

A Similarity-based Cooperative Co-evolutionary Algorithm for Dynamic Interval Multi-objective Optimization Problems

Supplementary Material

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This is a supplementary material of "A Similarity-based Cooperative Co-evolutionary Algorithm for Dynamic Interval Multi-objective Optimization Problems".

I. PROOFS

A. Proofs for the observations of Definition 3

In addition, we have the following observations.

- (1) $s(a, b) = 1$ if and only if $a = b$, which is called complete.
- (2) $s(a, a) = 1$, which is called reflexive.
- (3) $s(a, b) = s(b, a)$, which is called symmetric.
- (4) If $s(a, b) = 1$ and $s(b, c) = 1$ are held, one has $s(a, c) = 1$, which is called transitive.

Proof: (1) Without loss of generality, let $\max\{l_a, l_b\} = l_a$.

(i) If $l_a \neq 0$, then

$$s(a, b) = 1 \Leftrightarrow \frac{l_{a \cap b}}{l_a} = 1 \Leftrightarrow l_{a \cap b} = l_a \Leftrightarrow a \cap b = a \Leftrightarrow a \subseteq b \Leftrightarrow a = b \text{ (for } l_a \geq l_b \text{)}.$$

(ii) If $l_a = 0, a \neq 0$ or $b \neq 0$, then

$$s(a, b) = 1 \Leftrightarrow 1 - \frac{|b-a|}{\max\{|a|, |b|, \chi(|a|+|b|)\}} = 1 \\ \Leftrightarrow |b-a| = 0 \text{ and } \max\{|a|, |b|, \chi(|a|+|b|)\} \neq 0 \\ \Leftrightarrow a = b$$

(iii) If $l_a = 0, a = b = 0$

According to the third expression of Definition 3, $s(a, b) = 1 \Leftrightarrow a = b$.

Therefore, $s(a, b) = 1$ if and only if $a = b$.

(2)(3) From Definition 3, one easily has the two observations.

(4) According to (1), $s(a, b) = 1$ and $s(b, c) = 1$, one has that $a = b$ and $b = c$, therefore $a = c$, i.e. $s(a, c) = 1$.

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B. Proof for the Theorem 1

Theorem 1 For an additively separable interval function, $f(\mathbf{x}, \mathbf{c}(t))$, its difference, denoted as $\Delta f(\mathbf{x}, \mathbf{c})$, will be as follows when x_k has a disturbance of δ .

$$\Delta f(\mathbf{x}, \mathbf{c}) = [\Delta f(\mathbf{x}, \mathbf{c}), \Delta \overline{f(\mathbf{x}, \mathbf{c})}], \text{ where} \\ \Delta f(\mathbf{x}, \mathbf{c}) = \overline{f(\cdots, x_k + \delta, \cdots, \mathbf{c})} - \overline{f(\cdots, x_k, \cdots, \mathbf{c})}, \\ \Delta \overline{f(\mathbf{x}, \mathbf{c})} = \overline{f(\cdots, x_k + \delta, \cdots, \mathbf{c})} - \overline{f(\cdots, x_k, \cdots, \mathbf{c})}. \quad (1)$$

$\forall a, [\underline{c}_p, \overline{c}_p] \neq [\underline{c}'_p, \overline{c}'_p], \delta \in R, \delta \neq 0$, if the interval similarity

$$s(\Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}_p, \overline{c}_p]}, \Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}'_p, \overline{c}'_p]}) < 1$$

is held, x_k and $c_p(c_p \in \mathbf{c})$ are said inseparable.

Proof. We prove the theorem by contradiction. To fulfill this task, it is enough to prove that its contrapositive is wrong, which states that if variable, x_k , and parameter, c_p , are separable, for any two different values of c_p , the following result will be obtained.

$$s(\Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}_p, \overline{c}_p]}, \Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}'_p, \overline{c}'_p]}) = 1$$

Since $f(\mathbf{x}, \mathbf{c})$ is additively separable, for $\forall x_k \in \mathbf{x}^i$, one has

$$\frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} = \frac{\partial \sum_{j=1}^J f_j(x^j, c^j)}{\partial x_k} = \frac{\partial f_i(x^i, c^i)}{\partial x_k}.$$

