Robot Autonomy

Lecture 14: Grasp Synthesis and Quality Metrics

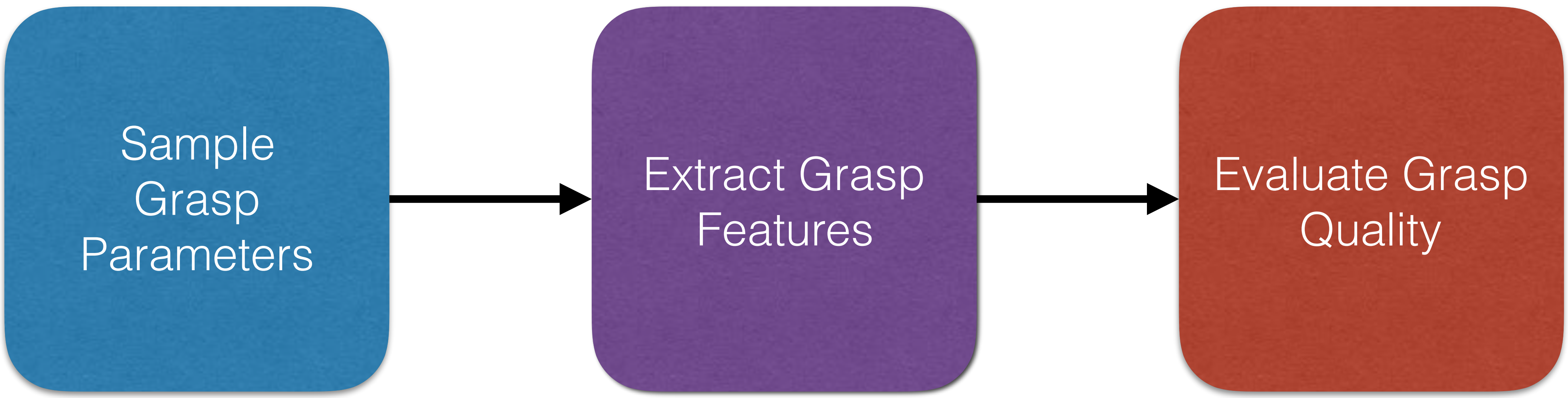
Oliver Kroemer

Grasp Synthesis Problem

- Where should I place hand to achieve a successful grasp?
  - Map object model to distribution of grasp parameters
  - Difficult to model and generalize
  - Procedures exist for certain sets of objects and contacts
    e.g. polygonal object and three contacts
  - Cache grasps for specific models
- Would placing my hand here result in a successful grasp?
  - Map a specific grasp to yes/no prediction
  - Easier to model and generalize across objects
  - Sample and evaluate various grasps

Sample and Evaluate

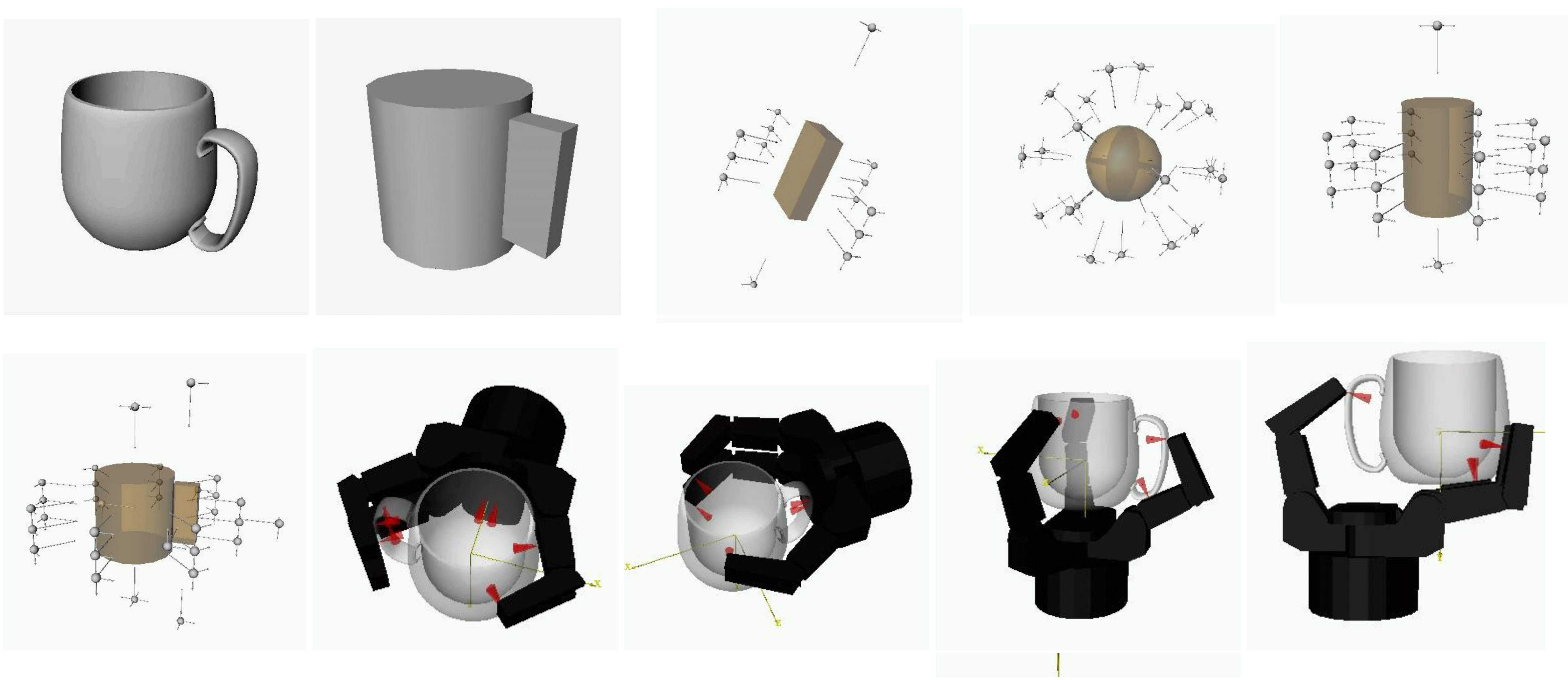
- Find suitable grasp through random sampling



- Do not need to find all grasps, just one that works
- Bias sampling towards regions more likely to succeed
  - Sample grasps near objects
  - Reduce dimensionality of sampling space

Sample Near Objects

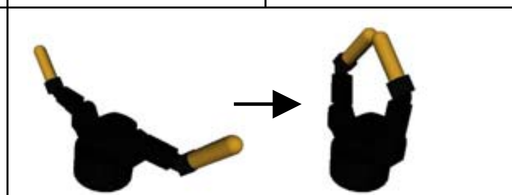
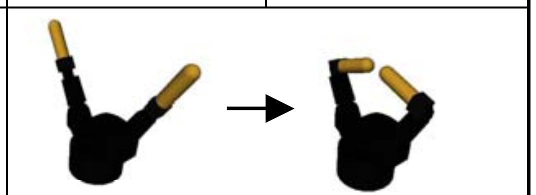
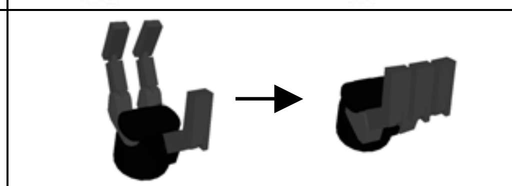
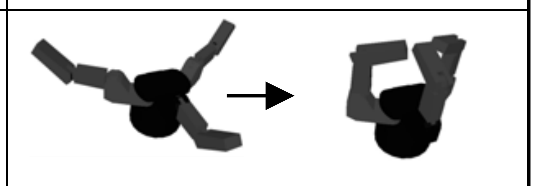
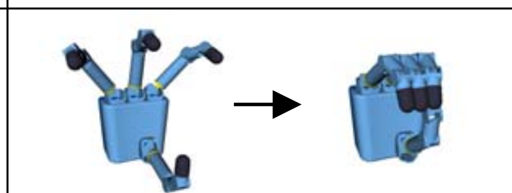
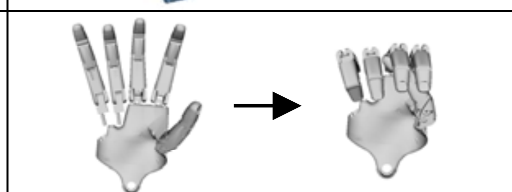
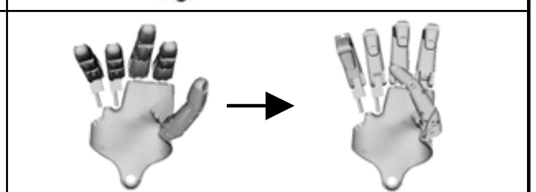
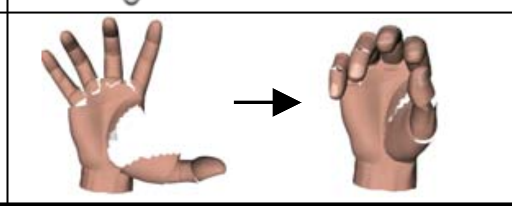
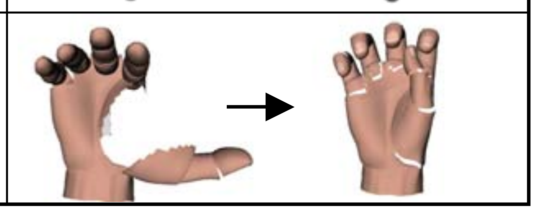
- Segment out supporting surfaces and focus on objects
  - Avoid sampling grasps at empty table locations
- Decompose objects into simple shapes for sampling



Miller et al.:Automatic Grasp Planning Using Shape Primitives, ICRA 2003

Reduce the Dimensionality

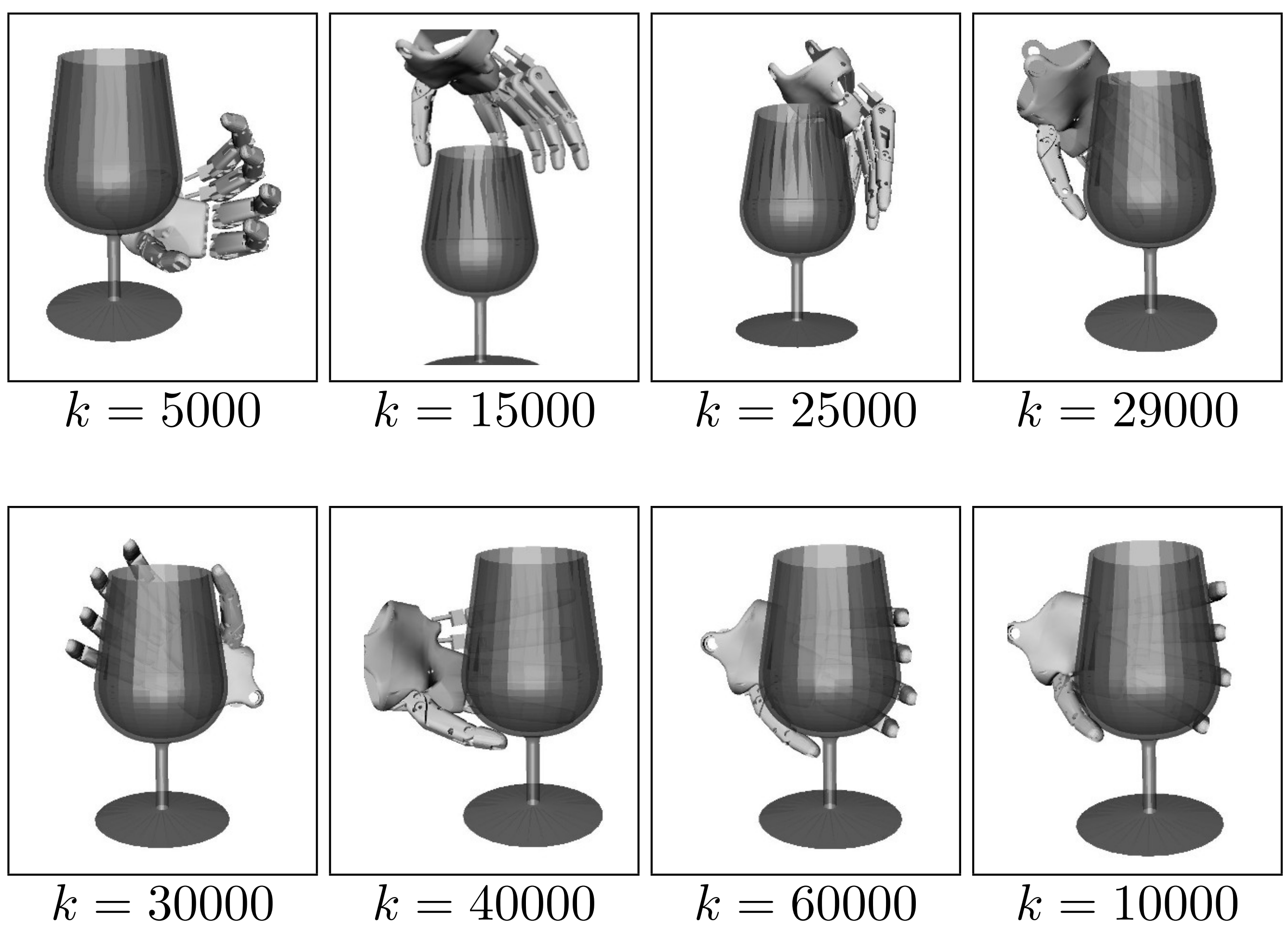
- Use planar top-down grasps for table top scenarios
  - Robust grasps that can exploit compliance of object
- Apply PCA to hand joints for grasping (Eigengrasps)

Model	DOFs	Eigengrasp 1			Eigengrasp 2		
		Description	min	max	Description	min	max
Gripper	4	Prox. joints flexion			Dist. joints flexion		
Barrett	4	Spread angle opening			Finger flexion		
DLR	12	Prox. joints flexion Finger abduction			Dist. joints flexion Thumb flexion		
Robonaut	14	Thumb flexion MCP flexion Index abduction			Thumb flexion MCP extension PIP flexion		
Human	20	Thumb rotation Thumb flexion MCP flexion Index abduction			Thumb flexion MCP extension PIP flexion		

Ciocarlie et al.:Dimensionality reduction for hand-independent dexterous robotic grasping, IROS 2007

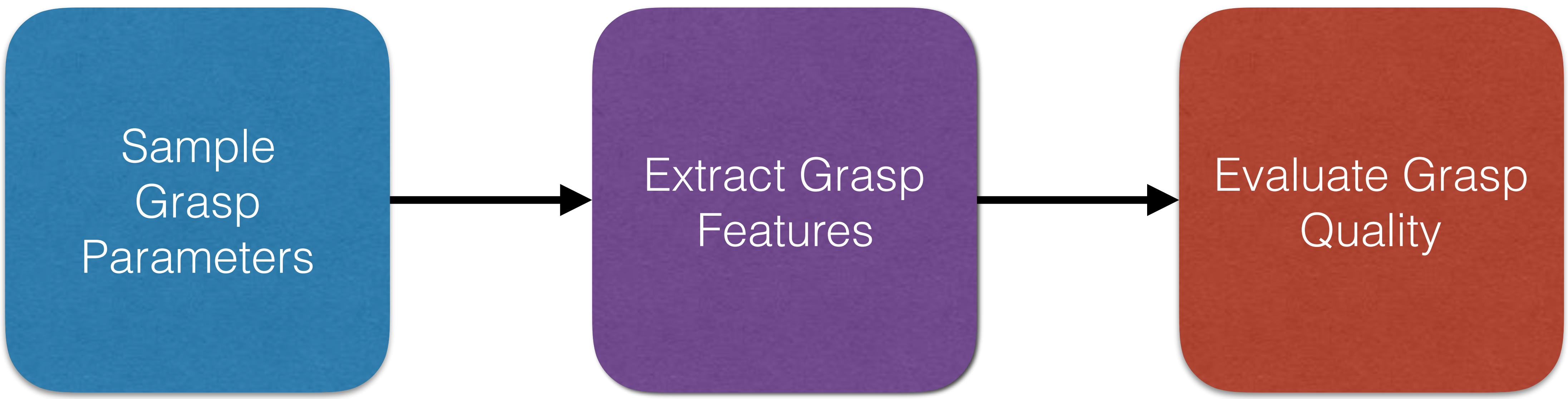
Optimization

- Search for high quality grasps by iterative sampling
  - Cross entropy method
  - Simulated annealing
- Best grasps after k steps of simulated annealing



Ciocarlie et al.:Dimensionality reduction for hand-independent dexterous robotic grasping, IROS 2007

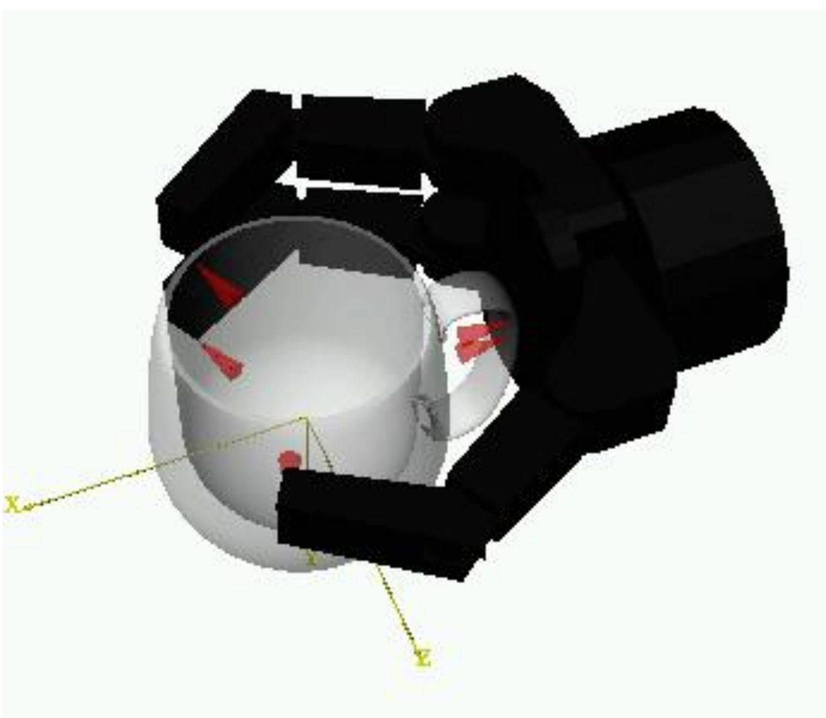
Sample and Evaluate Grasp Synthesis



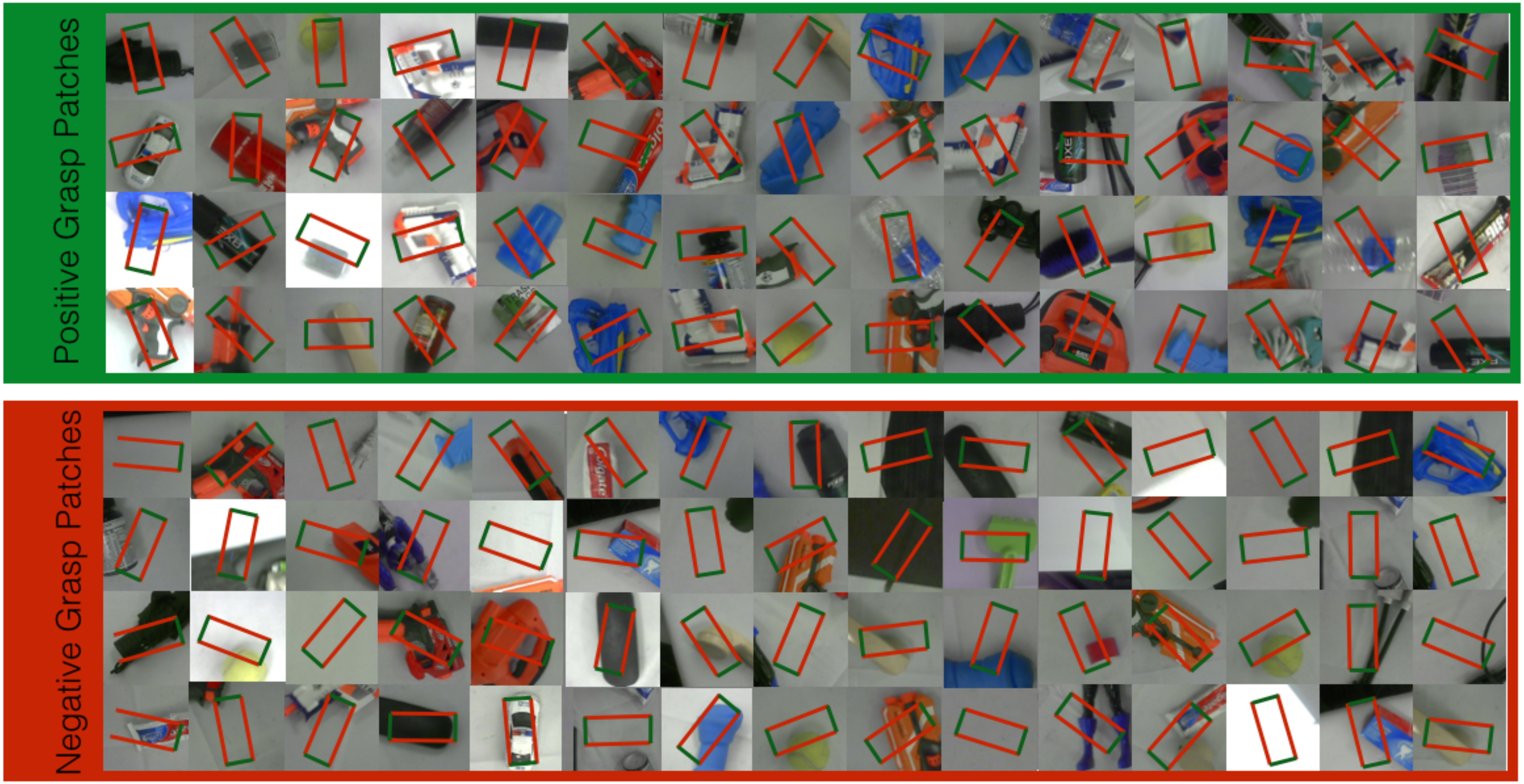
- Analytical model-based approaches
- Data-driven model-free approaches

