

# Solving sudoku's with R and C++

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# What is a sudoku

The goal of a sudoku puzzle is to fill a  $9 \times 9$  grid of cells with numbers. At the start the grid will be partially filled and the solver must fill all the empty cells according to a set of simple rules:

- ▶ Cells may only be filled by the numbers 1 to 9.
- ▶ A number may appear only once in each row.
- ▶ A number may appear only once in each column.
- ▶ The grid is divided in a  $3 \times 3$  grid of  $3 \times 3$  cells each. In each smaller grid a number may only appear once.

# Solving Sudoku's in R and C++

In this presentation two approaches are compared to solve sudoku's:

- ▶ R
- ▶ Hybrid of R and C++

## Representing a sudoku

A sudoku will be represented by a space separated numbers in 9 rows and 9 columns. For example (taken from Wikipedia):

```
sudokuTxt <- "  
1 0 0 0 0 0 0 0 6  
0 0 6 0 2 0 7 0 0  
7 8 9 4 5 0 1 0 3  
  
0 0 0 8 0 7 0 0 4  
0 0 0 0 3 0 0 0 0  
0 9 0 0 0 4 2 0 1  
  
3 1 2 9 7 0 0 4 0  
0 4 0 0 1 2 0 7 8  
9 0 8 0 0 0 0 0 0"
```

0 signifies an empty cell.

## Loading the sudoku

Reading in the sudoku and storing it in a matrix using R is simple:

```
sudoku <- as.matrix(  
  read.table(text = sudokuTxt,  
             col.names = letters[1:9]))
```

# General solution strategy

A simple idea to algorithm to solve sudoku's is to pick an empty cell fill it with a valid choice and then repeat this process.

- ▶ `findChoices()` is a function that finds possible values for empty cells given the current grid of numbers.
- ▶ A contradiction is found when for a partial solution a cell has *no* valid choices. In this case, this partial solution can never be made a correct solution by filling in further cells.

## In R code

```
solve <- function(partialSolution, choicesFUN) {  
  # Eliminate impossible values, give some suggestions  
  # and flag contradictions.  
  c(partialSolution, suggestions, contradiction) %<-%  
    eliminate(partialSolution, choicesFUN)  
  # If dead end FALSE to trace back, if finished TRUE.  
  if (contradiction) return(list(FALSE, NULL))  
  if (all(partialSolution %in% 1:9))  
    return(list(TRUE, partialSolution))  
  # Branching, exit when the solution is found.  
  for (suggestion in suggestions) {  
    c(result, solution) %<-% solve(suggestion,  
                                   choicesFUN)  
    if (result) return(list(result, solution))  
  }  
  list(FALSE, NULL)  
}
```

## R Implementation - `eliminate()`

The function `eliminate()` checks for every empty cell which values are possible, returns a contradiction when no possible values for a cell are found and keeps track of the cell with the least possibilities which can be used as a pivot. If only one value is possible it is filled in directly.



## R Implementation - eliminate()

```
eliminate <- function(grid, choicesFUN) {  
  suggestions <- 0:9  
  for (i in 1:nrow(grid)) { for (j in 1:ncol(grid)) {  
    if (grid[i, j] == 0L) {  
      choices <- choicesFUN(grid, i, j)  
      if (length(choices) == 0L) {  
        return(list(NULL, NULL, TRUE))  
      } else if (length(choices) == 1L) {  
        grid[i, j] <- choices  
        return(list(grid, list(grid), FALSE))  
      } else  
        suggestions <- updateSuggestions(  
          choices, grid, i, j, suggestions)  
    }  
  }  
  list(grid, suggestions, FALSE)  
}
```

## R Implementation - Helpers

```
# Find all the choices allowed by the rules.
findChoices <- function(grid, i, j) {
  1:9 %>% setdiff(grid[i, ]) %>%
    setdiff(grid[ , j]) %>%
    setdiff(grid[i - (i - 1) %% 3L + 0:2,
              j - (j - 1) %% 3L + 0:2])
}

# Create a list of grids with suggested next moves.
updateSuggestions <- function(choices, grid, i, j,
                              lastBest) {
  if (length(choices) < length(lastBest))
    lapply(choices, function(choice) {
      grid[i, j] <- choice; grid
    })
  else
    lastBest
}
```

