1. Background and Setup

Let Z ⊆ ■■ denote the latent space of a pretrained generative model G: Z → X, where X is the data space (e.g.

Let {z■} for i = 1 to N be a finite set of latent vectors. These may correspond to real images or synthetic data.

We are given a target latent vector z ∈ Z. Our goal is to interpolate or reconstruct z as a linear combination of th

■ = ∑ (α■ * z■), where ∑ α■ = 1, and α■ ∈ ■

with indices corresponding to the k nearest neighbors to z.

2. Nearest Neighbor Selection

Define a distance metric d: Z × Z → ■■. We use the Euclidean distance:

d(z, z■) = ||z - z■||■

Let I■(z) ⊆ {1,...,N} denote indices of the k closest latent vectors to z:

I■(z) = argmin over I: |I| = k of ∑ ||z - z■||■ for i in I

3. Linear Reconstruction via Least Squares

We want to find weights α■ ∈ ■, for i ∈ I■(z), such that:

■ = ∑ α■ * z■ ≈ z, with ∑ α■ = 1

Define matrix Z■ ∈ ■■■■ with columns z■ and let α ∈ ■■ be the weights. Then:

minimize ||Z■α - z||■² subject to 1■α = 1

4. Solution via Lagrange Multipliers

Form the Lagrangian:

L(α, λ) = ||Z■α - z||■² + λ(1■α - 1)

Set gradients to zero:

∇α L = 2 Z■■(Z■α - z) + λ1 = 0
∂L/∂λ = 1■α - 1 = 0

This gives a system of k+1 equations solvable with standard methods.

5. Reconstruction and Interpolation

The solution ■ = Z■α* is a convex (if α■ ≥ 0) or affine (if some α■ < 0) combination of the nearest latent vector

This is a locally linear approximation similar to Locally Linear Embedding (LLE).

6. Summary

- Nearest neighbors are chosen by Euclidean distance.
- Linear model with weights summing to 1 approximates z.
- Reconstruction lies in a local linear manifold in latent space.
- Final ■ can be passed to G to approximate G(z).