8.4 (1,3) , 8.5 (1,2,10), 8.6 (1,2,8)

8.4.1

$Y \sim b(100, p)$ $H_0: p = 0.08$

H_1 $p < 0.08$

a) $Y \leq 6$ so $\overline{y} = 0.06$

C.R. $(-\infty, 0.a]$

$$0.06 = p_0 - z_\alpha \sqrt{\frac{p_0(1-p_0)}{n}}$$

$p_0 = 0.08$ $n = 100$

$$0.06 = 0.08 - z_\alpha (0.027129)$$

$$z_\alpha = 0.516$$

$\alpha = 1 - 0.695 \approx 0.30$

b) $p = 0.04$

$$CR = \left[\frac{6.5}{100}, \infty \right)$$

$$= \left[p + z_\alpha \sqrt{\frac{p(1-p)}{n}}, \infty \right)$$

$$p + z_\alpha \sqrt{\frac{p(1-p)}{n}} = 0.065$$

$$0.04 + z_\alpha (0.0196) = 0.065$$

$$z_\alpha = 1.276$$

$$\alpha = 0.8997$$

Significance $= 1 - \alpha \simeq 0.1003$

8.4.3 $Y \sim b(192, p)$

$H_0: p = 0.75$

$H_1: p > 0.75$ iff $Y \geq 152$

$$\alpha = P(Y \geq 152 : p = 0.75)$$

$\mu = 192 \cdot 0.75 = 144$

$$\frac{Y - \mu}{\sigma} = \frac{Y - 144}{\sigma}$$

$\sigma^2 = np(1-p)$

$$= P\left(Z \geq \frac{Y - \mu - 0.05}{\sigma}\right)$$

$$= P(Z \geq 1.25)$$

$$= 1 - 0.8944$$

$$= 0.1056$$

$\beta = P(Y < 152)$ when $p = 0.8$

$\mu = 0.8 \cdot 192 = 153.6$

$\sigma^2 = np(1-p)$

$$\frac{\bar{Y} - \bar{P} - 0.5}{\sigma}$$

$$P\left(Z < \frac{\bar{Y} - \bar{P} - 0.5}{\sigma}\right)$$

$$= P(Z < -0.3784)$$

$$\approx 0.352$$

8.5 (1, 2, 10),

1 $H_0: m = 40{,}000$

$H_1: m < 40{,}000$

-1	-12	-11	-10
30 906,	31 031,	34 930,	36 840,

-8.5	-8.5	-7	-2	-1	3
38 050,	38 090,	38 345,	39 245,	39 485,	40 780

4	5	6
40 890,	41 195,	41 229

$$W = \sum Rank(sign(i)) = -55$$

$$C.R. = \left(-\infty, -z_\alpha \sqrt{\frac{n(n+1)(2n+1)}{6}}\right)$$

$$z_{0.05} = 1.645 \qquad \sqrt{\frac{n(n+1)(2n+1)}{6}} = 28.62$$

$$n = 13$$

$$C.R. = (-\infty, -47.07)$$

reject null Hypothesis

$-55 < -47.07$

$-55 = -z_\alpha \left[28.62 \right.$

$z_\alpha = 1.92$

$P(z_\alpha > 1.920) = 0.0274$

8.5.7

$H_0: m = 1.14$

$H_1: m > 1.14$

−9	−4.5	−2.5	−2.5	−1	4.5	6	7
1.06,	1.11,	1.12,	1.12,	1.13,	1.17,	1.18,	1.19,

8	10.5	10.5	12	13	14
1.20,	1.23,	1.23,	1.25,	1.29,	1.31

$W = 66$

$\alpha = 0.05$

$C.R. = \left[z_\alpha \sqrt{\frac{n(n+1)(2n+1)}{6}}, \infty \right)$

$= [52.408, \infty)$ 66 ∈ 52.408

Reject null hypothesis

$$p = 1 - \Phi\left(\frac{w}{s}\right) = 1 - \Phi$$

$$= 1 - \Phi(2.07) \approx 0.0192$$

8.5.10

$\alpha \approx 0.05$

$H_0 \quad m_x = m_y \qquad n_x = 12$

$H_1 \quad m_x < m_y \qquad n_y = 12$

Data ordered $rel = X$

1	2	3	
67.4	69.3	72.7	73.1
5	6	7	
75.9	77.2	77.6	78.9
9	10		12
83.2	83.3	82.5	84.0
	14		16
84.7	87.6	87.5	80.2
	17		
90.4	90.4	88.3	87.5
86.6	92.7	94.4	95.8