note: feature extraction + evaluation also applies for grasp outcome

Analytical Model-based Approach

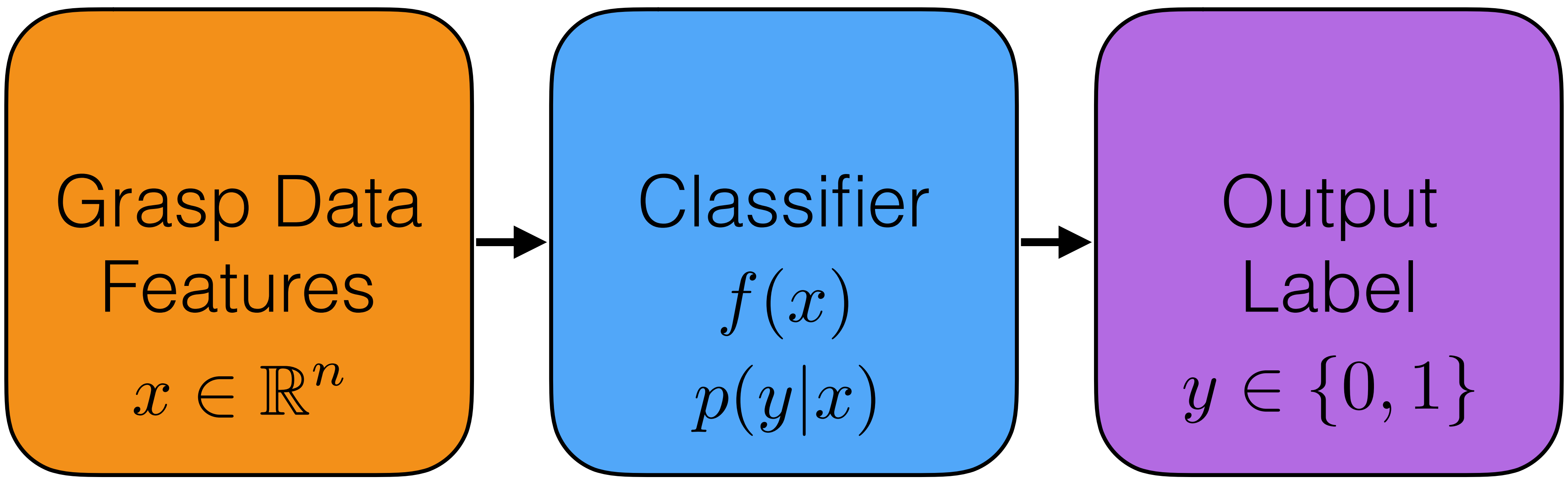


Model-free Data Driven Approach



Classifying Grasp Candidates

- Classification problem from machine learning viewpoint



- Discriminant function defines decision boundary

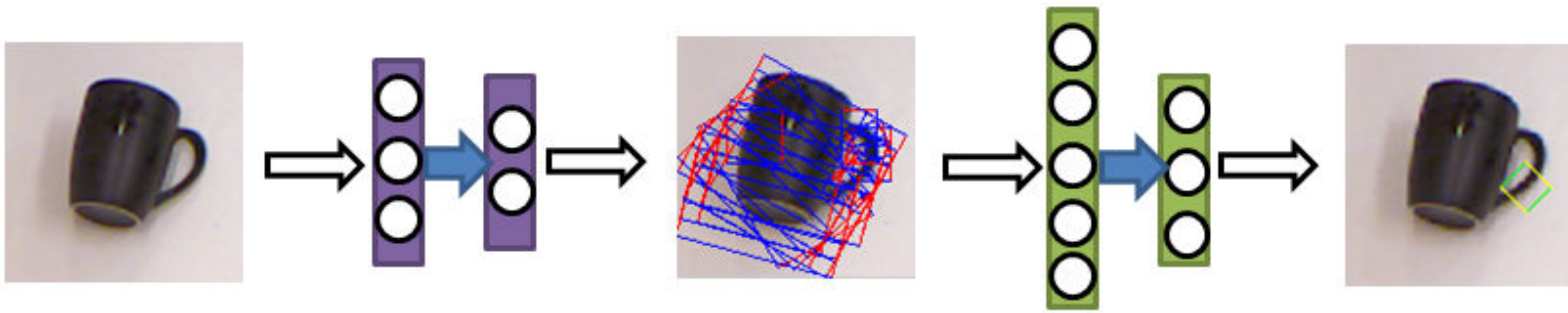
$$f(x) = 0$$

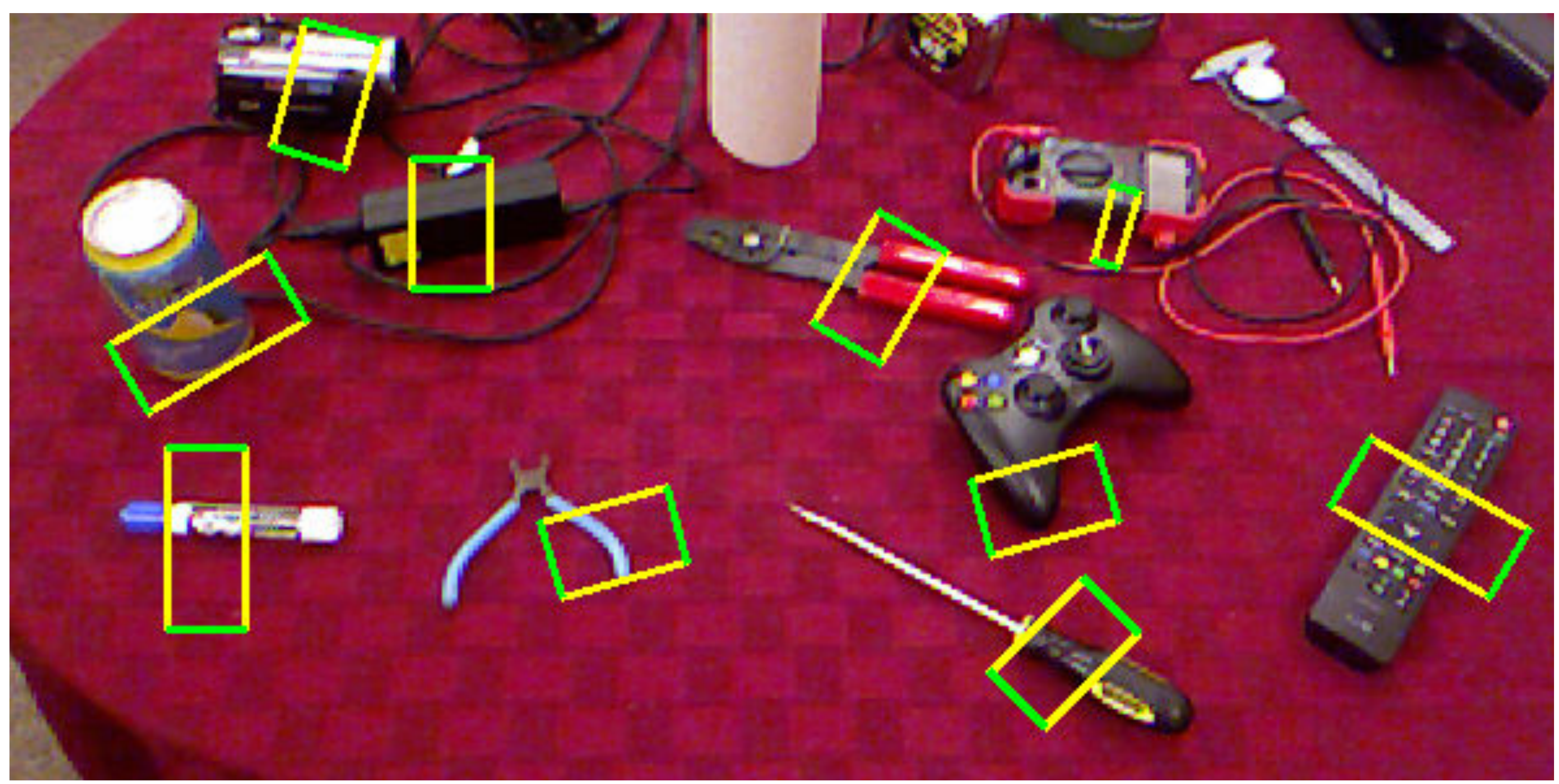
- Probabilistic classifier models posterior distribution

$$p(y|x)$$

Neural Networks

- Learn neural network classifier with RGB-D inputs
- Two-stage selection process

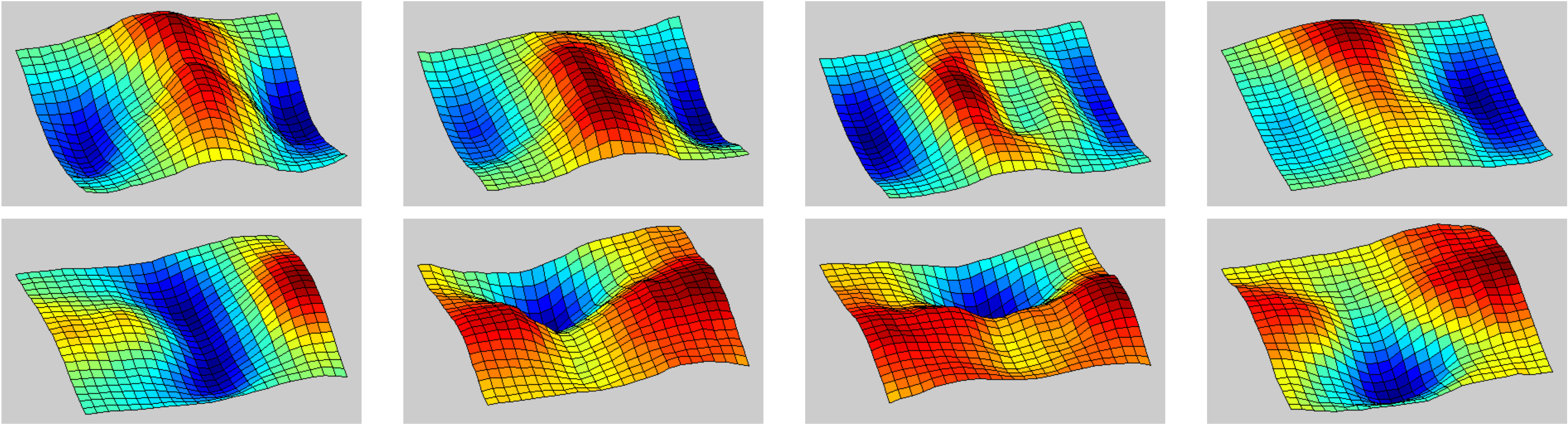




Lenz et al., Deep Learning for Detecting Robotic Grasps, International Journal of Robotics Research 2015

Neural Networks

- Learned depth features correlated with successful (top) and unsuccessful (bottom) grasps

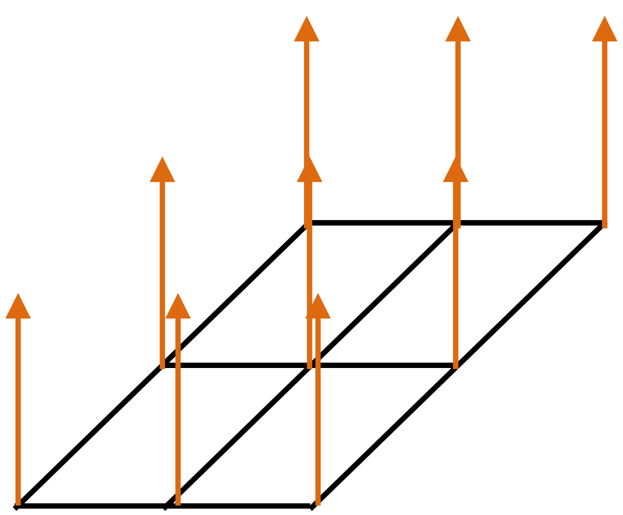


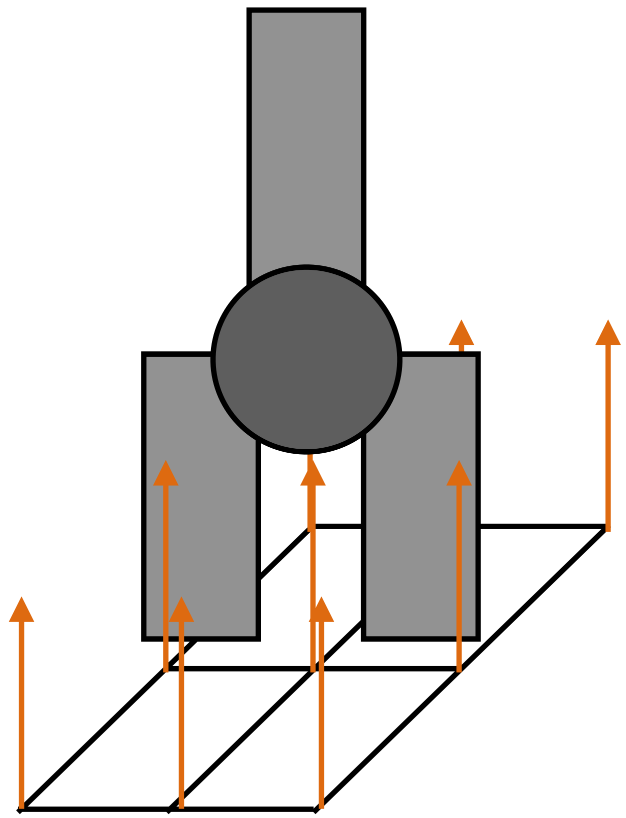
Lenz et al., Deep Learning for Detecting Robotic Grasps, International Journal of Robotics Research 2015

Consider Simple Grid of Features

- Define a set of features x as height points in a grid
  - Grid defined relative to gripper pose being evaluated

$x \in \mathbb{R}^9$





0	7	0.1
0	4	0
0	0.4	0

4.1	1.4	0.1
6	4	5
0	0.4	0

?

0	0.2	4.1
1.1	4	0.5
0	4.1	0

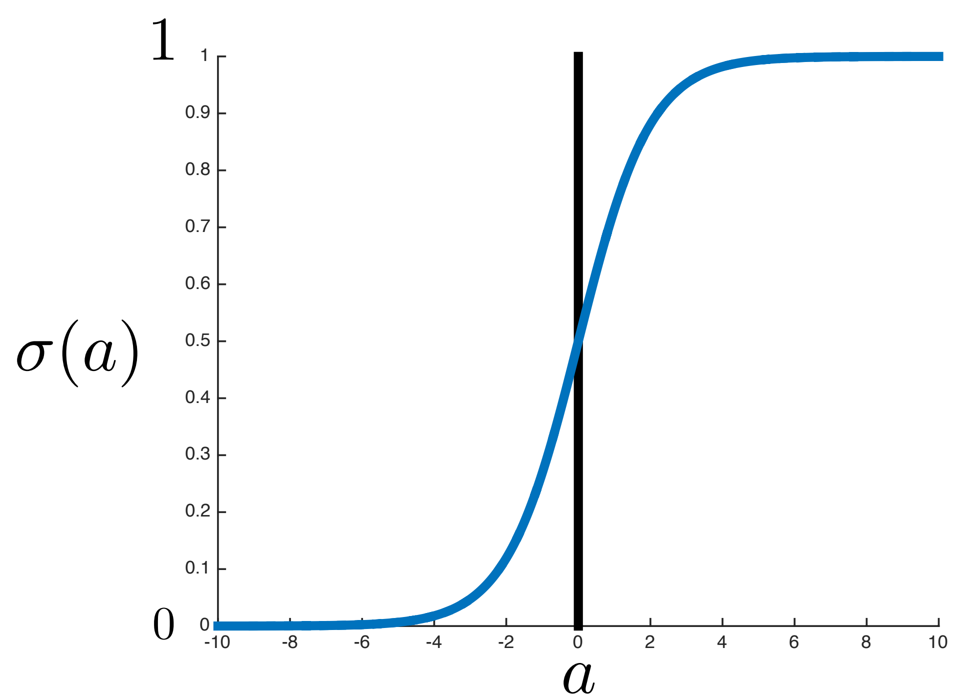
  - Could add additional feature indicate grasping heigh
- Output is binary success: $y = 1$, failure: $y = 0$

Logistic Regression

- Model posterior distribution using sigmoid function

$$p(y=1|x) = \sigma(\theta^T x) = \frac{1}{1+\exp(-\theta^T x)}$$

$$p(y=0|x) = 1 - \sigma(\theta^T x)$$



- Assume x contains a constant bias term of 1
- Need to learn the parameters θ from training data

Logistic Regression Training

- **Training data** consists of N labeled data points

$$y^{(1)}, y^{(2)}, \ldots, y^{(N)} \qquad\qquad x^{(1)}, x^{(2)}, \ldots, x^{(N)}$$

- **Likelihood** of the model parameters

$$L(\theta) = \prod_{i=1}^{N} \sigma(\theta^T x^{(i)})^{y^{(i)}} (1 - \sigma(\theta^T x^{(i)}))^{1-y^{(i)}}$$

