MAE 6225
Homework 2

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The aim of this assignment was to numerically approximate solutions to the Poisson equation with some stuff.

0.1 Discretization of the Poisson Equation

The Poisson equation, $\nabla^2 u = f(x,y)$, was discretized as follows:

$$u_{xx} + u_{yy} = f(x,y)$$

$$u_{xx} \approx \frac{1}{\Delta x^2}\left(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right)$$

$$u_{yy} \approx \frac{1}{\Delta y^2}\left(u_{i,j+1} - 2u_{i,j} + u_{i,j-1}\right)$$

For a uniform grid, $\Delta y = \Delta x = h$:

$$\frac{1}{h^2}\left(u_{i+1,j} - 4u_{i,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}\right) = f(x_i, y_j)$$

$$A\mathbf{u} = \mathbf{f}$$

$$\mathbf{u} = \left(u_{1,1}, ..., u_{I,1}, ..., u_{i,j}, ..., u_{1,J}, ..., u_{I,J}\right)$$

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right] - \frac{h^2}{4} f(i,j)$$

0.2 Comparison with Exact Solution, Dirichlet Boundary Conditions

Given the exact solution:

$$u_{ex}(x,y) = \sin(2\pi n x)\sin(2\pi n y)$$

0.2.1 Jacobi Method

stuff.

0.2.2 Successive Over Relaxation (SOR)

other stuff

0.3 Grid Refinement Study

This is where the grid refinement study results will go.

0.4 Neumann Boundary Conditions

Do it all again!