Hw 1 due 10/4 5pm

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17.1

a) $y'' + y' - 6y = 0$

$r^2 + r - 6 = 0$

$(r+3)(r-2)$

$y_1 = ce^{-3t}$

$y_2 = de^{2t}$

$y = c_1 e^{-3t} + c_2 e^{2t}$

b) $y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0$

$(r+1)(r+1) = 0$

$r = -1$

$y_1 = ce^{-t}$

$y_2 = cte^{-t}$

$y = c_1 e^{-t} + c_2 t e^{-t}$

$y_2' = ce^{-t} - cte^{-t}$

$y_2'' = -ce^{-t} - ce^{-t} + cte^{-t} = -2ce^{-t} + cte^{-t}$

c) $y'' + 8y = 0$

$e^{i\theta} = \cos\theta + i\sin\theta$

$r^2 + 8 = 0$

$r = \pm 2\sqrt{2}\, i$

$y = c_1 e^{2\sqrt{2} i t} + c_2 e^{-2\sqrt{2} i t}$

$c_1 + c_2 \qquad c_1 - c_2$

$= c_1 (\cos 2\sqrt{2} t + i \sin 2\sqrt{2} t) + c_2 (\cos -2\sqrt{2}\, t)$

$+ i \sin -2\sqrt{2} t$

$y = c_1 \cos 2\sqrt{2} t + c_2 \sin -2\sqrt{2} t$

17.2

a) $y'' - 5y' + 6y = 0 \qquad y(1) = e^2$

$y'(1) = 3e^2$

$r^2 - 5r + 6 = 0$

$(r-2)(r-3) = 0$

$y = c_1 e^{2t} + c_2 e^{3t}$

$\cancel{e^2} = c_1 \cancel{e^2} + c_2 e^{\cancel{3}}$

$3\cancel{e^2} = 2c_1 \cancel{e^2} + 3c_2 e^{\cancel{3}}$

$1 = c_1 + c_2 e$

$3 = 2c_1 + 3c_2 e$

$1 = c_2 e \qquad c_1 = 0$

$c_2 = \frac{1}{e}$

$y = e^{3t-1}$

b

$y'' - 6y' + 5y = 0 \qquad y(0) = 3$

$\qquad\qquad\qquad\qquad\qquad y'(0) = 11$

$r^2 - 6r + 5 = 0$

$(r-5)(r-1) = 0$

$y = c_1 e^{5t} + c_2 e^{t}$

$3 = c_1 + c_2$

$11 = 5c_1 + c_2$

$8 = 4c_1 \quad \Rightarrow c_1 = 2$

$c_2 = 1$

$y = 2e^{5t} + e^{t}$

3 show that $y'' + py' + qy = 0$

$p, q > 0$

$r^2 + pr + q = 0$

$r = \dfrac{-p \pm \sqrt{p^2 - 4q}}{2}$

$y = c_1 e^{\frac{-p}{2}} e^{\frac{\sqrt{p^2-4q}}{2}} + c_2 e^{\frac{-p}{2}} e^{\frac{\sqrt{p^2-4q}}{2}}$

assuming

$p > 0$

$q > 0$

$-p + \sqrt{p^2 - 4q} < -p + \sqrt{p^2}$

$< -p + p$

< 0

$p < 0$

$q < 0$

$-p + \sqrt{p^2 - 4q} > 0$

cand work

$p < 0$

$q > 0$

$-p + \sqrt{p^2 - 4q} > -p > 0$

$p < 0$

$q > 0$

$-p + \sqrt{p^2 - 4q} > -p + \sqrt{p^2}$

$> -p + p$

> 0

then $-p+\sqrt{p^2-4q}$ is only negative if p and q are positive and then

$e^{\frac{-p+\sqrt{p^2-4q}}{2}t}$ only approaches zero when $p, q > 0$

$\frac{-p-\sqrt{p^2-4q}}{2} < \frac{-p-\sqrt{p^2-4q}}{2}$ when $\sqrt{p^2-4q}$ is real

