# Homework I Due Wednesday Oct. 9 11 pm

1. The order of a finite group = the number of elements in the group. Find all the groups of order 4. (Suggestion: One is $\mathbb{Z}/4\mathbb{Z}$, operation $= +$. But there is another! One thing to think about is whether there can or cannot be a $g \in G$, $g \neq e$ such that $g \times g \times g = e$. Investigate that first, maybe.)

2. Find the greatest common divisor of 2124 and 1024, systematically

3. Show that the gcd of 111 and 113 is 1 and find $n, m \in \mathbb{Z}$ s.

$$m\,111 + n\,113 = 1$$

4. Assume (as we shall prove later) that if $g \in G$ and $n =$ the smallest positive integer such that $g^n = e$ ($g^n \overset{\text{def}}{=} g \times \cdots \times g$ $n$ times) then $n \mid \text{ord}(G)$ where $\text{ord}(G)$ = the order of $G$ = number of elements in $G$.

Prove: (a) If $G$ is a finite group, $g \in G$ then $\exists\ N_g > 0 \Rightarrow g^{N_g} = e$
so there is smallest such $N_g$. Notation ord(g)

(b) Use the given assumption to show that if $\text{ord}(G) = p$, $p$ prime then $G$ is the same group (except for notation) as $\mathbb{Z}/\mathbb{Z}_p$.

5. Let $S_3$ = the group of 1-1 functions from $\{1,2,3\}$ to itself.

(a) Show $S_3$ is a group where $\times$ = composition of functions (on the right) $(x)(f \times g) = g(f(x))$

(b) What is the order of $S_3$?

(c) Is $S_3$ the same group (except for notation) as $\mathbb{Z}/\text{ord}(S_3)$?

6. Suppose $N \in \mathbb{Z}^+$. Prove:
There exist only finitely many finite groups $G \ni \text{ord}(G) = N$.
(regarding $G_1$ and $G_2$ as the same if they are "isomorphic":
$\exists\ H: G_1 \to G_2$ 1-1 onto $\ni$ products, inverse and identity are

all preserved, i.e. $G_1$ and $G_2$ are the same group in different notations).

7. Look at

$$(N+1)^3 - N^3 = 3N^2 + 3N + 1$$

and sum the LHS from $N=1$ to $N=n$. Use the fact you know $\sum_1^n N$ and $\sum_1^n 1$ to figure out what $\sum_1^n N^2$ is!

8 Can you do the process in prob 7 for higher powers (inductively on the power)? How does it work for 3rd powers, i.e. $\sum_1^n N^3 = ?$