$\sum Rank = 103$

$CR: = (-\infty, \frac{n(n+m+1)}{2} - z_\alpha \sqrt{\frac{nm(n+m+1)}{12}}]$

$= (-\infty, 121.51]$ Reject null

8.6(1,2,8)

8.6.1

$X \sim N(\mu, 9)$

$H_0 : \mu = 25$

$H_1 : \mu < 25$

x_1, x_2, x_3, x_4 $\quad \bar{x}$

$C.R. = (-\infty, 22.5]$

$K(\mu) = P(\bar{x} < 22.5; \mu)$

$= \underline{\Phi}\left(\left(22.5 - \bar{x}\right)^2 / 3\right)$

a

$K(25) = \underline{\Phi}\left(\frac{5}{3}\right) = 0.0475$

b) $\bar{X} = 24.1225$

do not reject

$$p(\bar{X}; \mu) = 0.14$$

c p value of $\bar{X}$ =

$$P(\bar{X} \leq 24.1225; \mu = 25)$$

$$= P\left(\frac{\bar{X} - 25}{\sigma/2} \leq \frac{24.1225 - 25}{\sigma/2}\right)$$

$$= P(Z \leq -0.5685) = 0.2810$$

8.6.2

$X \sim N(\mu, 4)$ $\quad H_0 : \mu = 355$

$H_1 : \mu < 355$

$C = \{\bar{x} ; \bar{x} \leq 354.05\}$

$n = 12$

$K(\mu) = P(\bar{x} < 354.05 ; \mu)$

$$\frac{354.05 - \mu}{\sigma} < Z$$

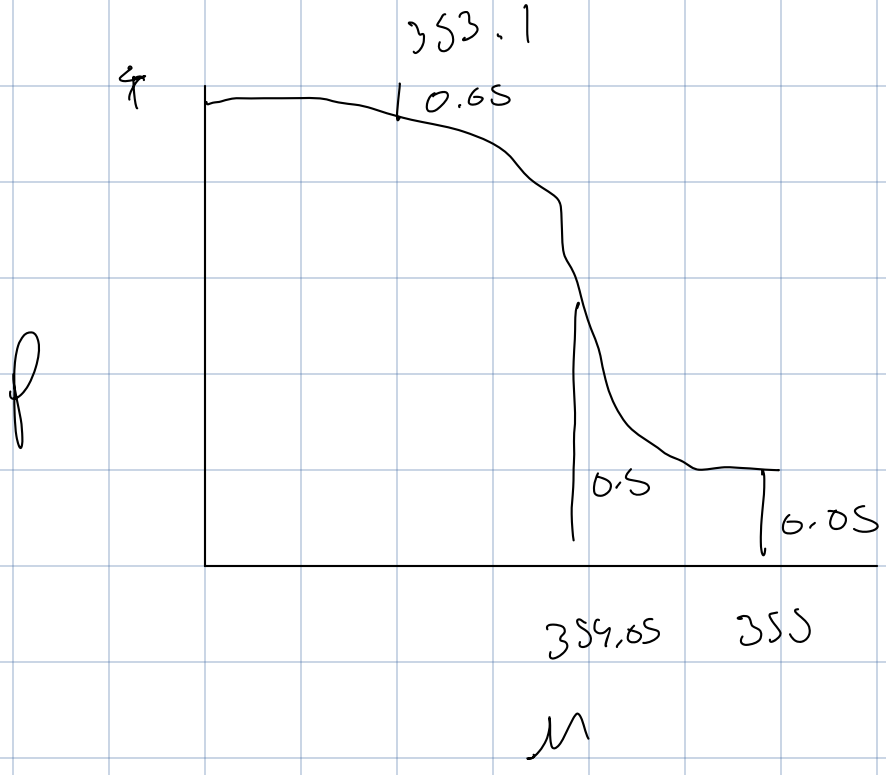
$$\frac{354.05 - \mu}{2/\sqrt{12}} < Z$$

b $\quad K(\mu) = \Phi\left(\frac{354.05 - \mu}{2/\sqrt{12}} \right)$

$K(355) \simeq 0.05$

$K(354.05) = 0.5$

$K(353.1) = 0.95$



(d) $\mu = 353.8$ falls in CI it is rejected

c $p(\bar{x} < 353.8; \mu = 355)$

$$p\left(\frac{\mu - 355}{\sqrt{\frac{5}{12}}} < \frac{353.8 - 355}{\sqrt{\frac{5}{12}}}\right)$$

