The Integrated Use of Analog and Digital Computing Machines for Aircraft Dynamic Load Problems

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***Summary*—This paper describes the techniques and methods used in investigating aircraft dynamic load problems such as vibration, flutter, gust loads, taxi and landing loads, based on an integration of both analog and digital facilities to provide maximum information on a problem in a minimum of time with a guarantee that the results are correct. The last requirement is an important consideration since high-speed computing machines not only provide the possibility of investigating far more complex dynamic problems but, unfortunately, also extend the probability of introducing errors due to the additional complexities of the problem.**

The advantages and disadvantages of both the digital and analog computers in terms of the needs specified for the aircraft dynamic problems are discussed; a method of analysis is suggested using the best features of each in a closely integrated manner. An example is presented of a typical dynamics problem for a transport type airframe.

INTRODUCTION

As a RESULT of higher airplane speeds due to more powerful engines, and increased flexibility due to thinner lifting surfaces necessary for optimum performance, the aircraft designer must give considerably more attention to aircraft dynamics problems such as flutter, gust, landing and taxi loads in airplane designs. In many cases the representation of the elastic structure for adequate predictions of the vibration, flutter, and dynamic load characteristics of the airplane must be considered to a higher degree of complexity. Prior to the final acceptance of an analysis requiring the increased complexity, two important problems must be considered:

1. The elapsed time involved in determining a network of solutions, which will be shown to be a definite characteristic of dynamics problems due to required variations of the structure and the aerodynamics, should be well within the time the airplane design is "frozen" in order that the results obtained may be of use.

2. The results should enjoy some reasonable guarantee that they are correct, as well as make "engineering sense."

The realization of treating problems of a high degree of complexity, prior to the development of high-speed computing machines—both digital and analog, appeared remote. However, in recent years the computing facilities necessary to overcome these difficulties have not only been developed but apparently have been incorporated in the engineering forces of major aircraft companies. The purpose of the present paper is to review the possible approaches to the various phases of aircraft dynamics problems and also to examine the optimum methods in terms of both the analog and digital facilities for successful solution of these highly complex problems. The optimum method should be based on integrating both the analog and digital facilities such that the best features of each would be available to overcome the two previously mentioned problems.

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AIRCRAFT DYNAMIC LOAD PROBLEMS AND SOLUTIONS AT VARIOUS AIRPLANE DESIGN STAGES

As implied by the word "dynamic," the aircraft problems of interest in this paper are those which involve the element of time. Specifically they are aeroelastic problems such as flutter, airplane dynamic stability as affected by wing and tail elasticity, gust loads, as well as vibrations problems which involve landing and taxi load investigations. Examination of the characteristics of each problem indicates that a major portion of each is common to all of the problems. As a result, the study of a variety of phases of airplane dynamics could be made with the same computer setup with relatively minor modifications. The study of these problems can be broken up into three airplane design stages, as follows:

1. Preliminary design.
2. Shake test.
3. Final analysis.

Preliminary Design Stage

The preliminary design stage of an airplane offers the dynamicist the opportunity of determining the adequacy of a design which is primarily set by performance, stability, and maneuverability criteria at a time when the design is still not frozen. In the past the role played by the dynamicist at this design stage has been to offer an opinion, on the basis of experience on previous designs, as to the possible difficulties the configuration would encounter. These difficulties, if still present after the configuration is built, could be counteracted by beefing up the structure or by judicious use of balance or ballast weights. On the present and future designs such a philosophy may not be adequate and could result in a final configuration that is unduly penalized by an unnecessary weight increase, since an easy "fix" may not be possible.

The principal difficulty in determining significant engineering results in the preliminary design stage is the restricted availability and accuracy of the data, both structural and perhaps aerodynamic, if the design is radically different, requiring experimental testing to help define the aerodynamic coefficients. Two possibilities exist at this point:

1. The analyst can direct his efforts principally to refine the data to a higher degree of accuracy and therefore postpone the preliminary analysis until such time as the data is accurate enough to warrant an analysis that would lead to significant engineering results.

