HW 7 135

4.1.7 (a, c)

$$l = 1 \qquad \gamma = 1$$

$$u(0, x) = \begin{cases} x & 0 \le x \le 1/2 \\ 1-x & 1/2 \le x \le 1 \end{cases}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(t, x) = X(x)\, T(t)$$

$$\frac{\partial u}{\partial x}(0, t) = 0 = \frac{\partial u}{\partial x}(1, t)$$

$$X(x)\, T'(t) = X''(x)\, T(t)$$

$$\frac{T'(t)}{T(t)} = \frac{X''(x)}{X(x)} = \lambda$$

$$T = d_1 e^{\lambda t}$$

$$X'' = \lambda X$$

if $\lambda > 0$

$$X = c_1 e^{\sqrt{\lambda} x} + c_2 e^{-\sqrt{\lambda} x}$$

if $d_1 \neq 0$

$$\frac{d}{dx}(TX) = d_1 e^{\lambda t} \left(c_1 \sqrt{\lambda} e^{\sqrt{\lambda} x} - c_2 \sqrt{\lambda} e^{\sqrt{\lambda} x} \right)$$

$$TX'(0,t) = 0 \Rightarrow \quad c_1 - c_2 = 0 \Rightarrow c_1 = c_2$$

$$TX'(1,t) = 0 \Rightarrow \quad c_1 e^{\sqrt{\lambda}} - c_1 e^{-\sqrt{\lambda}}$$

$$= c_1 \left(e^{\sqrt{\lambda}} - e^{-\sqrt{\lambda}} \right) \Rightarrow c_1 = 0$$

if $\lambda = 0$

$$X = ax + b$$

$$X' = ax + b$$

$b = 0$

$a = 0$

if $\lambda < 0$

$$X = c_1 \cos \lambda x + c_2 \sin \lambda x$$

$$X' = -c_1 \sin \lambda x + c_2 \cos \lambda x$$

$c_2 = 0$

$$-c_1 \sin \lambda = 0 \Rightarrow \lambda = n\pi$$

$$V(x,t) = \sum_{k=1}^{\infty} b_k e^{-k\pi t} \cos(k\pi x)$$

$$V(x,0) = \sum_{k=1}^{\infty} b_k \cos(k\pi x) = \begin{cases} x & 0 < x \le \frac{1}{2} \\ 1-x & \frac{1}{2} \le x \le 1 \end{cases}$$

$$b_k = 2\int_0^1 \cos(k\pi x)\, V(x,0)\, dx$$

$$= 2\left[\int_0^{\frac{1}{2}} x\cos(k\pi x) + \int_{1/2}^{1} \cos(k\pi x) - \int_{1/2}^{1} x\cos k\pi x\right.$$

$$\int x\cos(k\pi x) = \frac{x\sin k\pi x}{k\pi} - \int \frac{\sin k\pi x}{k\pi} = \frac{x\sin k\pi x}{k\pi} + \frac{\cos k\pi x}{(k\pi)^2}\Bigg|_0^{1/2}$$

$u = x \qquad v = \frac{\sin k\pi x}{k\pi}$

$du = 1 \qquad dv = \cos(k\pi x)$

$$= \frac{\overbrace{\tfrac{1}{2}\sin\left(\tfrac{k\pi}{2}\right)}^{(\pm 1)}}{k\pi} + \frac{\overbrace{\cos(k\pi/2)}^{0}}{(k\pi)^2} - \frac{1}{(k\pi)^2}$$

$$\int \cos(k\pi x)dx = \frac{\sin k\pi x}{k\pi}\Bigg|_{1/2}^{1} = \frac{\sin k\pi}{k\pi} - \frac{\sin\frac{k\pi}{2}}{k\pi} = -\frac{\sin k\pi/2}{k\pi}$$

$$\int x\cos k\pi x = \frac{x\sin k\pi x}{k\pi} + \frac{\cos k\pi x}{(k\pi)^2}\Bigg|_{1/2}^{1} = \frac{\cos k\pi}{(k\pi)^2} - \frac{\frac{1}{2}\sin k\pi/2}{k\pi} - \frac{\cos k\pi/2}{(k\pi)^2}$$