- Negative log likelihood **error function** (cross entropy)

$$E(\theta) = -\log(L(\theta)) = -\sum_{i=1}^{N} y^{(i)} \log(\sigma(\theta^T x^{(i)})) + (1 - y^{(i)}) \log(1 - \sigma(\theta^T x^{(i)}))$$

- Optimize parameters using **gradient decent**

$$\nabla E(\theta) = \sum_{i=1}^{N} (\sigma(\theta^T x^{(i)}) - y^{(i)}) x^{(i)} \qquad\qquad \theta_{\text{new}} = \theta_{\text{old}} - \beta \nabla E(\theta_{\text{old}})$$

learning rate

Logistic Regression Example

- Compute the updated parameters for the following data

$$x^{(1)} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \; y^{(1)} = 1 \qquad x^{(2)} \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}, \; y^{(2)} = 0 \qquad x^{(3)} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \; y^{(3)} = 1$$

$$\theta_{\text{old}} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}^T, \; \beta = 0.1$$

$$x^{(1)} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \ y^{(1)} = 1 \qquad x^{(2)} = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}, \ y^{(2)} = 0 \qquad x^{(3)} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \ y^{(3)} = 1$$

$$\theta_{\text{old}} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}^T, \ \beta = 0.1 \qquad \nabla E(\theta) = \sum_{i=1}^{N} (\sigma(\theta^T x^{(i)}) - y^{(i)}) x^{(i)}$$

- Compute current predictions

$$\sigma(0 + 2 - 1) = 0.7311 \qquad \sigma(1 + 0.5 - 1) = 0.6225 \qquad \sigma(-1 + 0 - 1) = 0.1192$$

- Compute errors

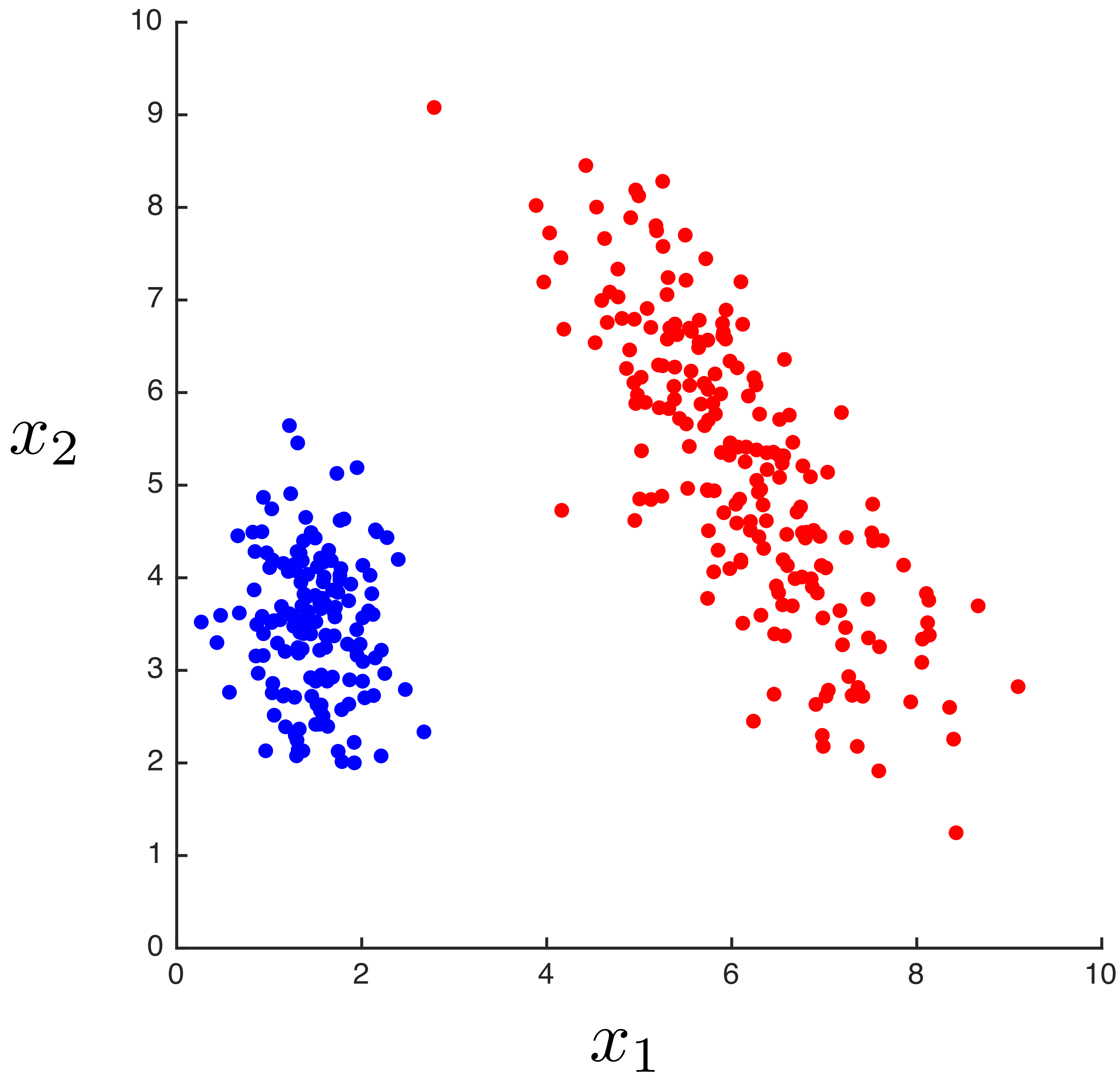
$$0.7311 - 1 = -0.2689 \qquad 0.6225 - 0 = 0.6225 \qquad 0.1192 - 1 = -0.8808$$

- Compute gradient and update parameters

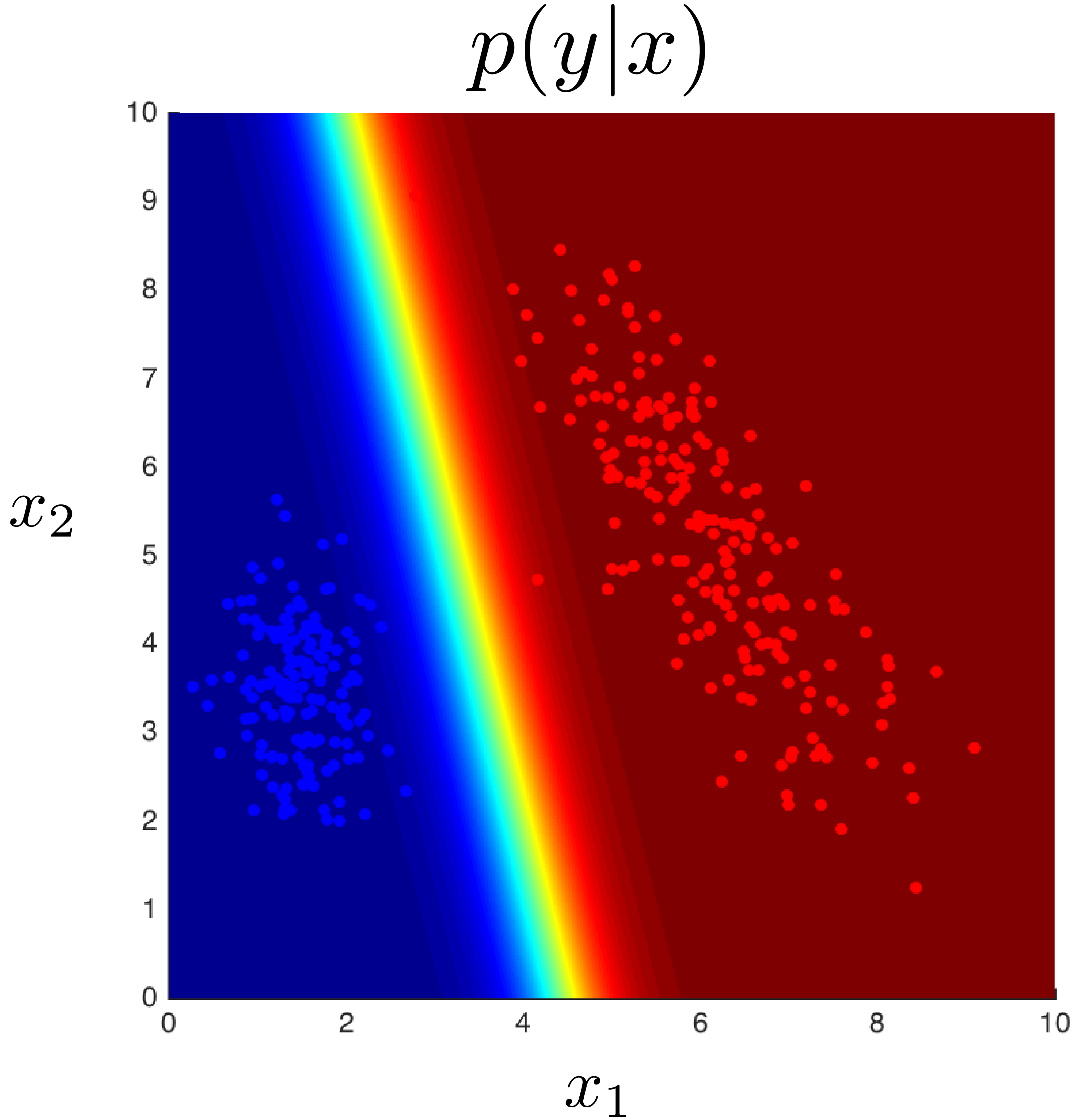
$$\nabla E = -0.2689 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + 0.6225 \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix} - 0.8808 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5033 \\ -0.2267 \\ -0.5273 \end{bmatrix}$$

$$\theta_{\text{new}} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - 0.1 \begin{bmatrix} 1.5033 \\ -0.2267 \\ -0.5273 \end{bmatrix} = \begin{bmatrix} 0.8497 \\ 1.0227 \\ -0.9473 \end{bmatrix}$$