2. An analysis can be made based on available data realizing its inadequacies and shortcomings with the intention of first getting a "feel" for the problem and of determining what structural or aerodynamic parameters have the most effect on the solutions. In contrast, those parameters which have a relatively minor effect can also be determined. At this point further investigation of those parameters which must be known to a higher degree of accuracy for adequate engineering results can be pursued by more detailed study or by experimental techniques involving carefully scaled models.

In view of the limited time available for performing a successful analysis while the airplane is in the preliminary design stage, the second choice appears to be the most logical. During the investigation of the preliminary design an unsatisfactory condition in terms of stability, such as flutter, may appear. Through the use of many parameter variations the most logical fix, that is, the one requiring the least weight increase and complexity, can be determined. Recommendations could then be given to the Design Group for possible incorporation in the design. Thus an analysis in this stage has many desirable features. However, the following requirements must be dealt with if the results of the analysis are to be useful:

1. Large number of degrees of freedom.
2. Large number of solutions.
3. Limited time for determining solutions.
4. Guarantee that the solutions are correct.

Shake Test Stage

The next stage of importance to the dynamicist involves the vibration or shake test of the airplane. As many dynamicists know, the results obtained in these tests play an important role in the studies that follow: consequently, careful attention should be given to these tests to insure adequate correlation with the calculations made in the Preliminary Design Stage. Correlation of the observed and calculated vibration results allows for a means of determining the validity of the mechanical representation used. Differences that do occur can be attributed to two primary reasons:

1. Selection of proper as well as adequate number of degrees of freedom.
2. Accuracy of structural data.

Detailed comparisons of the modes are usually sufficient for evaluation of the first item. The second item, however, is not readily evaluated by mere inspection but usually involves numerous calculations involving a large number of variations of structural parameters. Such a network of solutions supplies the dynamicist with a means of determining the optimum match of the observed shake test results requiring the most reasonable adjustment of the initial structural data. The optimization should be based on not only the match of frequencies but also mode shapes, node lines, and relative phasings and amplitudes among the modes. Of course, this stage also presents a crisis (except in those fortunate cases where it is obvious that no serious dynamic problems exist) in view of the allowable time required to match the observed shake tests, since the first flight date of the airplane is not far off. A final analysis check has to be made prior to the first flight date and time must be allowed for this investigation. Thus the success of refining the analysis at this stage depends again upon the four previously mentioned requirements.

Final Analysis Stage

The final analysis stage permits the dynamicist to determine the effects that the changes in structural representation necessary to match the shake tests have on the final results. In addition a reorganization of the pertinent degrees of freedom may appear warranted as a result of the knowledge obtained in matching the observed shake test results together with the aeroelastic stability or dynamic load results obtained in the preliminary stage. For instance, the flutter investigations may indicate that the critical flutter speed of the airplane involves the coupling of essentially two or three vibration modes on a particular surface. The modes on the remaining surfaces or structure may have a relatively minor effect on the critical mode. Consequently, a more refined definition of the critical modes could be made by redistributing the degrees of freedom. In addition, the pertinent degrees of freedom for gust, landing, and taxi loads may not necessarily be compatible for optimum representation for each problem. Thus a certain amount of further reorganization (much of it should be already planned from the results of the preliminary analysis) may be necessary and must, of course, be done rapidly and accurately. Again the success of this stage depends upon the four previously mentioned requirements.

Analog and Digital Methods of Analysis

In describing a complex physical structure for the purpose of analysis by means of any computing technique, a primary consideration is that of reducing the given physical system to a system having a finite number of degrees of freedom. In general, all computational methods are restricted to the consideration of a limited number of degrees of freedom. The problem then becomes one of retaining all of the significant characteristics of the physical system, within the limitations imposed by the particular method employed. Various representations have therefore been evolved which accomplish this reduction. Since each of these representations is necessarily an approximation, the final evaluation of each must be made in terms of the particular application. Certain conclusions, however, may be drawn from a consideration of the general characteristics of some of these methods.

Analog Methods

Modal: A common means of representation of a physical system is through the use of a modal type analysis. Here the motion of the physical system is assumed to be represented by the linear combination of a limited number of modes. These modes must be chosen to include all the relevant types of motion of all co-ordinates of significant importance in the system. In practice, the modes chosen are usually the lower-order vibration modes (either normal or uncoupled) in the important degrees of freedom of the system.