$$= 2\left[\cancel{\frac{\frac{1}{2}\sin\left(\frac{k\pi}{2}\right)}{k\pi}} + \frac{\cos(k\pi/2)-1}{(k\pi)^2} - \cancel{\frac{\sin k\pi/2}{k\pi}} - \frac{\cos k\pi}{(k\pi)^2} + \cancel{\frac{\frac{1}{2}\sin k\pi/2}{k\pi}} + \frac{\cos k\pi/2}{(k\pi)^2}\right]$$

$$= 2\left[\frac{2\cos(k\pi/2) - 1 - \cos k\pi}{(k\pi)^2}\right]$$

$$b_0 = 2\int_0^1 u(x,0)\,dx = 0.5$$

$$u(x,t) = \frac{b_0}{2} + \sum_{k=1}^{\infty} b_k e^{-k\pi t} \cos(k\pi x)$$

c) each $b_k \leq 1$ and $|\cos(k\pi x)| \leq 1$

So $u(x,t) \leq \left|\frac{b_0}{2}\right| + \sum_{k=1}^{\infty} |e^{-k\pi t}|$

if t big s.t. $e^{-k\pi t} < 1 \ \forall k$

then

$$u(x,t) \leq \left|\frac{b_0}{2}\right| + \frac{1}{1-e^{-\pi t}}$$

4.1.8 (a, c)

$$u_t = u_{xx} \qquad -2 < x < 2 \qquad t > 0$$

$$u(t,-2) = u(t,2) = 0 \qquad \text{let } v(t,x) = u(t, x-2)$$

$$v(t,0) = v(t,4) = 0$$

$$u(0,x) = \begin{cases} x, & |x| < 1 \\ 0, & \text{otherwise} \end{cases} \qquad v(0,x) = \begin{cases} x-2 & |x-2| < 1 \\ 0 \end{cases}$$

$$v = X(x)\,T(t) \qquad \lambda < 0$$

$$X(x)T'(t) = X''(x)T(t) \Rightarrow \frac{X''(x)}{X(x)} = \frac{T'(t)}{T(t)} = \lambda$$

$$X(x) = c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x$$

$$0 = X(0) = c_1 + 0 \qquad c_1 = 0$$

$$0 = X(4) = (c_2 \sin\sqrt{\lambda} \cdot 4)$$

$$4\sqrt{\lambda} = n\pi$$

$$\sqrt{\lambda} = \frac{n\pi}{4}$$

$$\lambda = \frac{(n\pi)^2}{4}$$

$$v(x,t) = \sum_{k=1}^{\infty} b_k e^{-\frac{(k\pi)^2}{4}t} \sin\left(\frac{k\pi}{2}x\right)$$

$$v(0,x) = \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi}{2}x\right) \approx \begin{cases} x-2 & |x-2| < 1 \\ 0 \end{cases}$$

$$b_k = \frac{2}{4}\int_0^4 \sin\left(\frac{k\pi}{2}x\right) V(0,x)\,dx$$

$$= \frac{1}{2}\int_1^3 (x-2)\sin\frac{k\pi}{2}x\,dx = \frac{1}{2}\left[\int_1^3 x\sin\frac{k\pi}{2}x\,dx - 2\int_1^3 \sin\frac{k\pi}{2}x\,dx\right.$$

$$\int x\sin\frac{k\pi}{2}x\,dx = \frac{-x\cos\frac{k\pi}{2}x}{\left(\frac{k\pi}{2}\right)} + \int\frac{\cos\frac{k\pi}{2}x}{\frac{k\pi}{2}} = \frac{-x\cos\frac{k\pi}{2}x}{\frac{k\pi}{2}} + \frac{\sin\frac{k\pi}{2}x}{\left(\frac{k\pi}{2}\right)^2}\Bigg|_1^3$$

$$= \frac{\sin\frac{3k\pi}{2}}{\left(\frac{k\pi}{2}\right)^2} - \frac{3\cos\frac{3k\pi}{2}}{\frac{k\pi}{2}} - \frac{\sin\frac{k\pi}{2}}{\left(\frac{k\pi}{2}\right)^2} + \frac{\cos\left(\frac{k\pi}{2}\right)}{\frac{k\pi}{2}}$$

