Hw 5 1705 Asher

8.1

1

$N(\mu, 100)$ $H_0: \mu = 110$

$H_1: \mu > 110$ $n = 16$

$\bar{x} = 113.5$

$\alpha = 0.05$

a) Critical $[\mu_0 + z_\alpha(\frac{\sigma}{\sqrt{n}}), \infty)$

$z_\alpha = 1.645$

$\sigma^2 = 100$ $\sigma = 10$

$\sqrt{n} = 4$

$[\mu_0 + 4.1125, \infty)$

$[114.1125, \infty)$

do not reject null hypothesis at 5%.

b) $z_{0,1} = 1.282$

$[\mu_0 + 3.205, \infty)$

$[113.205, \infty)$

Reject null hypotho

p valu

c) $P(\bar{X} \geq 113.5)$

$$P\left(\frac{|\bar{x} - 110|}{10/\sqrt{10}} \geq z_\alpha\right)$$

$$P(1.4 \geq z_\alpha) = 1 - 0.9192$$

$$= 0.0808$$

(3)

$X \sim N(\mu, 100)$ $\quad H_0 : \mu = 170$

$H_1 : \mu \cdot 170$ $\quad n = 25$ $\quad \sqrt{n} = 5$

$\alpha = 0.05$

$$P(T \geq z_{0.05}) = 0.05$$

$$T = \frac{\bar{x} - 170}{\frac{10}{5}}$$

$$p\left(\bar{x} \geq z_{0.05} \frac{10}{5} + 170\right)$$

$$p(\bar{x} > 173.29) = 0.05$$

$\bar{X} = 165.36$

Do not reject H_0

$$P\left(\frac{165.36 - 170}{2} \geq Z_\alpha \right) = 0.0102$$

1.26

-5 $\mu = 3315$ $\sigma = 575$

$H_0 : \mu = 3315$

$H_1 : \mu < 3315$ $n = 30$

$$P\left(\frac{\bar{X} - 3315}{\frac{s}{\sqrt{n}}} \le -Z_{0.05} \right) = 0.05$$

$$P\left(\bar{X} \le -Z_{0.05} \frac{s}{\sqrt{30}} + 3315 \right) = 0.05$$

$\bar{X} = 3184$

$s = 488$

$P(\bar{X} \le 3168) = 0.05$

$\bar{X} > 3168.47$

so fail to reject null

$$\frac{\bar{X} - 3315}{\frac{488}{\sqrt{30}}} = -1.414$$

$P(-1.414 \leq Z) = .0775$

-9

$n = 33$

$H_0 : \mu_0 = 15.7$

$H_1 : \mu < 15.7$

$S = 2.543$

$\bar{x} = 13.66$

$$P\left(\frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{33}}} \leq -Z_\alpha \right) = \alpha$$

$$\frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{33}}} = -4.608$$

$P(-4.608 \leq z) < 0.0001$

reject H_0

$$P\left(\bar{X} \leq -Z_{0.02}\left(\frac{s}{\sqrt{33}}\right) + \mu_0\right) = 0.0\cancel{8}$$

$P(\bar{x} \in [14,788)) = 0,02$

$[0, 14,788)$

8.2

-1 $X = X_1 + X_2$

$X \sim N(\mu_x, \sigma^2)$
$Y \sim N(\mu_y, \sigma^2)$

$H_0 : \mu_x = \mu_y$

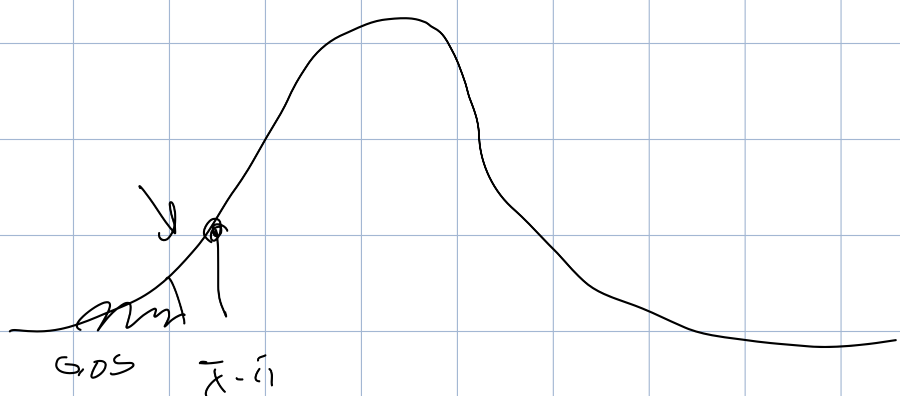
$H_1 : \mu_x < \mu_y$

$n_x = n_y = 10$

$\bar{D} = \frac{1}{10} \sum x_i - y_i$

$C.R. = \left(-\infty, -t_\alpha (n+m-2) s_p \sqrt{\frac{1}{n} + \frac{1}{m}}\right]$