For $\forall c_p \notin \mathbf{c}^i, [\underline{c}_p, \overline{c}_p] \neq [\underline{c}'_p, \overline{c}'_p]$,

$$\frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} \Big|_{c_p=[\underline{c}_p, \overline{c}_p]} = \frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} \Big|_{c_p=[\underline{c}'_p, \overline{c}'_p]} = \frac{\partial f_i(\mathbf{x}^i, \mathbf{c}^i)}{\partial x_k},$$

$$\int_a^{a+\delta} \frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} dx_k \Big|_{c_p=[\underline{c}_p, \overline{c}_p]} = \int_a^{a+\delta} \frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} dx_k \Big|_{c_p=[\underline{c}'_p, \overline{c}'_p]},$$

$$\left[\int_a^{a+\delta} \frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} dx_k, \int_a^{a+\delta} \frac{\partial \overline{f(\mathbf{x}, \mathbf{c})}}{\partial x_k} dx_k \right] \Big|_{c_p=[\underline{c}_p, \overline{c}_p]} = \\ \left[\int_a^{a+\delta} \frac{\partial f(\mathbf{x}, \mathbf{c})}{\partial x_k} dx_k, \int_a^{a+\delta} \frac{\partial \overline{f(\mathbf{x}, \mathbf{c})}}{\partial x_k} dx_k \right] \Big|_{c_p=[\underline{c}'_p, \overline{c}'_p]}.$$

namely,

$$\Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}_p, \overline{c}_p]} = \Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}'_p, \overline{c}'_p]}.$$

One has

$$s(\Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}_p, \overline{c}_p]}, \Delta f(\mathbf{x}, \mathbf{c}) \Big|_{x_k=a, c_p=[\underline{c}'_p, \overline{c}'_p]}) = 1.$$

■

II. BENCHMARK OPTIMIZATION PROBLEMS

Table I lists the benchmark optimization problems along with their characteristics. In this table, $\text{mid}(c_i(t))$ means the midpoint of $c_i(t)$, and the last column shows the separability between each decision variable and interval parameters as well as the change type of $\text{PS}(t)$ and $\text{PF}(t)$, i.e. Type I-Type IV [1]. Among them, Type I means that $\text{PS}(t)$ changes, whereas $\text{PF}(t)$ remains unchanged. For Type II, both $\text{PS}(t)$ and $\text{PF}(t)$ change. Type III implies that $\text{PF}(t)$ changes, whereas $\text{PS}(t)$ remains unchanged. Regarding the last type, neither $\text{PF}(t)$ nor $\text{PS}(t)$ changes.

The decision variables of ZDT3_{DI} , FDA1_{DI} , and FDA2_{DI} are inseparable with interval parameters, whereas the rest, FDA4_{DI} , FDA5_{DI} , and DSW1_{DI} - DSW3_{DI} , are separable. ZDT3_{DI} has a discontinuous $\text{PF}(t)$, which can test the capability of different algorithms in maintaining the population diversity on multiple Pareto fronts. The $\text{PF}(t)$ s of FDA1_{DI} and FDA2_{DI} are convex, and the $\text{PF}(t)$ of FDA2_{DI} changes over time. Therefore, FDA2_{DI} can evaluate the capability of different algorithms in tracking optimal solutions. For FDA4_{DI} and FDA5_{DI} , they have non-convex $\text{PF}(t)$ s with Type I and Type II, respectively. Accordingly, these two optimization problems are difficult to solve. The feasible regions of DSW1_{DI} - DSW3_{DI} are large due to each decision variable being in the range of $[-50, 50]$, which raises the difficulty in handling them. Especially for DSW3_{DI} , it belongs to Type II, which is beneficial to detecting the capability of different algorithms in convergence. In addition, these optimization problems have either 20- or 31-dimensional decision variables.