$$-2\int\sin\frac{k\pi}{2}x\,dx = \frac{2\cos\frac{k\pi}{2}x}{\frac{k\pi}{2}}\Bigg|_1^3 = -2\frac{\cos\frac{k\pi}{2}}{\frac{k\pi}{2}} + 2\frac{\cos\frac{3k\pi}{2}}{\frac{k\pi}{2}}$$

$$b(k) = \frac{1}{2}\left[\frac{\sin\frac{3k\pi}{2}}{\left(\frac{k\pi}{2}\right)^2} - \frac{\cos\frac{3k\pi}{2}}{\frac{k\pi}{2}} - \frac{\sin\frac{k\pi}{2}}{\left(\frac{k\pi}{2}\right)^2} - \frac{\cos\left(\frac{k\pi}{2}\right)}{\frac{k\pi}{2}}\right]$$

$$u(t,x) = V(t,x+2)$$

$$= \sum_{k=1}^{\infty} b(k)\, e^{-\left(\frac{k\pi}{2}\right)^2 t}\sin\left(\frac{k\pi}{2}(x+2)\right)$$

c the solutin approaches equilibrium

at an exponential rate because

each $|b_n| \le 1$ & each sin curve

≤ 1 in absolute value

thus exponential decay

among all elements

4.1.10

heated ring w/

(a) $\cos x$, (b) $\sin^3 x$, (c) $|x|$, (d) $\begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$

$\ell = 2\pi$

$u_t = u_{xx}$

$u(t, -\pi) = u(t, \pi)$ $\frac{\partial u}{\partial x}(t, -\pi) = \frac{\partial u}{\partial x}(t, \pi)$

$u(0, x) = f(x)$

$$u(t, x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n e^{-n^2 t} \cos nx + b_n e^{-n^2 t} \sin nx$$

$$u(0, x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

(a)

(i) $u(t,x) = e^{-t}\cos x$

(ii) as $t \to \infty$ $u(t,x) \to 0$

(b) $\sin^3 x = \dfrac{3\sin x - \sin 3x}{4}$

(i) $u(t,x) = \frac{3}{4}e^{-t}\sin x - \frac{1}{4}e^{-36}\sin 3x$

ii $u(t,x) \to 0$

c $|x|$

$$a_k = \frac{2}{\pi}\int_0^\pi x\cos kx \qquad b_k = 0$$

$$\int x\cos kx = \frac{x\sin kx}{k} - \int \frac{\sin kx}{k}\,dx$$

$$= \frac{x\sin kx}{k} + \frac{\cos kx}{k^2}\Big|_0^\pi$$

$$= \frac{\cos k\pi}{k^2} - \frac{1}{k^2} = \frac{(-1)^k - 1}{k^2}$$

$$a_k = \frac{2}{\pi}\left(\frac{(-1)^k - 1}{k^2}\right)$$

$$a_0 = \frac{2}{\pi}\int_0^\pi x\,dx = \pi$$

(i) $u(t,x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2 t} \cos(nx) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right) e^{-n^2 t} \cos nx$

(ii) $u(t,x) \to \frac{\pi}{2}$ as $t \to \infty$

d $\begin{cases} 1 & x<0 \\ 0 & x>0 \end{cases}$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{0} \cos kx \, dx = \frac{\sin kx}{k}\Big|_{-\pi}^{0} = 0$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{0} \sin(kx)\, dx = -\frac{\cos kx}{k}\Big|_{-\pi}^{0} = \frac{-1}{k} + \frac{\cos k\pi}{k}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} dx = 1 \qquad = \frac{(-1)^k - 1}{\pi k}$$

(i) $u(t,x) = \frac{1}{2} + \sum_{n=1}^{\infty} \left(\frac{(-1)^n - 1}{\pi n}\right) e^{-n^2 t} \sin nx$