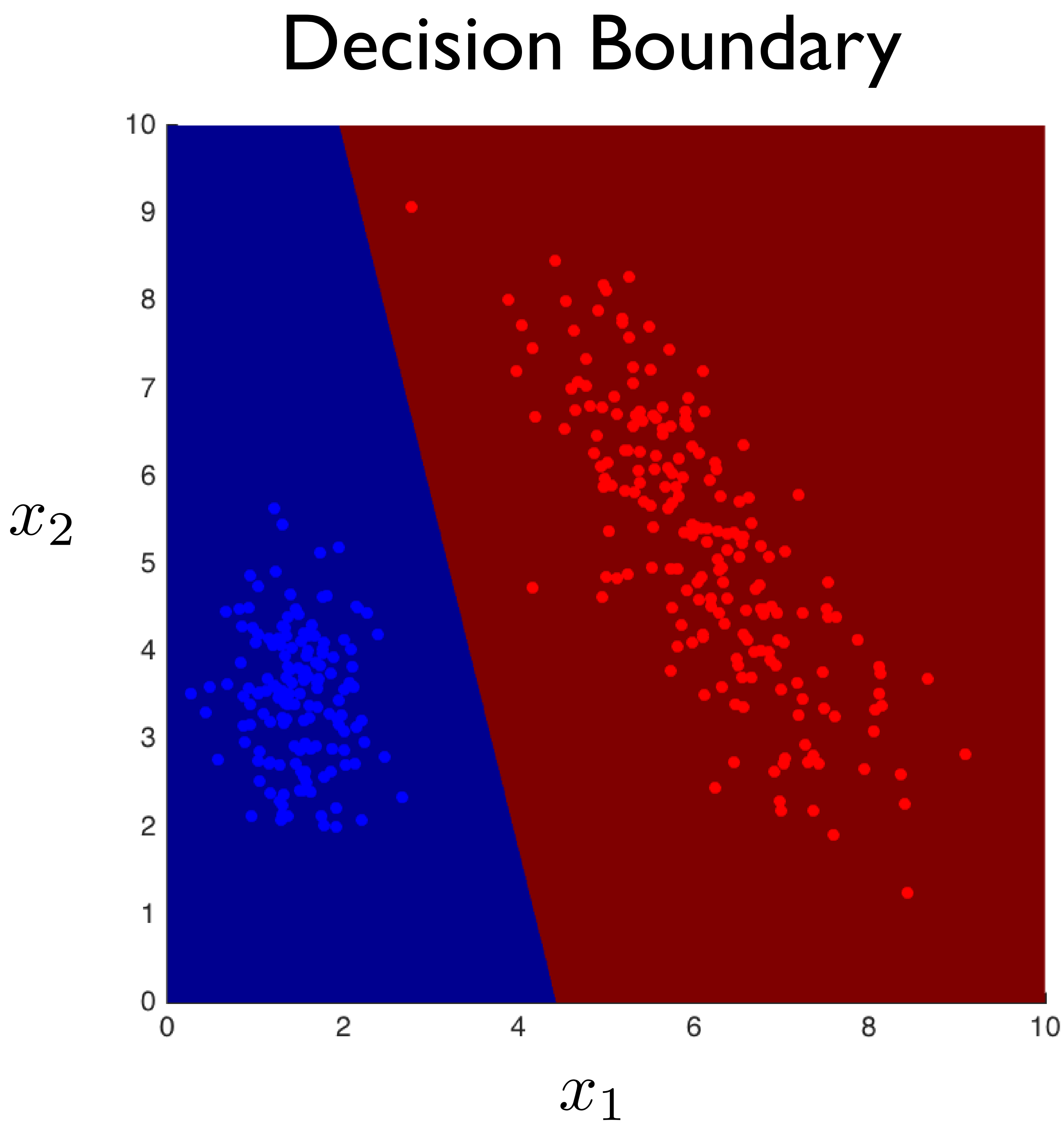
Logistic Regression Example



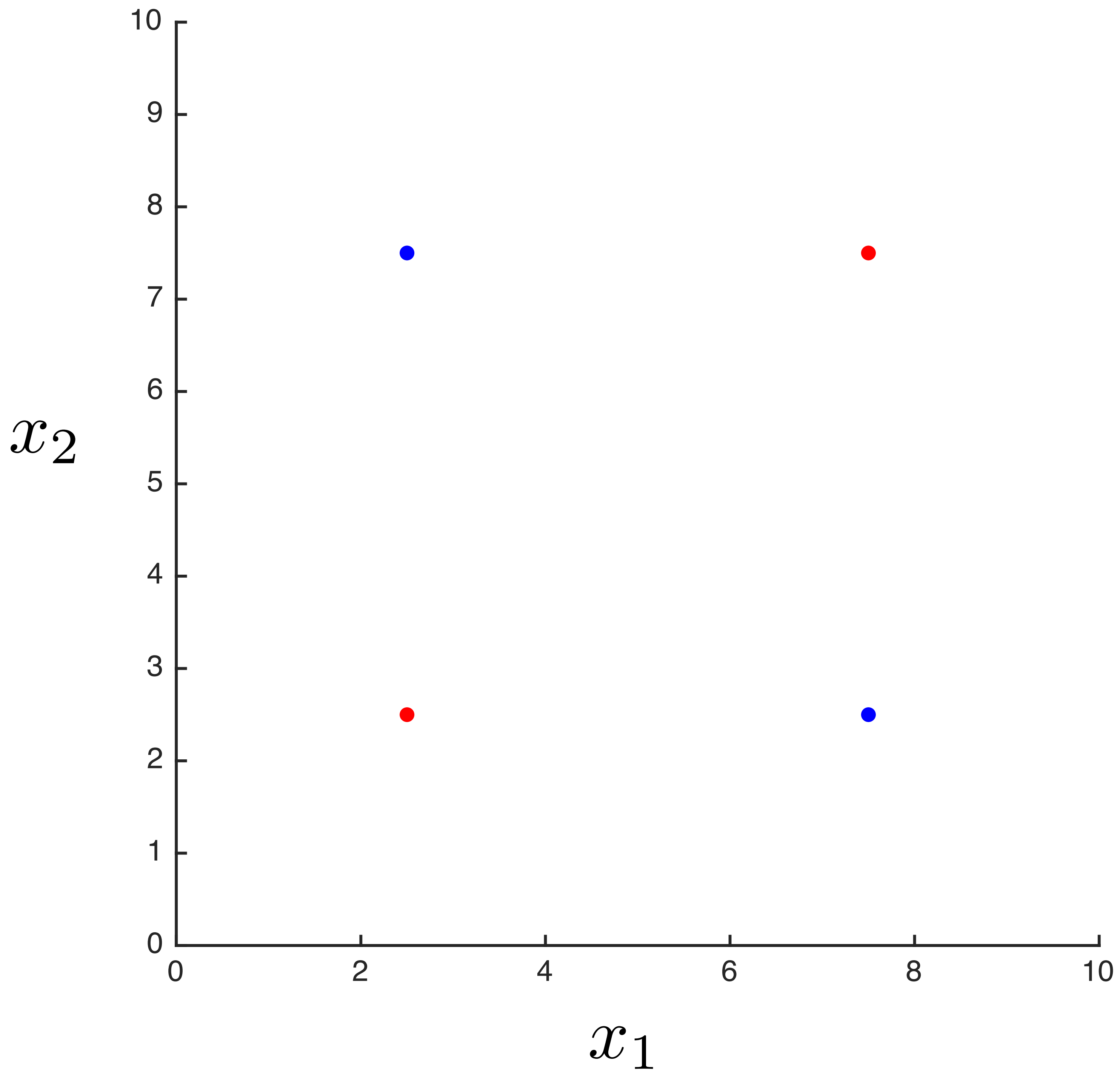
Logistic Regression Example



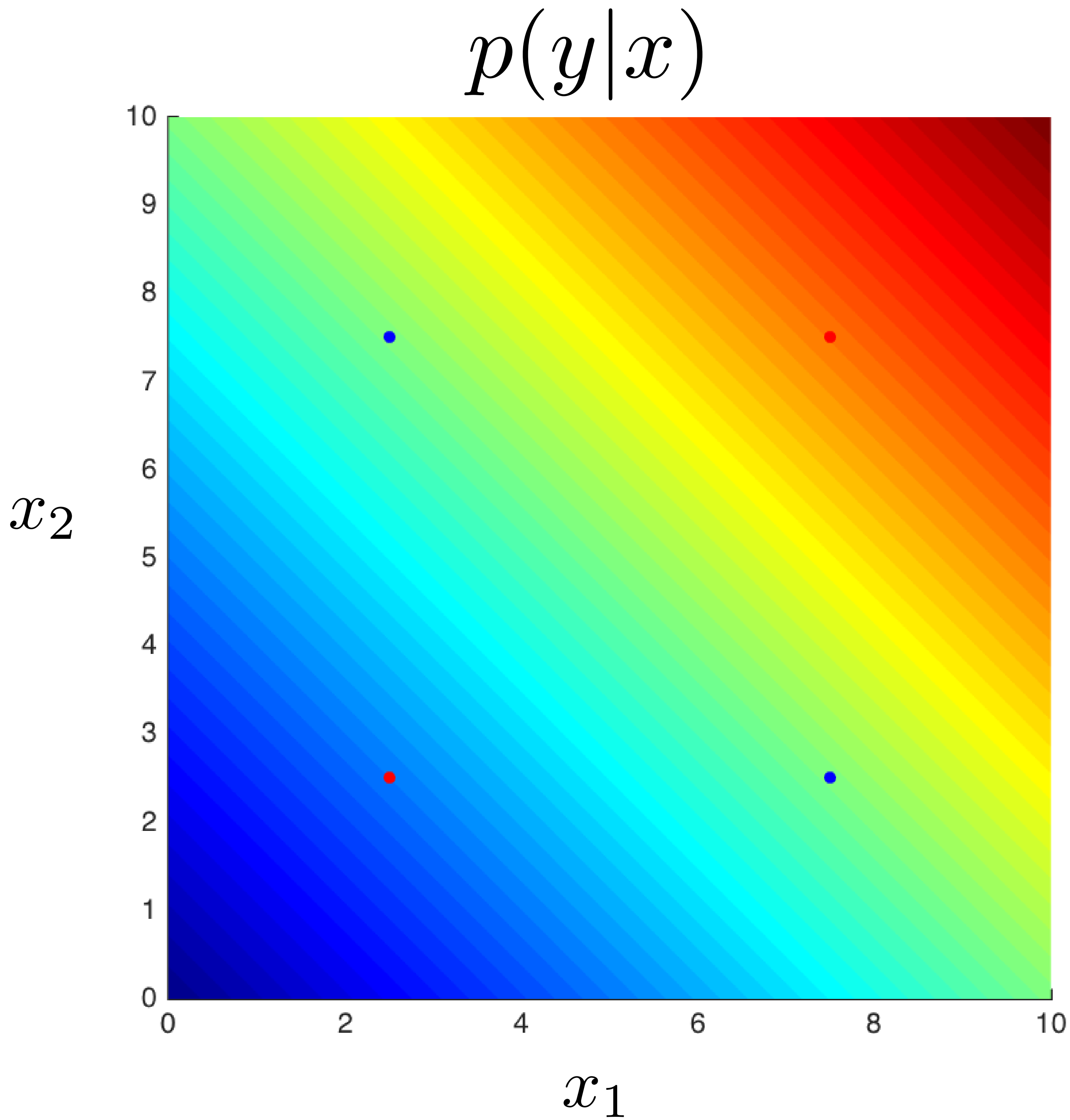
Logistic Regression Example



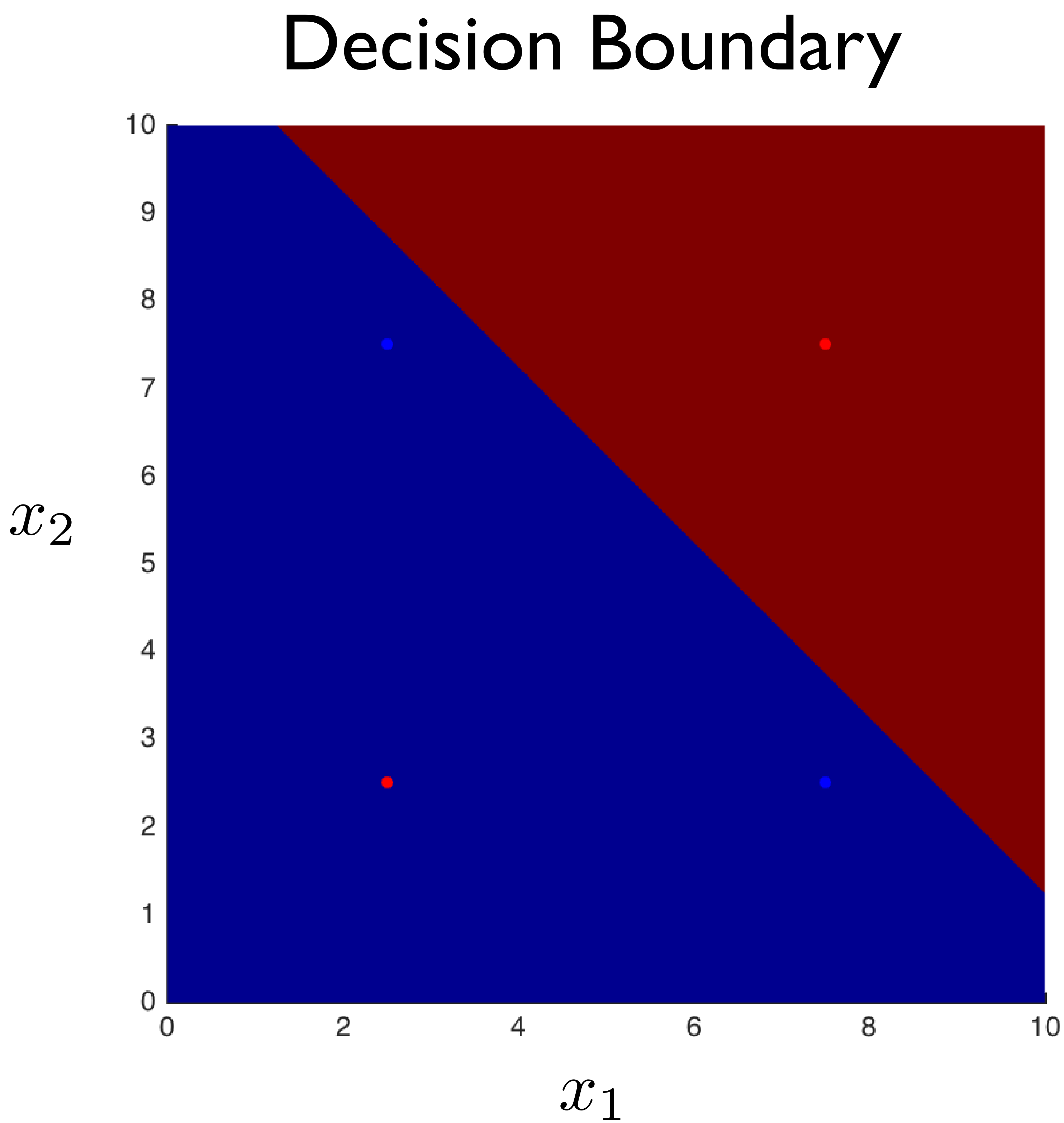
Logistic Regression XOR Example



Logistic Regression XOR Example



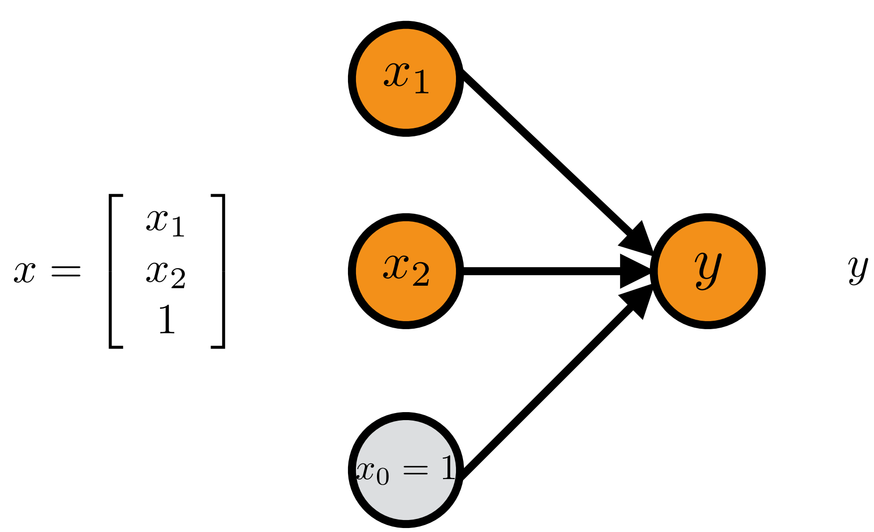
Logistic Regression XOR Example

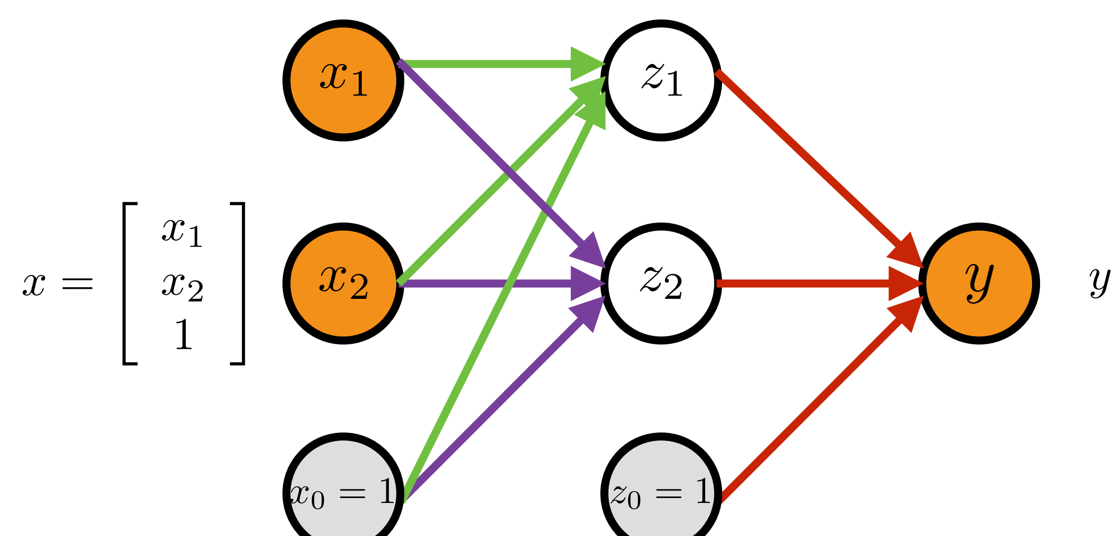


Not linearly separable

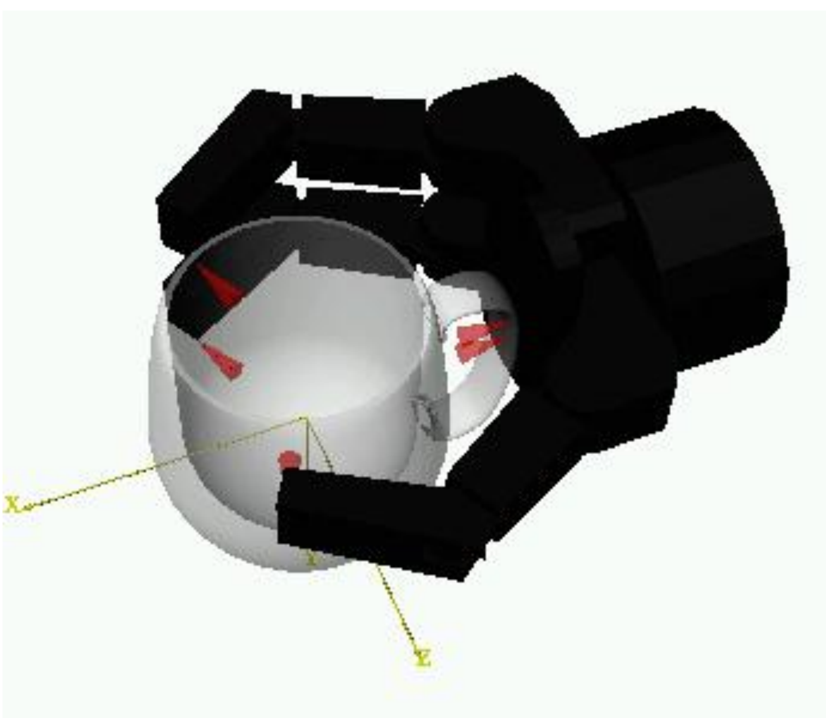
Need Better Features

- Finding suitable features is a key challenge
- Variety of approaches to creating features:
- Engineer features using prior domain knowledge
- Use kernels to implicitly capture infinite-dimensional features
- Use unsupervised learning methods
- Use neural networks to learn features

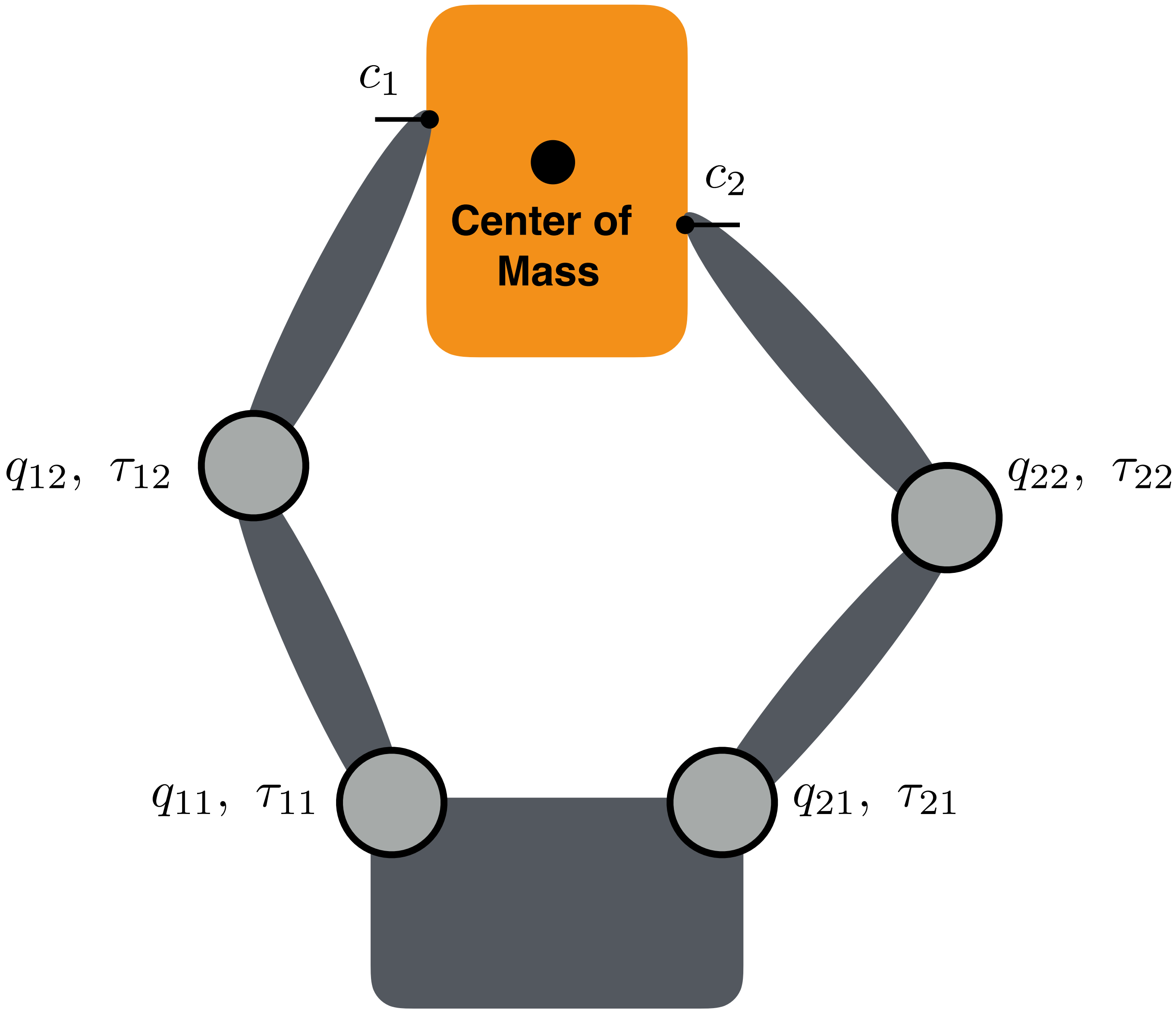




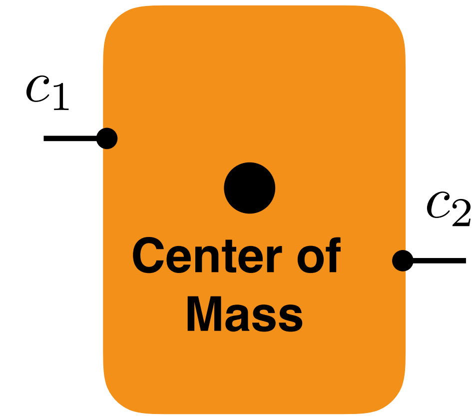
Analytical Model-based Approach



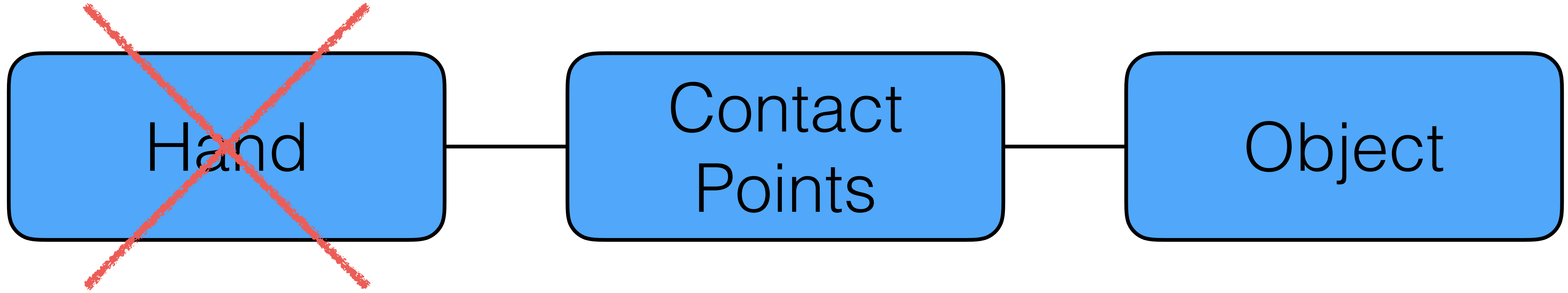
Modeling Grasps



Modeling Grasps



- Consider only object and contact points
- Contacts treated as idealized contact points
- Assume fingers can apply necessary forces
- Contact interactions apply forces and torques to object



Wrenches

- Interaction results in forces and torques (wrenches)