Fig. 1 depicts the true $\text{PF}(t)$ s on different environments of these benchmark problems. In these figures, rectangular means interval objectives, and red lines refer to the centers of these interval objectives. Taking FDA2_{DI} in Fig. 1c as an example, its true $\text{PF}(t)$ is that $f_1 = c_1(t)x_1, f_2 = 1 - (f_1)^{(H_i(t)+c_i(t) \cdot |X_{III}| \cdot (1+H_i(t))^2)^{-1}}$, which is periodic and affected by interval parameters, $c_i(t), i = 1, 2, \dots, n$. The labels (i.e., $t=1, 3, 5 \dots$) imply that $\text{PF}(t)$ remains unchanged for the environment time instance of 1, 3, 5, and so on.

III. EXPERIMENTS

A. Time consumption for each comparative algorithm

Table II lists the average time consumption for each algorithm on the eight benchmark problems, where the unit is second and the boldface ones are the best among these algorithms. From Table II, each of the three IP-MOEAs with cooperative co-evolution has significantly longer time consumption than the others. The reason lies in that cooperative co-evolution is generally time-consuming. However, all the three IP-MOEAs almost achieve the best performance in terms of AH and AI .

B. Effectiveness of the archive set

In this subsection, we discuss the influences of the archive size on the proposed method. In the experiments, there are

five levels of the archive size, 25, 50, 75, 100, and 150, on the proposed method when handling FDA2_{DI} for six environmental changes, i.e., $t=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$. The sub-population size and the maximal number of evolutionary generations on each environment are set to 50 and 20, respectively. Besides, the other parameters have the same setting as Subsection IV.B.

Fig. 2 shows the changes of \bar{H} with respect to the number of generations on different environments. We have the following observations from Fig. 2:

(1) At the first five generations, the \bar{H} value of each algorithm is small, and rapidly increases, suggesting that each algorithm has a capability in rapidly converging and maintaining the diversity when seeking the optimal solutions in the beginning of a new evolutionary process.

(2) However, after ten generations, the \bar{H} value shows a gradual reduction with the number of generations when the archive size is 25. The reason lies in that the archive size is too small to provide more historical information for a new environment. The \bar{H} values have a slight difference between each pair of the other archive sizes, and the larger the archive size, the better the performance of the corresponding algorithm. On the whole, the proposed algorithm obtains the largest \bar{H} value when the archive size is 100 or 150, with however a limited promotion in performance. To balance the performance and complexity of the proposed algorithm, we suggest that the archive size is set to 100."

REFERENCES

- [1] M. Farina, K. Deb, and P. Amato, "Dynamic multiobjective optimization problems: test cases, approximations, and applications," *IEEE Transactions on Evolutionary Computation*, vol. 8, no. 5, pp. 425–442, 2004.

