Homework 2

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1 Problem 1

Suppose we flip a fiar coin 100 times independently

1. What is the Probability we get 50 heads? Sample space = 2^{100} possible outcomes all equally likely. Of those $\binom{100}{50}$ is the number in which 50 are heads.
$$\frac{\binom{100}{50}}{2^{100}} = \frac{100!}{50!50!2^{100}} \approx 0.07958923738717877.$$
2. Let X be the number of heads. What is $P(40 \leq X \leq 60)$? Same sample space as before: 2^{100} However the Event space is larger it is
$$\bigcup_{40 \leq i \leq 60} \{X = i\}$$
Because each occurance is mutually exclusive (i.e. the space of runs where heads = 50 has no intersection with the space of runs where heads = 60) It is enough to add the respective probabilities
$$\sum_{i=40}^{60} P(X = i) = \sum_{i=40}^{60} \frac{100!}{(100-i)!i!2^{100}} \approx 0.964799799782$$
3. Let
$$Z_i = \begin{cases} 1 & \text{if the } i\text{-th flip is heads} \\ 0 & \text{if the } i\text{-th flip is tails} \end{cases}.$$
Express X in terms of Z_i.
$$X = \sum_{i=0}^{100} Z_i.$$

2 Problem 2

Draw a Galton board with 5 Layers

1. Draw the corresponding Pascal's triangle

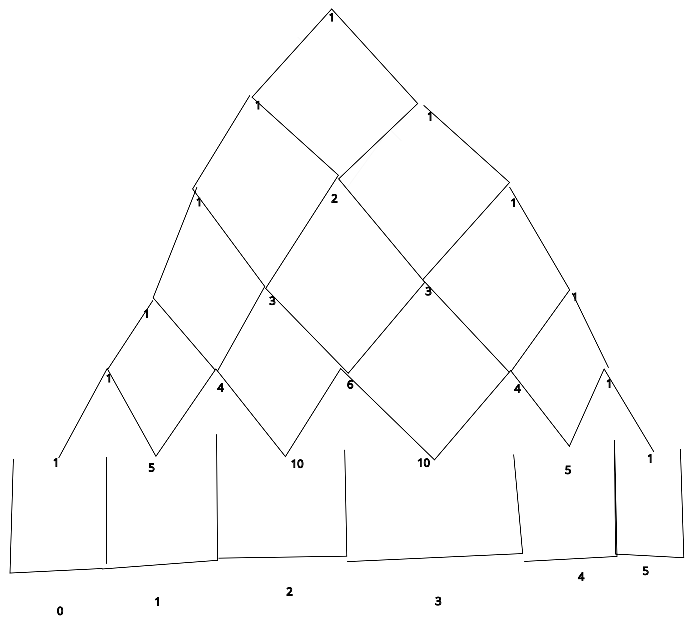

Figure 1: Galton Board With Pascal's triangle overlay

2. Suppose we label the bins by 0, 1, 2, 3, 4, 5. Calculate the probability that the ball drops into each bin

$$P(0) = \frac{1}{32}, P(1) = \frac{5}{32}, P(2) = \frac{10}{32}, P(3) = \frac{10}{32}, P(4) = \frac{5}{32}, P(5) = \frac{1}{32}, .$$

3. Suppose we drop 1 million balls. What are the proportions of balls in these bins? The proportions will correspond to the above results. Most balls will land in the second and third buckets with slightly less in the 1st and 4th and very few in the 0th and 5th buckets.

