, \text{Closed}), \text{OpenDoor}) = \text{Open}$$

- Mode Graphs**

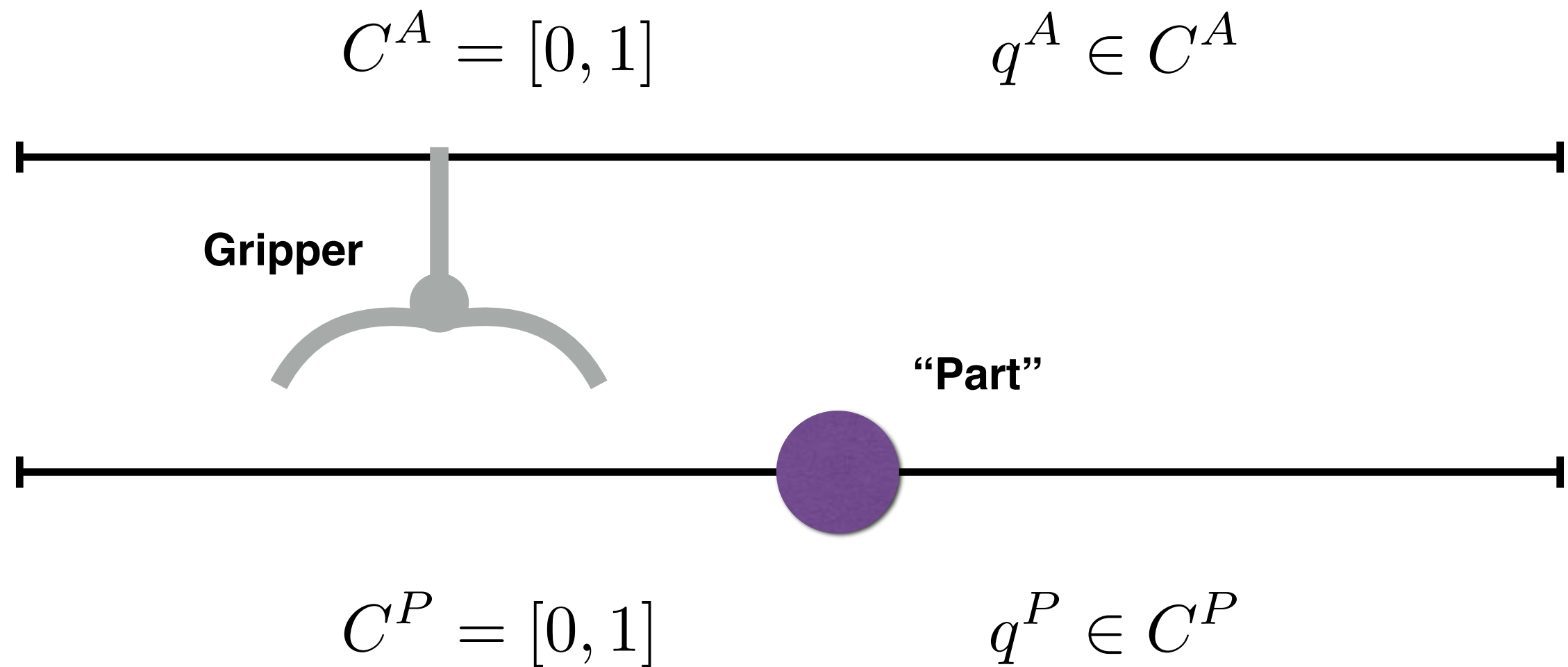


- Modes must be **discrete**, but can have many of them



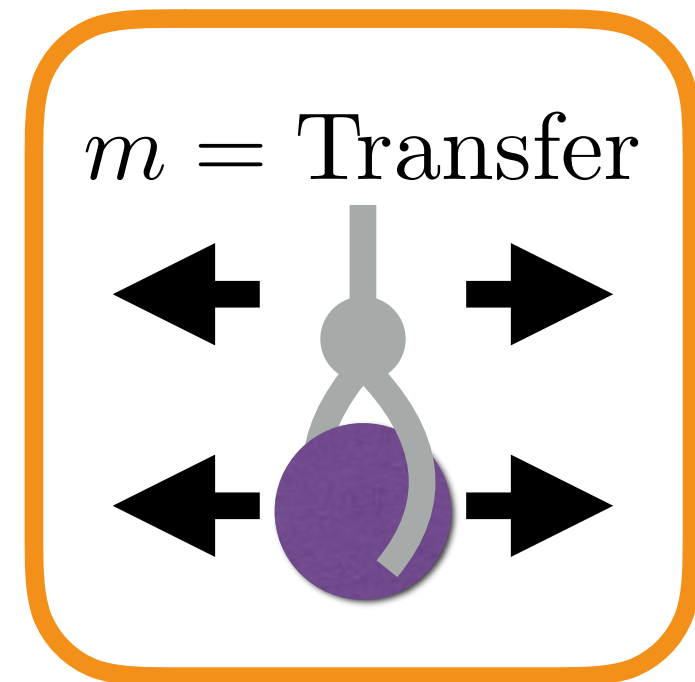
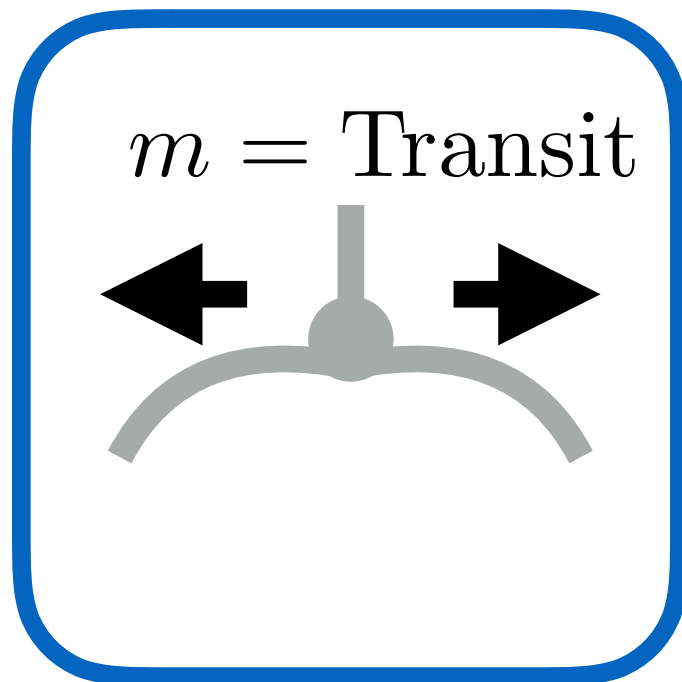
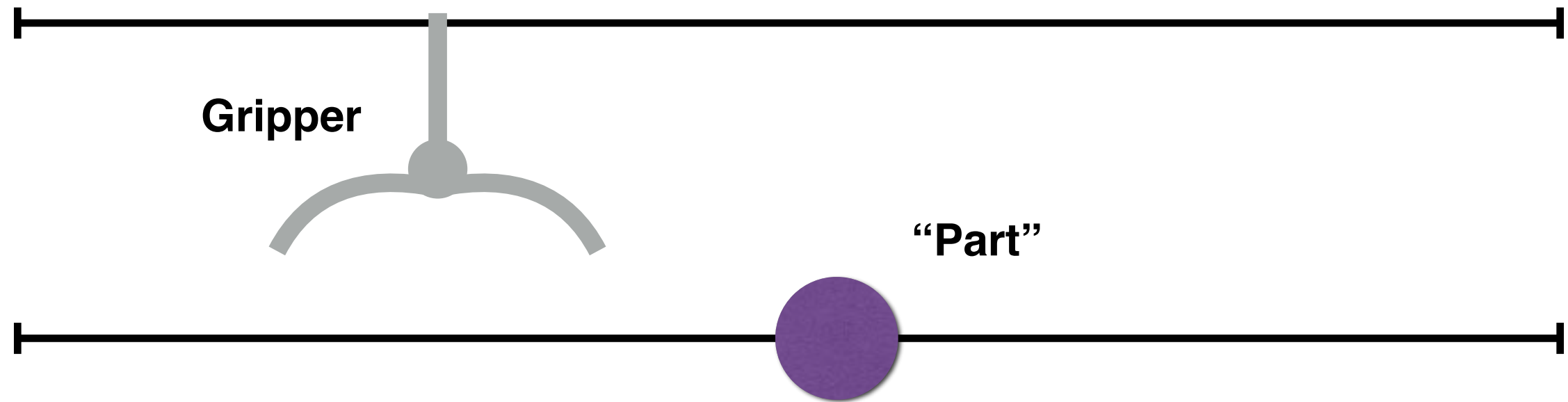
- Well behaved c-spaces with portals between them

Bead on a Wire Manipulation

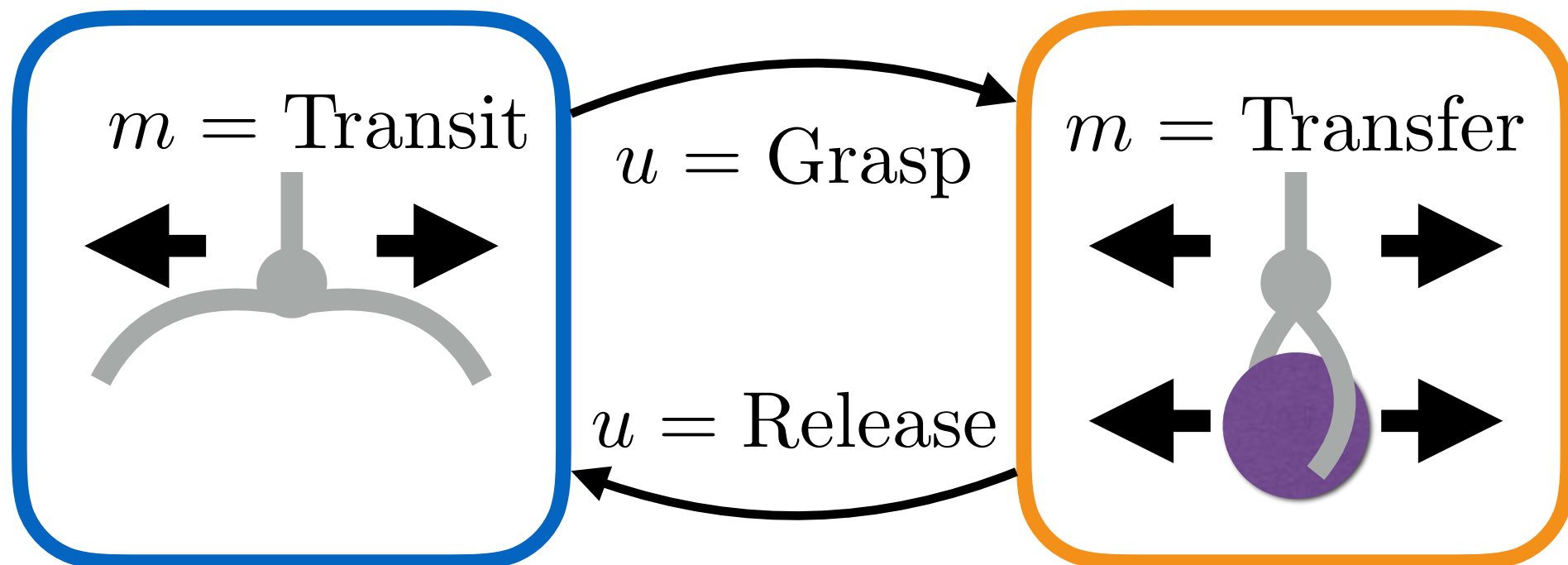
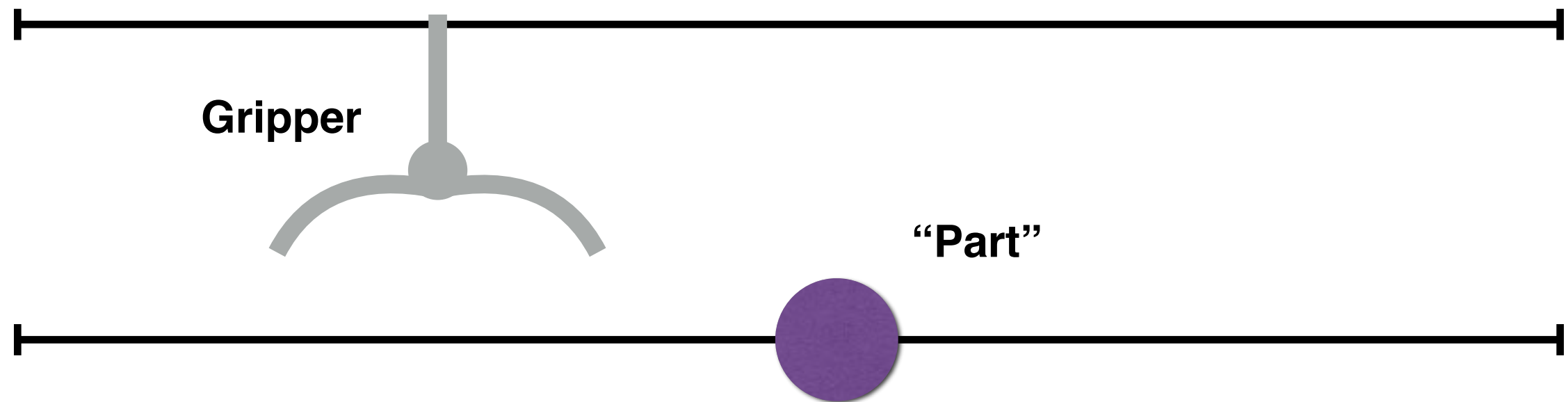