The description of a mode of a distributed physical system involves the assumption of a deflection curve which is a function of a space-wise dimension. The deflection curve should approximate the true deflection of the system in the particular mode, and is chosen either on the basis of analytical or experimental results or from previous experience. Once this deflection curve has been prescribed, the generalized inertial, elastic, and aerodynamic properties of the system may be determined for motions of this mode. The determination of these generalized properties involves an integration with respect to a space-wise dimension. The differential equations of the modal system are then derived from the Lagrangian energy principles as applied to these generalized properties. Electronic differential analyzers such as BEAC or REEVES computers are normally used to simulate the resulting differential equations.

It will be noted that the modal approach results in a two-fold simplification of the distributed physical system:

1. The number of degrees of freedom is restricted to those of importance to the particular analysis.
2. The three-dimensional properties of the distributed physical system are replaced by equivalent two-dimensional properties.

Finite-Difference: As an alternative analog method, a distributed physical system may be represented by assuming the properties of the system to be concentrated at discrete intervals. This lumped-parameter system is then described by means of finite-difference equations, which are in turn represented by an analog computer. The computer which most commonly utilizes this method of representation is the network analyzer, or direct analog computer. The network analyzer utilizes passive electrical circuit elements to represent the discrete structural parameters of the approximate physical system. In the direct analog computer, both the electrical quantities, voltage and current, are analogous to quantities of the mechanical system. Two analogies are widely used—the loop analogy, wherein electrical voltage is analogous to mechanical force; and the nodal analogy, wherein electrical current is analogous to mechanical force. The nodal analogy is generally preferred in the representation of structural systems; this is due in part to the close topological similarity between the resulting electrical circuit and the structural system. Since mechanical quantities are usually translated directly into electrical quantities without reference to the differential equations of either the mechanical or the electrical system, this topological similarity is an important characteristic of the nodal analogy. The external forces applied to the mechanical system, e.g., aerodynamic forces, are represented by the analogs of the differential equations of these forces, much as is done in the case of the differential analyzer. In this case, however, current generators are required as well as voltage amplifiers, since current and voltage both represent mechanial quantities.

Although each of the two methods, modal and finite-difference, has been identified with a particular type of analog computer, variations of these methods are used in conjunction with either type. Differential analyzers have frequently been used to represent the finite-difference equations of distributed mechanical systems. More recently, a method has been devised wherein mechanical properties are simulated in differential form by elements of the differential analyzer, and these elements are interconnected in accordance with finite-difference equations of the mechanical system. In the case of the network analyzer, frequent use is made of modal representation in order to achieve a saving of electrical circuit elements. This is usually done in representing a subsidiary portion of the structure, such as a fuselage, where detailed information is not a required part of the solution.

Digital Methods

The methods used in the solution of aircraft dynamics problems with high-speed digital computers parallel quite closely those described in conjunction with analog computers. Modal type analyses are commonly used with digital computers in much the same manner as with analog computers, with one principal difference—the generalized co-ordinates in the digital analysis are usually assumed to be prescribed functions of time, i.e., to have only simple harmonic motions. In some cases, as for landing or taxi loads, limited time histories are computed for discrete forcing functions. The assumption of harmonic motion is certainly true in the case of the free vibrations of a conservative system, and while it is not in general true of a mechanical system under the influence of aerodynamic forces, the assumption is valid for those critical cases where the aeroelastic system becomes neutrally stable. In other cases, harmonic motions are obtained by the addition of an appropriate damping or driving force to the mechanical system. The differential equations of the modal system are then expressed as homogeneous equations containing an unknown frequency. The solutions of the characteristic equation of the system are then obtained by numerous methods such as matrix iteration, or fraction series, etc., and the frequencies so obtained are the desired vibration frequencies or flutter modes, as the case may be. The magnitude and sign of the required damping (if any) is then taken as a measure of the stability of the given root of the system.

Finite-difference techniques may also be applied to the solution of dynamics problems on digital computers. The mechanical and aerodynamic properties of the system are described in terms of discrete stations or panels of the distributed structure. The equations of motion of each station are expressed in matrix form in terms of the summation of forces at each station. The summation of these forces—elastic, inertial and aerodynamic—is performed by the use of integrating matrices which prescribe the distribution of these forces. This formulation results in equations of motion of finite-difference form which replace the partial differential equations of the distributed physical system. The equations obtained in this manner are time-dependent functions. As such, they are not in an expedient form for solution by digital means, especially when aerodynamic forces must be considered. The motions of the system are, therefore, normally restricted to harmonic oscillations and the equations written as functions of frequency. This set of equations is then solved as an eigenvalue problem by means of matrix iteration.

