Homework 1

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N people, N_1 males, N_0 females, T_1 males above 6 feet, T_0 females above 6 feet A event person is male, B event person is above 6 feet.

1. Calculations

    a. $P(A) = \frac{N_1}{N}$

    b. $P(B) = \frac{T_1+T_0}{N}$

    c. $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{T_1}{N}}{\frac{T_1+T_0}{N}} = \frac{T_1}{T_1+T_0}$

    d. $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{\frac{T_1}{N}}{\frac{N_1}{N}} = \frac{T_1}{N_1}$

    e. $P(A\cap B) = \frac{T_1}{N}$

2. Verify $P(A\cap B) = P(A)P(B|A) = P(B)P(A|B)$

    - This can be verified directly from rearranging the conditional Probability formula which has been shown in class $P(A|B) = \frac{P(A\cap B)}{P(B)} \rightarrow P(A|B)P(B) = \frac{P(A\cap B)}{P(B)}P(B) = P(A\cap B)$. Because $P(A\cap B) = P(B\cap A)$ Swapping A for B results in the same equation.
    - $P(A\cap B) = \frac{T_1}{N} = P(A)P(B|A) = \frac{N_1}{N}\frac{T_1}{N_1} = \frac{T_1}{N} = P(B)P(A|B) = \frac{T_1+T_0}{N}\frac{T_1}{T_1+T_0} = \frac{T_1}{N}$

3. Verify $P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$.

    - $P(A)P(B|A) + P(A^c)P(B|A^c) = P(A\cap B) + P(A^c\cap B) = P(B\cap A) + P(B\cap A^c) = P(B)$
    - $P(B) = \frac{T_1+T_0}{N} \rightarrow P(A)P(B|A) + P(A^c)P(B|A^c) = \frac{N_1}{N}\frac{T_1}{N_1} + \frac{N_0}{N}\frac{T_0}{N_0} = \frac{T_1}{N} + \frac{T_0}{N} = \frac{T_1+T_0}{N}$

4. Verify $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A)+P(A^c)P(B|A^c)}$

- The first equality was verified in class by the venn diagram. $P(A)P(B|A) = P(A \cap B)$ was proved in question 2. $P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$ was verified directly in question 3
- We know $P(A|B) = \frac{T_1}{T_1+T_0}$ so we show $\frac{P(A)P(B|A)}{P(A)P(B|A)+P(A^c)P(B|A^c)} = \frac{\frac{N_1}{N}\frac{T_1}{N_1}}{\frac{N_1}{N}\frac{T_1}{N_1}+\frac{N_0}{N}\frac{T_0}{N_0}} = \frac{T_1}{T_1+T_0}$

2

X and Y are generated from a uniform distribution over [0,1] independently.

1. find $P(X^2 + Y^2 \leq 1)$ $\frac{\text{Area of quarter circle with radius one}}{\text{Area of unit square}} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$
2. For large n with m random points landing inside the area. Then $\frac{m}{n} \approx \pi$ so $m \approx n\pi$
3. Calculate $P(X \geq \frac{1}{2})$and$P(X \geq \frac{1}{2}|X + Y \geq 1)$:
   $P(X \geq \frac{1}{2}) = \frac{1}{2}$
   $P(X \geq \frac{1}{2}|X + Y \geq 1) = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{4}$
4. area of $P(A \cap B) = \frac{\text{Area of } A \cap B}{\text{Area of square}} = \frac{2}{25} = \frac{2}{5} * \frac{1}{5} = P(A)P(B)$

3

In the simplest case all drops can be generalized to a needle dropping in the upper half (by switching our orientation if it lands below) with an angle between the horizontal and the upper line. Assuming all orientations are equally likely this is equivalent to finding the area in which the distance D between the midpoint and the top is less than $\frac{1}{2} * \sin\theta$ with $D \in [0, \frac{h}{2}]$ and $\theta \in [0, \frac{\pi}{2}]$ this results in

$$P(\text{needle doesn't hit the edge}) = \frac{\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin\theta d\theta}{\int_0^{\frac{\pi}{2}} \frac{1}{2}d\theta}.$$

Which equals $\frac{2}{\pi}$ In a Monte-Carlo situation $\frac{m}{n} = P(A) = \frac{2}{\pi} \to \pi = \frac{2n}{m}$