$$X = C^P \times C^A \times M$$

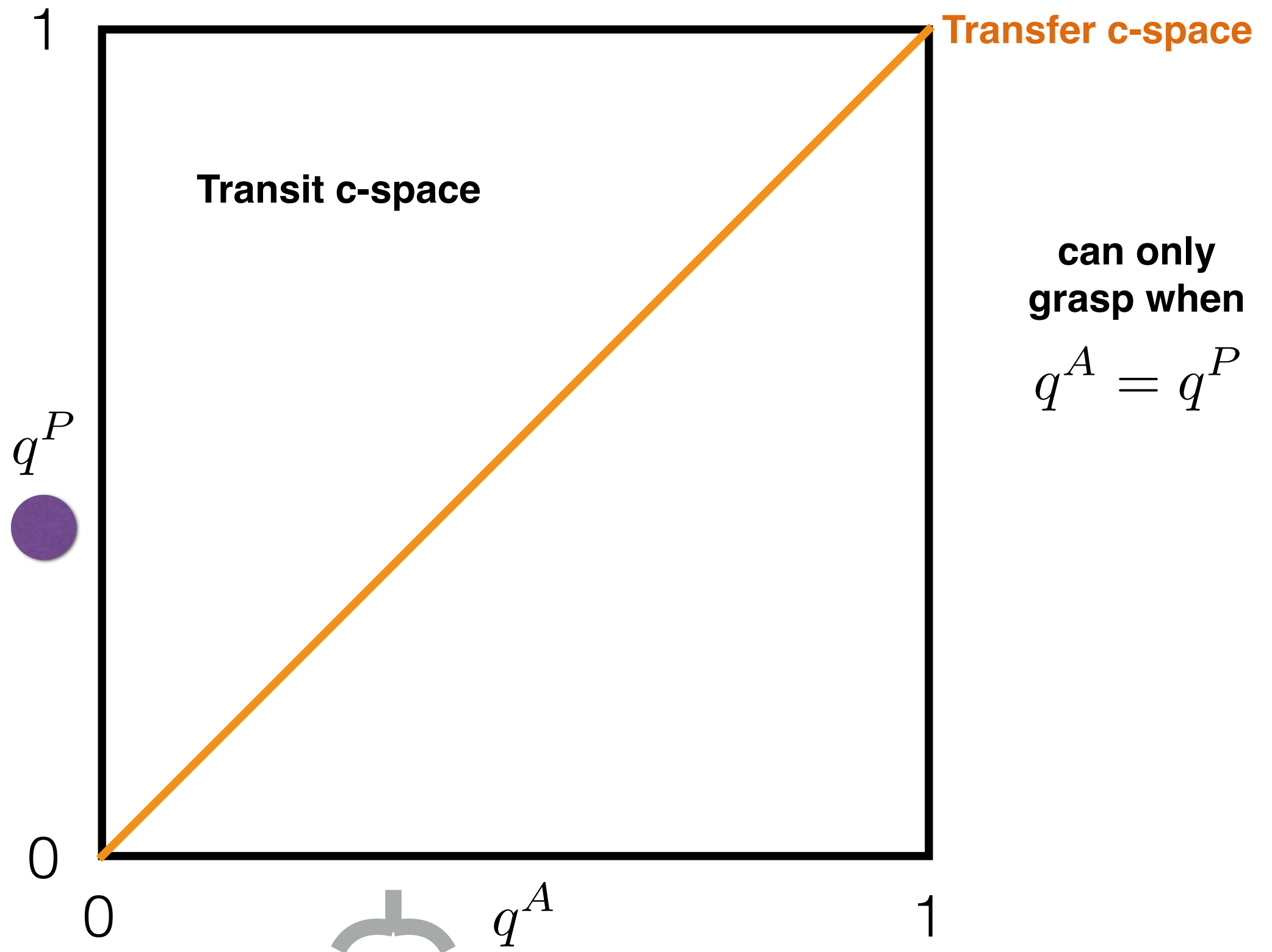
Bead on a Wire Manipulation



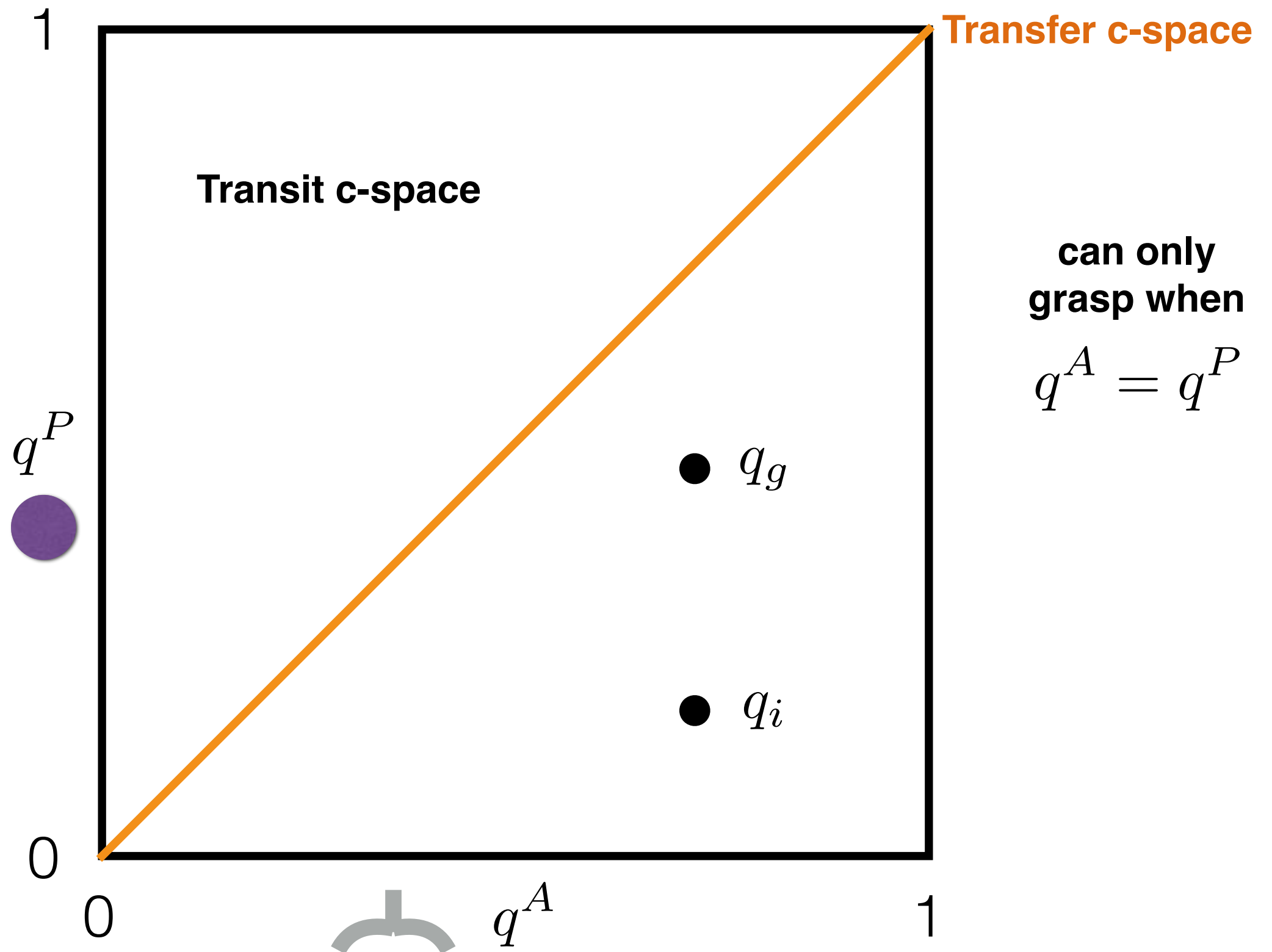
Bead on a Wire Manipulation



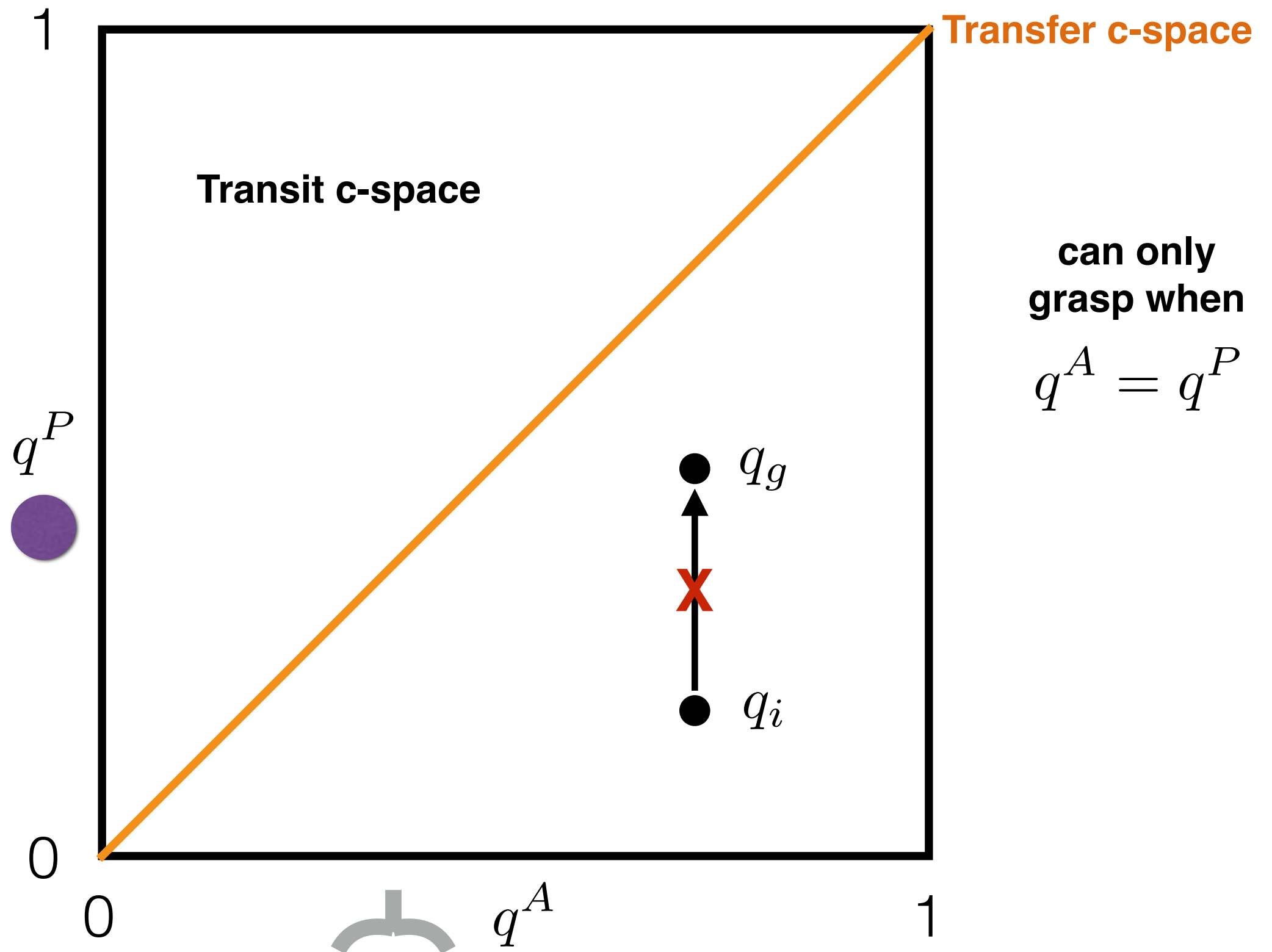
Bead on a Wire C-Space