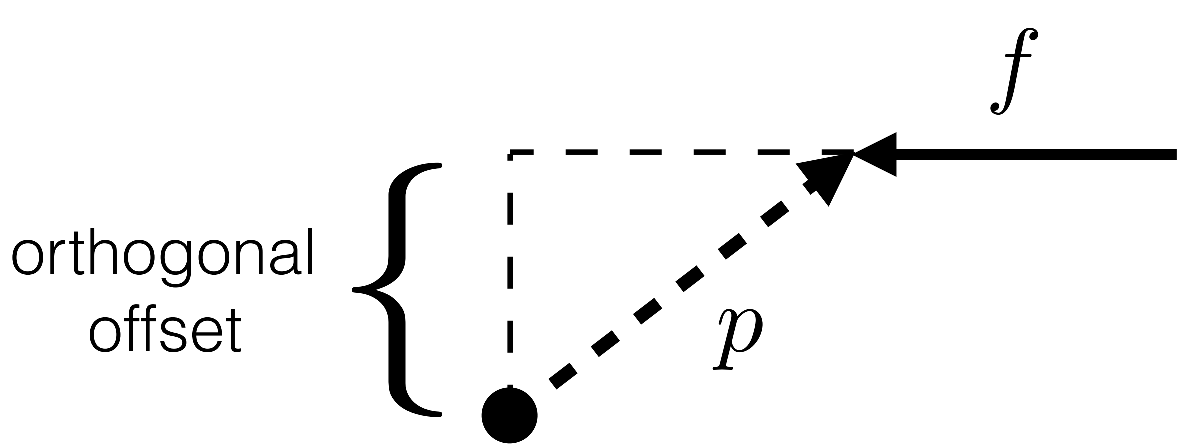
3D

$$w = \begin{bmatrix} f_x \\ f_y \\ f_z \\ t_x \\ t_y \\ t_z \end{bmatrix}$$

Planar

$$w = \begin{bmatrix} f_x \\ f_y \\ t_z \end{bmatrix}$$

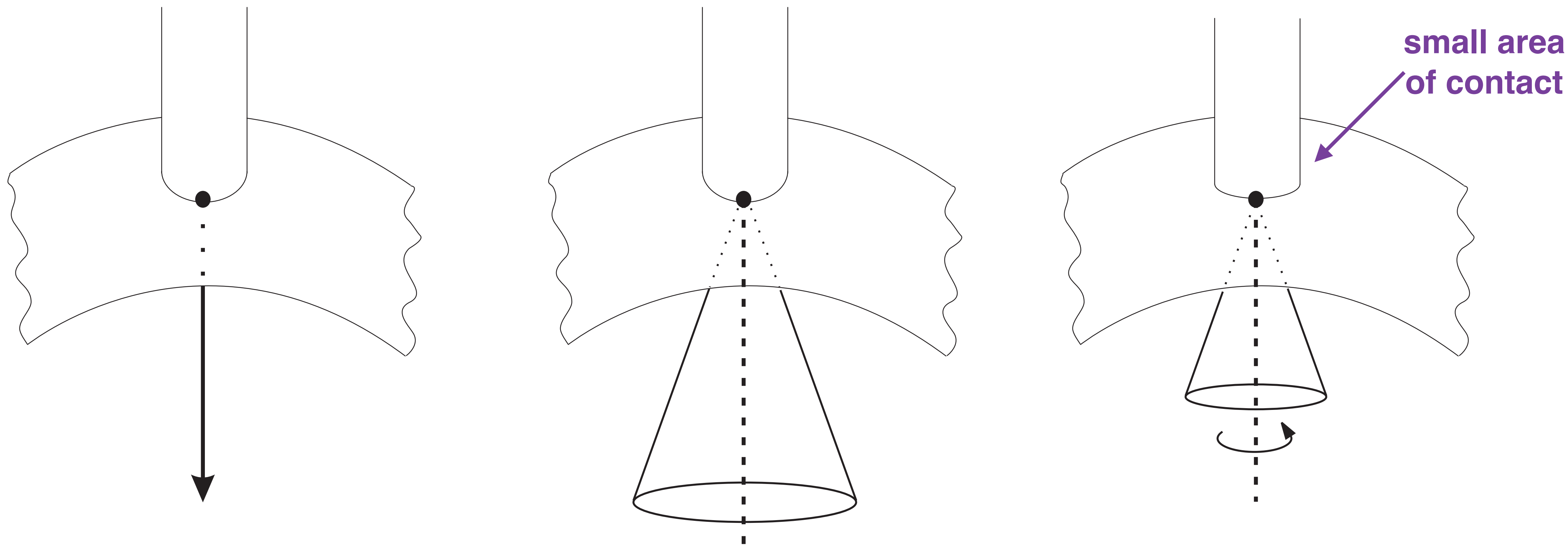
- Force with an orthogonal offset results in a torque



$$t = p \times f$$

Types of Contacts

Models for contacts between object and hand:



Point Contact without Friction

- Normal force

Hard Finger

- Normal force
- Friction cone

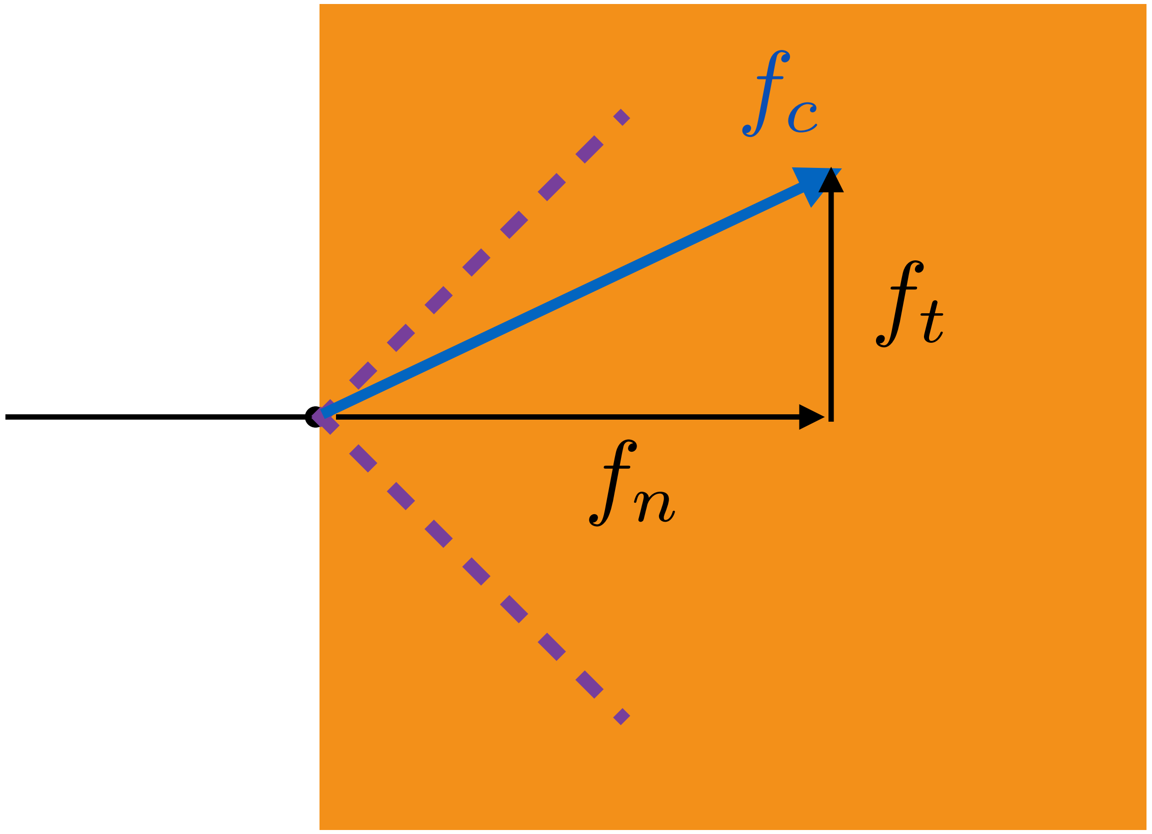
Soft Finger

- Normal force
- Friction cone
- Torque <u>about normal</u>

Maximo Roa and Raúl Suárez, Grasp Quality Measures: Review and Performance, Autonomous Robots 2015

Coulomb Friction Model

- Contact force must be within the friction cone



$$f_c = f_t + f_n$$

$$|f_t| \leq |\mu f_n|$$

coefficient of static friction

- Soft contact - torsional friction constraint is given by

$$|t_n| \leq |\gamma f_n|$$

Grasp Example

Consider the following grasp with frictionless contacts:

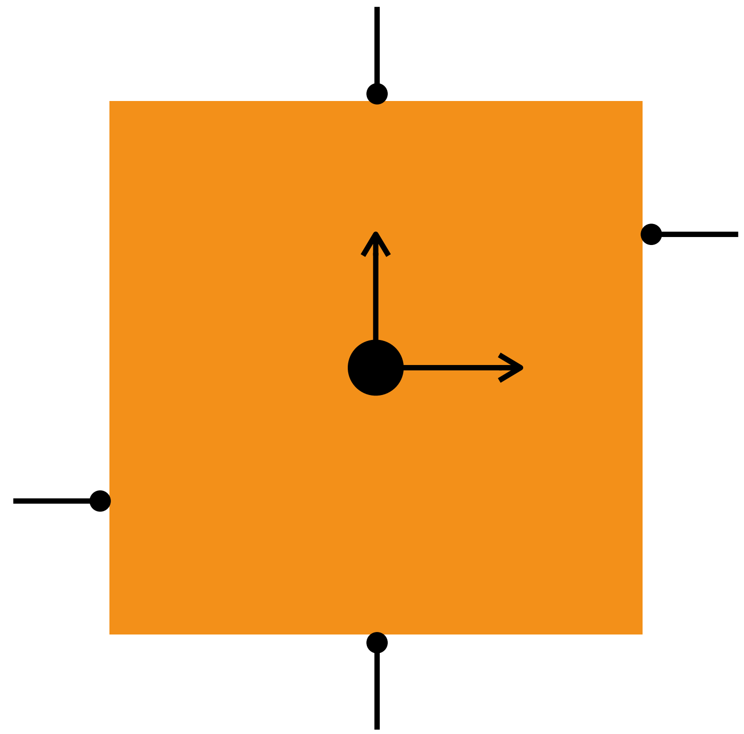
f_4

$p_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad n_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$p_3 = \begin{bmatrix} -1 \\ -0.5 \end{bmatrix}$

f_3

$n_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



$p_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

f_1

$n_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$f_1 \geq 0, \ f_2 \geq 0, \ f_3 \geq 0, \ f_4 \geq 0$$

$p_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad n_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

f_2

Can the grasp resist external wrenches to immobilize the object?

Grasp Wrenches

- The wrench applied by the grasp is given by

$$w = \begin{bmatrix} n_1 \\ p_1 \times n_1 \end{bmatrix} f_1 + \begin{bmatrix} n_2 \\ p_2 \times n_2 \end{bmatrix} f_2 + \begin{bmatrix} n_3 \\ p_3 \times n_3 \end{bmatrix} f_3 + \begin{bmatrix} n_4 \\ p_4 \times n_4 \end{bmatrix} f_4 = Gf$$

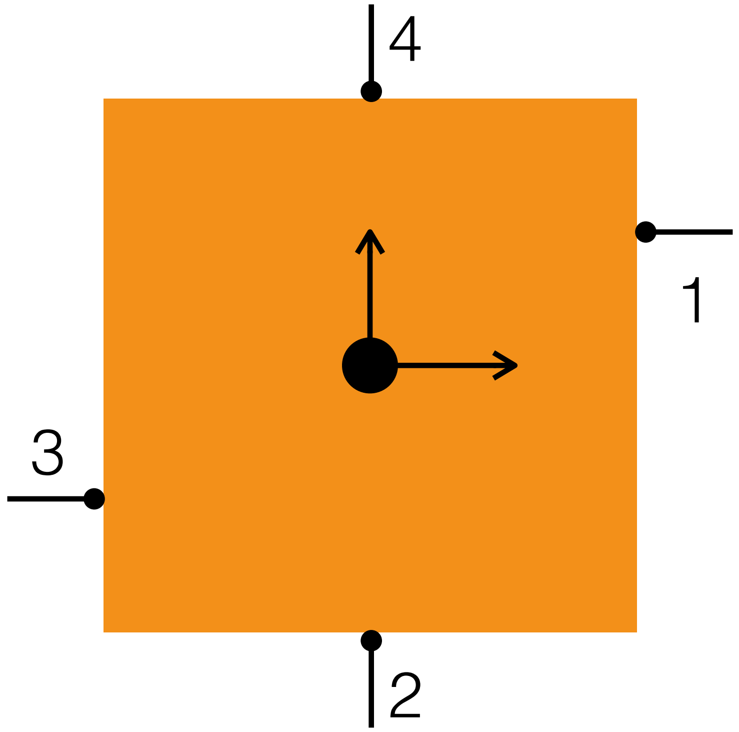
$$w = Gf$$

Grasp Matrix

$$G = \begin{bmatrix} n_1 & n_2 & n_3 & n_4 \\ p_1 \times n_1 & p_2 \times n_2 & p_3 \times n_3 & p_4 \times n_4 \end{bmatrix} \in \mathbb{R}^{3\times 4}, f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \in \mathbb{R}^4$$

- The grasp immobilizes the object if and only if the columns of G positively span the wrench space
- Equivalently, the convex hull of the columns of G includes a neighbourhood around the origin

Grasp Example

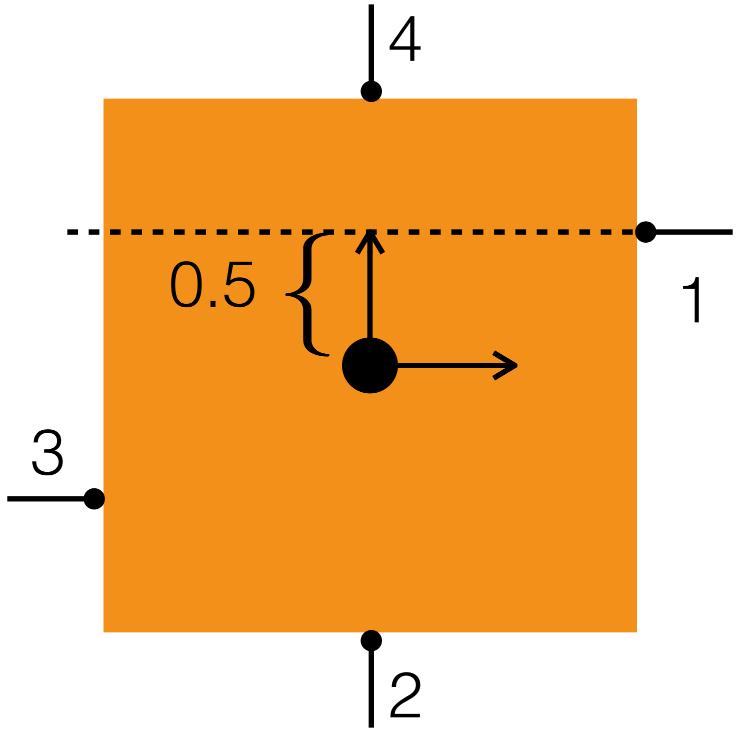


$$n_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad n_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad n_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad n_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$p_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \qquad p_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad p_3 = \begin{bmatrix} -1 \\ -0.5 \end{bmatrix} \qquad p_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$g_i = \begin{bmatrix} n_i \\ p_i \times n_i \end{bmatrix} \qquad G = \begin{bmatrix} g_1 & g_2 & g_3 & g_4 \end{bmatrix}$$

Grasp Example



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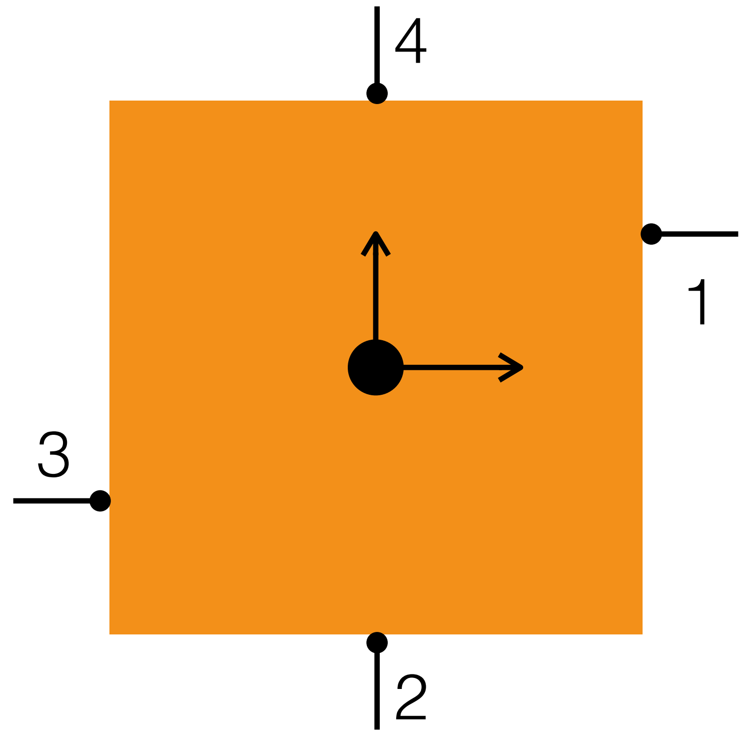
Grasp Example



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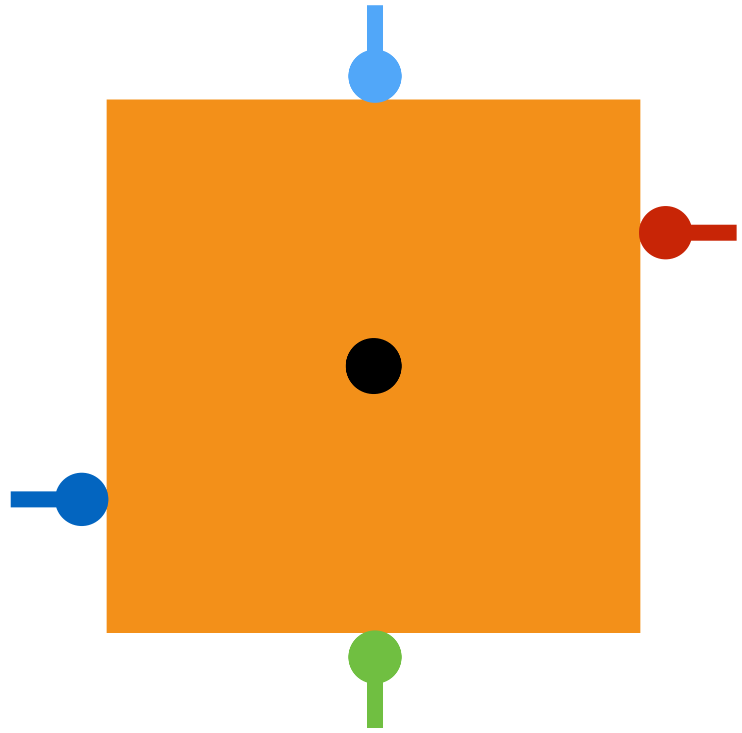
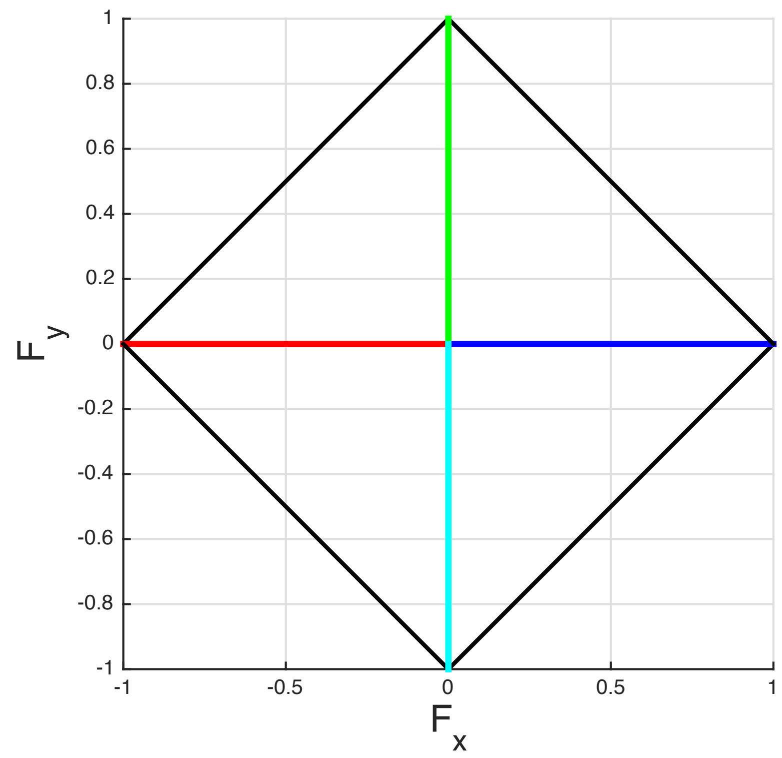
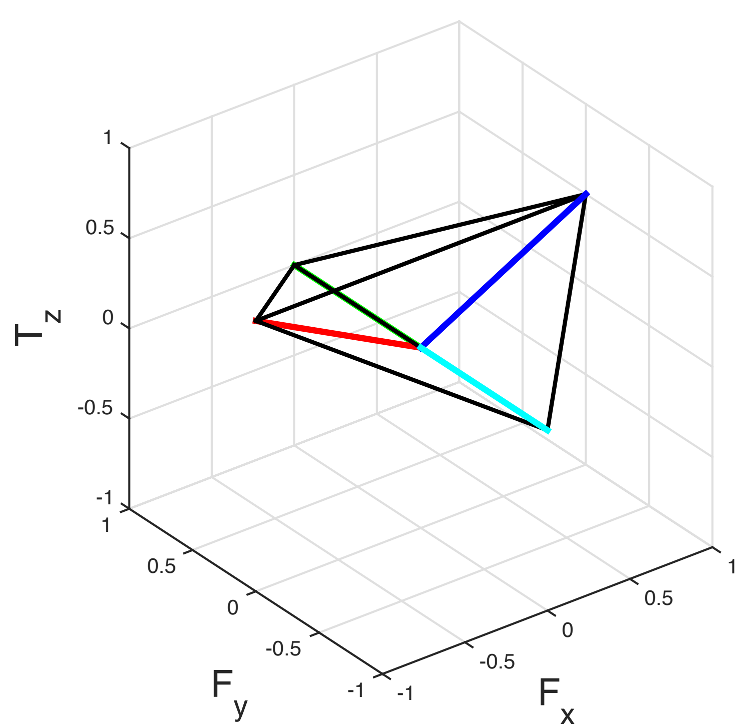
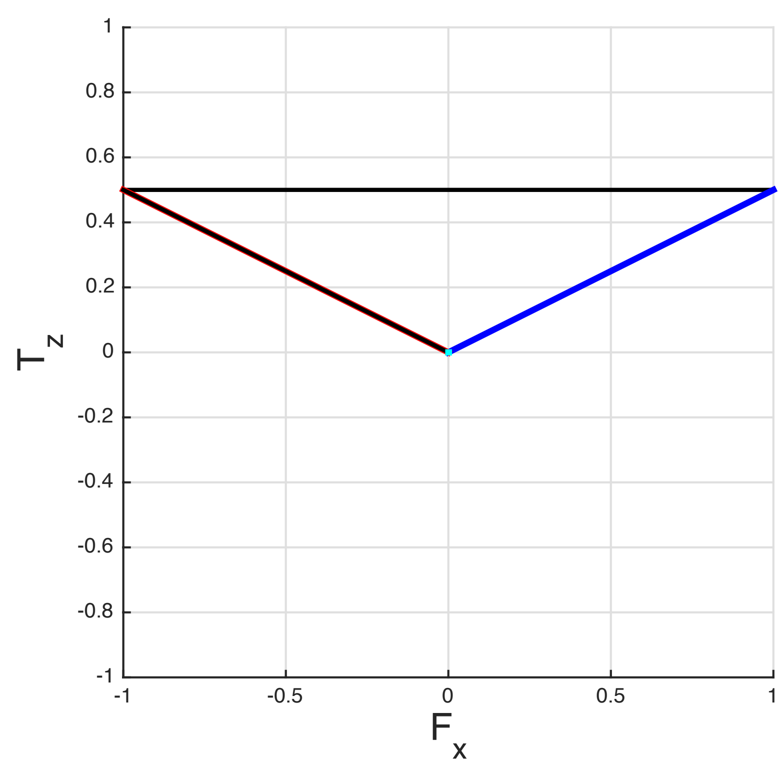
Cannot apply a negative torque

