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inverted pendulum on cart

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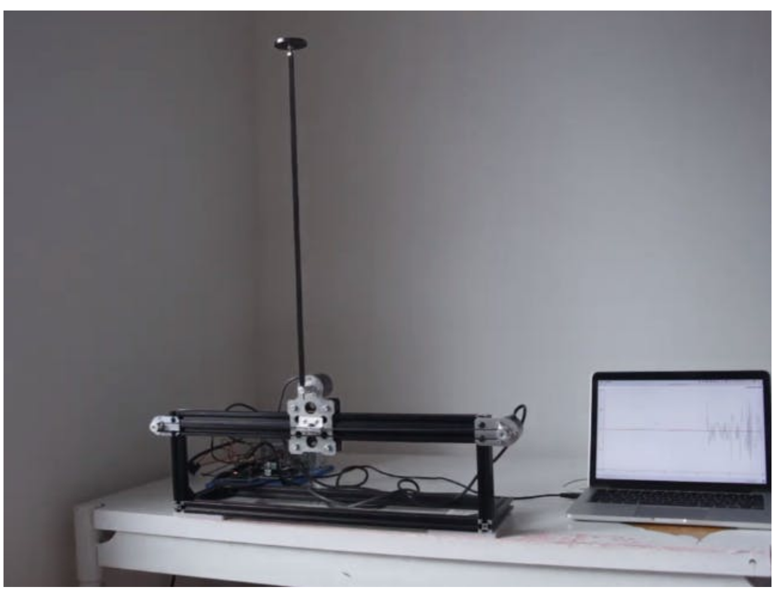
Definition

An inverted pendulum consists of a mass (the pendulum) mounted on a pivot point. Mounted on a moving cart along a slider the pendulum is free to be in the gravitation direction The challenge is to keep the pendulum balanced in an inverted (upright) position, despite external disturbances.

Introduction:

The system in this example consists of an inverted pendulum mounted to a motorized cart. The inverted pendulum system is an example commonly found in control system textbooks and research literature. Its popularity derives in part from the fact that it is unstable without control, that is, the pendulum will simply fall over if the cart isn't moved to balance it. Additionally, the dynamics of the system are nonlinear. The objective of the control system is to balance the inverted pendulum by applying a force to the cart that the pendulum is attached to. A real-world example that relates directly to this inverted pendulum system is the attitude control of a booster rocket at takeoff.



free-body diagrams of the two elements of the inverted pendulum system.

- the two Free Body Diagrams of the system. Summing the forces in the Free Body Diagram of the cart in the horizontal direction, we get the following equation of motion:

$$M\ddot{x} + b\dot{x} + N = F \qquad (1)$$

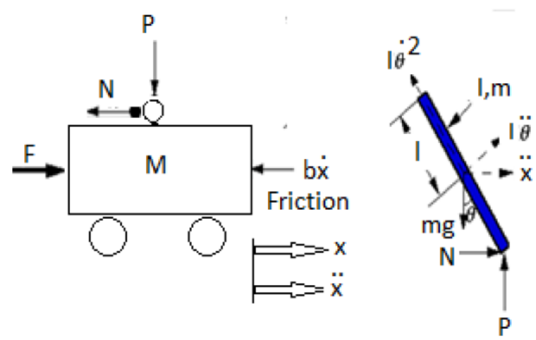


Fig.1.Free Body Diagram of Inverted Pendulum

Note that the forces can be sum in the vertical direction, but no useful information would be gained. Summing the forces in the Free Body Diagram of the pendulum in the horizontal direction, we can get an equation for N

$$N = m\ddot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta \qquad (2)$$

By substituting (2) equation into the (1) equation, we get the equation of motion for this system

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = F \qquad (3)$$

To get the second equation of motion, sum the forces perpendicular to the pendulum

$$Psin\theta + Ncos\theta - mgsin\theta = ml\ddot{\theta} + m\ddot{x}cos\theta (4)$$