$= \left(-\infty, -t_{0.05}(18) s_p \sqrt{\frac{2}{10}}\right]$



$\bar{x} = 2.31$ $\quad$ $\bar{x} - \bar{y} = -0.55$

$\bar{y} = 2.86$

$S_x = 0.59848$

$S_y = 0.6022$

$$S_p = \sqrt{\frac{9 S_x^2 + 9 S_y^2}{18}} = 0.600397$$

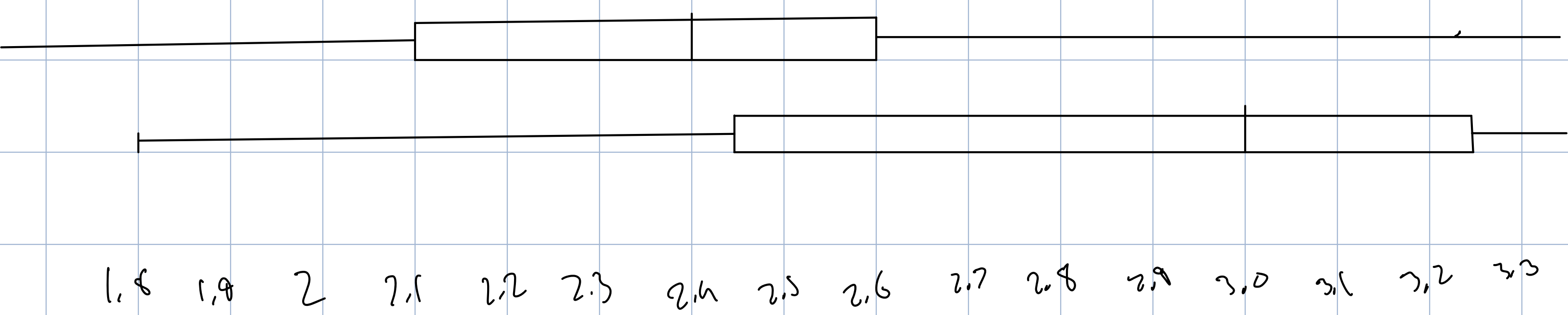
$$t_{0.05}(18) = 1.734$$

$$C.R. = (-\infty, -0.4656]$$

reject Null Hypothesis

$m_x = 2.4$ $\quad$ $\Pi_{0.25}(x) = 2.1$ $\quad$ $\Pi_{0.75}(x) = 2.6$

$m_y = 3.0$ $\quad$ $\Pi_{0.25}(y) = 2.45$ $\quad$ $\Pi_{0.75}(y) = 3.25$



-3

$X \sim N(\mu_x, \sigma_x^2)$
$Y \sim N(\mu_y, \sigma_y^2)$

$n_x = 9 \qquad n_y = 13$

assuming $\sigma_x = \sigma_y^2$

$H_0: \mu_x = \mu_y$

$H_1: \mu_x \neq \mu_y$

$$\left(-\infty, -t_{\alpha/2}(9+13-2)\, S_p \sqrt{\frac{1}{13} + \frac{1}{9}}\right) \cup \left(t_{\alpha/2}(28) S_p \sqrt{\frac{1}{13} + \frac{1}{9}}\right.$$