(ii) $u(t,x) \to \frac{1}{2}$ as $t \to \infty$

4.1.15

$v(t,x)$ 4.30 – 4.31

$$N(t) = \int_{-\pi}^{\pi} u(t,x)^2 \, dx$$

decreasing in t

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(t,-\pi) = u(t,\pi)$$

$$\frac{\partial u}{\partial x}(t,-\pi) = \frac{\partial u}{\partial x}(t,\pi)$$

if $t_2 > t_1$

$$L^2 = \int_{-\pi}^{\pi} u(t,x)^2 \, dx$$

$$N(t_2) - N(t_1) < 0$$

$$\int_{-\pi}^{\pi} v(t_2,x)^2 - \int_{-\pi}^{\pi} v(t_1,x)^2$$

$$= \int_{-\pi}^{\pi} v(t_2,x)^2 - v(t_1,x)^2 \, dx$$

$$\frac{\partial v^2}{\partial t} = 2\, v(t,x) \cdot \frac{\partial v}{\partial t} = 2\, v(t,x) \frac{\partial^2 (v)}{\partial x^2}$$

$$\frac{d}{dt}\int_{-\pi}^{\pi} u(t,x)^2\,dx = \int_{-\pi}^{\pi} \frac{d}{dt} u(t,x)^2\,dx$$

$$= \int_{-\pi}^{\pi} 2u(t,x)\frac{du}{dt}\,dx$$

$$= \int_{-\pi}^{\pi} 2\,u\cdot u_{xx}\,dx = 2\left(\left[u\cdot u_x\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} u_x^2\,dx\right)$$

$u(\pi)\,u_x(\pi) = u(-\pi)\,u_x(-\pi)$

$$= -2\int_{-\pi}^{\pi} u_x^2\,dx$$

Sin u_x is real

u_x^2 is positive

and $\int_{-\pi}^{\pi} u_x^2 > 0$

So $-2\int_{-\pi}^{\pi} u_x^2 < 0$

and so $\frac{d}{dt} L^2 < 0$

and L^2 is decreasin

4, 2, 3

(a) (f)

4) $U_{tt} = U_{xx}$ $\quad U(t,0) = U(t,\pi) = 0$

$U(0,x) = 1$

$U_t(0,x) = 6$

$U = X(x)T(t)$

$X(x)T''(t) = X''(x)T(t)$

$$\frac{X''(x)}{X(x)} = \frac{T''(t)}{T(t)} = \lambda$$

$\lambda < 0$

$X = \cancel{c_1 \cos \lambda x} + c_2 \sin \lambda x$ $\qquad \lambda = n\pi$

$T = c_3 \cos \lambda t + c_4 \sin \lambda t$

$XT = (c_1 \cos \lambda x + c_2 \sin \lambda x)(c_3 \cos \lambda t + c_4 \sin \lambda t)$

$c_1 \qquad = 0 = \qquad c_2 \sin \lambda \pi)$

$c_1 = -c_1 \Rightarrow c_1 = 0$ $\qquad \lambda = 1, \dots, n$

$X = c_2 \sin nt$

$U(0,x) = c_2 \sin nx \, (c_3) = 1$

$U_t(0,x) =$

$$XT = \sin nx \left(c_1 \cos nt + c_2 \sin nt \right)$$

$$u = \sum_n \sin nx \left(c_1 \cos nt + c_2 \sin nt \right)$$

$$u_t = \sum_n \sin nx \left(-c_1 \sin nt + c_2 \cos nt \right)$$

$$u_t(0) = 0 = \sum_n \sin nx \; c_2 \Rightarrow c_2 = 0$$

$$u = \sum_{n=1}^{\infty} c_1 \sin nx \cos nt$$

$$u(0, x) = \sum_{n=1}^{\infty} c_1 \sin nx \approx 1$$

$$c_n = \frac{2}{\pi} \int_0^{\pi} \sin nx \, dx = \left. \frac{-2\cos nx}{\pi n} \right|_0^{\pi} = -\frac{2\cos n\pi}{n\pi} + \frac{2}{n\pi}$$

$$= \frac{2(1-(-1)^n)}{n\pi}$$

$$u = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left(1 - (-1)^n \right) \sin nx \cos nt$$