Comparison and Evaluation of Methods

Analog: 1. Modal Systems—The use of modal systems with analog computers has the advantage that it requires a minimum of electrical circuit parameters for a given complexity of structure. This feature is, indeed, the principal attraction of modal analyses. As might be expected, however, this simplicity of representation gives rise to other characteristics which are less desirable:

(a) The fact that the generalized structural parameters are obtained from space-wise integrations of the original system parameters means that localized structural or inertial changes are not simple to perform.

(b) The deflection curves which are assumed in modal analyses represent constraints on the system, which may have a significant effect unless the advantage of simplicity is lost by using many modes. The inclusion of these constraints assumes that external forces have no significant effect on the mode shapes.

2. Direct Analog Systems—Direct analog representations as applied to the network analyzer offer several formidable advantages:

(a) Complex structural systems may usually be analyzed without specific reference to the differential equations of the systems when the modal analogy is used, as is customary.

(b) Since the mechanical system is represented directly on the computer, the complete aircraft dynamics problem may be solved with one general setup. The changes in setup from static deflection tests to free vibrations, to flutter analysis, to gust response studies, etc., are accomplished in a very short time.

(c) The high degree of identification between electrical quantities results in extreme convenience in making localized changes of structural parameters. To expedite these parameter changes, the passive electrical circuit components of the network analyzer are variable over a wide range of values in small increments.

(d) The short time required for the actual machine solution, coupled with the ease of making parameter changes, makes possible the investigation of a wide range of structural configurations in a relatively short time. In this way, the significant parameters in a particular analysis are readily ascertained and investigated in detail.

The use of the network analyzer in the solution of dynamics problems is not, however, free of disadvantages:

The analysis of complex structural systems necessarily results in complex electrical circuits. Each element of the circuit representing the structure must be set to the appropriate value and correctly connected into the structure. The aerodynamic forces are simulated through the use of active elements, and these electronic devices are, of course, subject to failure or malfunction. One of the most difficult phases of the analysis is that of ascertaining that the setup is correct.

Although the electrical elements used in the network analyzer are the best quality obtainable, they are not perfect. The parasitic effects associated with these imperfections may be minimized by the proper choice of element values, but in some cases these effects are difficult to evaluate. This is particularly true of the small amount of damping which is necessarily present in any circuit containing inductors or transformers.

Since the network analyzer has been used principally in analyses employing classic two-dimensional incompressible theory, the use of other aerodynamic theories generally requires some development of the associated circuits. Although this is not an inherent limitation of the computer, it may be a serious difficulty if close cooperation between the aerodynamics group and the computer group is not maintained.

Digital: 1. Modal Systems—The use of digital computers in modal analyses is characterized by the same general advantages and disadvantages as were mentioned in conjunction with modal analyses with analog computers. The principal differences between these two methods are:

(a) The accuracy of solution is inherently better in the case of the digital computer.

(b) The digital computer requires a solution time which is considerably longer than that of the analog computer (that is, when many parameter variations must be considered).

2. Finite-Difference Methods—The use of finite-difference techniques with high-speed digital computers is in many respects similar to the use of direct analog representation. This method of formulation eliminates the necessity of prescribing deflection shapes; the motions which are represented are not limited by artificial constraint except due to the number of panels which may be included in the analysis. Successive stages of the

analysis—static deflection tests, vibration test, flutter—may be performed using digital programs which are very similar to those required for the analog. The resulting eigenvalue problem is solved by means of matrix iteration; this technique provides the means of discontinuing the solution after a specified number of roots are obtained. This number, of course, depends on the problem and its counterpart solution obtained on the analog for comparisons.