# Solving sudoku's

With this code solving a sudoku is simple:

```
solution <- solve(sudoku, findChoices)
if (!solution[[1]]) { cat('Solution not found\n')
} else { print(as.data.frame(solution[[2]])) }
```

```
##   a b c d e f g h i
## 1 1 2 3 7 8 9 4 5 6
## 2 4 5 6 1 2 3 7 8 9
## 3 7 8 9 4 5 6 1 2 3
## 4 2 3 1 8 9 7 5 6 4
## 5 5 6 4 2 3 1 8 9 7
## 6 8 9 7 5 6 4 2 3 1
## 7 3 1 2 9 7 8 6 4 5
## 8 6 4 5 3 1 2 9 7 8
## 9 9 7 8 6 4 5 3 1 2
```

or as a picture

1	2	3	7	8	9	4	5	6
4	5	6	1	2	3	7	8	9
7	8	9	4	5	6	1	2	3
2	3	1	8	9	7	5	6	4
5	6	4	2	3	1	8	9	7
8	9	7	5	6	4	2	3	1
3	1	2	9	7	8	6	4	5
6	4	5	3	1	2	9	7	8
9	7	8	6	4	5	3	1	2

# Solving harder sudoku's

This has been called “World’s hardest sudoku” (see <https://puzzling.stackexchange.com/a/389/7698>) and we will solve it.

```
sudokuTxt <- "  
8 0 0 0 0 0 0 0 0  
0 0 3 6 0 0 0 0 0  
0 7 0 0 9 0 2 0 0  
0 5 0 0 0 7 0 0 0  
0 0 0 0 4 5 7 0 0  
0 0 0 1 0 0 0 3 0  
0 0 1 0 0 0 0 6 8  
0 0 8 5 0 0 0 1 0  
0 9 0 0 0 0 4 0 0"  
sudoku <- as.matrix(  
  read.table(text = sudokuTxt,  
             col.names = letters[1:9]))
```

# Solution to the “World’s hardest sudoku”

8	1	2	7	5	3	6	4	9
9	4	3	6	8	2	1	7	5
6	7	5	4	9	1	2	8	3
1	5	4	2	3	7	8	9	6
3	6	9	8	4	5	7	2	1
2	8	7	1	6	9	5	3	4
5	2	1	9	7	4	3	6	8
4	3	8	5	2	6	9	1	7
7	9	6	3	1	8	4	5	2

## Some benchmarking

Benchmarking using `profvis` is not so easy due to recursion and the number of function calls in `zeallot` but `Rprof` in base works fine:

```
Rprof(tmp <- tempfile())  
solution <- solve(sudoku, findChoices)  
Rprof()  
summaryRprof(tmp)
```

and produces lots of output.

The operators `%<-%` and `%>%` are called a lot and bring a bit of overhead and we don't really need them. The code without is shown on the next slides.

## In R code (2), removing %<-%

```
solve2 <- function(partialSolution, choicesFUN) {  
  # Eliminate impossible values, give some suggestions  
  # and flag contradictions.  
  elStep <- eliminate(partialSolution, choicesFUN)  
  # If dead end FALSE to trace back, if finished TRUE.  
  if (elStep[[3]]) return(list(res = FALSE,  
                               sol = NULL))  
  if (all(elStep[[1]] %in% 1:9))  
    return(list(res = TRUE, sol = elStep[[1]]))  
  # Branching, exit when the solution is found.  
  for (suggestion in elStep[[2]]) {  
    ans <- solve2(suggestion, choicesFUN)  
    if (ans$res) return(ans)  
  }  
  list(res = FALSE, sol = NULL)  
}
```



## In R code (2), removing %>%

The profiling also shows that the use of the pipe operator slows the code down. It is not necessary either so we sacrifice some readability to get:

```
findChoices2 <- function(grid, i, j) {  
  setdiff(setdiff(setdiff(1:9,  
                        grid[i, ]),  
                  grid[ , j])),  
  grid[i - (i - 1) %% 3L + 0:2,  
        j - (j - 1) %% 3L + 0:2])  
}
```

# OK, enough talk

show me a benchmark:

```
options(digits = 2)
microbenchmark::microbenchmark(
  pipe = solve(sudoku, findChoices),
  `no operators` = solve2(sudoku, findChoices2),
  times = 5)
```

## Unit: seconds

##	expr	min	lq	mean	median	uq	max	neval
##	pipe	37	37.1	37.2	37.2	37.2	37.2	5
##	no operators	5	5.1	5.1	5.1	5.1	5.2	5

This helps but can we do better?

