Subgroups, normal subgroups, Homomorphisms

$H \subset G$ is a subgroup if H is a group with $h_1 \times h_2 =$ product of h_1, h_2 as elements of G

$H \subset G$ is <u>normal</u> in G (notation $H \triangleleft G$) if H is a subgroup and $gHg^{-1} = H$ $\forall g \in G$.

Example of S_3 (see homework)

If $H \triangleleft G$ then a group structure on

$G/H =$ set of cosets $gH, g \in H$ (<u>not</u> a subgroup of G!)

by setting

$$g_1 H \times g_2 H = (g_1 g_2) H.$$

Reason

$$g_1 H \times g_2 H = g_1 g_2 H \quad \text{since}$$

$Hg_2 = g_2 H$ (because $g_2^{-1} H g_2 = H$)

so $g_1 H \times g_2 H = g_1 (H \times g_2) H$

$= g_1 (g_2 H) H = g_1 g_2 H.$ Exercise, check grp properties

Think about how this works and experiment with S_3 to see why e.g. the cosets of $H = \{e, \text{interchange of } 1 \text{ and } 2\}$ do not form a subgroup if you try this definition on them. (exercise)