MAE 6225 Homework 1

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1.

$$\frac{\delta\phi}{\delta x} = \frac{-\phi_{j+2} + 8\phi_{j+1} - 8\phi_{j-1} + \phi_{j-2}}{12h} :$$

$$\phi_{j-2} = \phi_j - 2h{\phi_j}' + \frac{4h^2}{2!}{\phi_j}'' - \frac{8h^3}{3!}{\phi_j}''' + \frac{16h^4}{4!}{\phi_j}'''' + o\left(h^5\right)$$

$$\phi_{j-1} = \phi_j - h{\phi_j}' + \frac{h^2}{2!}{\phi_j}'' - \frac{h^3}{3!}{\phi_j}''' + \frac{h^4}{4!}{\phi_j}'''' + o\left(h^5\right)$$

$$\phi_j = \phi_j$$

$$\phi_{j+1} = \phi_j + h{\phi_j}' + \frac{h^2}{2!}{\phi_j}'' + \frac{h^3}{3!}{\phi_j}''' + \frac{h^4}{4!}{\phi_j}'''' + o\left(h^5\right)$$

$$\phi_{j+2} = \phi_j + 2h{\phi_j}' + \frac{4h^2}{2!}{\phi_j}'' + \frac{8h^3}{3!}{\phi_j}''' + \frac{16h^4}{4!}{\phi_j}'''' + o\left(h^5\right)$$

$$\frac{\delta\phi}{\delta x} = \alpha_1\phi_{j-2} + \alpha_2\phi_{j-1} + \alpha_3\phi_j + \alpha_4\phi_{j+1} + \alpha_5\phi_{j+2}$$

$$= \left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5\right)\phi_j$$

$$+ \left(-2\alpha_1 - \alpha_2 + \alpha_4 + 2\alpha_5\right)h{\phi_j}'$$

$$+ \left(4\alpha_1 + \alpha_2 + \alpha_4 + 4\alpha_5\right)\frac{h^2}{2}{\phi_j}''$$

$$+ \left(-8\alpha_1 - \alpha_2 + \alpha_4 + 8\alpha_5\right)\frac{h^3}{6}{\phi_j}'''$$

$$+ \left(16\alpha_1 + \alpha_2 + \alpha_4 + 16\alpha_5\right)\frac{h^4}{24}{\phi_j}'''' + o\left(h^5\right)$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0$$

$$\left(-2\alpha_1 - \alpha_2 + \alpha_4 + 2\alpha_5\right) h = 1$$

$$4\alpha_1 + \alpha_2 + \alpha_4 + 4\alpha_5 = 0$$

$$-8\alpha_1 - \alpha_2 + \alpha_4 + 8\alpha_5 = 0$$

$$16\alpha_1 + \alpha_2 + \alpha_4 + 16\alpha_5 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & 1 & 0 & 1 & 2 \\ 4 & 1 & 0 & 1 & 4 \\ -8 & 1 & 0 & 1 & 8 \\ 16 & 1 & 0 & 1 & 16 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{h} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{12h} \\ -\frac{2}{3h} \\ 0 \\ \frac{2}{3h} \\ -\frac{1}{12h} \end{bmatrix}$$

$$\frac{\delta\phi}{\delta x} = \frac{-\phi_{j+2} + 8\phi_{j+1} - 8\phi_{j-1} + \phi_{j-2}}{12h} + o\left(h^4\right)$$

This scheme is **4th order accurate**.