TABLE I
BENCHMARK OPTIMIZATION PROBLEMS

Problem	Definition	Characteristic
ZDT3 _{DI}	$f_1(X_I, t) = c_1 x_1, f_2(X_{II}, t) = g(2 - \sqrt{(f_1/g)} - (f_1/g) \sin(10\pi f_1))$ $g(X_{II}, t) = 1 + 9 \sum_{x_i \in X_{II}} c_i(t)(x_i - G_i(t))^2 / (m - 1)$ $c_1 = [0.9, 1], c_i(t) = [0.45 \sin(0.5i\pi t) , 0.5 + 0.45 \sin(0.5i\pi t)],$ $G_i(t) = \text{mid}(c_i(t)), i = 2, \dots, m; t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $m = 20, X = (X_I, X_{II}), X_I = (x_1), X_{II} = (x_2, \dots, x_m).$ $0 \leq x_i \leq 1, i = 1, 2, \dots, m.$	Type I Disconnected, inseparable PS(t) : $x_1 \in [0, 1], x_i = G(t), x_i \in X_{II};$ PF(t) : $f_1 = c_1 x_1,$ $f_2 = 2 - \sqrt{f_1} - f_1 \sin(10\pi f_1).$
FDA1 _{DI}	$f_1(X_I, t) = c_1 x_1, f_2(X, t) = g \cdot h$ $g(X_{II}, t) = 1 + \sum_{x_i \in X_{II}} c_i(t)(x_i - G_i(t))^2, h(f_1, g, t) = 1 - \sqrt{\frac{f_1}{g}}$ $c_1 = [0.9, 1], c_i(t) = [0.45 \sin(0.5i\pi t) , 0.5 + 0.45 \sin(0.5i\pi t)],$ $G_i(t) = \text{mid}(c_i(t)), i = 2, \dots, m; t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ $X = (X_I, X_{II}), X_I = (x_1) \in [0, 1]; X_{II} = (x_2, \dots, x_{20}) \in [-1, 1].$	Type I Convex, inseparable PS(t) : $x_1 \in [0, 1]; x_i = G(t), x_i \in X_{II};$ PF(t) : $f_1 = c_1 x_1, f_2 = 1 - \sqrt{f_1}.$
FDA2 _{DI}	$f_1(X_I, t) = c_1 x_1, f_2(X, t) = g \cdot h$ $g(X_{II}, t) = 1 + \sum_{x_i \in X_{II}} x_i^2$ $h(X_{III}, f_1, g) = 1 - \left(\frac{f_1}{g} \right)^{\left(\sum_{x_i \in X_{III}} (x_i - H_i(t))^2 \right)^{-1}}$ $c_1 = [0.9, 1], c_i(t) = [0.45 \sin(0.5i\pi t) , 0.5 + 0.45 \sin(0.5i\pi t)],$ $H_i(t) = \text{mid}(c_i(t)), t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ where : $X = (X_I, X_{II}, X_{III}), X_I = (x_1) \in [0, 1];$ $X_{II} = (x_2, \dots, x_{16}) \in [-1, 1]; X_{III} = (x_{17}, \dots, x_{31}) \in [-1, 1].$	Type III Non - convex, inseparable PS(t) : $x_1 \in [0, 1]; x_i = 0, x_i \in X_{II};$ $x_j = -1, x_j \in X_{III};$ PF(t) : $f_1 = c_1 x_1,$ $f_2 = 1 - (f_1)^{(H_i(t) + c_i(t) \cdot X_{III} \cdot (1 + H_i(t))^2)^{-1}}.$
FDA4 _{DI}	$f_1(X, t) = (1 + g_1 + g_2) \cos(0.5\pi c_1 X_I)$ $f_2(X, t) = (1 + g_1 + g_2) \sin(0.5\pi c_1 X_I)$ $g_1(x) = \sum_{x \in X_{II}} c_i(t)(x_i - 0.5)^2, g_2(x) = \sum_{x \in X_{III}} c_i(t)(x_i - G(t))^2$ $c_1 = [0.9, 1], c_i(t) = 1, i = 2, \dots, 10;$ $c_i(t) = [0.45 \sin(0.5i\pi t) , 0.5 + 0.45 \sin(0.5i\pi t)],$ $G_i(t) = \text{mid}(c_i(t)), i = 11, \dots, 20; t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor$ where : $x_i \in [0, 1], i = 1, 2, \dots, 20;$ $X_I = (x_1), X_{II} = (x_2, \dots, x_{10}), X_{III} = (x_{11}, \dots, x_{20}).$	Type I Concave, separable PS(t) : $x_1 \in [0, 1]; x_i = 0.5, x_i \in X_{II};$ $x_i = G_i(t), x_i \in X_{III};$ PF(t) : $f_1(x) = \cos(0.5\pi c_1 x_1),$ $f_2(x) = \sin(0.5\pi c_1 x_1).$
