Assignment 2

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Question 1

Please respond to the following prompt:

Understand the {M, I, U}-system well enough to state and prove a metatheorem about what strings are in S. One way to think of this is as though you are creating some sort of litmus test to determine whether a given string is a theorem of the {M, I, U}-system. Your statement should be able to be used in the following way: If someone presents you with a string of Ms, Is, and Us and asks \Is this an MIU-theorem?" you might be able to answer deni-tively. I don't expect you to, because you won't be able to, classify all strings of the {M, I, U}-system.

Metatheorem: A string with never not start with a M.

To answer a question about understanding the {M, I U} - system, we must first define the axioms of the M I U syestem, They are as follows

#	Axiom
0	MI $\in$ S
1	xI $\in$ S $\Rightarrow$ xIU
2	Mx $\in$ S $\Rightarrow$ Mxx $\in$ S
3	xIIIy $\in$ S $\Rightarrow$ xUy $\in$ S
4	xUUy $\in$ S $\Rightarrow$ xy $\in$ S

Proof: If we observe axiom 0 we see that each axiom that follows axiom 0 is implied from a derived axiom 0. This implies that every M I U axiom after axiom 0 cannot exist without the definition of axiom 0, also we notice that axiom 0 is the only axiom of the 5 M I U axioms that does not contain any sort of variable (such as x), this allows for expansion on axiom 0. Therefore with the use of a variable, we see that for each axiom, axiom 0 implies another axiom, this proves that all M I U axioms will always start with an M.

Question 2

Let P = {(a,b,c) : a, b, c $\in \mathbb{Z}$, and $a^2 + b^2 = c^2$}, and T = {(p, q, r) : $p = x^2 - y^2$, $q = 2xy$, and $r = x^2 + y^2$, where x, y $\in \mathbb{Z}$}.

Claim: $T \subseteq P$

Proof: To prove that $T \subseteq P$, we have to show that the ordered triple of $p^2 + q^2 = r^2$.

Computing $r^2 = x^4 + 2x^2y^2 + y^4$, $p^2 = x^4 - 2x^2y^2 + y^4$, $q^2 = 4x^2y^2$. If we replace $p^2 + q^2 = r^2$ into the from $a^2 + b^2 = c^2$, this shows that $(4x^2y^2) + (x^4 - 2x^2y^2 + y^4) = x^4 + 2x^2y^2 + y^4$ which is true and of the form $a^2 + b^2 = c^2$, Therefore $T \subseteq P$.

Question 3

Determine, with proof, the number of ordered triples (A_1, A_2, A_3) of sets which have the property that

(i) $A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and

(ii) $A_1 \cap A_2 \cap A_3 = \oslash$ express the answer in the form $2^a 3^a 5^c 7^d$

Claim: There are 60,466,176, (6^{10}) different ordered triples in $A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and that 60,466,176 can also be expressed in the form $2^a 3^a 5^c 7^d$ where a = 10, b = 10, c = 0, and d = 0

Proof: Let i $\in$ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, We need to compute how many variations of $A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ exist to make the statement $A_1 \cap A_2 \cap A_3 = \oslash$ true. To do this task we can derive that a single instance of A_1, A_2, A_3 can contain, at least 1 instance of i and at most 2 instance of i, for each instance of i we need to compute how many variations each instance of i can have. Therefore I have constucted a table below to signigy the possibilities each instance of i can have.

A_1	A_2	A_3
i	-	-
-	i	-
-	-	i
i	i	-
i	-	i
-	i	i

for i at 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Each one holds 6^1 possiblities to keep the statement $A_1 \cap A_2 \cap A_3 = \oslash$ true. But since i $\in$ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, we can say that $6^1 * 6^1$ ..... 6^1 for however many elements i could be, which is 10. This can be rewritten to be 6^{10} and in terms of $2^a 3^a 5^c 7^d$, a = 10, b = 0, c = 0, and d = 0, which is the same as 60,466,176.