Grasp Example - Grasp Wrench Space



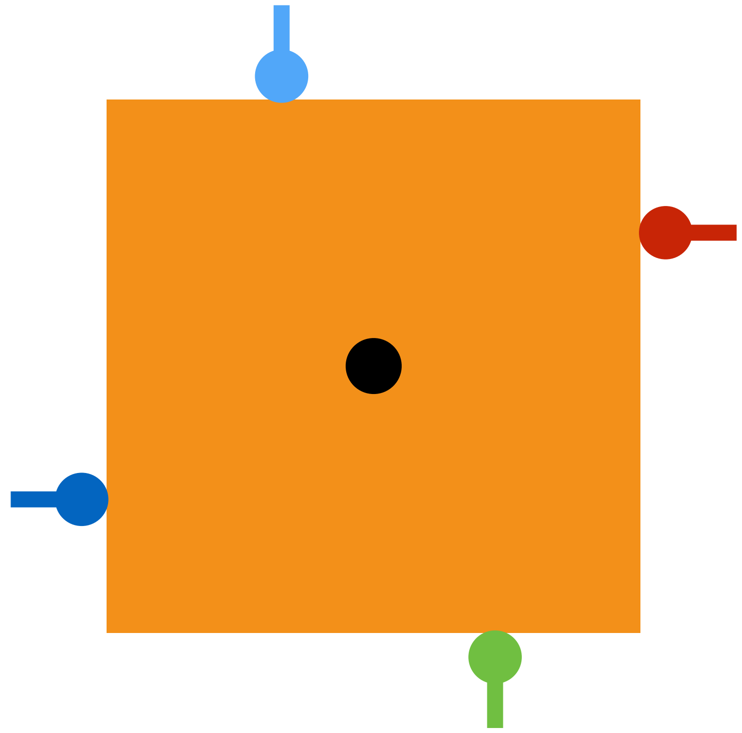


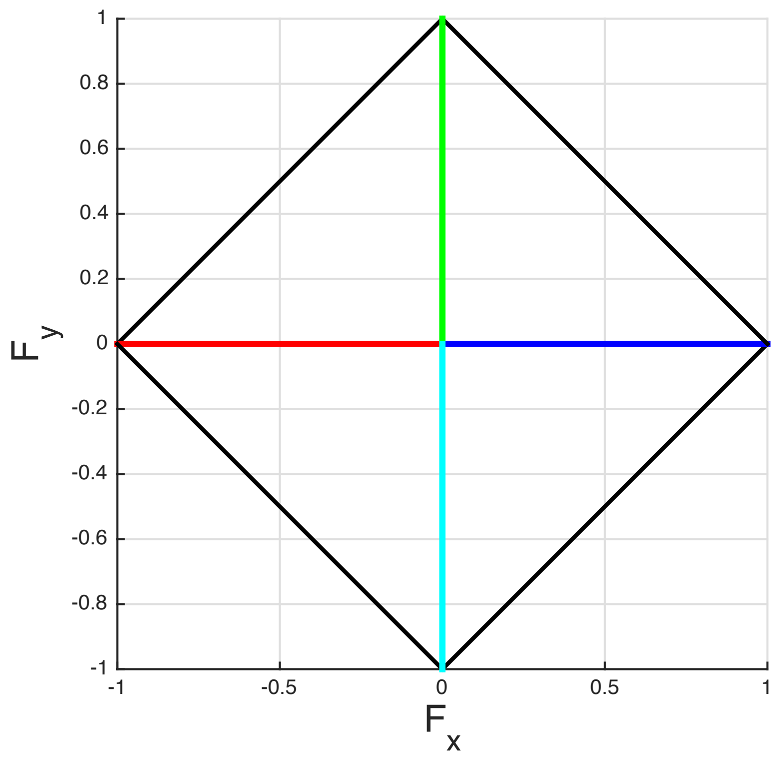


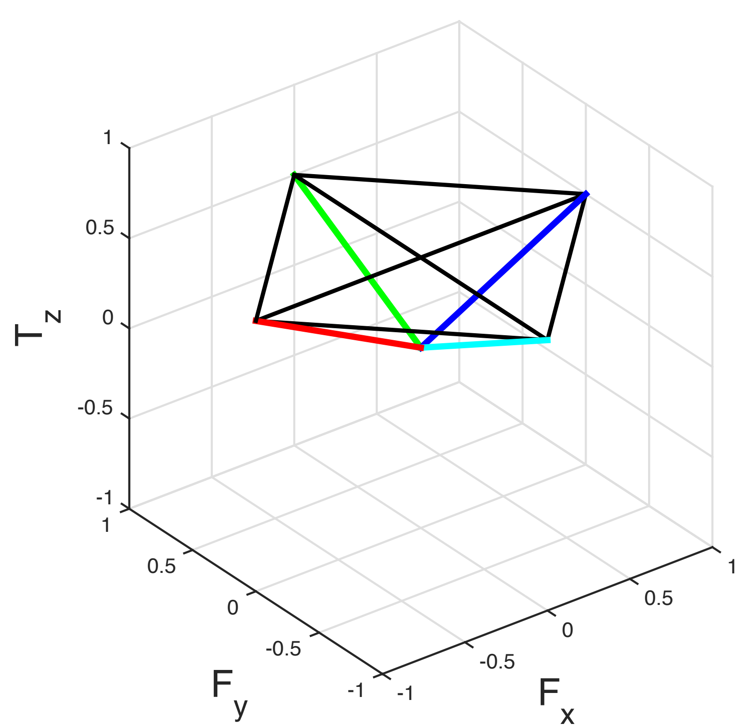


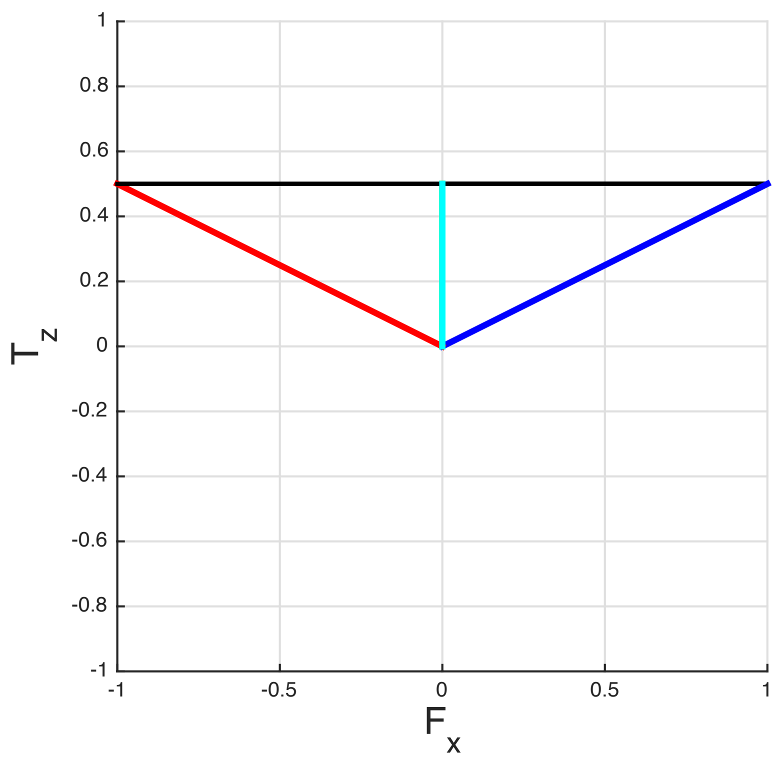
Failure

Grasp Example - Grasp Wrench Space



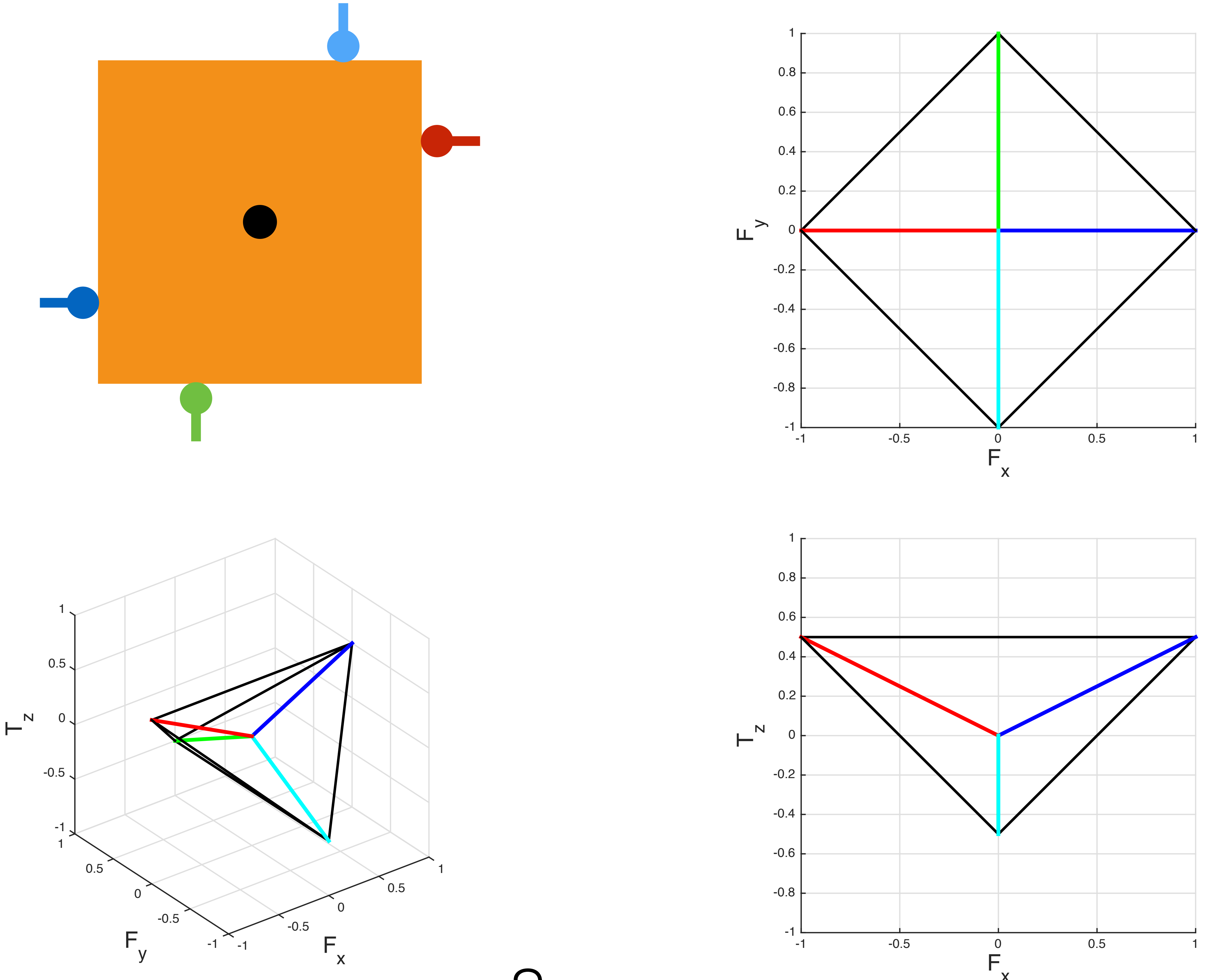






Failure

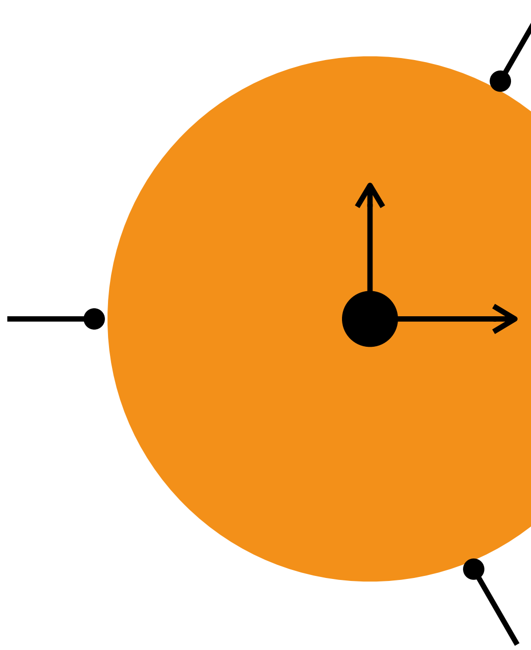
Grasp Example - Grasp Wrench Space



Success

Grasp Example

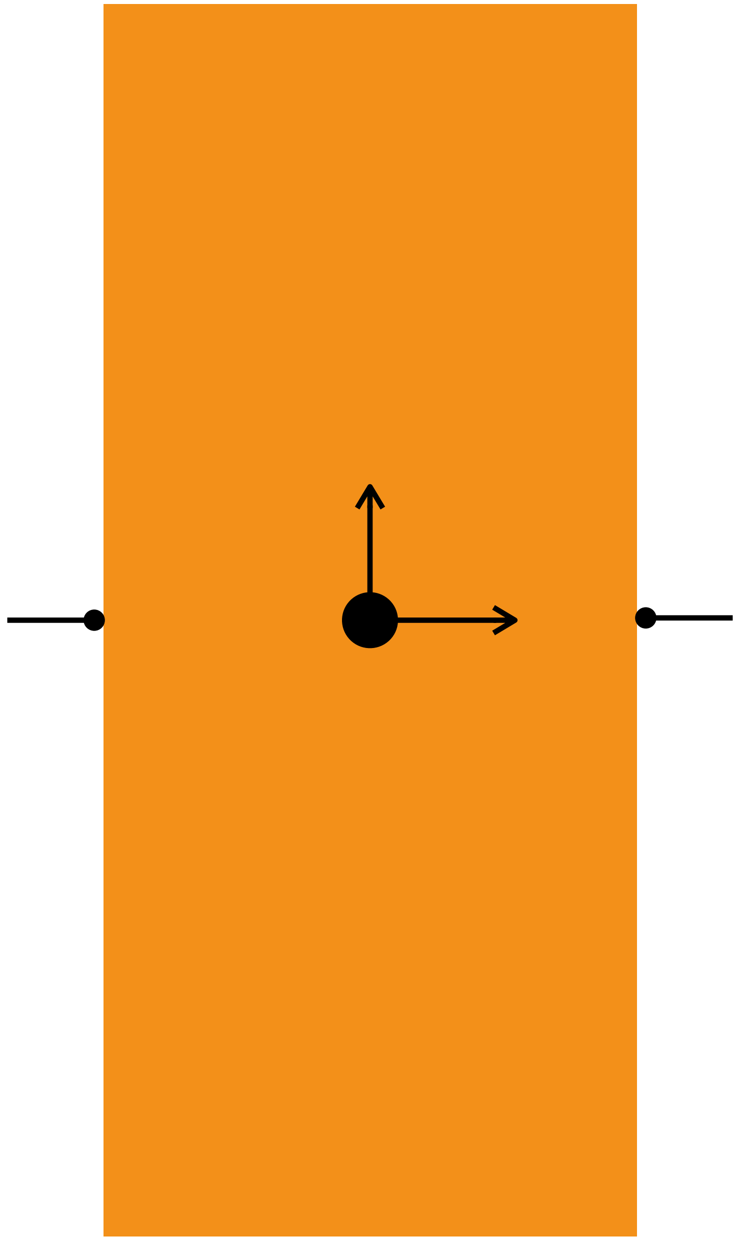
Does this grasp immobilise the objects?