FDA5 _{DI}	$f_1(x) = (1 + g_1) \cos(0.5\pi c_1 x_1) + (1 + g_2) \cos(0.5\pi c_1 y_1)$ $f_2(x) = (1 + g_1) \sin(0.5\pi c_1 x_1) + (1 + g_2) \sin(0.5\pi c_1 y_1)$ $g_1(x) = \sum_{x \in X_{II}} c_i(t)(x_i - 0.5)^2, g_2(x) = G(t) + \sum_{x \in X_{III}} c_i(t)(x_i - G_i(t))^2,$ $c_1 = [0.9, 1], c_i(t) = 1, i = 2, \dots, 16;$ $c_i(t) = [0.45 \sin(0.5i\pi t) , 0.5 + 0.45 \sin(0.5i\pi t)],$ $G(t) = \sin(0.5\pi t) , G_i(t) = \text{mid}(c_i(t)), i = 17, \dots, 31;$ $y_1 = x_1^{F(t)}, F(t) = 1 + 100 \sin^4(0.5\pi t), t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor,$ where : $x_i \in [0, 1],$ $i = 1, 2, \dots, n; X_I = (x_1), X_{II} = (x_2, \dots, x_{16}), X_{III} = (x_{17}, \dots, x_{31}).$	Type II Non - concave, separable PS(t) : $x_1 \in [0, 1]; x_i = 0.5, x_i \in X_{II};$ $x_i = G_i(t), x_i \in X_{III};$ PF(t) : $f_1(x) = \cos(0.5\pi c_1 x_1) +$ $(1 + G(t)) \cos(0.5\pi c_1 y_1),$ $f_2(x) = \sin(0.5\pi c_1 x_1) +$ $(1 + G(t)) \sin(0.5\pi c_1 y_1).$
DSW1 _{DI}	$f_1(X, t) = (a_{11} c_1 x_1 + a_{12} c_1 x_1 - b_1 G_i(t))^2 + \sum_{x_i \in X_{II}} x_i^2$ $+ c_i(t) \sum_{x_i \in X_{III}} (x_i - G_i(t))^2$ $f_2(X, t) = (a_{21} c_1 x_1 + a_{22} c_1 x_1 - b_2 G_i(t) - 2)^2 + \sum_{x_i \in X_{II}} x_i^2$ $+ c_i(t) \sum_{x_i \in X_{III}} (x_i - G_i(t))^2$ $a_{11} = 1, a_{12} = 0, a_{21} = 1, a_{22} = 0, b_1 = 1, b_2 = 1; c_1 = [0.9, 1],$ $c_i(t) = [0.45 \sin(0.5i\pi t) , 0.5 + 0.45 \sin(0.5i\pi t)], G_i(t) = \text{mid}(c_i(t))$ $x_i \in [-50, 50]^{20}, X_I = (x_1), X_{II} = (x_2, \dots, x_{10}), X_{III} = (x_{11}, \dots, x_{20}).$	Type I Convex, separable PS(t) : $x_1 \in [G(t) - 2, G(t) + 2];$ $x_i = 0, x_i \in X_{II};$ $x_i = G(t), x_i \in X_{III};$ PF(t) : $f_1 = (c_1 x_1 - G(t))^2,$ $f_2 = (\sqrt{f_1} - 2)^2.$
DSW2 _{DI}	$f_1(X, t), f_2(X, t), c_1, c_i(t),$ and $G(t)$ are the same as DSW _{DI} 1; $a_{11} = 0, a_{12} = 1, a_{21} = 0, a_{22} = 1, b_1 = 1, b_2 = 1$ $X = (X_I, X_{II}), X_I = (x_1), X_{II} = (x_2, \dots, x_{20}), x_i \in [-50, 50]^{20}.$	Type I, convex, separable PS(t) : $x_1 \in [-G(t) - 2, -G(t) + 2] \cup$ $[G(t) - 2, G(t) + 2];$ $x_i = 0, x_i \in X_{II}; x_i = G(t), x_i \in X_{III};$ PF(t) : $f_1 = (c_1 x_1 - G(t))^2,$ $f_2 = (\sqrt{f_1} - 2)^2.$
DSW3 _{DI}	$f_1(X, t), f_2(X, t), c_1, c_i(t),$ and $G(t)$ are the same as DSW _{DI} 1; $a_{11} = 0, a_{12} = 1, a_{21} = 0, a_{22} = 1, b_1 = 0, b_2 = 1$ $X = (X_I, X_{II}), X_I = (x_1), X_{II} = (x_2, \dots, x_{20}), x_i \in [-50, 50]^{20}.$	Type II, Convex, separable PS(t) : $x_1 \in [-G(t) - 2, G(t) + 2]; x_i = 0,$ $x_i \in X_{II}; x_i = G(t), x_i \in X_{III};$ PF(t) : $f_1 = (c_1 x_1)^2,$ $f_2 = (\sqrt{f_1} - G(t) - 2)^2.$