(g) $\quad u_{tt} = u_{xx} \qquad u_x(t,0) = u_x(t,1) = 0$

$u(0,x) = x(1-x) \qquad u_t(0,x) = 0$

$$X(x) = c_1 \sin n\pi x$$
$$T(t) = c_1 \cos n\pi t$$

$$u(t,x) = \sum_{n=1}^{\infty} c_n \sin n\pi x \cos n\pi t$$

$$u(0,x) = \sum_{n=1}^{\infty} c_n \sin n\pi x \simeq x(1-x)$$

$$c_n = \frac{2}{1} \int_0^1 (x - x^2) \sin n\pi x \, dx$$

$$\int_0^1 x \sin n\pi x = \frac{-x\cos n\pi x}{n\pi}\Big|_0^1 + \int_0^1 \frac{\cos n\pi x}{n\pi} = \frac{\sin n\pi x}{(n\pi)^2} - \frac{x\cos n\pi x}{n\pi}\Big|_0^1$$

$$= \frac{-\cos n\pi}{n\pi} = \frac{(-1)^{n+1}}{n\pi}$$

$$\int_0^1 x^2 \sin n\pi x = \frac{-\overset{(-1)}{x^2} \overset{(-1)^n}{\cos n\pi x}}{n\pi}\Big|_0^1 + \frac{2}{n\pi}\int_0^1 x\cos n\pi x \, dx = \frac{(-1)^{n+1}}{n\pi} + \frac{2}{n\pi}\int_0^1 x\cos n\pi x$$

$$\int_0^1 x\cos n\pi x \, dx = \frac{x\sin n\pi x}{n\pi}\Big|_0^1 - \int_0^1 \frac{\sin n\pi x}{n\pi} = \frac{x\sin n\pi x}{n\pi} + \frac{\cos n\pi x}{(n\pi)^2}\Big|_0^1 = \frac{\cos n\pi}{(n\pi)^2} - \frac{1}{(n\pi)^2} = \frac{(-1)^n - 1}{(n\pi)^2}$$

$$= \frac{(-1)^{n+1}}{n\pi} + \frac{2((-1)^n - 1)}{(n\pi)^3}$$

$$c_n = 2\left(\cancel{\frac{(-1)^{n+1}}{n\pi}} - \cancel{\frac{(-1)^{n+1}}{n\pi}} - \frac{2((-1)^n - 1)}{(n\pi)^3} \right)$$

$$= \frac{4 - 4(-1)^n}{(n\pi)^3}$$

$$u(t,x) = 4\sum_{n=1}^{\infty} \frac{1-(-1)^n}{(n\pi)^3} \sin n\pi x \cos n\pi t$$

f

$u_{tt} = u_{xx}$ $\quad u_x(t,0) = u_x(t,2\pi) = 0, \quad u(0,x) = -1 \quad u_t(0,x) = 1$

$X(x) = c_1 \cos\lambda x + c_2 \sin\lambda x$ $\quad u_x = X'(x)T(t)$ $\quad X'(x) = \lambda - c_1 \sin\lambda x + c_2\lambda\cos\lambda x$

$T(t) = c_3 \cos\lambda t + c_4 \sin\lambda t$

$u_x(t,0) = c_2\lambda T(t) = 0 \Rightarrow c_2 = 0 \Rightarrow X = c_1 \cos\lambda x$

$u_x(t,2\pi) = c_1 \sin 2\pi\lambda \Rightarrow \lambda = \frac{n}{2} \quad n \in \{0,1,\dots\}$

$u(t,x) = \sum_{n=0}^{\infty} \cos\frac{n}{2}x \left(a_n \cos\frac{n}{2}t + b_n \sin\frac{n}{2}t\right)$

$u(0,x) = -1 = \sum_{n=0}^{\infty} a_n \cos\frac{n}{2}x \Rightarrow a_n = \frac{2}{2\pi}\int_0^{2\pi} -1 \cdot \cos\frac{nx}{2}\,dx = \frac{1}{n}\sin\frac{nx}{2}\Big|_0^{2\pi} = 0$