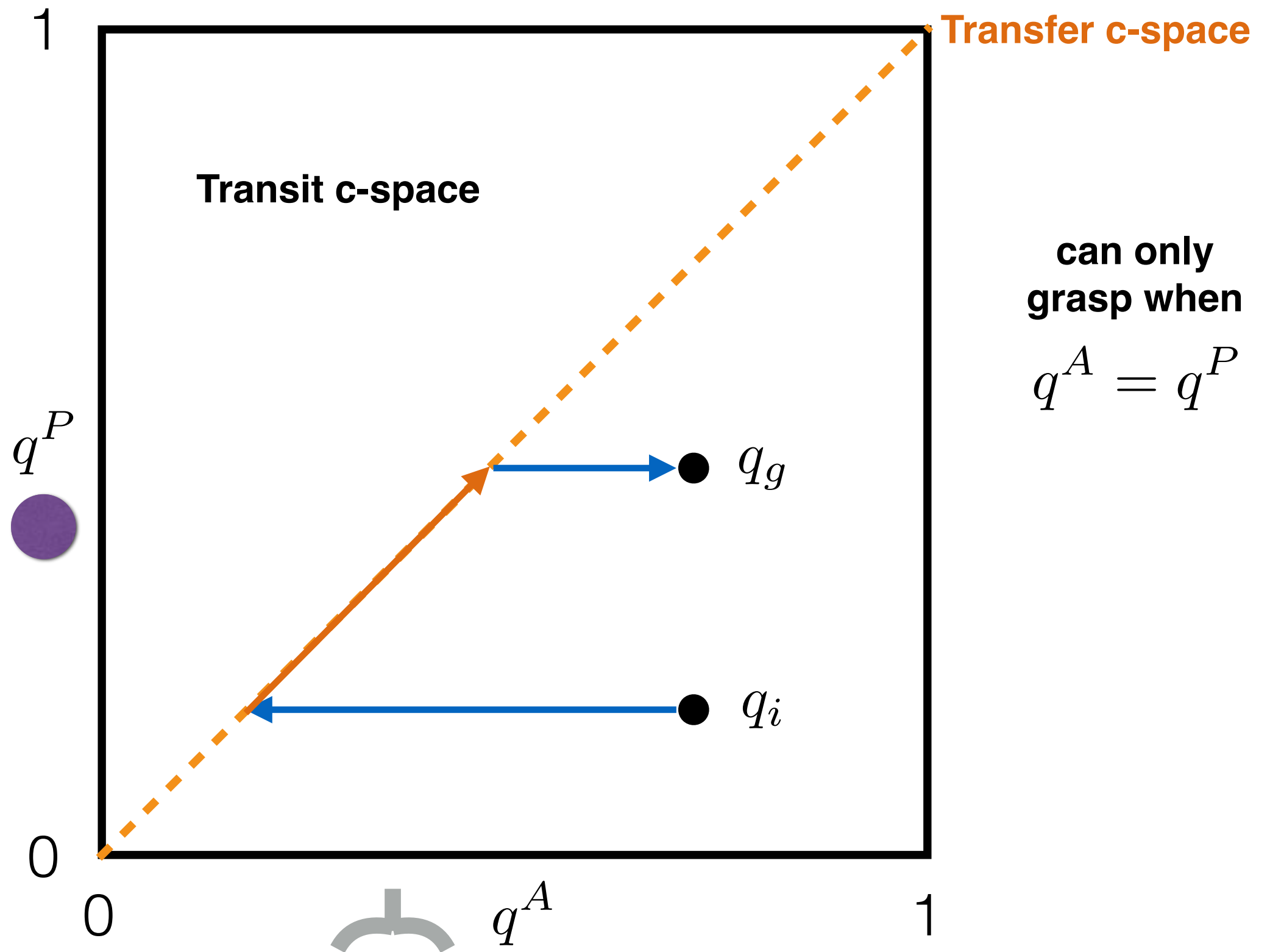
Bead on a Wire C-Space



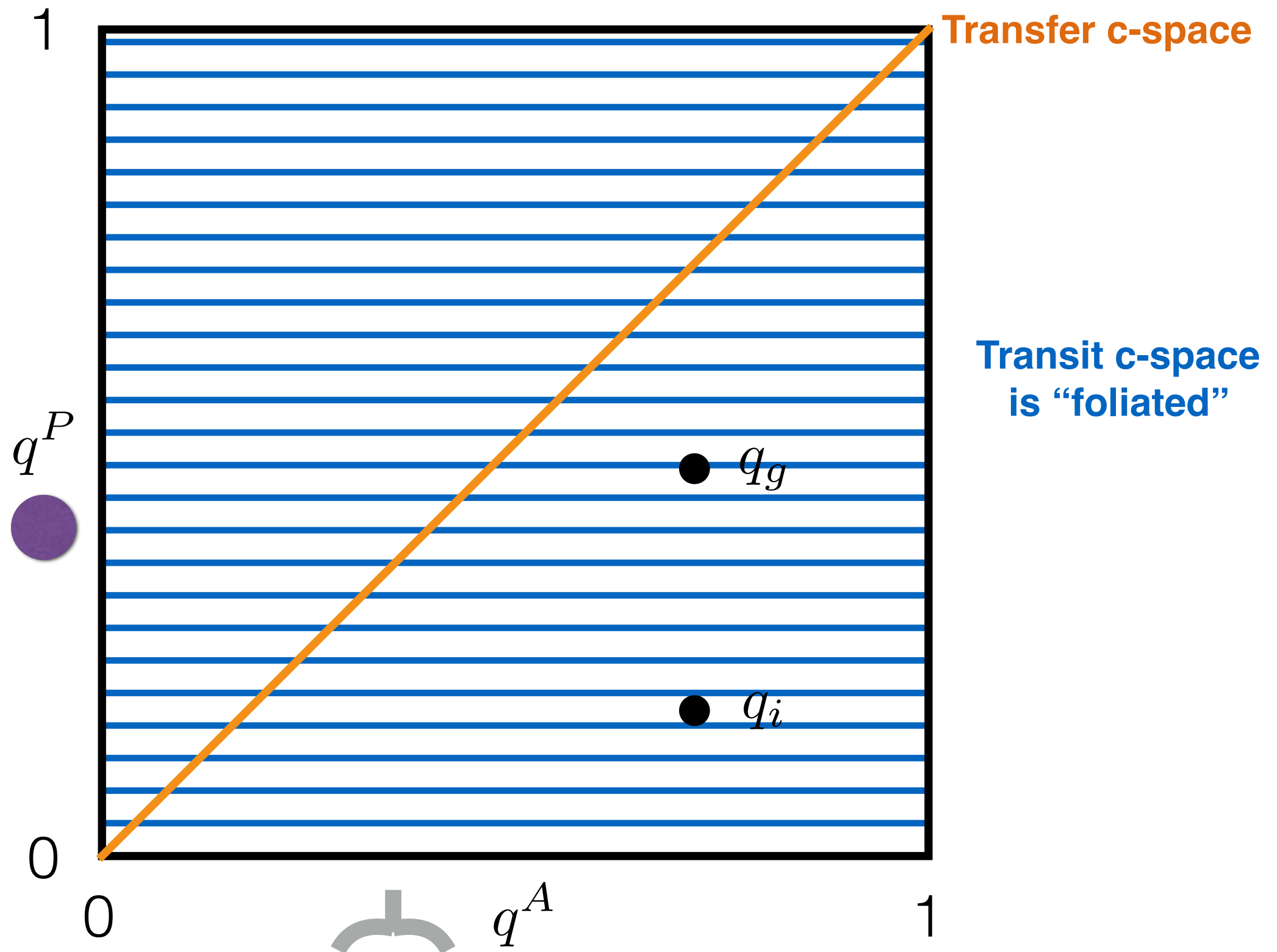
Bead on a Wire C-Space



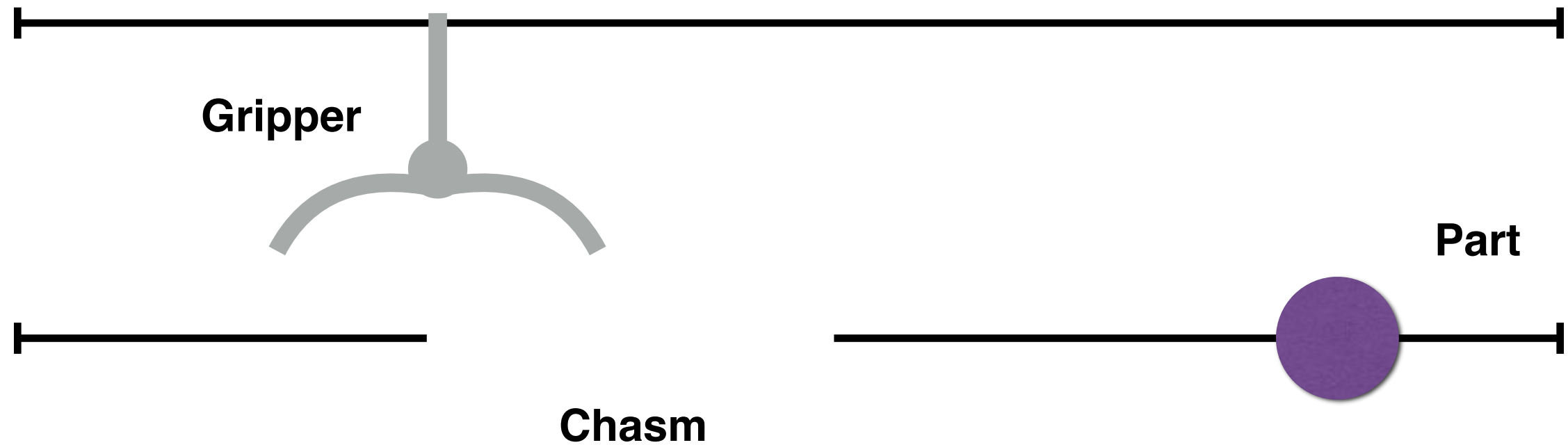
Bead on a Wire C-Space



Bead on a Wire C-Space



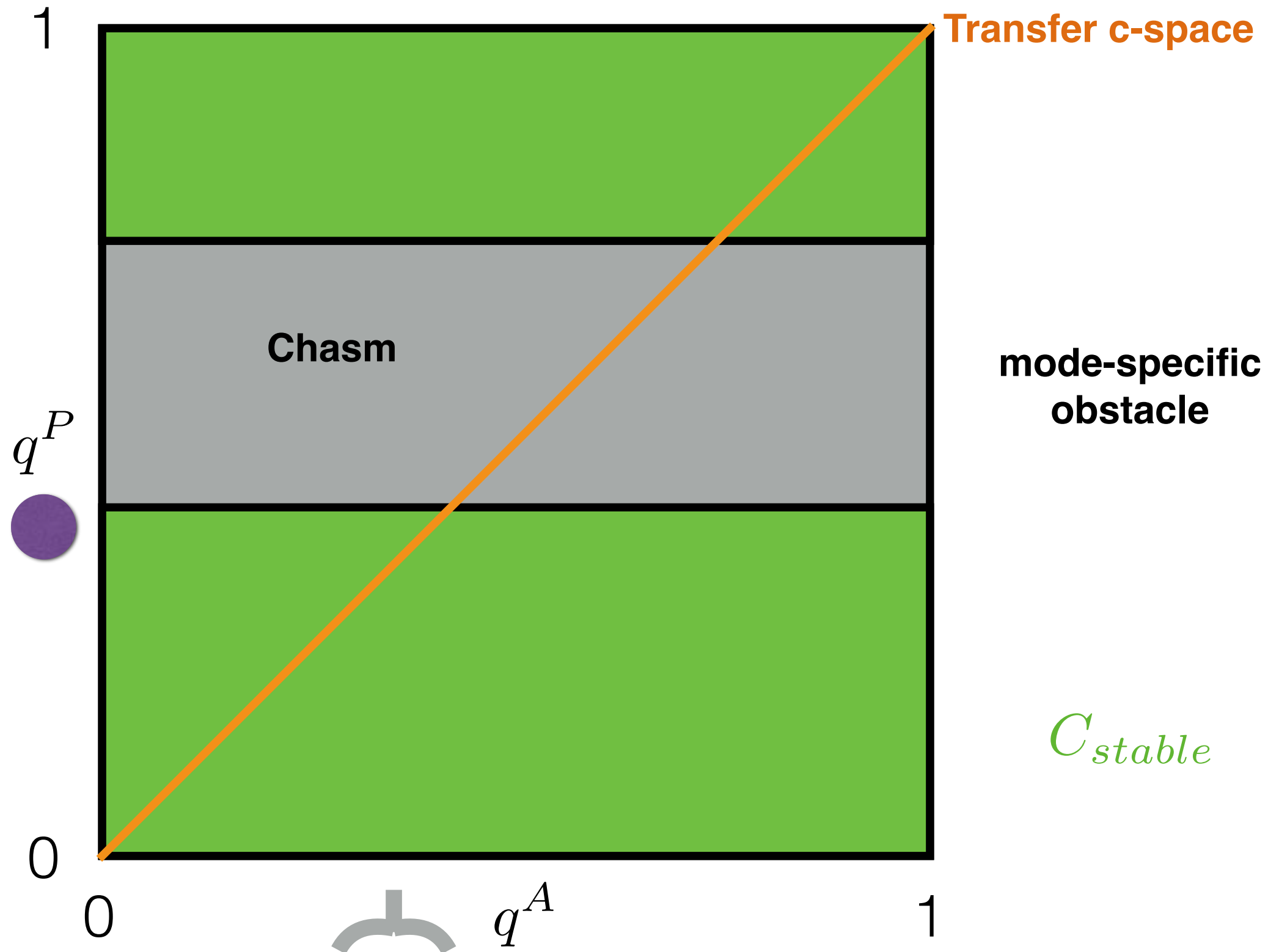