Question 4

Claim: The system $\{\mu, \Phi, \neg, \vee, \wedge, \Rightarrow\}$ can be reduced to $\{\mu, \Phi, \bigtriangledown\}$, where x $\bigtriangledown$ y is equivalent to $\neg$(x $\vee$ y)

Proof: In order to get the system $\{\mu, \Phi, \neg, \vee, \wedge, \Rightarrow\}$ to reduce to $\{\mu, \Phi, \bigtriangledown\}$ where $\neg$ is "not", $\vee$ is "or", $\wedge$ is "and", $\Rightarrow$ is "if/implies".
First we must prove a way to resolve each symbol in the set $\{\neg, \vee, \wedge, \Rightarrow\}$, to be equivelant to a set using the symbol $\bigtriangledown$. Given to us in the prompt x $\bigtriangledown$ y $\equiv$ $\neg$(x $\vee$ y). I show the equivelancy of x $\bigtriangledown$ y $\equiv$ $\neg$(x $\vee$ y) in the truth table below.

x	y	$\neg$(x $\vee$ y)	x $\bigtriangledown$ y
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

The truth table above proves x $\bigtriangledown$ y $\equiv$ $\neg$(x $\vee$ y) because the results of the truth table above are exactly the same, (FTTT). Following, we can now define the symbol $\bigtriangledown$ as being nor, nor meaning "not or", however "not or" is difficult for me to think about in terms of a truth table so I prefer to evalaute or, then not. Since we now have nor defined we can use nor to mean the symbol $\bigtriangledown$.

The next thing we can interpret is the that $\neg x \equiv x \bigtriangledown x$ as I show in the truth table below which allows us to safely remove $\neg$. This is an important definition because it will be used in later truth tables.

x	**¬x**	**$x \bigtriangledown x$**
T	F	F
T	F	F
F	T	T
F	T	T

Next we will prove that $\vee$ is removable. See the truth table below.

x	**y**	**$x \vee y$**	**$(x \bigtriangledown y) \bigtriangledown (x \bigtriangledown y)$**
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Notice that we can explicetly say now that $x \vee y \equiv (x \bigtriangledown y) \bigtriangledown (x \bigtriangledown y)$ because we proved that for possible situation of x and y, $x \vee y \equiv (x \bigtriangledown y) \bigtriangledown (x \bigtriangledown y)$ holds true.

Next we will prove that $\wedge$ can be shown via terms of $\bigtriangledown$.

x	**y**	**$(x \wedge y)$**	**$\neg(\neg x \vee \neg y)$**	**$((x \bigtriangledown x) \bigtriangledown (y \bigtriangledown y)) \bigtriangledown ((x \bigtriangledown x) \bigtriangledown (y \bigtriangledown y))$**
T	T	T	T	T
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

Now we say that $(x \wedge y) \equiv \neg(\neg x \vee \neg y) \equiv ((x \bigtriangledown x) \bigtriangledown (y \bigtriangledown y)) \bigtriangledown ((x \bigtriangledown x) \bigtriangledown (y \bigtriangledown y))$ because of the definition we established in a previous truth table that $\neg x \equiv x \bigtriangledown x$. Therefore have proved we can remove $\vee$ from the set in question.

Lastly, we need to prove that $\Rightarrow$ can be removed from the set in question.

x	**y**	**$x \Rightarrow y$**	**$\neg$ (y $\bigtriangledown$(x $\bigtriangledown$ y))**	**(y $\bigtriangledown$(x $\bigtriangledown$ y)) $\bigtriangledown$ (y $\bigtriangledown$(x $\bigtriangledown$ y))**
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Referring to the truth table above we can now prove that $x \Rightarrow y \equiv$ (y $\bigtriangledown$(x $\bigtriangledown$ y)) $\bigtriangledown$ (y $\bigtriangledown$(x $\bigtriangledown$ y) because of the definition we established in a previous truth table that $\neg x \equiv x \bigtriangledown x$ and using nor which we previously defined.

Finally we have proved that he system $\{\mu, \Phi, \neg, \vee, \wedge, \Rightarrow\}$ can be reduced to $\{\mu, \Phi, \bigtriangledown\}$, where $x \bigtriangledown y$ is equivalent to $\neg(x \vee y)$.