$p(Z < -2.078) = 0.0192$

8.6-8

$$X \sim N(\mu, 140^2)$$

$H_1: \mu = 715$

$H_1: \mu < 715$

$$C = \{\bar{x} : \bar{x} \leq 668.84\}$$

$n = 25$

$$K(\mu) = P(\bar{X} < 668.84; \mu$$

$$= \Phi\left(\frac{668.84 - \mu}{\frac{140}{5}}\right)$$

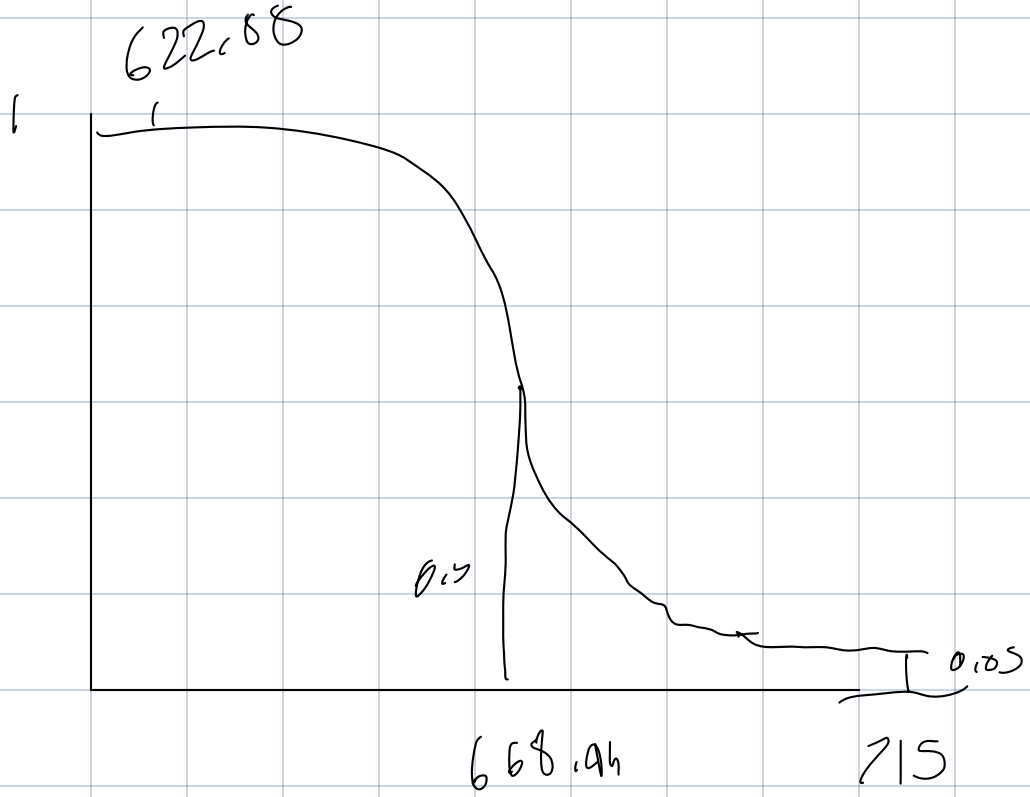
b $K(715) = \Phi(-1.645)$

$= 0.05$

c $K(668.94) = 0.5$

$K(622.88) = 0.95$

d



e $M = 667.92$ in critical

so reject H_0

f

$$P(\bar{x} < 667{,}92 ; H_1)$$

$$P\left(\frac{\bar{x} - 715}{\sigma/\sqrt{n}} < \frac{667{,}92 - 715}{(40/\sqrt{25})} ; H_0\right)$$

$$P(Z < -1{,}68)$$

$$= 0{,}05$$