if $\sqrt{p^2-4q}$ is imaginary

then will be in form $e^{-\frac{p}{2}x}(\cos + \sin)$

which approaches zero iff $p > 0$

17.4 $\quad y'' + py' + qy = 0$

Show that $y' = y$ is a solution

$$\frac{d}{dt}\left(y'' + py' + qy\right) = \frac{d}{dt}(0)$$

$$y''' + py'' + qy' = 0$$

Set $y = y'$ $\Rightarrow$ $y''' + p y'' + q y' = 0$
is true

18.1

(a) $y'' + 3y' - 10y = 6e^{4x}$

$(r+5)(r-2)$ $\quad y_H = c_1 e^{-5x} + c_2 e^{2x}$

$y_p = Ae^{4x}$

$4^2 A\cancel{e^{4x}} + 12A\cancel{e^{4x}} - 10A\cancel{e^{4x}} = 6\cancel{e^{4x}}$

$A(16 + 12 - 10) = 6$

$A = \frac{6}{18} = \frac{1}{3}$

$y = \frac{1}{3}e^{4x} + c_1 e^{-5x} + c_2 e^{2x}$

b)

$y'' + 4y = 3\sin x$

$r = \pm 2i$

$$y_H = C_1 \cos 2x + C_2 \sin 2x$$

$$y_p = A \sin x + B \cos x \qquad A = 1$$

$$B = 0$$

$$-A \sin x - B \cos x + 4A \sin x + 4B \cos x = 3 \sin x$$

$$y = C_1 \cos 2x + C_2 \sin 2x + \sin x$$

c)

$$y'' + 10y' + 25y = 14e^{-5x}$$

$$(r+5)(r+5)$$

$$y_H = C_1 e^{-5x} + C_2 x e^{-5x}$$

$$y_p = Ax^2 e^{-5x}$$

$$y_p' = 2Axe^{-5x} - 5Ax^2 e^{-5x}$$

$$y_p'' = 2Ae^{-5x} - 10Axe^{-5x}$$

$$-10Axe^{-5x} + 25Ax^2 e^{-5x}$$

$$2Ae^{-5x} \cancel{- 20Axe^{-5x}} \cancel{+ 25Ax^2e^{-5x}}$$
$$+ \quad \cancel{20Axe^{-5x}} \cancel{- 50Ax^2e^{-5x}}$$
$$+ \quad \cancel{25Ax^2e^{-5x}}$$

$$2Ae^{-5x} = 14e^{-5x}$$

$$A = 7$$

$$y = C_1 e^{-5x} + C_2 x e^{-5x} + 7x^2 e^{-5x}$$

g) $$y'' + y = 2\cos x$$

$$y_H = C_1 \cos x + C_2 \sin x$$

$$y_p = Ax\cos x + Bx\sin x$$

$$y' = A\cos x - Ax\sin x + B\sin x + Bx\cos x$$

$$y'' = \underbrace{-A\sin x - A\sin x}\; - Ax\cos x$$
$$\underbrace{+ B\cos x + B\cos x}\; - Bx\sin x$$

$$2B\cos x - 2A\sin x - \cancel{Ax\cos x} - \cancel{Bx\sin x}$$

$$+ \quad \cancel{Ax\cos x} + \cancel{Bx\sin x}$$

$$2B\cos x - 2A\sin x = 2\cos x$$

$$B = 1 \quad A = 0$$

$$y = c_1\cos x + c_2\sin x + x\sin x$$

4 $\quad y'' - 2y' = 12x - 10$

$$r = 2, 0$$

$$y_H = c_1 e^{2t} + c_2$$

$$y_p = Ax^2 + Bx + C$$

$$y' = 2Ax + B$$

$$y'' = 2A$$

$$A = -3$$

$$B = 2$$

$$2A - 4Ax - 2B = 12x - 10$$

$y = c_1 e^{2t} + c_2 \quad - 3x^2 + 2x + $