$$\frac{\delta\phi}{\delta x} = \frac{2\phi_{j+1} + 3\phi_j - 6\phi_{j-1} + \phi_{j-2}}{6h} :$$

$$\phi_{j+1} = \phi_j + h{\phi_j}' + \frac{h^2}{2!}{\phi_j}'' + \frac{h^3}{3!}{\phi_j}''' + o\left(h^4\right)$$

$$\phi_j = \phi_j$$

$$\phi_{j-1} = \phi_j - h{\phi_j}' + \frac{h^2}{2!}{\phi_j}'' - \frac{h^3}{3!}{\phi_j}''' + o\left(h^4\right)$$

$$\phi_{j-2} = \phi_j - 2h{\phi_j}' + \frac{4h^2}{2!}{\phi_j}'' - \frac{8h^3}{3!}{\phi_j}''' + o\left(h^4\right)$$

$$\frac{\delta\phi}{\delta x} = \alpha_1\phi_{j+1} + \alpha_2\phi_j + \alpha_3\phi_{j-1} + \alpha_4\phi_{j-2}$$

$$= \left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4\right)\phi_j$$

$$+ \left(\alpha_1 - \alpha_3 - 2\alpha_4\right) h{\phi_j}'$$

$$+ \left(\alpha_1 + \alpha_3 + 4\alpha_4\right) \frac{h^2}{2}{\phi_j}''$$

$$+ \left(\alpha_1 - \alpha_3 - 8\alpha_4\right) \frac{h^3}{6}{\phi_j}''' + o\left(h^4\right)$$

$$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0$$

$$\alpha_1 - \alpha_3 - 2\alpha_4 = \frac{1}{h}$$

$$\alpha_1 + \alpha_3 + 4\alpha_4 = 0$$

$$\alpha_1 - \alpha_3 - 8\alpha_4 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ 1 & 0 & 1 & 4 \\ 1 & 0 & -1 & -8 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{h} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{3h} \\ \frac{1}{2h} \\ -\frac{1}{h} \\ \frac{1}{6h} \end{bmatrix}$$

$$\frac{\delta \phi}{\delta x} = \frac{2\phi_{j+1} + 3\phi_j - 6\phi_{j-1} + \phi_{j-2}}{6h} + o\left(h^3\right)$$

This scheme is **3rd order accurate**.

2. Construct a one-sided scheme which is $3^{\mathbf{rd}}$ **order accurate** to approximate $\frac{\partial \phi}{\partial x}$:

$$\frac{\partial \phi}{\partial x} \approx \frac{\delta \phi}{\delta x} = \alpha_1 \phi_{j+4} + \alpha_2 \phi_{j+3} + \alpha_3 \phi_{j+2} + \alpha_4 \phi_{j+1} + \alpha_5 \phi_j + o\left(h^4\right)$$

$$\phi_{j+4} = \phi_j + 4h_1 \phi_j{}' + \frac{16h_1\left(h_1+h_2\right)}{2!}\phi_j{}'' + \frac{64h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)}{3!}\phi_j{}'''$$

$$+\frac{256h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)\left(h_1+h_2+h_3+h_4\right)}{4!}\phi_j{}''''$$

$$\phi_{j+3} = \phi_j + 3h_1 \phi_j{}' + \frac{9h_1\left(h_1+h_2\right)}{2!}\phi_j{}'' + \frac{27h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)}{3!}\phi_j{}'''$$

$$+\frac{81h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)\left(h_1+h_2+h_3+h_4\right)}{4!}\phi_j{}''''$$

$$\phi_{j+2} = \phi_j + 2h_1 \phi_j{}' + \frac{4h_1\left(h_1+h_2\right)}{2!}\phi_j{}'' + \frac{8h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)}{3!}\phi_j{}'''$$

$$+\frac{16h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)\left(h_1+h_2+h_3+h_4\right)}{4!}\phi_j{}''''$$

$$\phi_{j+1} = \phi_j + h_1 \phi_j{}' + \frac{h_1\left(h_1+h_2\right)}{2!}\phi_j{}'' + \frac{h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)}{3!}\phi_j{}'''$$

$$+\frac{h_1\left(h_1+h_2\right)\left(h_1+h_2+h_3\right)\left(h_1+h_2+h_3+h_4\right)}{4!}\phi_j{}''''$$

$$\phi_j = \phi_j$$

$$\frac{\delta\phi}{\delta x} =$$