To get rid of the P and N terms in the equation above, sum the moments around the centroid of the pendulum to get the following equation

$$-Plsin\theta - Nlcos\theta = I\ddot{\theta} \qquad (5)$$

Combining equation (4) & (5), we get the second dynamic equation:

$$(I + ml^2)\dot{\theta} + mglsin\theta = -ml\ddot{x}cos\dot{\theta} \qquad (6)$$

These set of equations (3) & (6) should be linearized about = ∅,Assume that theta = + ∅ (represents a small angle from the vertical upward direction). Therefore, cos θ = -1, sin θ = -1 and (d θ /dt) 2 = 0. After linearization the two equations of motion become (where u represents the input):

$$(I + ml^2)\ddot{\emptyset} - mgl\emptyset = ml\ddot{x} \qquad (7)$$

$$(M + m)\ddot{x} + b\dot{x} - \quad ml\ddot{\emptyset} = u \qquad (8)$$

Transfer Function of Pendulum Model:

To obtain the transfer function of the linearized system equations analytically, we must first take the Laplace transform of the system equations (7) & (8). The Laplace transforms are:

$$(I + ml^2)\emptyset(s)s^2 - mgl\emptyset(s) = mlX(s)s^2 \quad (9)$$

$$(M + m)X(s)s^2 + bX(s)s - ml\emptyset(s)s^2 = U(s)(10)$$

Since we will be looking at the angle, Ø as the output of interest, solve the equation (9) for X(s),

$$X(s) = \left[\frac{(I+ml^2)}{ml} - \frac{g}{s^2}\right]\emptyset(s)(11)$$

Substitute value of X(s) from equation (11) to (10) and re-arrange. The transfer function is:

$$\frac{\emptyset(s)}{U(s)} = \frac{\frac{ml\,s^2}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad (12)$$

where,

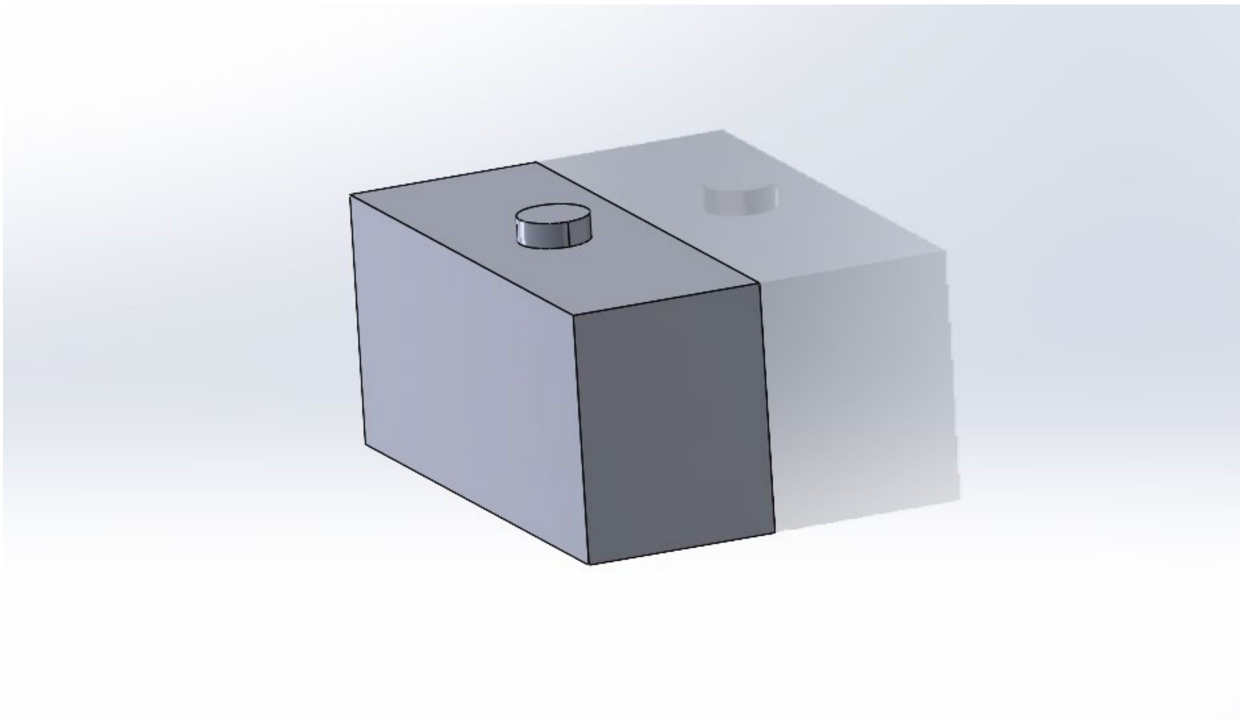
$$q = [(M + m)(I + ml^2) - (ml)^2] \qquad (13)$$