Bead on a Wire Manipulation



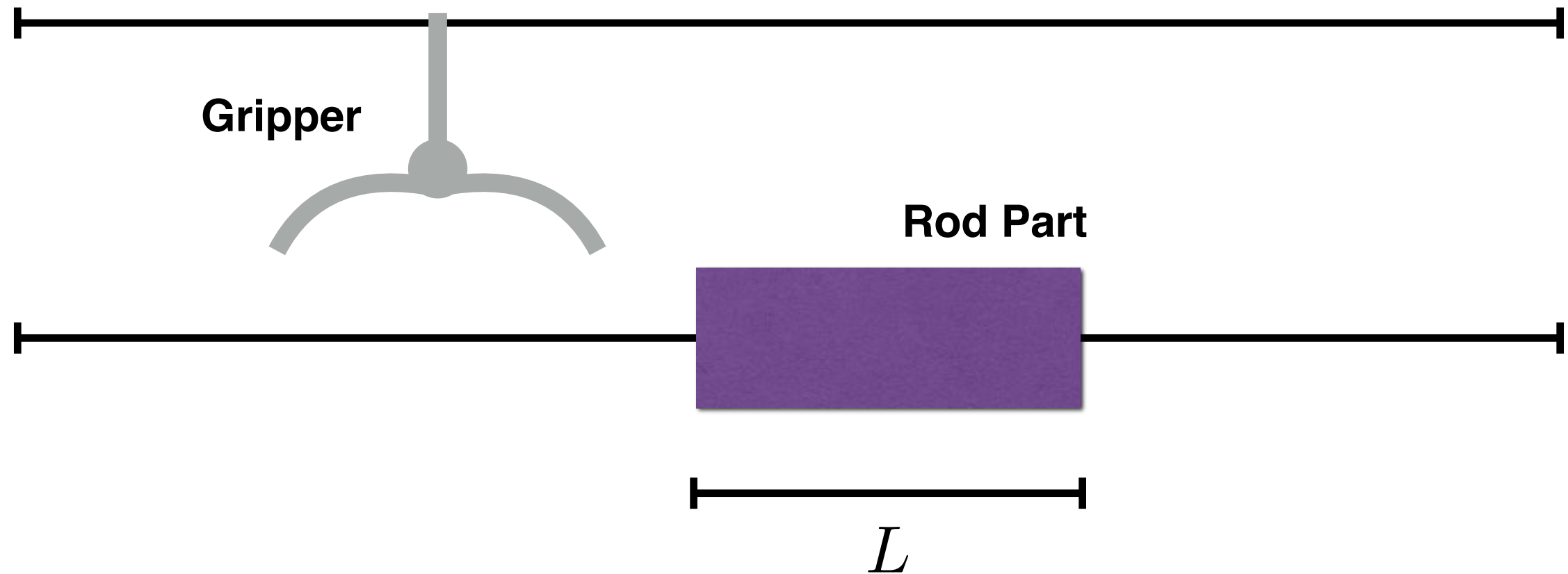
part can be carried across chasm

Bead on a Wire C-Space

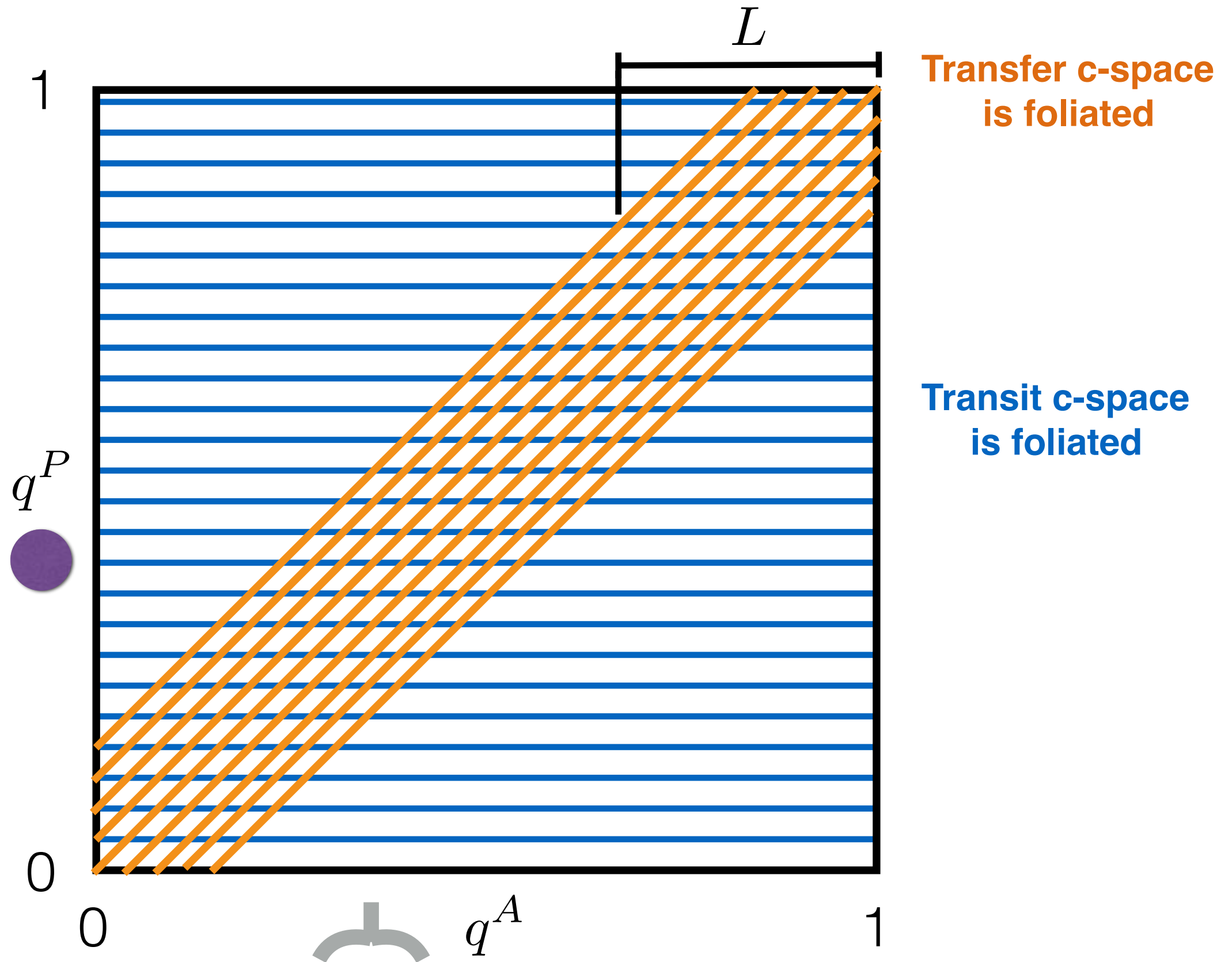
$$X_{obs}(\text{Transit}) = \{(q^A, q^P, m) | q^P \in \text{chasm} \text{ AND } m = \text{Transit}\}$$