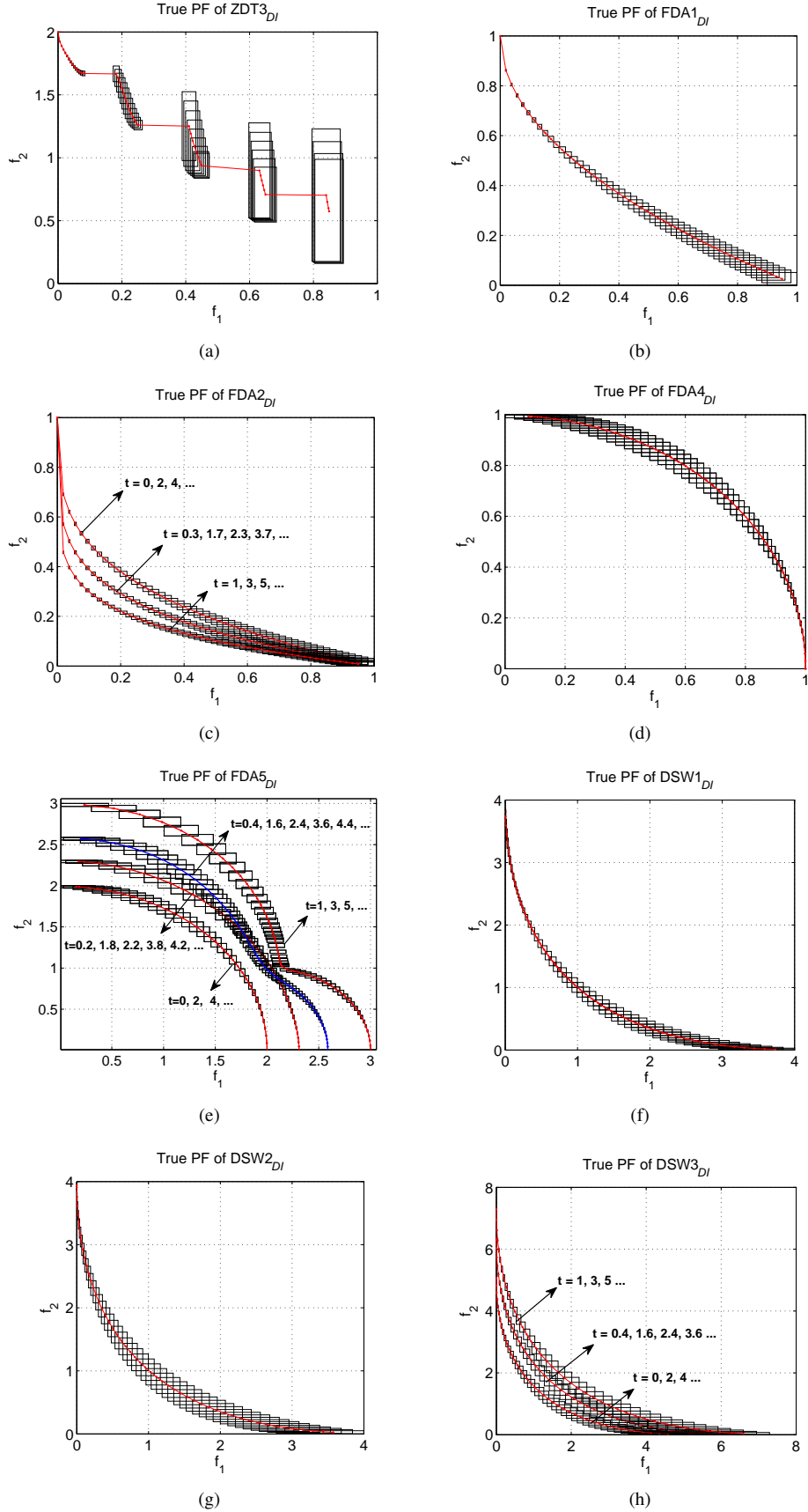
Fig. 1. The true PF(t)s of the benchmark optimization problems

