part 3

Asher Christian 006-150-286

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1 Discrete

X : eye color of random person
Y : hair
$N(x, y) = \#$ of people of eye color x and hair color y
$p(x, y) = P(X = x, Y = y) = \frac{N(x,y)}{N}$
$N = $ total $\#$
$N(x) = \sum_y N(x, y)$
$N(x, \cdot) = \sum_y N(x, y)$
$N(\cdot, y) = \sum_x N(x, y)$

$$p(x) = P(X = x) = \frac{N(x, \cdot)}{N} = \frac{\sum_y N(x, y)}{N} = \sum_y \frac{N(x, y)}{N} = \sum_y p(x, y).$$

$$p(y) = \sum_x p(x, y).$$

Conditional:

$$p(y|x) = P(X = x|Y = y) = \frac{N(x, y)}{N(\cdot, y)} = (\frac{\frac{N(x,y)}{N}}{\frac{N(\cdot,y)}{N}}) = \frac{p(x, y)}{p(y)}.$$

$$p(y|x) = \frac{p(x, y)}{p(x)}.$$

$$p(x, y) = p(x)p(y|x) = p(y)p(x|y).$$

$$p(x|y) = \frac{p(x, y)}{p(y)} = \frac{p(x, y)}{\sum_x p(x, y)} = \frac{p(x)p(y|x)}{\sum_x p(x)p(y|x)}.$$

$$p(y) = \sum_x p(x, y) = \sum_x p(x)p(y|x).$$

$$E(h(x)) = \sum_x h(x)p(x).$$

$$E(h(x,y)) = \sum_x \sum_y h(x,y)p(x,y) = \sum_{x,y} h(x,y)\frac{N(x,y)}{N} = \frac{1}{N}\sum_{x,y} h(x,y)N(x,y).$$

The population average of h

Continuous random variables
X = height of random person
Y = weight

$$f(x,y) = \frac{\text{\# of points in}(x, x+\Delta x)\cdot(y, y+\Delta y)/N}{\Delta x \Delta y} = \frac{P(X \in (x, x+\Delta x) \& Y \in (y, y+\Delta y))}{\Delta x \Delta y}.$$

$$= \frac{\frac{N(x,y)}{N}}{\Delta x \Delta y}.$$

$$\begin{aligned} f(x) &= \frac{\text{Proportion}}{\text{size}} \\ &= \frac{N(X,\cdot)/N}{\Delta x} \\ &= \frac{P(X \in (x, x+\Delta x))}{\Delta x} \\ &= \frac{\sum_y \frac{N(x,y)}{N}}{\Delta x} \\ &= \frac{\sum_y f(x,y)\Delta x \Delta y}{\Delta x} \\ &= \sum_y f(x,y)\Delta y \\ &= \int f(x,y)dy \end{aligned}$$

$$f(y) = \int f(x,y)dx.$$

Conditional

$$
\begin{aligned}
f(x|y) &= \frac{\text{Proportion}}{\text{Size}} \\
&= \frac{P(X \in (x, x+\Delta x) | Y \in (y, y+\Delta y)}{\Delta x} \\
&= \frac{\frac{N(x,y)}{N(\cdot,y)}}{\Delta x} \\
&= \frac{\frac{N(x,y)}{N} / \frac{N(\cdot,y)}{N}}{\Delta x} \\
&= \frac{\frac{f(x,y)\Delta x \Delta y}{f(y)\Delta y}}{\Delta x} \\
&= \frac{f(x,y)}{f(y)}
\end{aligned}
$$

$$
\begin{aligned}
f(x,y) &= f(x)f(y|x) \\
&= f(y)f(x|y)
\end{aligned}
$$

$$
\begin{aligned}
E(h(x,y)) &= \int_x \int_y h(x,y)f(x,y)dxdy \\
&= \frac{1}{N} \sum_x \sum_y h(x,y)N(x,y)
\end{aligned}
$$

$p(x,y)$ prob mass function or probability desnity function
Rule 1: Marginalization

$$
\begin{aligned}
p(x) &= \sum_y p(x,y) \\
&= \int p(x,y)dy \\
p(y) &= \sum_x p(x,y) \\
&= \int p(x,y)dx
\end{aligned}
$$

Rule 2: Conditioning

$$
\begin{aligned}
p(x|y) &= \frac{p(x,y)}{p_y(y)} \\
p(y|x) &= \frac{p(x,y)}{P_x(x)}
\end{aligned}
$$

Rule 3: Chain

$$\begin{aligned} P(x,y) &= p(x)p(y|x) \\ &= P(y)p(x|y) \end{aligned}$$

Independence X orthogonal Y

$$P(x,y) = p_x(x)p_y(y), p(x|y) = p_x(x), p(y|x) = p_y(y).$$

2 Day 2

$$E(h(x,y)) = \int_y \int_x h(x,y)f(x,y)dxdy.$$

independence:

$$p(x,y) = p_x(x)p_y(y).$$

Standardization
$\mu = E(x), \sigma^2 = Var(x), X <- \frac{x-\mu}{\sigma}$

$$X \sim N(0,1).$$

$$f_x(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

$$[Y|X = x] \sim N(\rho x, 1-\rho^2) \text{ regression model.}$$

$$f(Y|x) = \frac{1}{\sqrt{2\pi(1-\rho^2)}}e^{\frac{-(y-\rho x)^2}{2(1-\rho^2)}}.$$

$\rho < 1$ $y = \rho x$ is the regression line

1. Conditional expectation & variance
   $E(Y|X = x) = \rho x$
   $Var(Y|X = x) = 1 - \rho^2$ regression towards the mean
   $Y = \rho X + \epsilon$ and epsilon is indep of x
   $\epsilon \sim N(0, 1-\rho^2)$ ϵ is independant of X
   $Y = \rho x + \epsilon \sim N(\rho x, 1-\rho^2)$
   $E(Y) = E(\rho X + \epsilon) = \rho E(x) + E(\epsilon) = \rho$
   $Var(Y) = Var(\rho X) + Var(\epsilon) = \rho^2 Var(X) + (1-\rho^2) = 1$

2. Modern meaning of regression $E(Y|X) = f(x)$

3. $$(x,y) \sim f(x,y).$$

   $$Cov(x,y) = E((X - E(X))(Y - E(Y)).$$

   $$Cov(x,x) = E((X - E(X))^2) = Var(X).$$

Covariance changes units depending on the measure - to fix that we standardize

$$Cov(\frac{X-\mu_X}{\sigma_X}, \frac{Y-\mu_Y}{\sigma_Y}) = E((X_s)(Y_s)).$$
$$= \frac{E((X-\mu_X)(Y-\mu_Y))}{\sigma_X \sigma_Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}.$$
$$= \rho = Corr(X,Y).$$