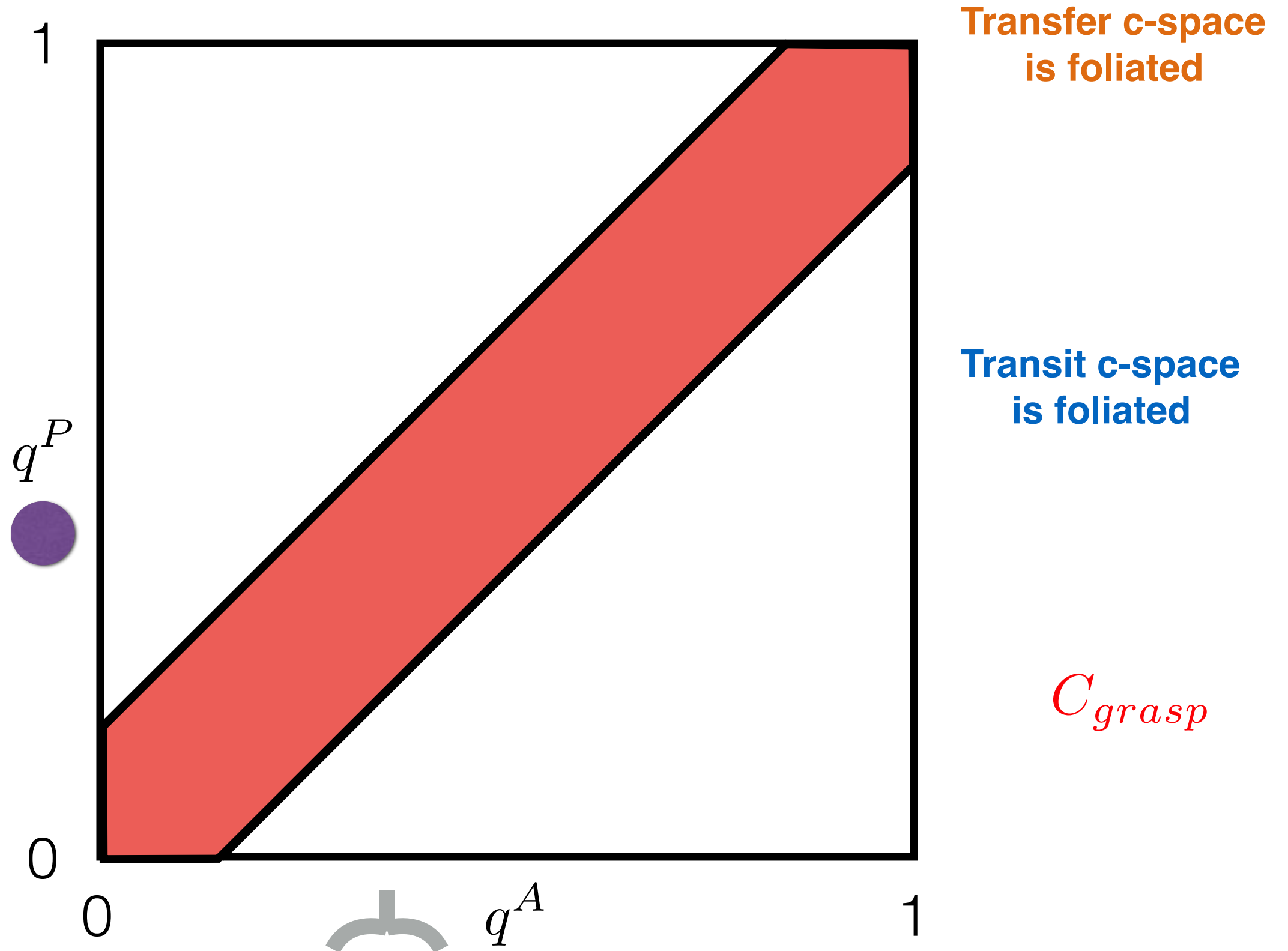
Rod on a Wire Manipulation



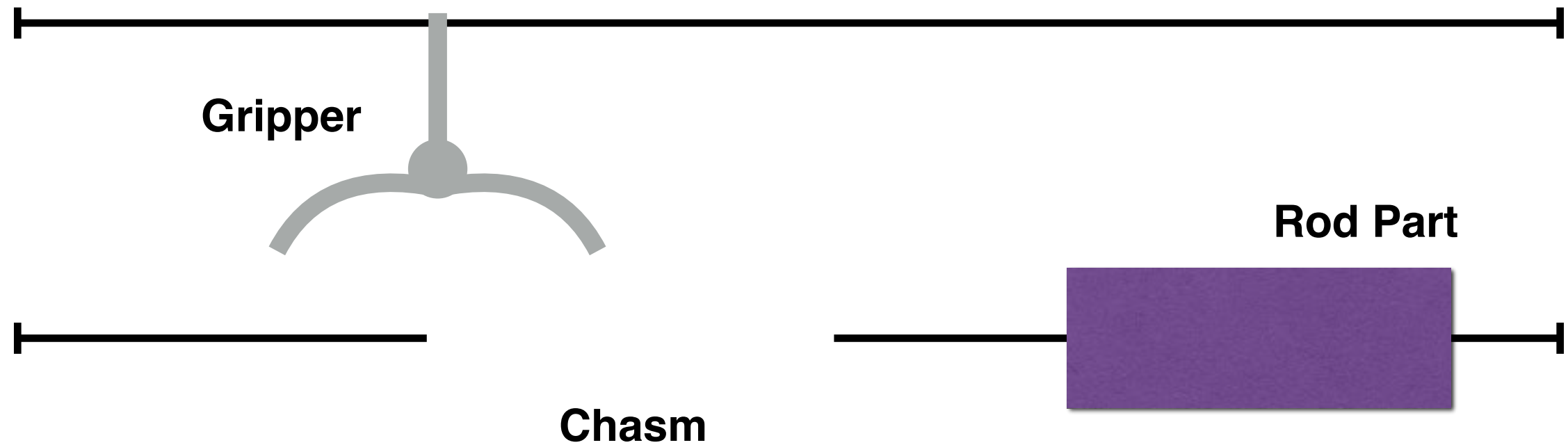
Bead on a Wire C-Space



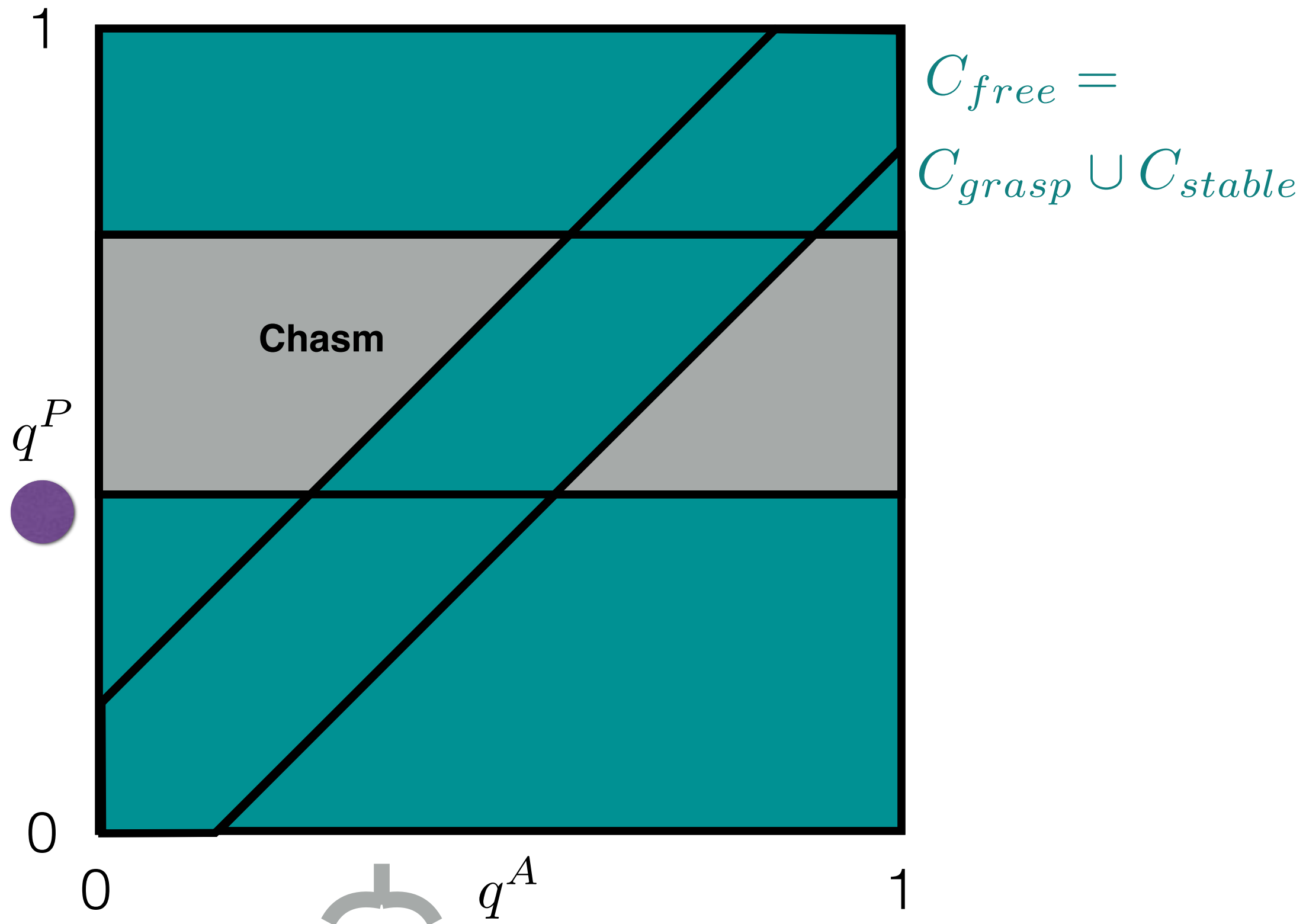
Bead on a Wire C-Space



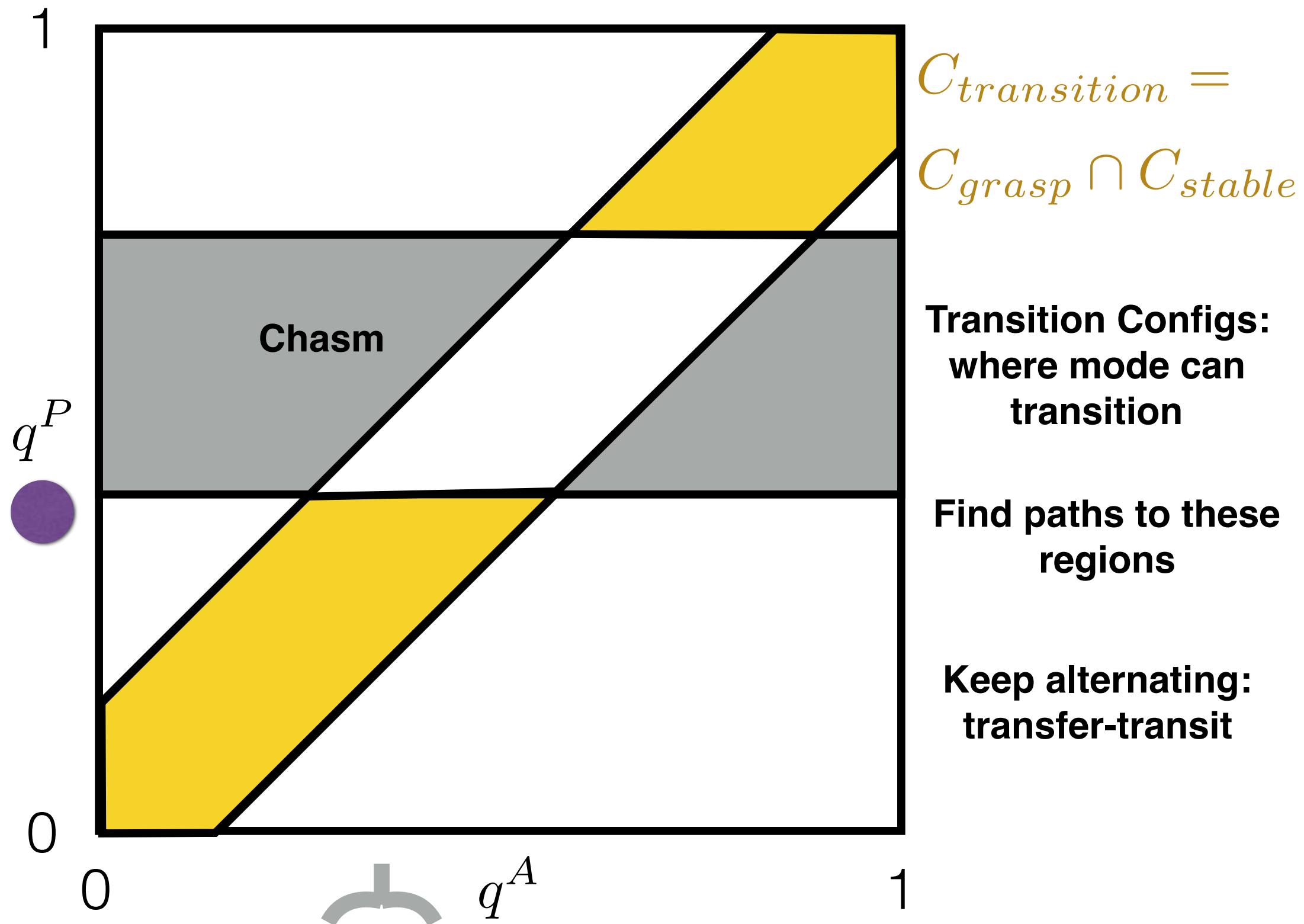
Bead on a Wire Manipulation



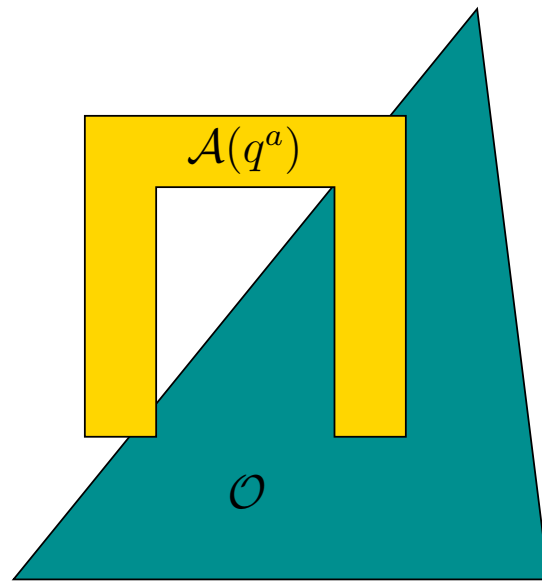
Bead on a Wire C-Space



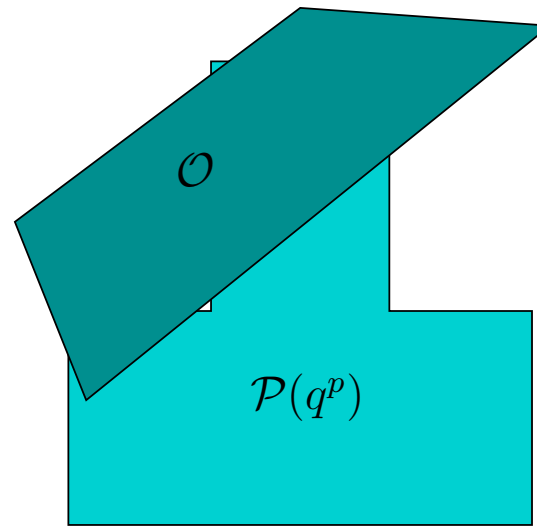
Bead on a Wire C-Space



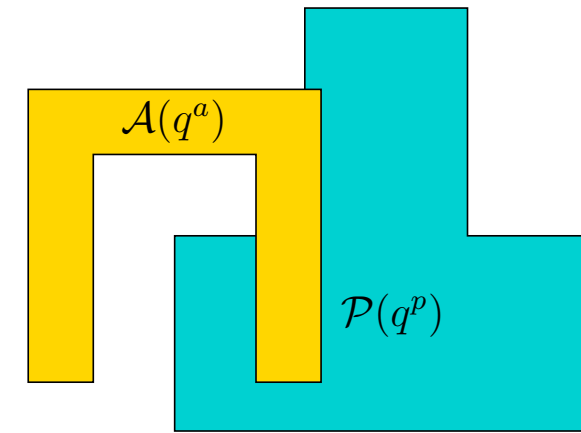
C-space Regions



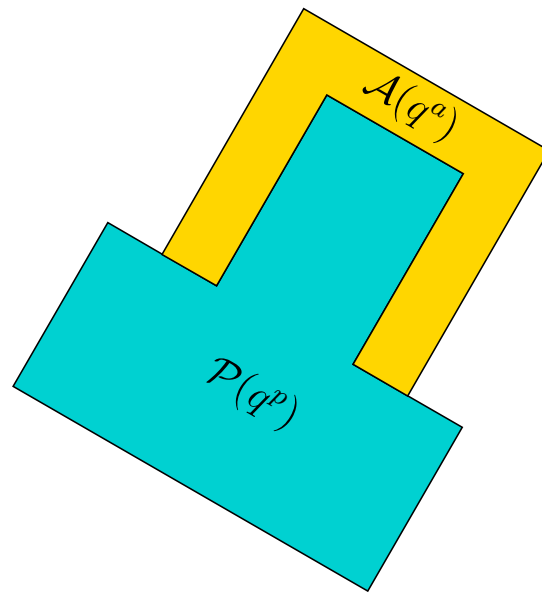
$$q \in \mathcal{C}_{obs}^a$$