$u(t,x) = \sum_{n=1}^{\infty} b_n \cos\frac{n}{2}x \sin\frac{n}{2}t$

$u_t = \sum_{n=1}^{\infty} \frac{n}{2} b_n \cos\frac{n}{2}x \cos\frac{n}{2}t$

$u_t(0,x) = 1 = \sum_{n=1}^{\infty} \frac{n}{2} b_n \cos\frac{n}{2}x$

$\frac{n}{2} b_n = \frac{2}{2\pi}\int_0^{2\pi} \cos\frac{n}{2}x = 0$

impossible unless $\frac{n}{2} = 0$

then $\lambda = 0$ $\quad X(x) = a + Bx$

$T(t) = C + Dt$

then $u(t,x) = (a+Bx)(C+Dt)$

$u(0,x) = -1 = Ca + BCx \Rightarrow B = 0 \Rightarrow Ca = -1$

$u(t,x) = -1 + Dt$

$u_t(t,x) = D = 1$ so $D = 1$

So $u(t,x) = t - 1$

4.24 (a) $\qquad u_{tt} = u_{xx} \qquad 0 \le x \le \pi$

(a) $\quad u(t,0) = 0 \qquad u_x(t,\pi) = 0$

$u(x,t) = X(x)T(t) \qquad u_{xx} = X''(x)T(t) \qquad u_{tt} = X(x)T''(t)$

$$\frac{X''}{X} = \frac{T''}{T} = \lambda$$

if $\lambda > 0$ then $\quad X = c_1 e^{\sqrt{\lambda}x} + c_2 e^{-\sqrt{\lambda}x}$

$T = c_3 e^{\sqrt{\lambda}t} + c_4 e^{-\sqrt{\lambda}t}$

$u(t,0) = 0 = (c_1 + c_2)(c_3 e^{\sqrt{\lambda}t} + c_4 e^{-\sqrt{\lambda}t}) = 0 \Rightarrow c_1 = -c_2$

$u(t,\pi) = 0 = c_1(e^{\sqrt{\lambda}\pi} - e^{-\sqrt{\lambda}\pi})(c_3 e^{\sqrt{\lambda}t} + c_4 e^{-\sqrt{\lambda}t}) \Rightarrow c_1 = 0$

if $\lambda = 0$ then $X(x) = A + Bx \qquad T(t) = C + Dt$

$u(t,0) = A(C+Dt) = 0 \Rightarrow A = 0$

$u(t,\pi) = 0 = Bx(C+Dt) \Rightarrow B = 0 \qquad$ DNE

if $\lambda < 0 \qquad X = c_1 \cos\lambda x + c_2 \sin\lambda x \qquad T(t) = c_3 \cos\lambda t + c_4 \sin\lambda t$

$u(t,0) = \quad c_1 T(t) = 0 \Rightarrow c_1 = 0$

$u(t,\pi) = \quad c_2 \sin(\lambda\pi)T(t) = 0 \Rightarrow -\lambda = n \in \{1, 2, \ldots\}$

$$u(x,t) = \sum_{n=1}^{\infty} \sin nx \left(a_n \cos nt + b_n \sin nt\right)$$

(b) $u_x(t,0)=0 \qquad u_x(t,\pi)=0$

if $\lambda > 0$

$$u_x = X'(x)T(t) = \left(c_1\sqrt{\lambda}e^{\sqrt{\lambda}x} - c_2\sqrt{\lambda}e^{-\sqrt{\lambda}x}\right)\left(c_3 e^{\sqrt{\lambda}t} + c_4 e^{-\sqrt{\lambda}t}\right)$$

$$u_x(t,0) = \sqrt{\lambda}(c_1 - c_2)T(t) \Rightarrow c_1 = c_2$$

$$u_x(t,\pi) = \sqrt{\lambda}c_1\left(e^{\sqrt{\lambda}\pi} - e^{-\sqrt{\lambda}\pi}\right)T(t) \Rightarrow c_1 = 0 \quad \text{DNE}$$

if $\lambda = 0$

$$X(x) = A + Bx \qquad T(t) = C + Dt$$

$$u_x = B(C+Dt)$$

$$u_x(t,0) = BC + BDt \Rightarrow B = 0$$

$$u_x(t,\pi) = 0 = 0$$

$$\text{S.} \quad u(x,t) = C + Dt$$

$\lambda < 0$

$$X(x) = c_1\cos\lambda x + c_2\sin\lambda x$$

$$u_x = \left(-c_1\lambda\sin\lambda x + c_2\lambda\cos\lambda x\right)T(t)$$

$$u_x(t,0) = 0 \Rightarrow c_2\lambda T(t) \Rightarrow c_2 = 0$$

$$u_x(t,\pi) = 0 = \left(-c_1\lambda\sin\pi\lambda\right)T(t) \Rightarrow \lambda = n = \{1, \dots\}$$

$$u(t,x) = C + Dt + \sum_{n=1}^{\infty}\cos nx\left(a_n\cos nt + b_n\sin nt\right)$$