No - revolute object can rotate

Grasp Example

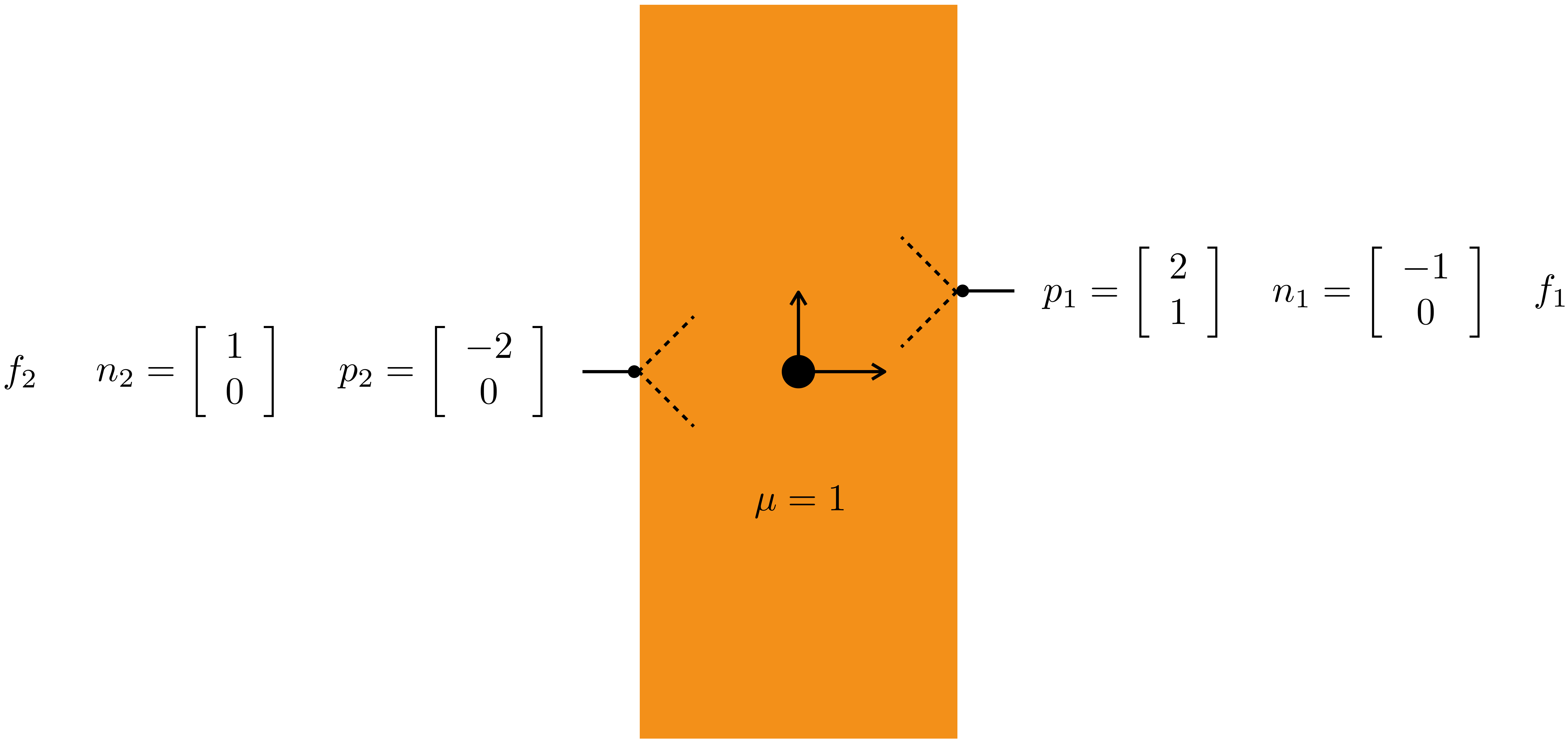
Does this grasp immobilise the objects?



No - no constraint in the vertical or rotational directions

Grasp Example

Does this grasp with hard fingers immobilise the objects?



Grasp Example

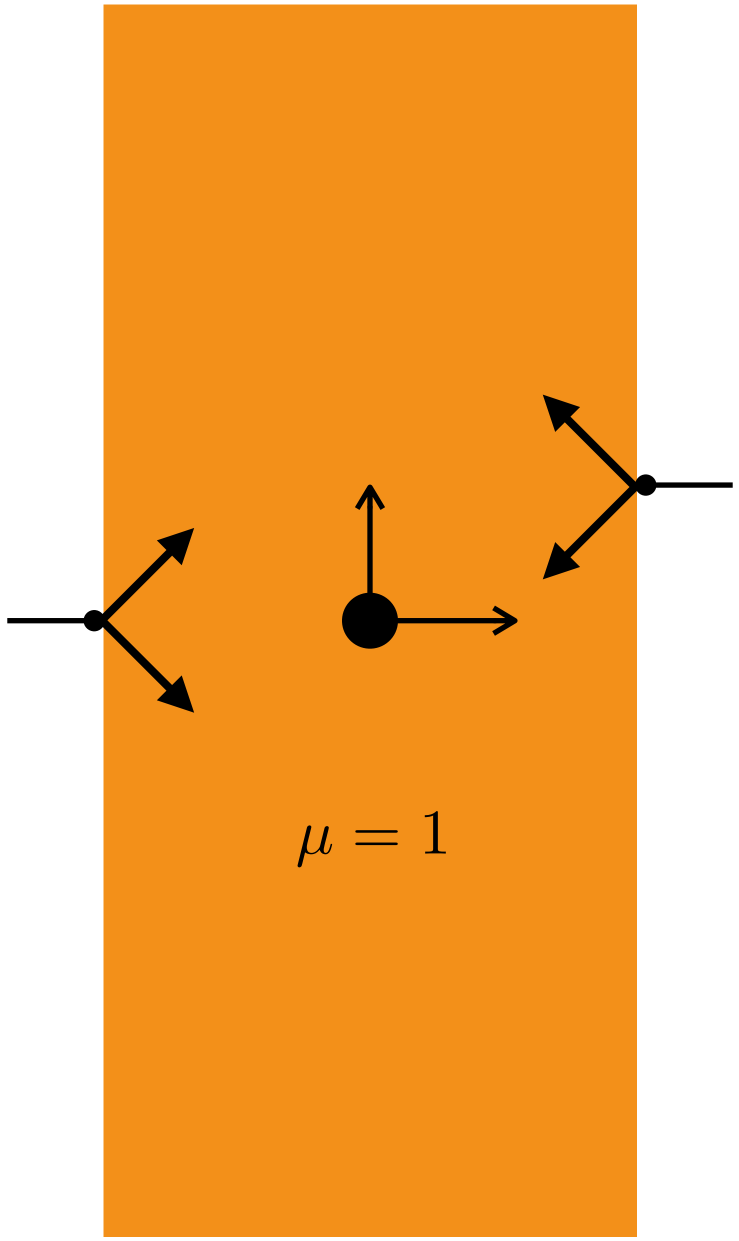
Convexity: represent cone by positive sum of edge vectors

$$f_i n_i = f_{i1} n_{i1} + f_{i2} n_{i2}$$

f_{22} $\quad n_{22} = \begin{bmatrix} 1 \\ \mu \end{bmatrix}$

$$p_{21} = p_{22} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

f_{21} $\quad n_{21} = \begin{bmatrix} 1 \\ -\mu \end{bmatrix}$



$n_{12} = \begin{bmatrix} -1 \\ \mu \end{bmatrix}$ $\quad f_{12}$

$$p_{11} = p_{12} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$n_{11} = \begin{bmatrix} -1 \\ -\mu \end{bmatrix}$ $\quad f_{11}$

$$f_{11} \geq 0, \;\; f_{12} \geq 0, \;\; f_{21} \geq 0, \;\; f_{22} \geq 0$$

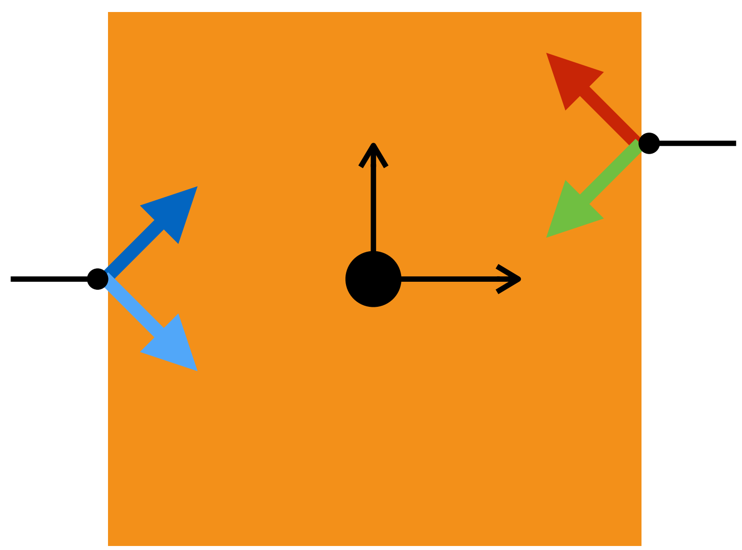
Grasp Example

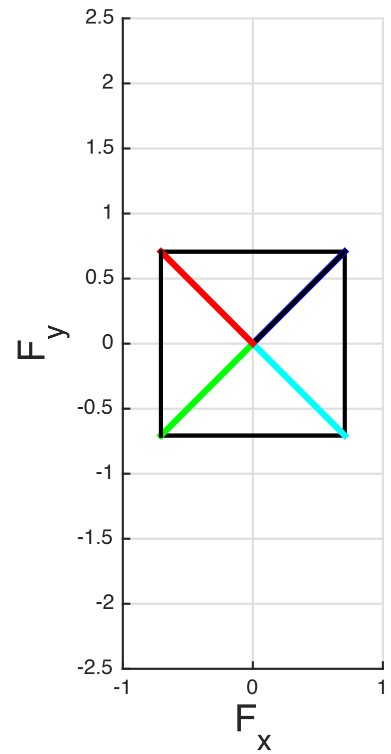
$$n_{11} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{2}} \qquad n_{12} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}} \qquad n_{21} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{2}} \qquad n_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}}$$

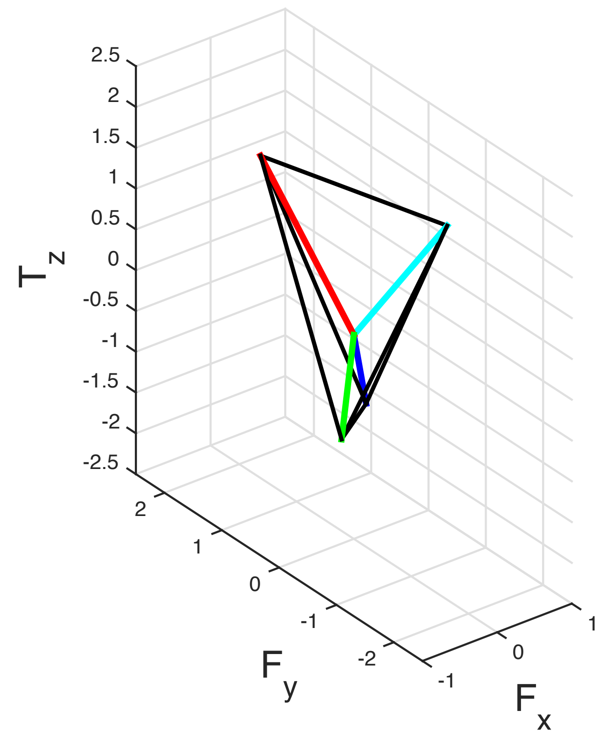
$$p_{11} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad p_{12} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad p_{21} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \qquad p_{22} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

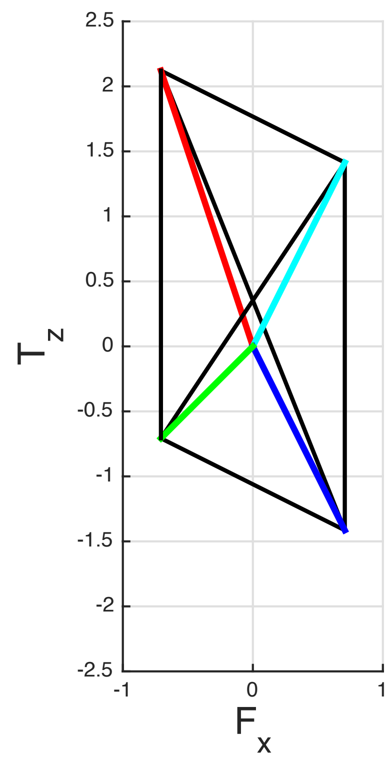
$$g_{11} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{2}} \qquad g_{12} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \frac{1}{\sqrt{2}} \qquad g_{21} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \frac{1}{\sqrt{2}} \qquad g_{22} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \frac{1}{\sqrt{2}}$$

Grasp Example - Grasp Wrench Space





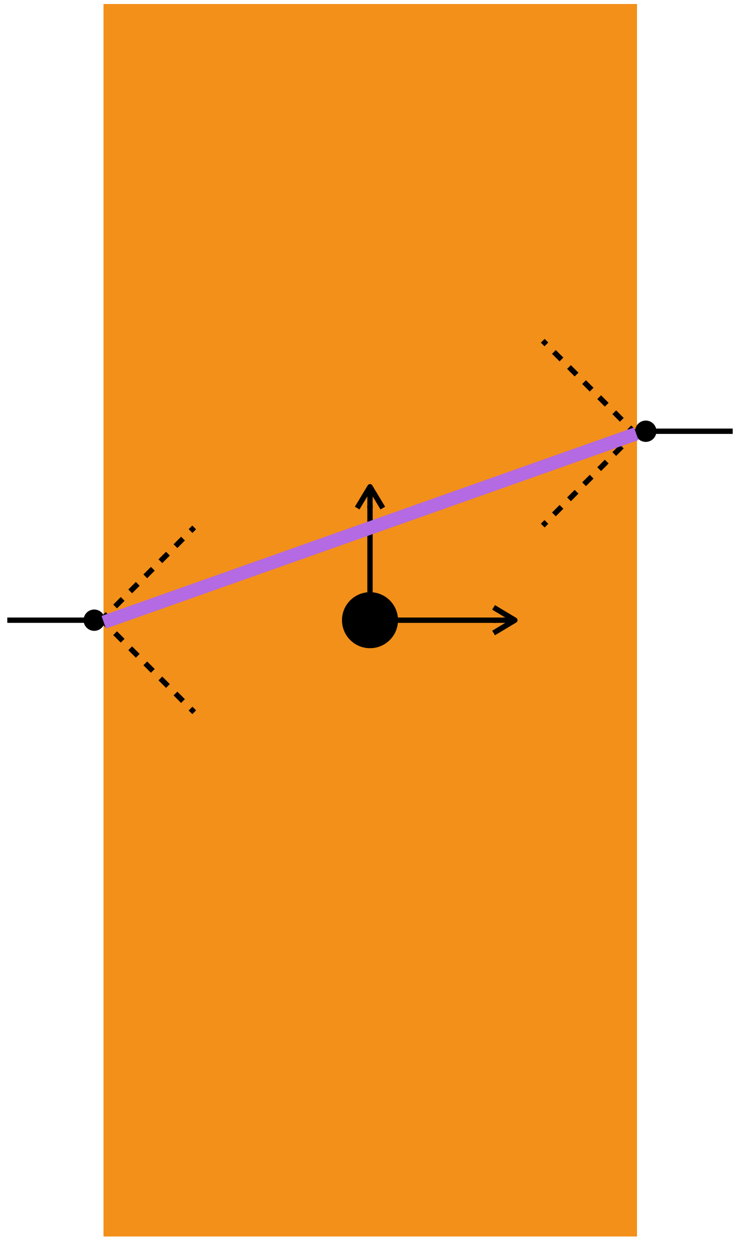




Success

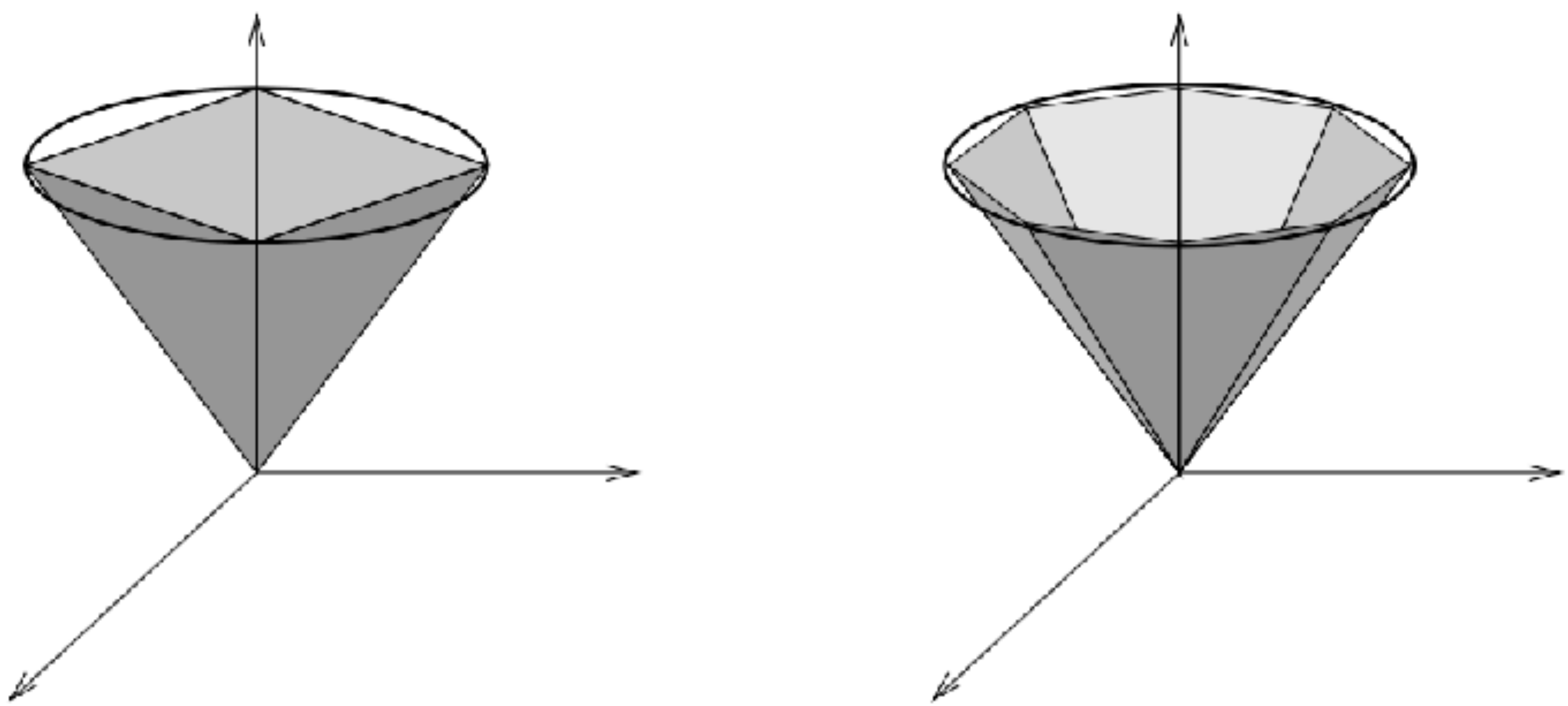
Antipodal Grasp

Immobilize object if the connecting line is within friction cones



Friction Cones in 3D

- Planar analysis relied on decomposing the contact forces into vectors along the edges of the friction cone
- Cannot define 3D cone as a finite sum of vectors



- Approximate the cone using a finite set of vectors

Murray et al., A Mathematical Introduction to Robotic Manipulation, CRC Press, 1994

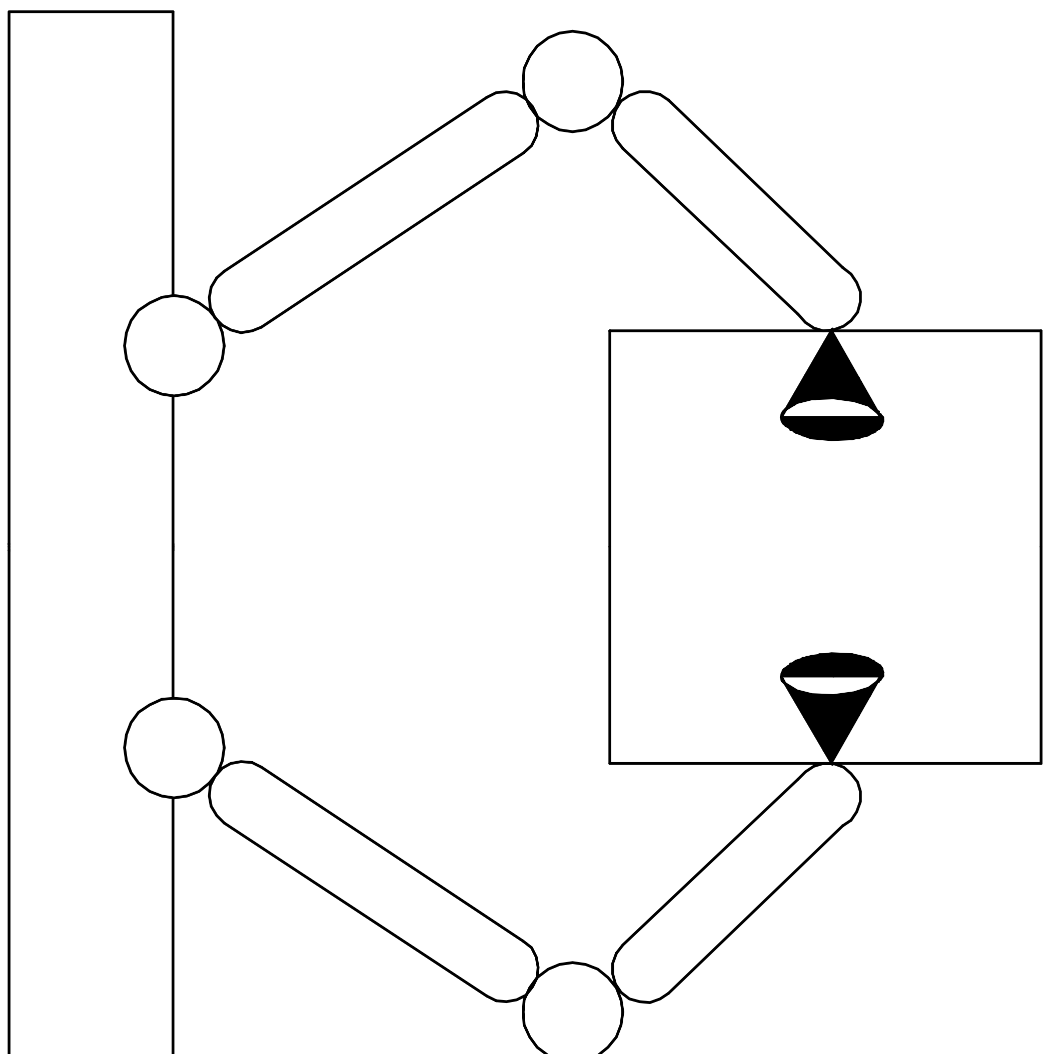
Form- and Force- Closure Grasps

- Form-Closure
  The positions of the contacts ensure object immobility, with any movement resulting in piercing a contact.