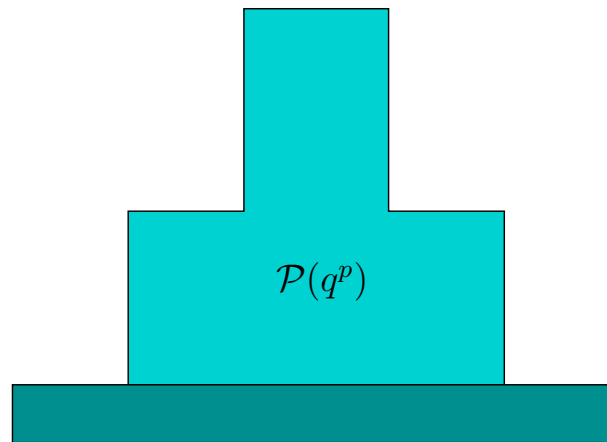
$$q \in \mathcal{C}_{obs}^p$$



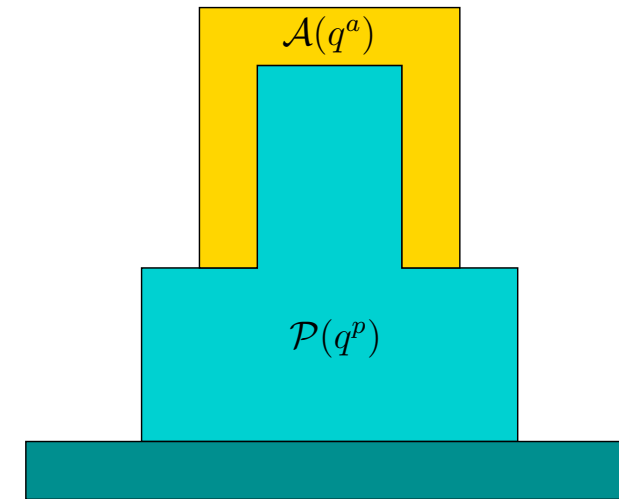
$$q \in \mathcal{C}_{obs}^{ap}$$



$$q \in \mathcal{C}_{gr}$$



$$q \in \mathcal{C}_{sta}$$

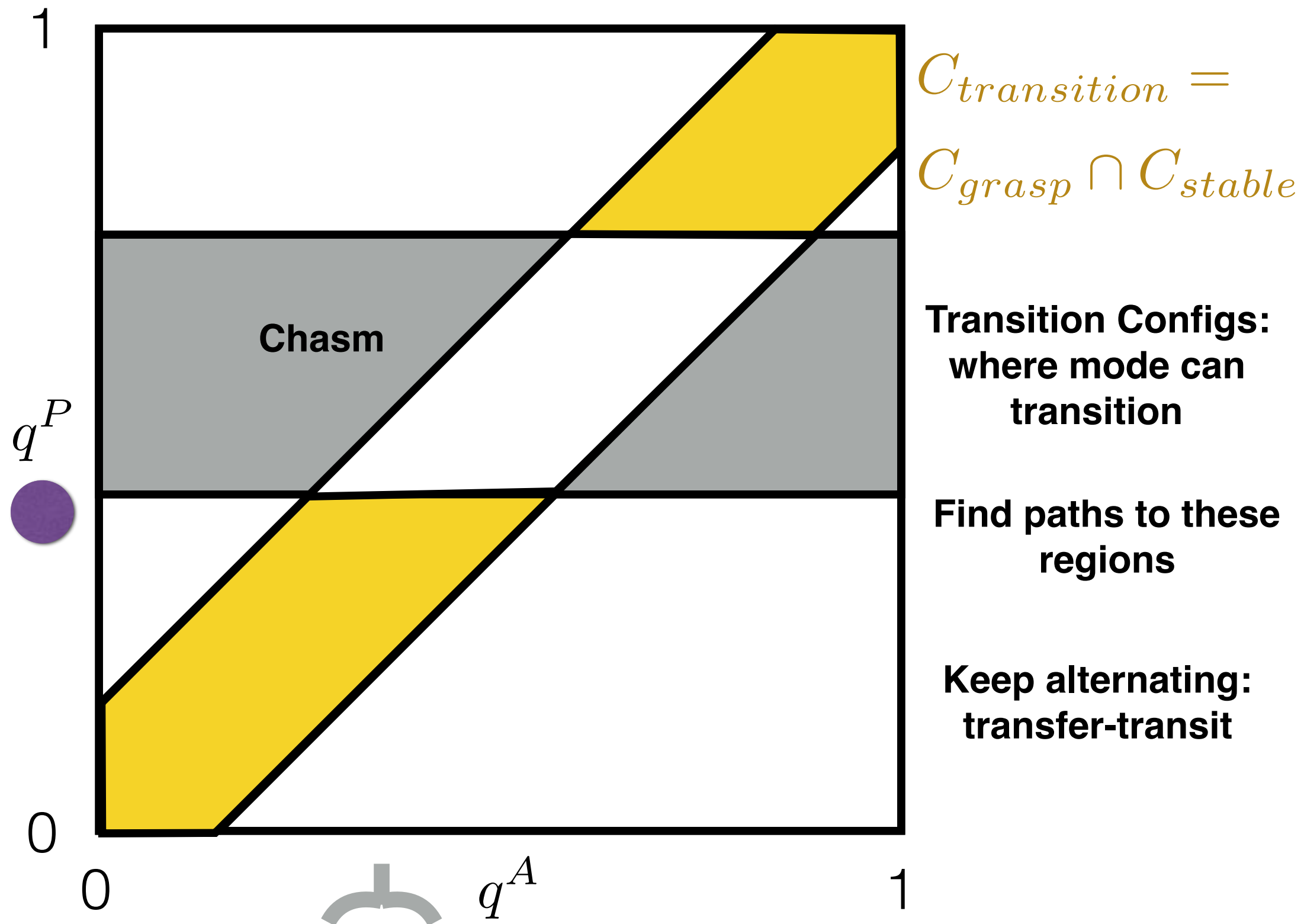


$$q \in \mathcal{C}_{tra}$$

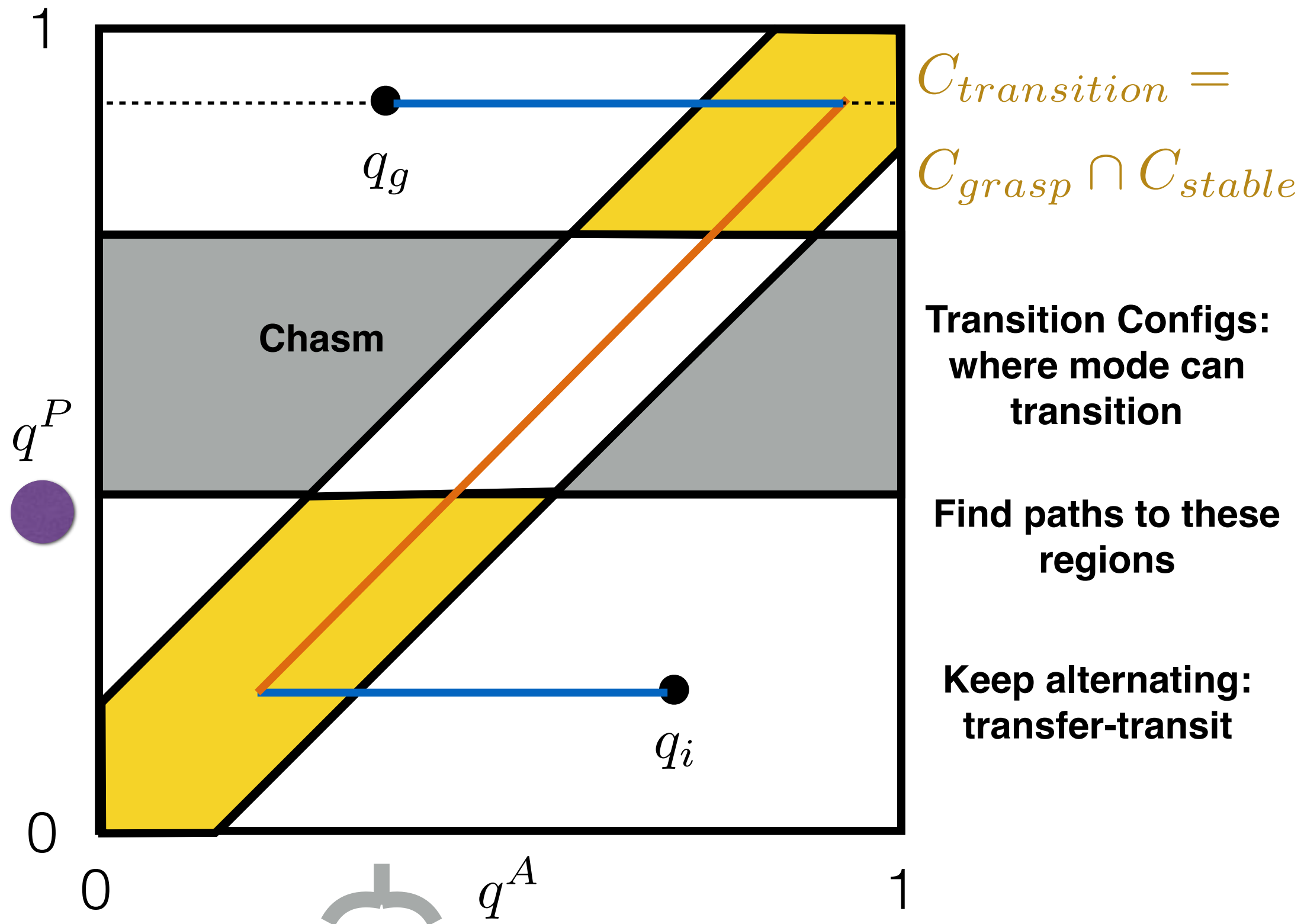
$$\mathcal{C}_{adm} = \mathcal{C} \setminus (\mathcal{C}_{obs}^a \cup \mathcal{C}_{obs}^p \cup \mathcal{C}_{obs}^{ap}),$$

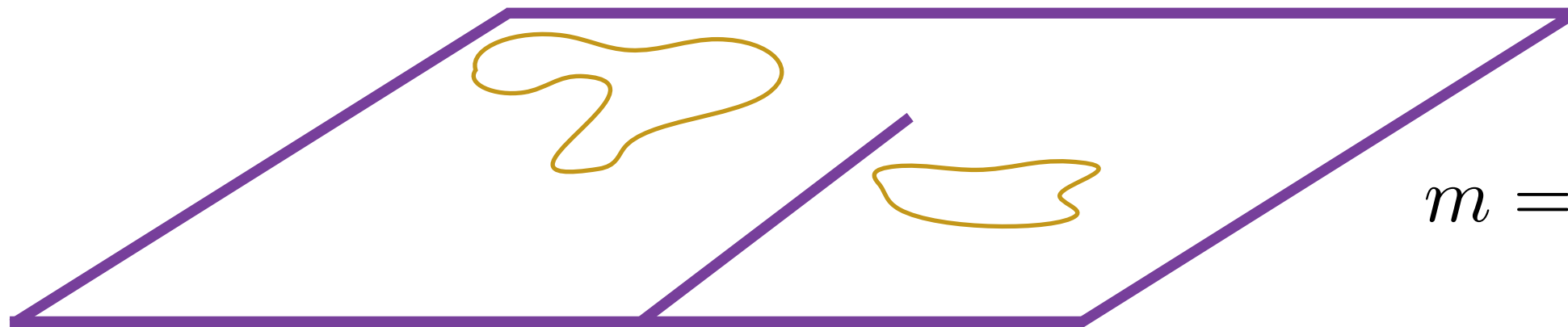
$$\mathcal{C}_{free} \subset \mathcal{C}_{admissible}$$