The time required for the formulation and setup of the problem is comparable to the time required for the same operation using the network analyzer. The principal advantage of the digital method over the corresponding analog method is due to the inherent accuracy of the digital computer. In general, the digital computer may be considered to solve exactly the problem which is programmed, while the network analyzer involves certain inaccuracies even under optimum conditions. The digital method, however, requires an appreciably longer solution time than the analog for repetitive solutions of the kind required by parameter studies. Although this longer solution time may not be significant if the number of solutions is small, the nature of the dynamics problem indicates that the number of solutions required is large. Therefore, the use of the digital method by itself is seriously restricted.

Suggested Method Based on Integration of Analog and Digital Machines

As noted previously, the analysis of a complex dynamic system usually requires a large number of solutions involving wide variations of the significant parameters of the system. These solutions, however, must often be obtained with a minimum of elapsed time, and must be free of both system errors and errors in machine setup. The finite-difference or "panel" method using the digital and analog computers in an integrated manner provides a means of achieving these desired results. The advantages of both computational systems are obtained—the ease of parameter variations and speed of solution using the direct analog computer as well as a continued check on the accuracy of the solution using the digital computer. The very fact that the analysis is performed by two independent methods is of course advantageous in assuring elimination of errors. The realization of the full capabilities of the integrated digital-analog method is most nearly attained when both computers use the finite-difference or "panel" approach. This formulation results in optimum compatibility between the digital and analog systems, and provides for comparative checks of the two systems at each stage of the analysis—elasticity, vibration, and aeroelastic stability checks.

Example of Integrated Analog and Digital Analyses

The integrated approach suggested in the preceding section was recently applied to the tail symmetric flutter analysis of a transport type airplane being built at Lockheed. The full capacity of the network analyzer was utilized as well as the maximum programming techniques available at the time of investigation on the IBM 701 computer. A sketch of the degrees of freedom is shown in Fig. 1. Six spanwise stations for the stabilizer

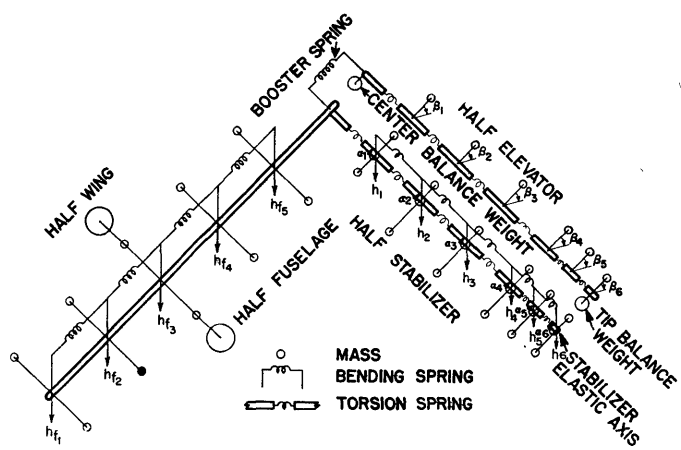

Fig. 1—Schematic of degrees of freedom for symmetric tail flutter.

and elevator combination were chosen as well as five sections for the fuselage. This combination provided six bending and six torsion modes on the stabilizer, six elevator flapping and torsion modes, as well as five fuselage vertical bending modes. Simple beam theory was used for representing the structure in bending and torsion. The elevator was assumed to have a flapping mode with a restraint determined by the boost system. The torsional rigidity of the torque tube required to transfer the elevator hinge moments into the fuselage through the boost system was also represented. In addition an elevator balance weight located at the fuselage centerline was assumed as a separate degree of freedom because of its large concentrated weight on the relatively flexible torque tube extending from the inboard end of the elevator to the balance weight linkage. The whole airplane was free to pitch and plunge. Unfortunately, the wing flexibility was not included due to the limited capacity of the computer. This problem illustrates very well the degree of complexity of airplane structure that can be handled by such a computing facility.