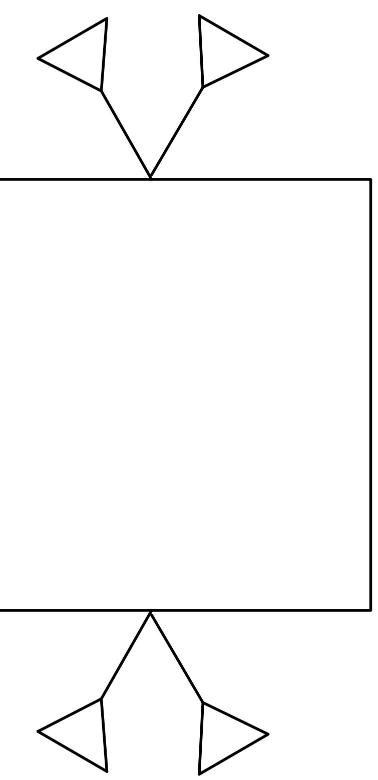
- Force-Closure
  The fingers can apply forces on the object to produce wrenches in any direction, allowing it to compensate for any external wrench on the object

Form- and Force- Closure

Equivalent 2D problems:



Force-Closure
Problem

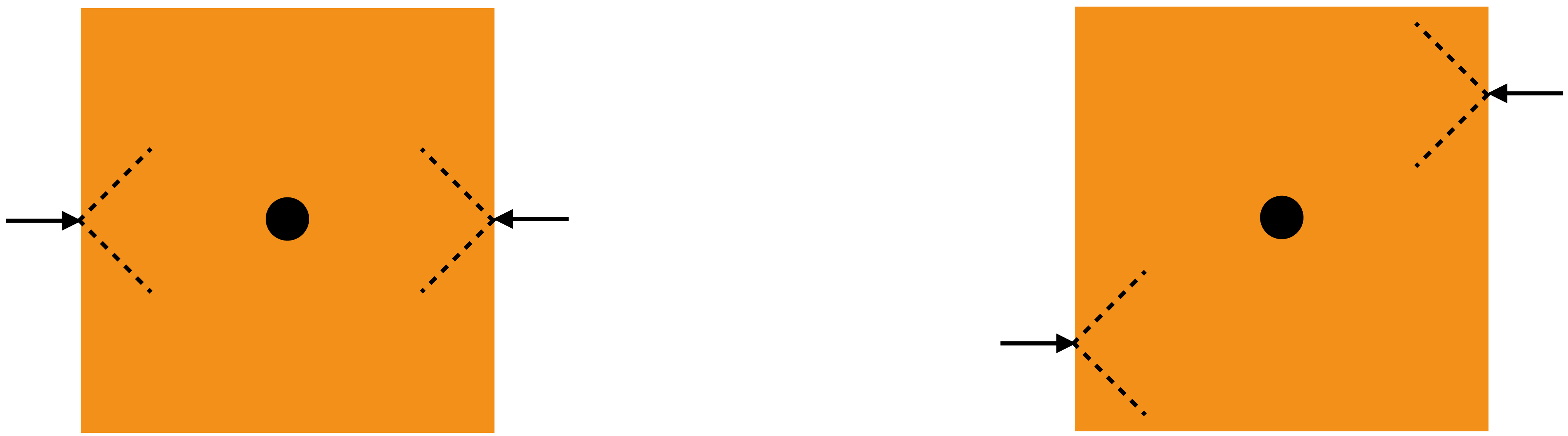


Form-Closure
Problem

Antonio Bicchi, On the Closure Properties of Robotic Grasping, International Journal of Robotics Research 1995

Grasp Quality Measures

- Form and force closure criterion give binary output



  - Both grasps provide force closure
  - Are they equally good grasps?
  - What makes one grasp better or worse than another?
- Quality of grasps is defined by a grasp quality metrics

Grasp Quality Measures

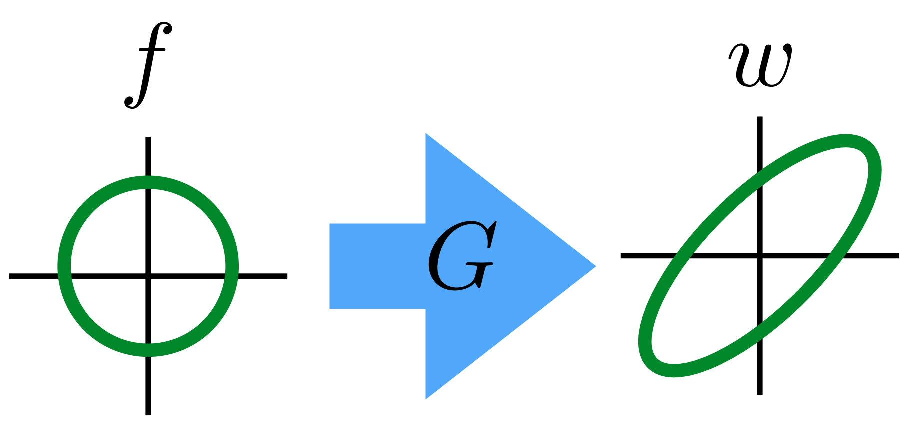
- Quality measures computed from Grasp Matrix G
- Minimum singular value of G

$$\{\sigma_1, \sigma_2, ...\} = \sqrt{\mathrm{eig}(GG^T)} \qquad Q = \min_i \sigma_i$$

  - Indicates how close G is to singular and cannot resist wrench

- Volume of ellipsoid in wrench space

$$Q = \sqrt{\det(GG^T)} = \sigma_1 \sigma_2 \sigma_3 ...$$

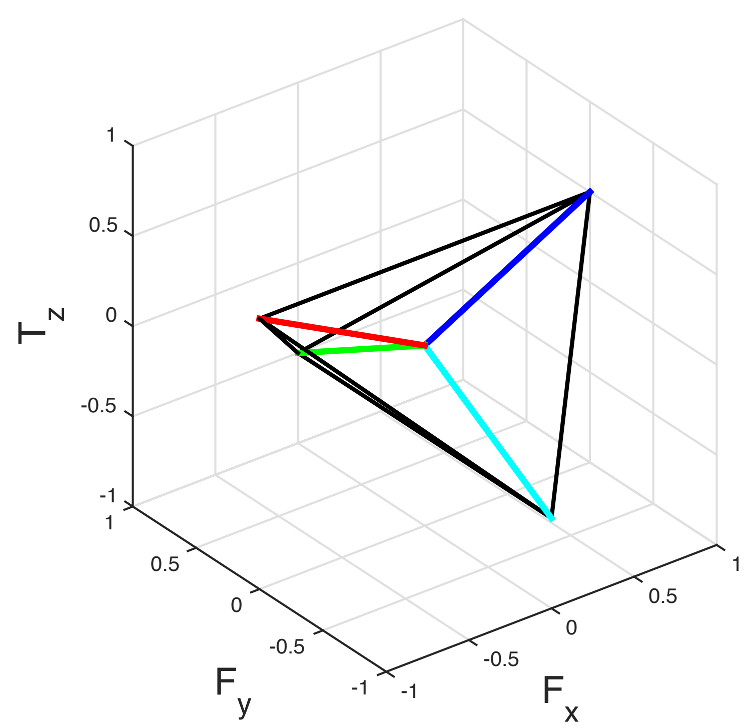


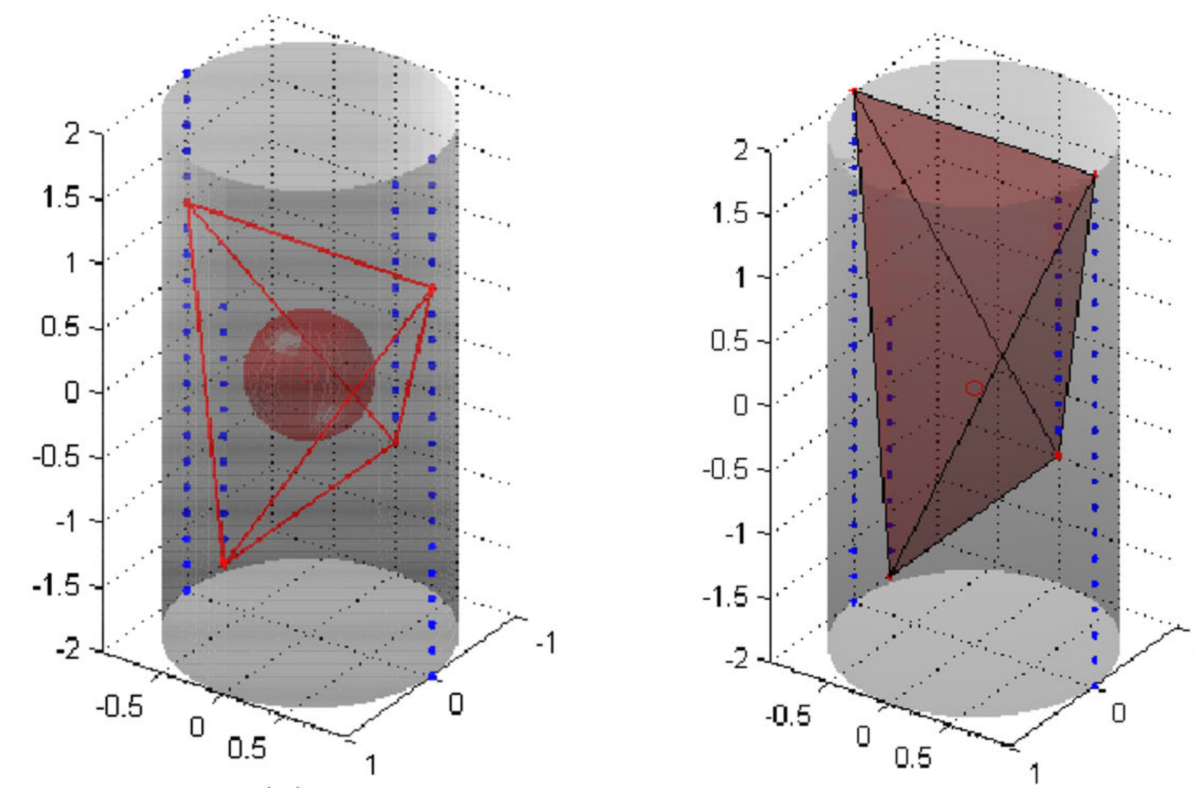
  - Volume of an ellipsoid resulting from mapping a unit sphere of forces into the wrench space

Maximo Roa and Raúl Suárez, Grasp Quality Measures: Review and Performance, Autonomous Robots 2015

Grasp Quality Measures

- Quality measures computed from Grasp Wrench Space



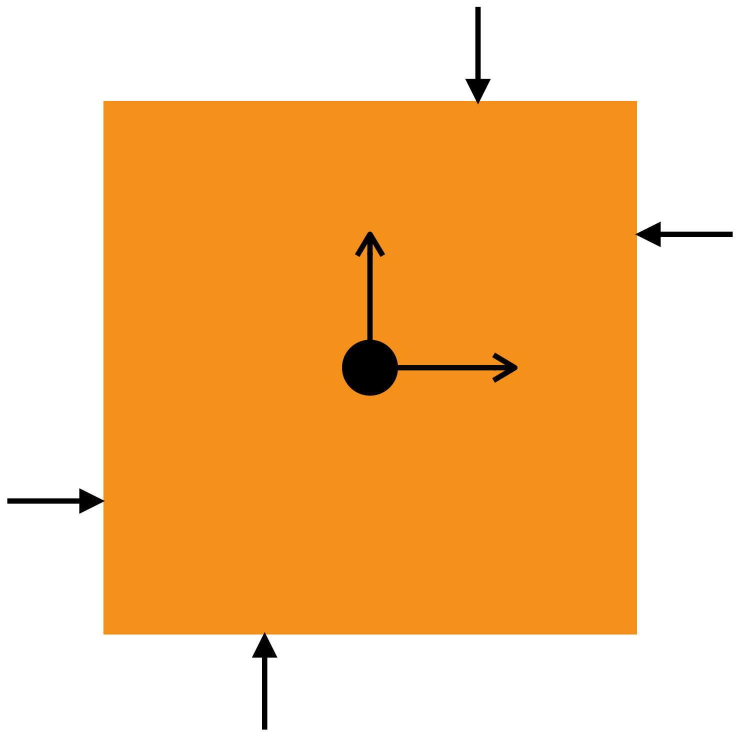


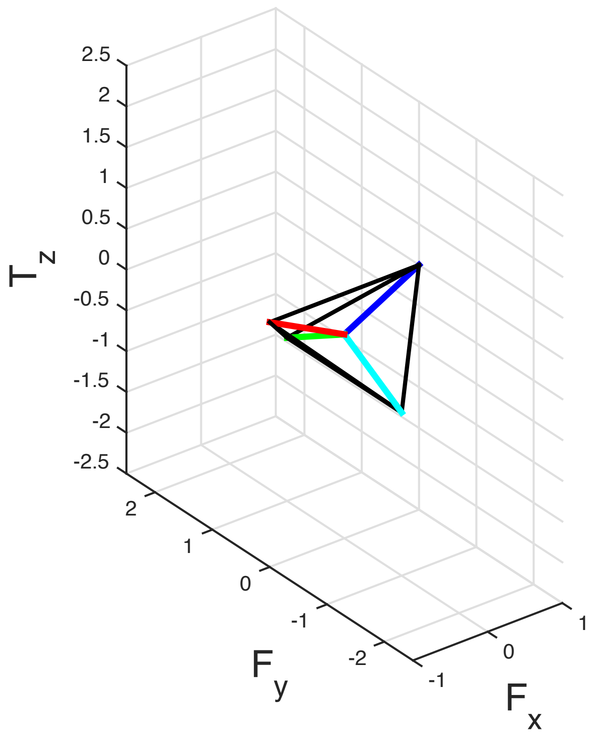
- Radius of largest centered sphere in the GWS
  - Smallest wrench that could break the grasp
- Volume of the grasp wrench space
  - Larger volumes implies that more wrenches could be resisted

Maximo Roa and Raúl Suárez, Grasp Quality Measures: Review and Performance, Autonomous Robots 2015

Grasp Example

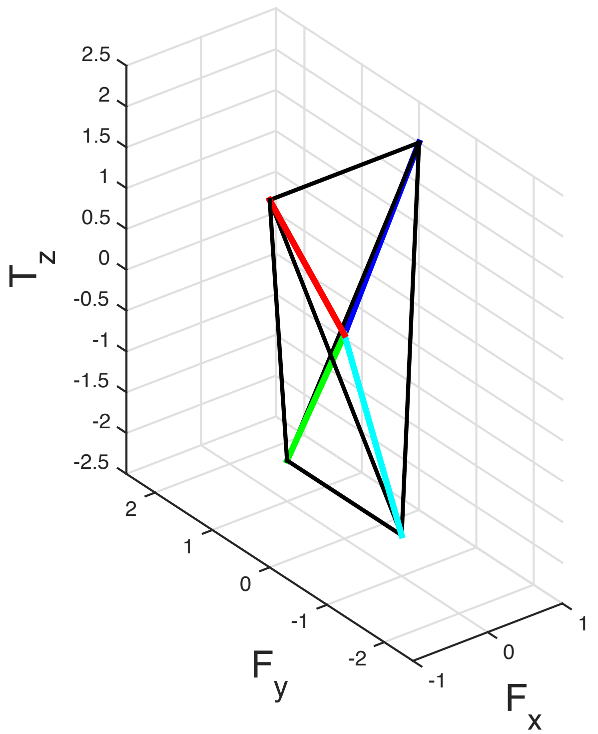
Which contacts positions would maximise the GWS volume?

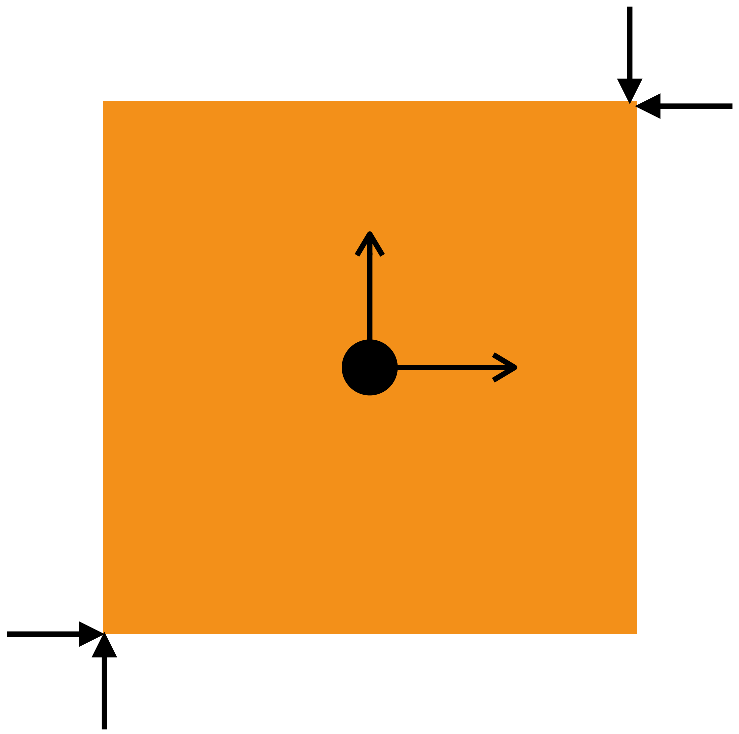




Grasp Example

Which contacts positions would maximise the GWS volume?





Questions?