Bead on a Wire C-Space



Bead on a Wire C-Space



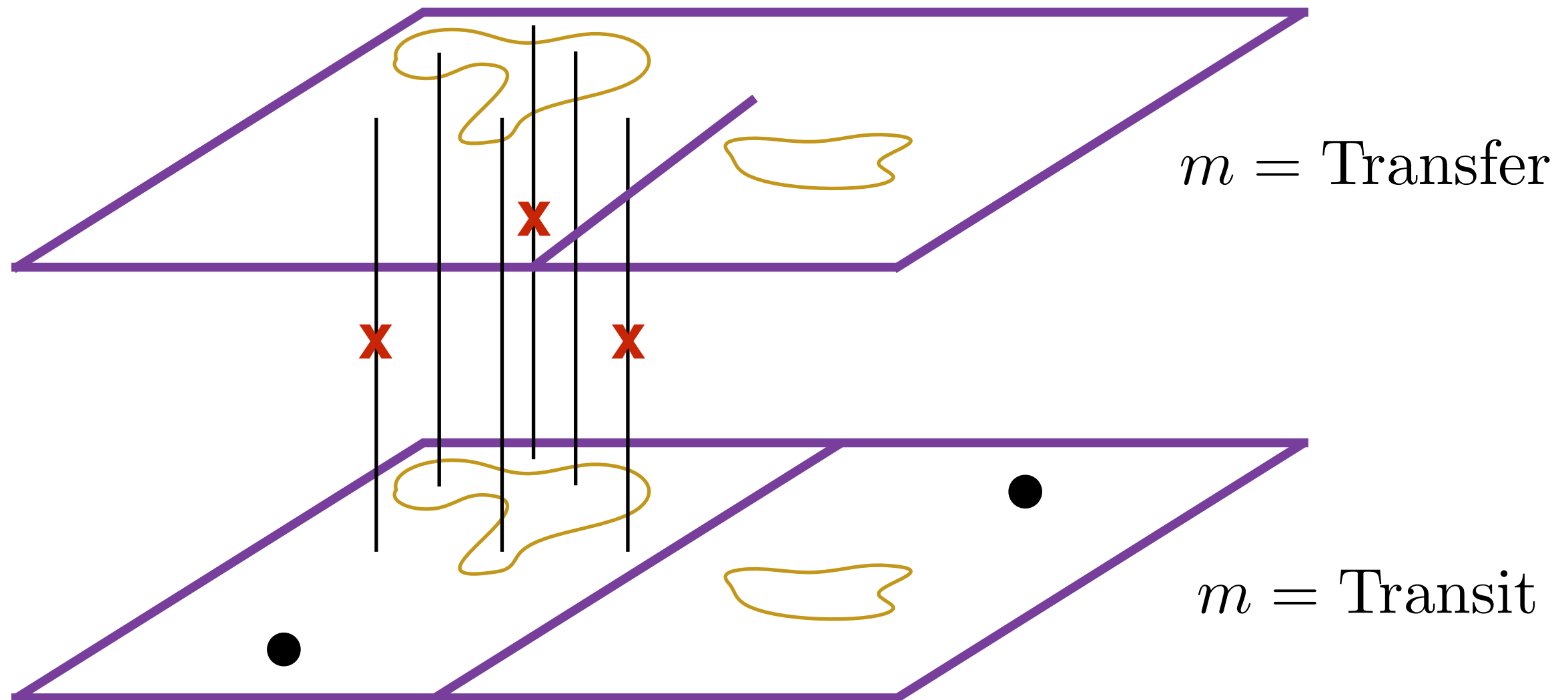


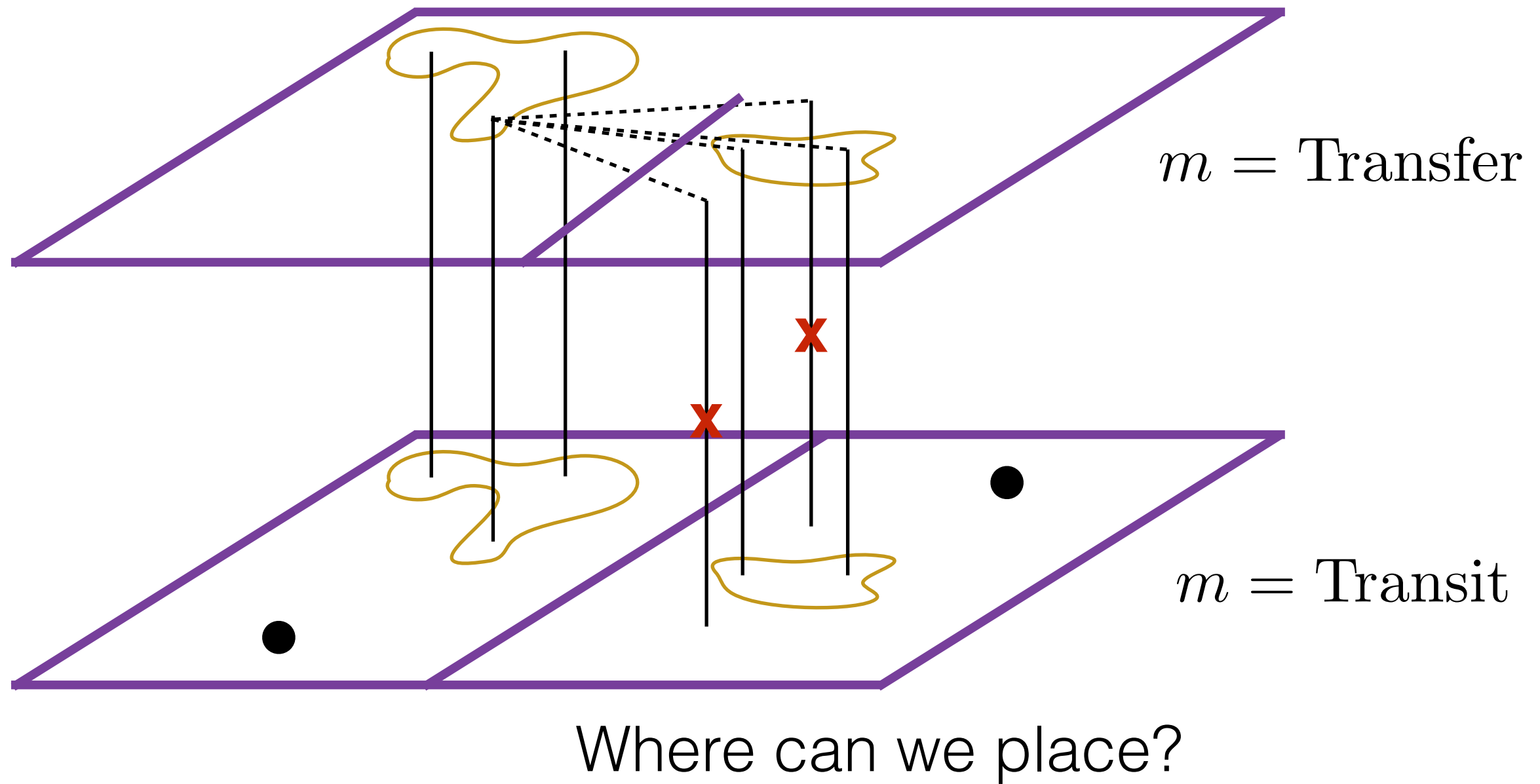
$m = \text{Transfer}$

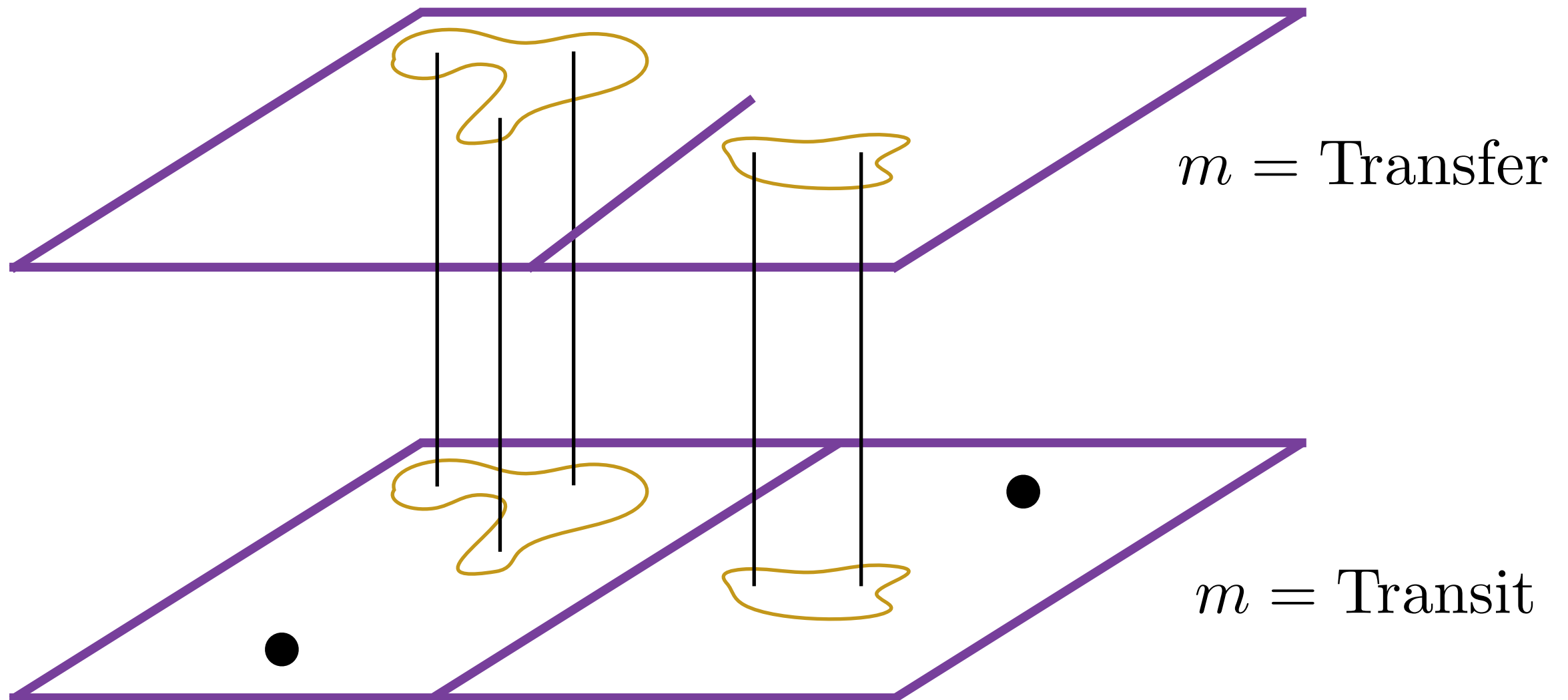


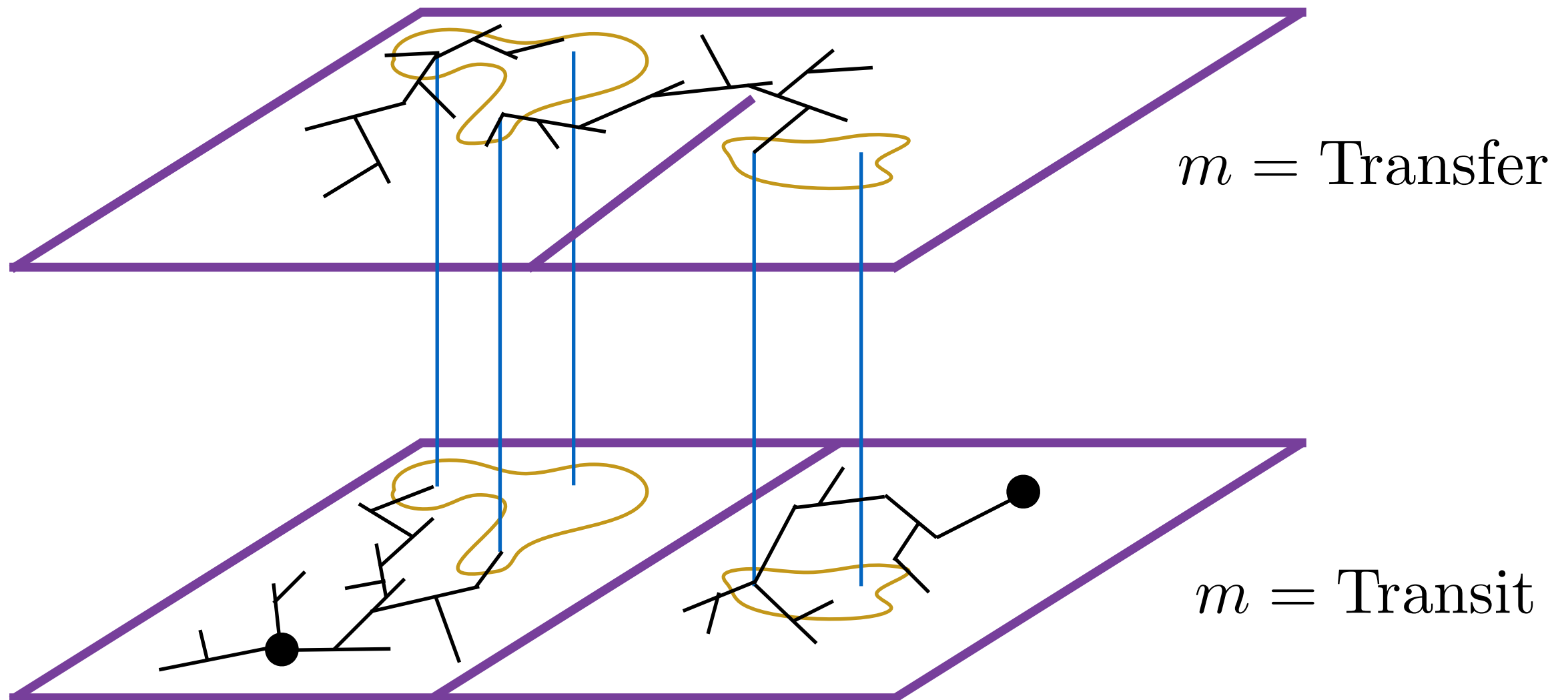
$m = \text{Transit}$

Where can we grasp?