TABLE II
TIME CONSUMPTION(S) OF DIFFERENT ALGORITHMS ON EIGHT BENCHMARK PROBLEMS

Problem	IP-MOEA	D-IP-MOEA-A	D-IP-MOEA-B	CC-IP-MOEA-A	CC-IP-MOEA-B	CC-IP-MOEA-IS
ZDT3 _{DI}	1.45* 0.0016	1.50* 0.0047	1.50* 0.0041	2.45 0.0136	2.42 0.013	2.41 0.0111
FDA1 _{DI}	1.12* 0.0011	1.15* 0.0028	1.19* 0.0031	1.44 0.0089	1.43 0.0089	1.44 0.0077
FDA2 _{DI}	1.03* 0.0009	1.07* 0.0016	1.13* 0.0024	1.46 0.0008	1.44 0.0005	1.48 0.0004
FDA4 _{DI}	1.14* 0.0012	1.18* 0.003	1.18* 0.0031	1.93 0.0035	1.89 0.0034	1.91 0.0004
FDA5 _{DI}	1.16* 0.0013	1.20* 0.0019	1.25* 0.0023	2.00 0.0021	1.97 0.0019	2.00 0.0006
DSW1 _{DI}	1.23* 0.0036	1.35* 0.0063	1.34* 0.0058	1.57 0.0051	1.60 0.0041	1.59 0.0042
DSW2 _{DI}	1.12* 0.0061	1.06* 0.0023	1.06* 0.0022	1.46 0.0047	1.50 0.0044	1.50 0.0038
DSW3 _{DI}	1.04* 0.0023	1.05* 0.0022	1.05* 0.0023	1.42 0.0045	1.44 0.0038	1.54 0.0059

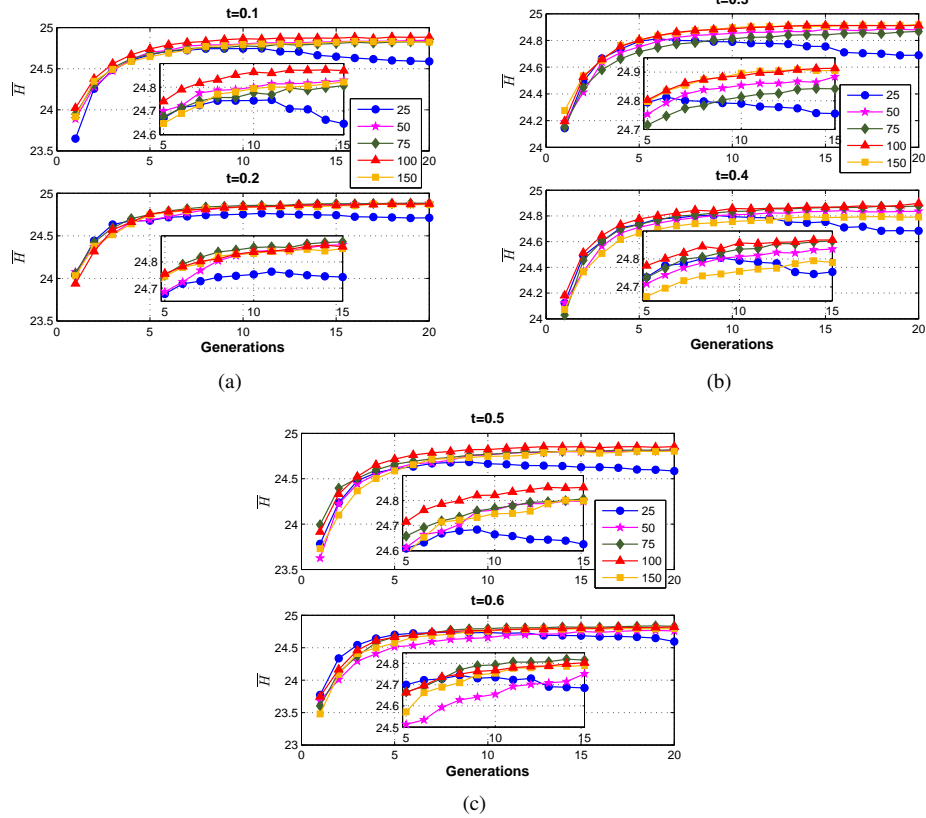


Fig. 2. The \bar{H} of different levels of archive size on FDA2_{DI}