Assignment 1

Bryant Oblad A01770171

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Question 1

- Please interpret the quotation of Bertrand Russell given below and write an explanation you would use to a non-Mathematically indoctrinated friend (who is curious and patient enough to listen to you).

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To choose one sock from each of innitely many pairs of socks
requires the Axiom of Choice, but for shoes the Axiom is not
needed." { Bertrand Russell
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answer: Suppose you have infinite socks and an infinite pair of shoes. For each set of shoes you can tell a difference between the left and the right shoe. But in a theoretical world there is absolutely no difference between a set of socks, therefore each set is exactly the same. The definition of Axiom of Choice states 'Every set with no empty elements has a choice function.' What this means in terms of infinate shoes and socks (the sets) means that if you must choose a sock that is completely identical to another, there is no logical reason to choose one sock over another, therefore a outside source of decision making (the function) must take place. This is the axium. Because in a real world we must choose a sock, an axium (or assumption) must take place.

Question 2

- Define the set E via the equation E = {x ∈ $\mathbb{Z}$: x is even}; dene the set F via F = {y ∈ $\mathbb{Z}$: y = a + b, where a and b are odd}. Please prove that E = F.

Claim: The set E = {x ∈ $\mathbb{Z}$: x is even} is equal to the set F = {y ∈ $\mathbb{Z}$: y = a + b, where a and b are odd} which implies E = F.

Proof: We know that the definition of an odd number lets say S_{odd} = 2k_0 + 1 where k_0 ∈ $\mathbb{Z}$. Also the defintion of an even number lets say S_{even} = 2k_1 + 1 where k_1 ∈ $\mathbb{Z}$. With these definitions we can restate the problem to be E = {x, k_0 ∈ $\mathbb{Z}$: x = 2k_0} is equal to F = {y, k_2, k_3 ∈ $\mathbb{Z}$: y = a + b where a = 2k_2 + 1, b = 2k_3 + 1}. y = 2(k_2 + k_3 + 1) let k_4 = k_2 + k_3 + 1 so y = 2k_4 which by definition is even, therefore F = E.

Question 3

- Suppose x is a positive integer with n digits, say x = d1d2d3 · · · dn. In other words, di ∈ {0, 1, 2, . . . , 9} for 1 ≤ i ≤ n, but d1 $\neq$ 0. Please prove the following. Recall that, for a, b ∈ Z, a is a divisor of b if b = ak, for some k ∈ Z.

(a) If 9 is a divisor of d1 + d2 + · · · + dn, then 9 is a divisor of x.

(b) If dn = 0 or dn = 5, then 5 is a divisor of x.

(a)

Claim: If 9 is a divisor of d_1 + d_2 + · · · + d_n, then 9 is a divisor of $d_1\ d_2\ d_3\ \cdots\ d_n$, where d_k is ∈ $\mathbb{Z}$ and 1 ≤ i ≤ n where d_i ∈ {0, 1, 2, . . . , 9}, let x = $d_1\ d_2\ d_3\ \cdots\ d_n$.

Proof: Lets say x = $d_1\ d_2\ d_3\ \cdots\ d_n$. This can be expanded to be the same as x = $d_1(10^{(n-1)})$ + $d_2(10^{(n-2)})$ + $d_3(10^{(n-3)})$ + · · · + $d_n(10^n)$ where 1 ≤ i ≤ n and d_i ∈ {0, 1, 2, . . . , 9} and $d_1 \neq$ 0... Therefore each term in $\frac{d_1(10^{(n-1)})-1}{9}$ + $\frac{d_2(10^{(n-2)})-1}{9}$ + $\frac{d_3(10^{(n-3)})-1}{9}$ + · · · + $\frac{d_n(10^{(n)})-1}{9}$ + $\sum_{i=0}^{n} d_i$ must be divisible by 9, which is equal to $d_1\ d_2\ d_3\ \cdots\ d_n$, this proves that 9 is a divisor of $d_1\ d_2\ d_3\ \cdots\ d_n$ when 1 ≤ i ≤ n and d_i ∈ {0, 1, 2, . . . , 9} and $d_1 \neq$ 0.

(b)

Claim: When d_n = 0 or d_n = 5, then 5 is a divisor of x.

Proof: Lets say x = $d_1\ d_2\ d_3\ \cdots\ d_n$, where x $\geq$ 5 and x ∈ $\mathbb{Z}$, because we are using base 10 to model our values, 10^n will always be divisiable by 10 excluding n = 0, this proves that any integer that ends in a 0 or 5 where x $\geq$ 5 will be divisable by 5 because 5 is always divisable by 10.

Question 4

- Please prove or disprove: If n ∈ Z+, then n^2 + n + 41 is prime.

Disproof: when n = 42, the result of 42^2 + 42 + 41 = 1763, which is not prime.