# Let's do another profile

With the zeallot and pipe overhead gone, profiling works again. It shows the following:

```
## ---- echo = TRUE-----
solve <- function(partialSolution, choicesFUN) {

  # Eliminate impossible values, give some suggestions
  # and flag contradictions.
  elStep <- eliminate(partialSolution, choicesFUN)
  # If dead end FALSE to trace back, if finished TRUE.
  if (elStep[[3]]) return(list(res = FALSE,
                              sol = NULL))
  if (all(elStep[[1]] %in% 1:9))
    return(list(res = TRUE, sol = elStep[[1]]))
  # Branching, exit when the solution is found.
  for (suggestion in elStep[[2]]) {
    ans <- solve(suggestion, choicesFUN)
    if (ans$res) return(ans)
  }
  list(res = FALSE, sol = NULL)
}

## ---- echo = TRUE-----
eliminate <- function(grid, choicesFUN) {
  suggestions <- 0:9
  for (i in 1:nrow(grid)) { for (j in 1:ncol(grid)) {
    if (grid[i, j] == 0L) {
      choices <- choicesFUN(grid, i, j)
      if (length(choices) == 0L) {
        return(list(NULL, NULL, TRUE))
      } else if (length(choices) == 1L) {
        grid[i, j] <- choices
        return(list(grid, list(grid), FALSE))
      } else
        suggestions <- updateSuggestions(
          choices, grid, i, j, suggestions)
    }
  }}
  list(grid, suggestions, FALSE)
}
```

-419.4	419.7	5180
-9446.1	9690.9	5180
-5.2	9.1	100
-35.5	31.0	390
-351.9	360.7	4400
-27.0	20.0	270
	1.0	10

## What now?

It seems hard to improve `findChoices2()` further but we can use C++. It is easier than you might expect.

## C++ code

```
#include <Rcpp.h>
using namespace Rcpp;
IntegerMatrix subGrid(IntegerMatrix& x, int i, int j) {
    i -= i % 3; j -= j % 3;
    return x(Range(i, i + 2), Range(j, j + 2));
}
// [[Rcpp::export]]
IntegerVector findChoicesCpp(IntegerMatrix& x,
                             int i, int j) {
    IntegerVector candidates(9);
    std::iota(candidates.begin(), candidates.end(), 1);
    // C++ is zero-indexed.
    return setdiff(setdiff(setdiff(candidates,
                                   IntegerVector(x(i - 1, _))),
                                   IntegerVector(x(_, j - 1))),
                  subGrid(x, i - 1, j - 1));
}
```

## For comparison

```
# Find all the choices allowed by the rules.
findChoices2 <- function(grid, i, j) {
  setdiff(setdiff(setdiff(1:9,
                        grid[i, ]),
                        grid[ , j]),
    grid[i - (i - 1) %% 3L + 0:2,
      j - (j - 1) %% 3L + 0:2])
}
```

## New benchmark

```
options(digits = 2)
microbenchmark::microbenchmark(
  `no operators` = solve2(sudoku, findChoices2),
  `C++` = solve2(sudoku, findChoicesCpp),
  times = 10)
```

## Unit: milliseconds

##	expr	min	lq	mean	median	uq	max	neval
##	no operators	4997	5010	5021	5025	5031	5037	10
##	C++	681	692	711	708	733	744	10

# This is great, how do I get started

Simply

```
install.packages('Rcpp')
```

and then start reading

```
vignette('Rcpp-introduction')
```



## CRAN shows a lot packages

[illegible]

but I believe a lot of Rcpp never makes it to CRAN.

# Questions?

Any questions, remarks or observations?

# On GitHub

You can find this presentation and related materials on

<https://github.com/bobjansen/RcppSudoku>