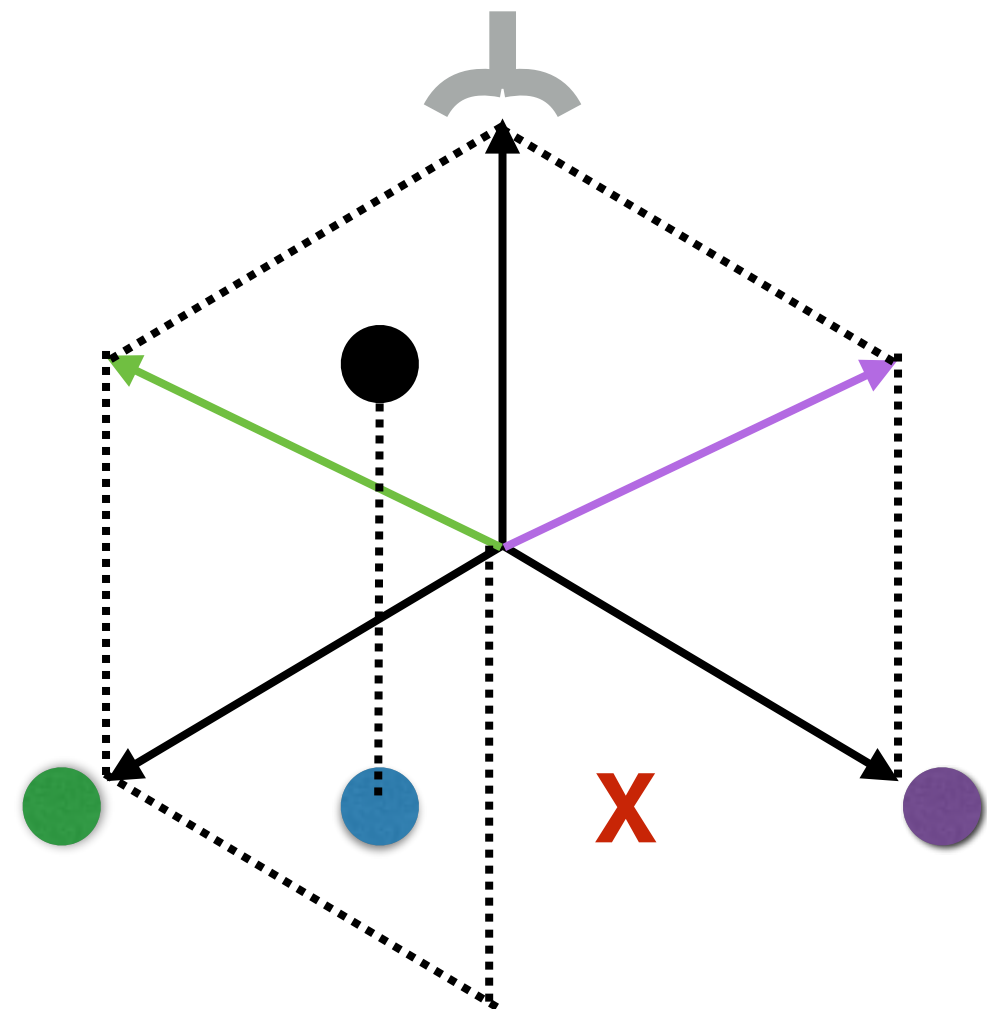
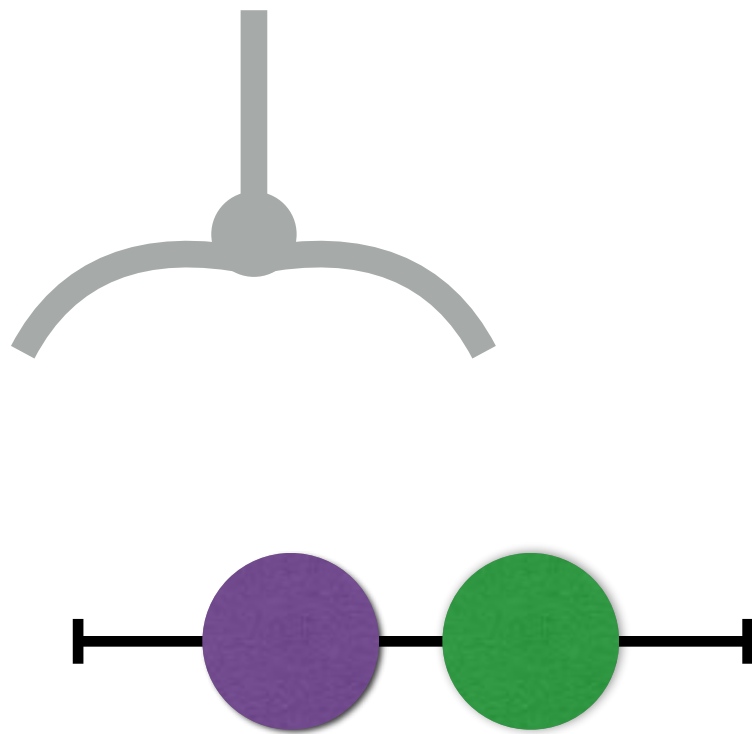






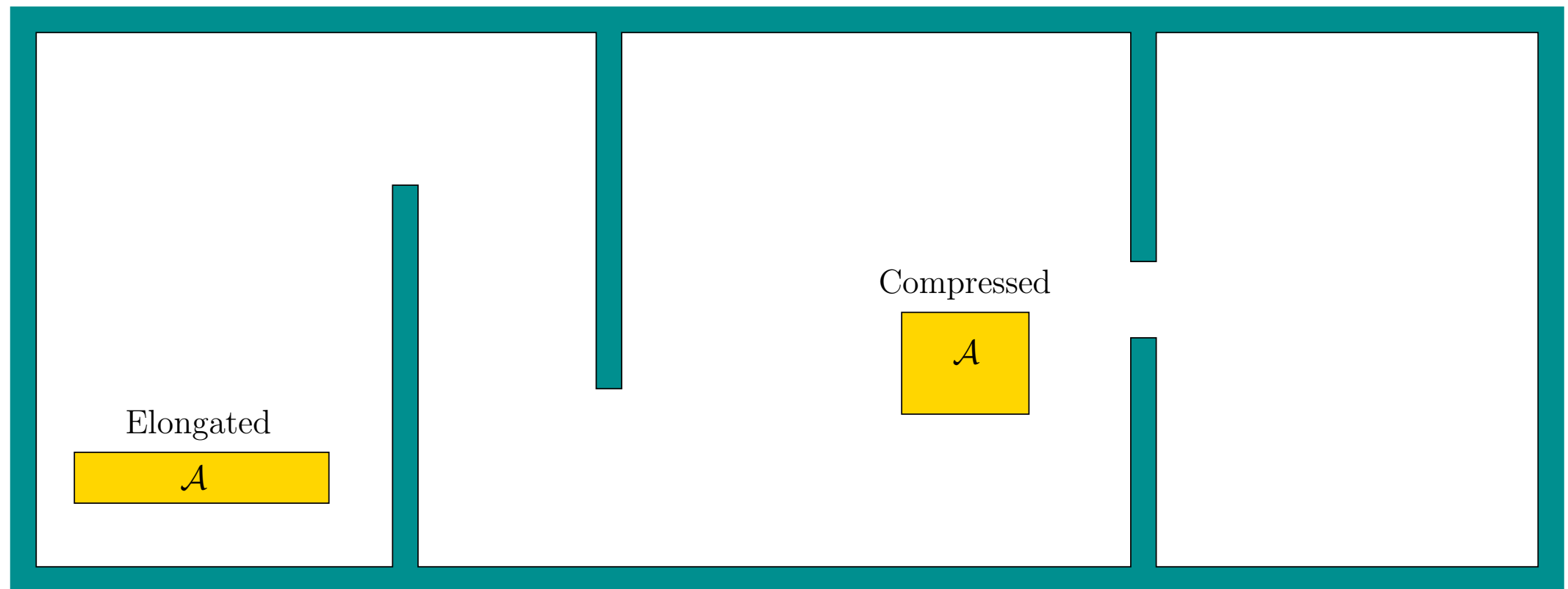
Multiple Objects

- Configuration space dimension increases with nr. objects
- Still have only two modes: **transfer** and **transit**
- Transfer lines are not parallel for different objects



Reconfigurable Robots

- Two modes to indicate discrete transformer modes



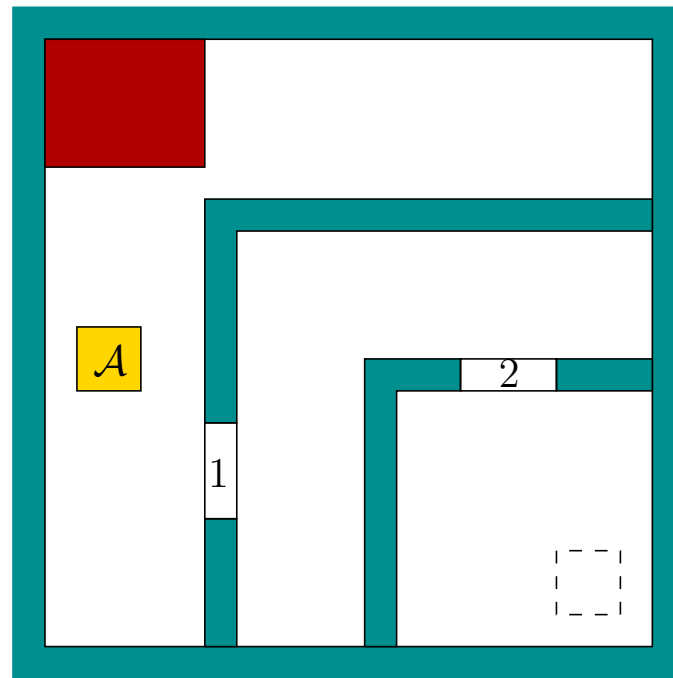
Elongated mode



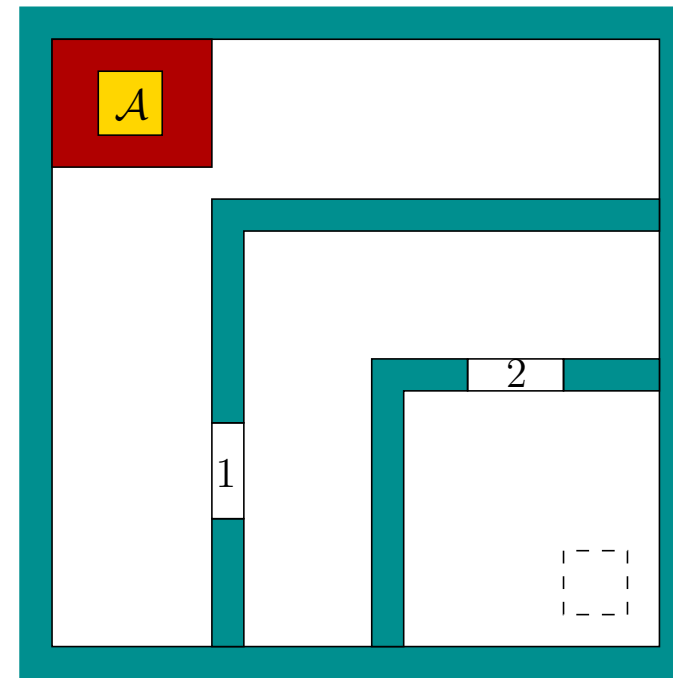
Compressed mode

Key Exchange

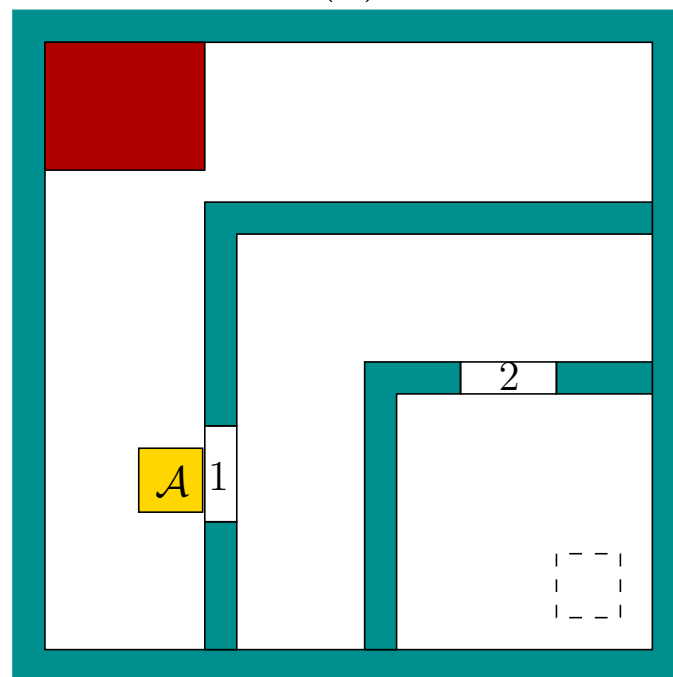
- 12 modes to indicate key holding (0,1,2) and door states



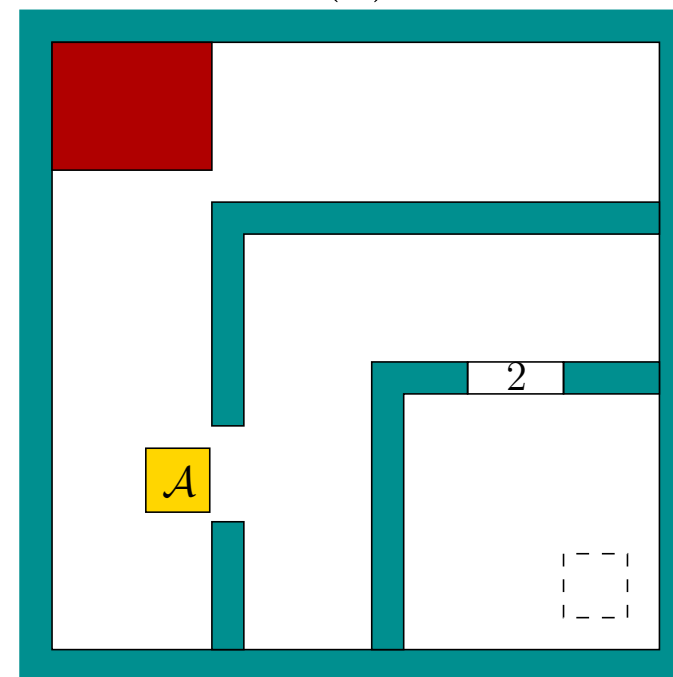
(a)



(b)



(c)



(d)

- Modes serve a **variety of roles** in robotics literature
 - ▶ May model problem in different ways (e.g., key exchange)
 - ▶ Some formulations create an infinite number of modes
- Modes provide **abstraction** and **structure** for planning
 - ▶ Don't need to consider details (e.g. grasping)
 - ▶ Result in subgoal bottle necks
 - ▶ Generate paths that go between the transition regions

Questions?