The first stage in the analysis was to check the stiffness circuits on the analog as well as the stiffness values used in both the analog and digital setups. This was accomplished by applying loads, moments, and hinge moments and recording the deflections. A comparison of the deflections was then made with those calculated from the digital finite-difference representations for the same loading conditions. These comparisons are shown in Fig. 2 for bending and torsion of the stabilizer as well as the elevator deflection. Note the remarkable agreement between the two approaches. No detailed struc-

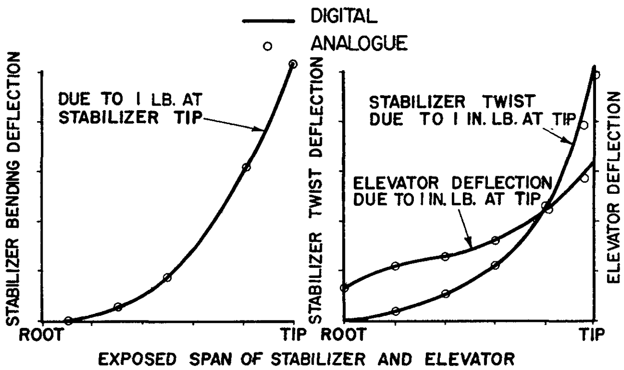

Fig. 2—Comparison of analog and digital static deflections due to unit tip load and moments.

tural deflection measurements were made for the fuselage. The elasticity checks, which were made concurrently on the digital and analog computers, took approximately one week from the time the structural data were made available to the engineers.

The next step in the integrated approach required the addition of the appropriate masses, inertias, together with the proper center-of-gravity locations of the masses in order that a vibration test correlation could be made. Since the fuselage modes were matched on the analog with observed shake tests, and no aerodynamics would be present on the fuselage, the correlations were made with the fuselage rigid and restrained in pitch and plunge. A summary of the natural frequencies obtained by the two computers is given in the following table.

TABLE I

Comparison of Analog and Digital Natural Frequencies

Analog (cps)	Mode	Digital (cps)
6.4	Stabilizer first bending	6.12
7.34	Elev. flapping (boost on)	7.30
16.8	Elevator first torsion	16.23
22.85	Elevator second torsion	23.1
28.0	Stabilizer second bending	26.4
32.1	Stabilizer first torsion	33.3

Note the close agreement between the two completely independent solutions. The differences in frequencies between the digital and analog are most likely due to the differences in finite-difference approximations used in the digital and analog representations. The analog representation implies trapezoidal type approximations while the digital consistently employed parabolic type approximations similar to Simpson's rule. It has been found that for the digital representation the use of higher order finite-difference approximations does not complicate the computations; for the analog, more refined finite-difference approximations, while possible, are usually not practical due to the excessive use of electrical components.

The corresponding mode shapes and relative phasing were also compared. A typical comparison is shown in Fig. 3 for the stabilizer mode. Examination of the results in Fig. 3 as well as those for the other modes indicated that, in general, the mode shapes and relative phasings agree remarkably well; the relative amplitudes, however, experience appreciable differences. These differences are primarily a result of the inherent "structural" damping of the order of one to two per cent present in the analog computer. The corresponding digital calculations were made, of course, with no structural damping.

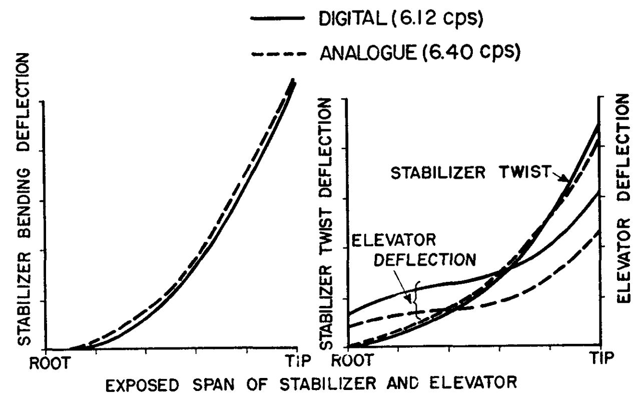

Fig. 3—Comparison of vibration modes for stabilizer first bending determined on digital and analog computers.

An estimate of the time involved to set up this problem on the analog and digital computer was made. Since both setups are made completely independent of each other, both problems were solved simultaneously and took approximately one week from the time the elasticity correlations were made. Parameter variations were made on the analog for several days to determine the adequacy of the degrees of freedom selected as well as the structural data determined from preliminary calculations.

