# HW 2 - 131BH

ASHER CHRISTIAN 006-150-286

## 1. Exercise 2.1

*Show that a power series* $\sum_{n=0}^{\infty} c_n(x-a)^n$ *has the same radius of convergence as* $\sum_{n=0}^{\infty} c_{n+m}(x-a)^n$*, for any positive integer* $m$. let $r$ be the radius of convergence of the original series, then

$$\limsup_{n\to\infty} |c_n|^{\frac{1}{n}} = \frac{1}{r}.$$

## 2. Exercise 2.2

*Let* $(c_n)_{n=0}^{\infty} \subset \mathbb{R}$ *with at least one non null term, let* $a \in \mathbb{R}$ *and let* $\sum_{n=0}^{\infty} c_n(x-a)^n$ *have radius of convergence* $r > 0$. *Show that there exists* $\delta \in (0, r)$ *such that the sum of the series is nonzero for every real number* $x$ *such that* $0 < |x-a| < \delta$
Pick $m$ such that $c_m$ is the first non-zero term of the power series. Then

$$\sum_{n=0}^{\infty} c_n(x-a)^n = (x-a)^m \sum_{n=m}^{\infty} c_n(x-a)^{n-m}.$$

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