From the transfer function above it can be seen that there is both a pole and a zero at the origins. These can be canceled and the transfer function becomes:

$$\frac{\emptyset(s)}{U(s)} = \frac{\frac{mls}{q}}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \qquad (14)$$

Inverted pendulum design:

Cart:



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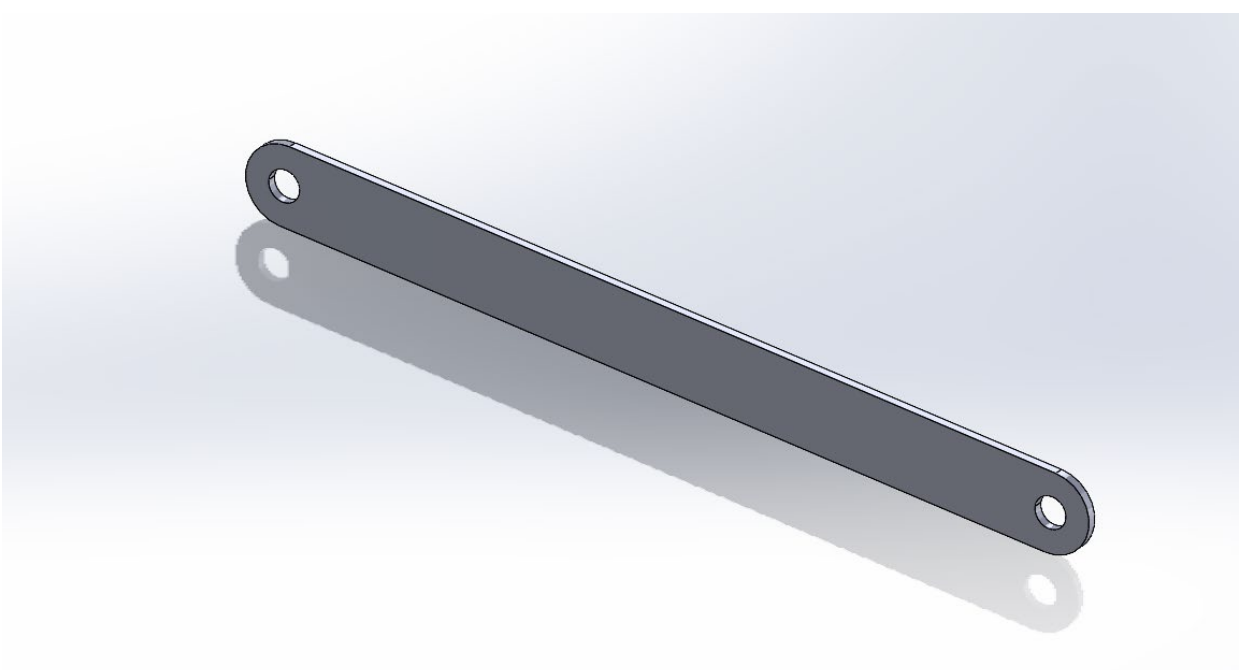
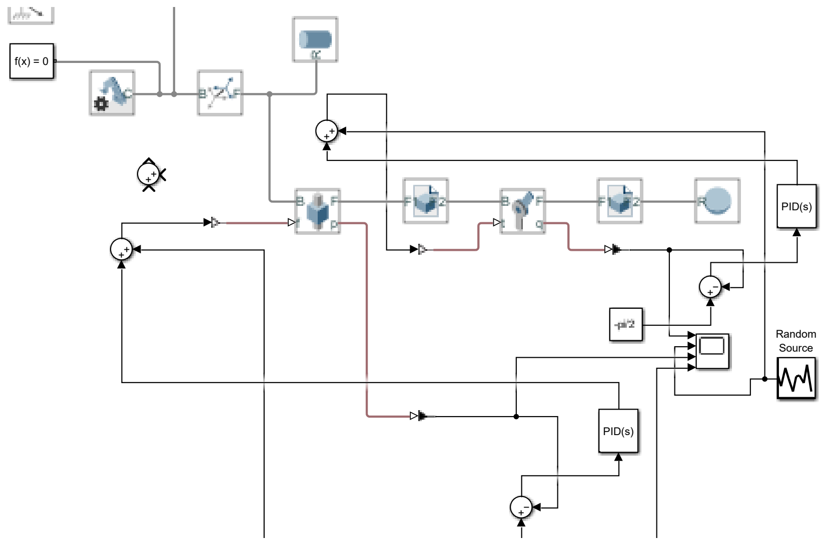
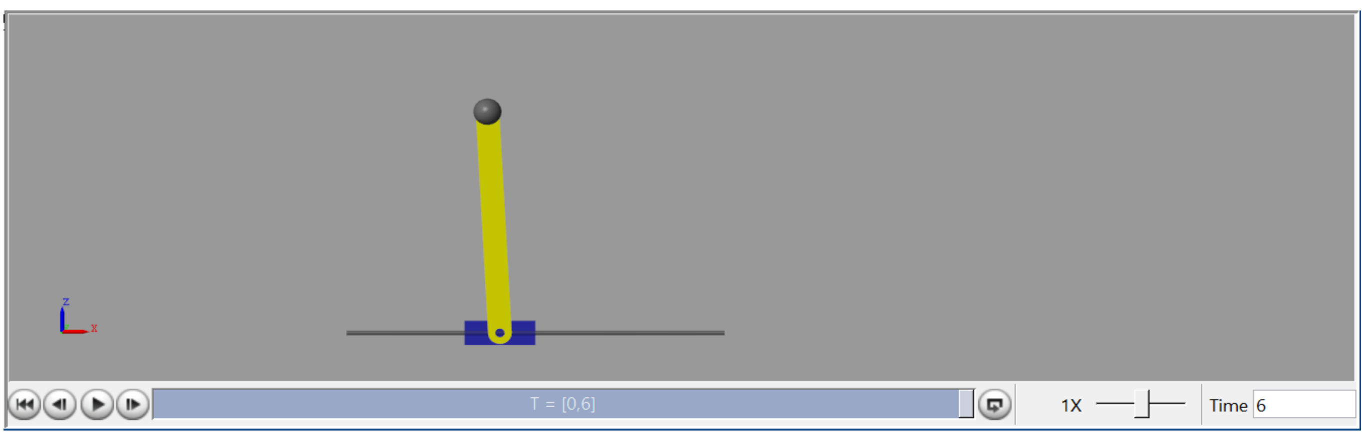


PID Model:





Reference:

1. https://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum&section=SystemModeling

2. https://www.youtube.com/watch?v=LcYd0goujWU&t=1888s

3. https://www.youtube.com/watch?v=hAI8Ag3bzeE