$\bar{X} = 21.033 \qquad \bar{X} - \bar{Y} = 0.141$

$\bar{Y} = 20.892$

$S_x = 0.606$

$S_y = 1.01$

$S_p = 0.8712$

$t_{0.025}(20) = 2.086$

$t_{0.025}(20)\, s_p \sqrt{\frac{1}{13}+\frac{1}{8}} = 0.788$

$(-\infty, -0.788] \cup [0.788, \infty)$

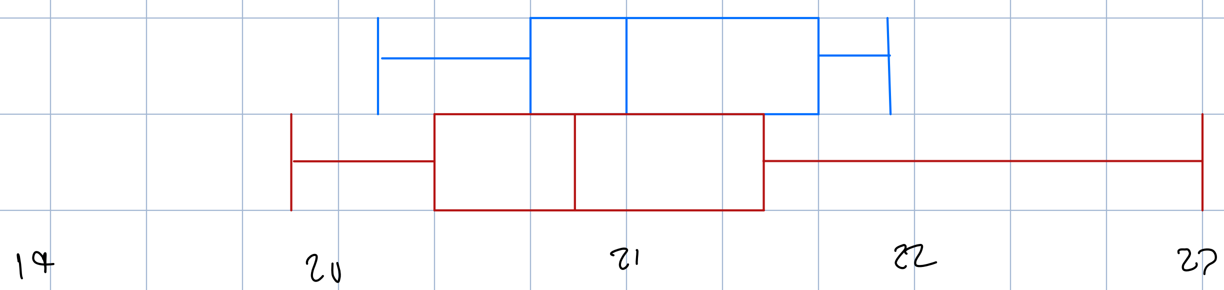
Do not reject null hypothesis

$m_x = 21$ $\quad \pi_{0.25}(x) = 20.6, \quad \pi_{.75} = 21.6$

$m_y = 20.8$ $\quad \pi_{0.25}(x) = 20.3, \quad \pi_{0.75\,y} = 21.5$

$\min_x = 20.2$ $\qquad \max x = 21.8$

$\min_y = 18.9$ $\qquad \max y = 23.0$



-6

$X \sim N(\mu_x, \sigma_x^2)$

$Y \sim N(\mu_y, \sigma_y^2)$

$n_x = n_y = 10$

$H_0: \mu_x = \mu_y$ $\alpha = 0.05$

$H_1: \mu_x \neq \mu_y$

$C.R. = (-\infty, -t_{\alpha/2}(18)\, s_p \sqrt{\frac{1}{5}}] \cup [t_{\alpha/2}(18)\, s_p \sqrt{\frac{1}{5}}, \infty)$

$\bar{X} = 131.5$ $\bar{X} - \bar{Y} = -12.7$

$\bar{Y} = 144.2$

$s_x = 14.081$

$s_y = 12.269$

$s_p = \sqrt{\frac{s_x^2 + s_y^2}{2}} = 13.204$

$t_{0.025}(10) = 2.101$

$C.R. = (-\infty, -12.47] \cup [12.47, \infty)$

reject null

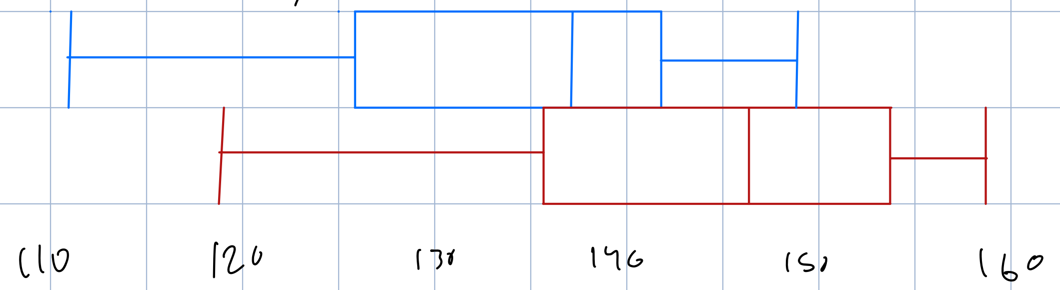
$t_{\beta/2}(18) = \left| \frac{-12.7}{s_p \sqrt{\frac{1}{5}}} \right| = 2.151$

$0.01 < p < 0.25$

(1)

min_x $max_x = 148$ $\quad m_x = 137$ $\quad q_{0,25x} = 126,75$ $\quad q_{0,75x} = 142,0$

$min = 118$ $\quad max = 157$ $\quad m_y = 147,5$ $\quad = 136,0$ $\quad = 154,25$



-11

$H_0 : \mu_x = \mu_y$

$H_1 : \mu_x \neq \mu_y$

$n_x = 90$

$\bar{x} - 8.1$

$s_x = 6.117$

$n_y = 110$

$\bar{y} = 8.07$

$s_y = 6.054$

$$C.R. = (-\infty, -Z_{\alpha/2}\, s_p \sqrt{\tfrac{1}{n_x} + \tfrac{1}{n_y}}\,] \cup [\,Z_{\alpha/2}\, s_p \sqrt{\tfrac{1}{n_x} + \tfrac{1}{n_y}}, \infty)$$

$$Z_{0.025} = 1.960 \qquad s_p = \sqrt{\frac{89(s_x^2) + 109(s_y^2)}{198}} = 0.088$$

$$C.R. = (-\infty, -0.025] \cup [0.025, \infty)$$

$$\bar{x} - \bar{y} = 0.03$$

Reject H_0

$$Z = \frac{0.03}{s_p \sqrt{\frac{1}{n} + \frac{1}{n}}} = 2.39$$

$$P(2.39 \geq Z) = \alpha/2$$

$= 0.016$

1.6%