Once the elasticity and vibration correlations were made, the transient aerodynamics were applied to the stabilizer and elevator to determine the flutter characteristics. The significant results are plotted in Fig. 4 as

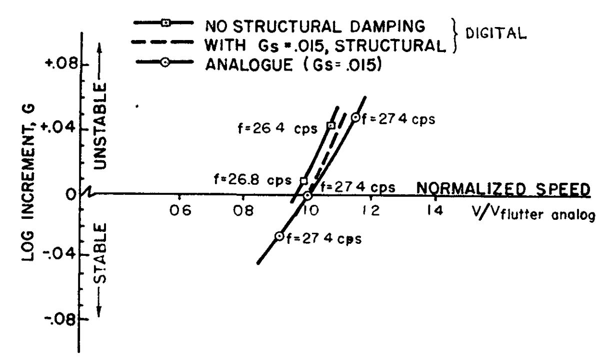

Fig. 4—Comparison of stability diagrams (*V-G*) determined by analog and digital computers.

a function of logarithmic increment of damping versus speed, normalized to the flutter speed obtained on the analog. Although other modes were present on both the analog and digital solutions, the one shown in this figure represents the most critical; that is, the mode having the lowest flutter speed. Also shown are the frequencies of oscillation for the analog and IBM at various stability and speed conditions. Comparison of the two solutions indicates that the frequency of oscillation at flutter agrees remarkably well. The flutter speed for the analog, however, is higher than the speed obtained by the digital setup. This difference can be explained by adjusting the digital calculations to correspond to the amount of inherent "structural" damping present in the analog. An estimate of the analog damping is approximately $g_s = 0.015$. The original digital calculations correspond, of course, to no structural damping. If a value of damping, $g_s = 0.015$, is used for the digital calculations, the revised digital stability curve corresponds almost identically to the analog curve. Thus the necessary correlation has been established. At this point the effect of numerous variations of structural or aerodynamic parameters on the critical flutter speed can be investigated in a relatively short time.

Approximately one to two weeks is required to obtain both the analog and digital correlations.

Concluding Remarks

This paper attempts to define the major phases of dynamic loads problems faced by the dynamicist at various design stages of the airplane. At each stage the nature of these problems was investigated and the following characteristics were found to exist for each:

1. The dynamic problems are becoming increasingly complex due to higher airplane speeds and increased flexibility due to thinner lifting surfaces necessary for optimum performance.
2. The amount of time required for obtaining accurate results which make good "engineering sense" is critically limited since accurate structural and aerodynamic data is not readily available.
3. As a result of the lack of data a large number of parameter variations are required in the preliminary design stage. Examination of the other two stages—shake test and final analysis—also indicates the requirement of a large number of parameter variations.
4. As a result of the increased complexity of the dynamics problem the results must be reasonably guaranteed to be free of errors.

In view of these characteristics a review of possible methods using high-speed computing machines—both digital and analog—was made and an approach was determined using both the analog and digital computing machines in an integrated manner. An example illustrating the use of both types of computers is given, indicating a means of rapidly correlating the results obtained as well as permitting the dynamicist to use the best features of each computer to meet the stringent requirements necessary for a successful analysis.

A General Digital Computer Program for Static Stress Analysis

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Summary—**In a digital computer installation devoted primarily to the solution of aircraft design problems, an attempt has been made to provide the engineer, whenever possible, with programs which are general enough to be applicable to a large class of the problems which he typically encounters. Such a general program has been devised for the stress and deformation analysis of aircraft structures. The mathematical formulation of the problem and the program for its solution are discussed, and some selected applications of the analysis to actual structures are presented.**

Introduction

THE EXISTENCE of the high performance digital computer and the introduction of matrix algebra into structural analysis have created conditions favoring the development of a general program for solving static stress problems. To be most useful such a program should be applicable to any structure without additional programming time, should require as input data no more than the basic geometric and elastic parameters which define the structure, should be expressed in terms familiar to the practicing stress analyst, should require a minimum of machine time with provision for rapidly taking into account changes of stiffness and loading, and should permit the accurate analysis of highly complex indeterminate structures. The following paper describes a program which was designed with these requirements in mind.

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Static Stress Analysis

The problem considered is the stress and deflection analysis of statically indeterminate structures. When either the Maxwell-Mohr or the least work approach is adopted, the solution of the problem usually requires the following steps: (1) Given the co-ordinates of the structure, compute certain geometric constants, such as direction cosines and moment arms of forces; (2) write