3 Problem 3

Random walk of integers with $X_0 = 0$ $X_{t+1} = X_t + \epsilon_t$

$$\epsilon_t = \begin{cases} 1 & P = \frac{1}{2} \\ -1 & P = \frac{1}{2} \end{cases}.$$

1. At time $t = 5$, what are the possible values for X_t? $X \in \{-5, -3, -1, 1, 3, 5\}$
2. What is the probability of each value in (1)?

$$P(-5) = \frac{1}{32}, P(-3) = \frac{5}{32}, P(-1) = \frac{10}{32}, P(1) = \frac{10}{32}, P(3) = \frac{5}{32}, P(5) = \frac{1}{32}, .$$

3. What is $P(X_{t+1} = j | X_t = i)$?

$$P = \begin{cases} \frac{1}{2} & |i - j| = 1 \\ 0 & |i - j| \neq 1 \end{cases}.$$

4. Interpret (2) in terms of 1 million people doing the random walk simultaneously and inde- pendently, all starting from 0. The probabilities are equivalent to that of a 5 level galton board. Thus, the various positions of the people should allign with those of the galton board with large masses of people at 1 and -1 and less at 3, -3 and fewest at 5, -5

4 Problem 4

Write R code to simulate flipping a fair coin 100 times independently. Let X be the number of heads. Repeat the experiment 1000 times. Plot the histogram of these 1000 X. Also plot the histogram of X/100.

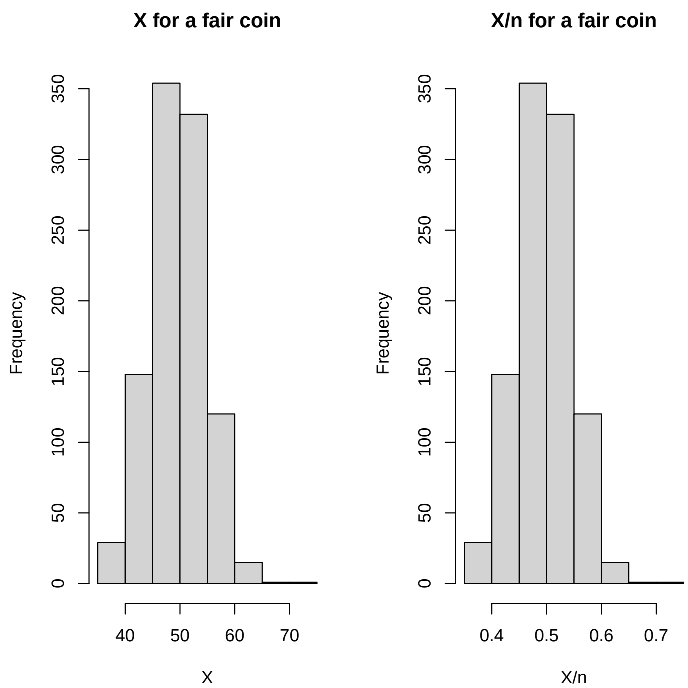

Figure 2: Flipping a fair coin

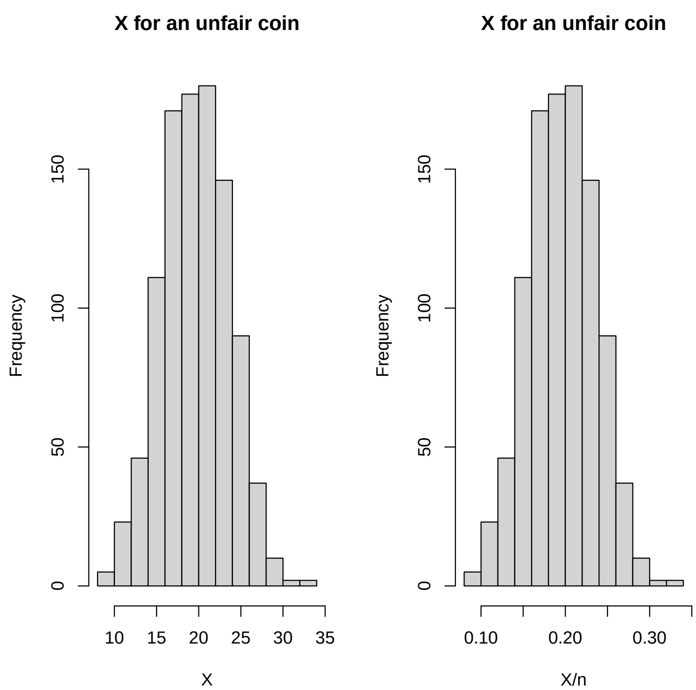

Figure 3: Flipping an unfair coin