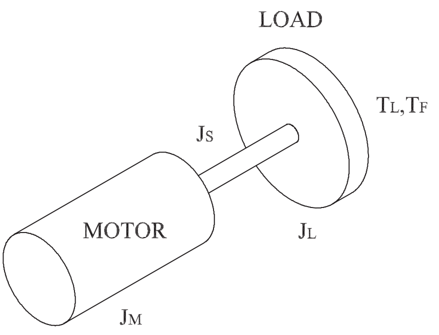

Figure 4.74 Rotary motion; direct coupled

System Data:

$$J_M = \text{motor inertia (g cm s}^2)$$
$$J_L = \text{load inertia (g cm s}^2)$$
$$J_S = \text{shaft inertia (g cm s}^2)$$
$$T_L = \text{load torque (g cm)}$$
$$T_F = \text{friction torque (g cm)}$$

Motion:

$$\text{Position: } \theta_M = \theta_L \text{ (rad)}$$
$$\text{Velocity: } \theta'_M = \theta'_L \text{ (rad s}^{-1})$$
$$\text{Acc/Dec: } \theta''_M = \theta''_L \text{ (rad s}^{-2})$$

At Motor:

$$J_T \text{ (total inertia)} = J_M + J_L + J_S \tag{4.71}$$
$$T_{ACC} = +J_T\theta''_M + T_F \pm T_L \tag{4.72}$$
$$T_{DEC} = -J_T\theta''_M + T_F \pm T_L \tag{4.73}$$
$$T_{RUN} = +T_F \pm T_L \tag{4.74}$$
$$T_{SS} = \pm(T_L - T_F) \quad \text{if} \quad T_L > T_F \tag{4.75}$$
$$= 0 \text{ if } T_L < T_F$$

The sign for T_L depends on whether it is bi-or unidirectional and whether it opposes or aids acceleration or deceleration. At zero velocity, if a load torque is present and it is greater than friction torque, then the motor must develop a counter torque to maintain zero velocity.

Example

A spindle drive is required for a disc memory tester. The spindle inertia (a cylinder) is 5.4 g cm s^2. There is a maximum load torque during rotation of 720 g cm opposing the rotation. The spindle will be accelerated and decelerated between 0 and 3000 rpm in 0.5 s, run for 5 s and rest at 0 velocity for 5 s, per a trapezoidal profile. A coupling/shaft assembly will be used to connect the motor to the spindle, with a total bearing torque of 144 g cm and inertia of 0.072 g cm s^2

$$\theta'_M = 3000 \text{ rpm} = 314 \text{ rad s}^{-1}$$

$$\theta''_M = 314/0.5 = 628 \text{ rad s}^{-2}$$

Since this is a fairly small system, in order to quickly get a sense of the accelerate torque involved, we will initially just use the known inertias.

$$\text{Initial inertia} = 5.4 + 0.072 = 5.5 \text{ g cm s}^2$$

$$\text{Initial } T_{ACC} = (5.5)(628) + 720 + 144 = 4318 \text{ g cm}$$

First motor choice: A NEMA size 23 brushless servo motor with the following specs.

$$T_{CONT} = 5616 \text{ g cm} \quad T_{PEAK} = 16\,920 \text{ g cm}$$

$$N_{MAX} = 4000 \text{ rpm} \quad R = 11.9\ \Omega$$

$$K_T = 2880 \text{ g cm A}^{-1} \quad J_M = 0.0013 \text{ oz in s}^2$$

$$J_T = 0.094 + 5.4 + 0.072 = 5.6 \text{ g cm s}^2$$

$$T_{ACC} = (5.6)(628) + 720 + 144 = 4381 \text{ g cm} \qquad I_{ACC} = 4381/2880 = 1.5 \text{ A}$$

$$T_{DEC} = -(5.6)(628) + 720 + 144 = -2653 \text{ g cm} \qquad I_{DEC} = 2653/2880 = 0.92 \text{ A}$$

$$T_{RUN} = 720 + 144 = 864 \text{ g cm} \qquad I_{RUN} = 864/2880 = 0.3 \text{ A}$$

$$T_{RMS} = \sqrt{\frac{4381^2(0.5) + 864^2\,(5)\, + 2653^2(0.5)}{0.5 + 5 + 0.5 + 5}} = 1237 \text{ g cm} \qquad I_{RMS} = 1237/2880 = 0.43 \text{ A}$$

Although this motor can easily do this application, it appears to be quite oversized and it would be advantageous to see if a smaller, less expensive motor is available.

Second motor choice: A NEMA size 16 brushless servo motor with the following specs.

$$T_{CONT} = 2664 \text{ g cm} \quad T_{PEAK} = 7992 \text{ g cm}$$

$$N_{MAX} = 5000 \text{ rpm} \quad R = 4.38\ \Omega$$

$$K_T = 1015 \text{ g cm A} \quad J_M = 0.022 \text{ g cm s}^2$$

Repeating the calculations results in:

$$
\begin{aligned}
&J_T = 5.5 \text{ g cm s}^2 && \\
&T_{ACC} = 4318 \text{ g cm} && I_{ACC} = 4.3 \text{ A} \\
&T_{DEC} = -2590 \text{ g cm} && I_{DEC} = 2.6 \text{ A} \\
&T_{RUN} = 864 \text{ g cm} && I_{RUN} = 0.85 \text{ A} \\
&T_{RMS} = 1221 \text{ g cm} && I_{RMS} = 1.2 \text{ A}
\end{aligned}
$$

This motor can also do the application, has an almost 2:1 safety margin for both the maximum and RMS currents, a 1.7:1 margin for the maximum speed and would be less expensive than the size 23 first chosen.

Table 4.3 outlines an Excel program for the above calculations and Table 4.4 shows the results of running this program for the first choice motor. It can easily be modified to develop a program for each of the remaining building blocks shown in this section.

Table 4.3 Excel program for direct drive calculations

	A	B
1		
2		**TABLE 4.3**
3		
4		**Excel Program for Example in 4.9.1**
5		
6	SYSTEM SPECS	
7		
8	Load Inertia	5.4
9	Shaft Inertia	0.072
10	Motor Velocity (max)	3000
11	Acc Time	0.5
12	Dec Time	0.5
13	Run Time	5
14	Stop Time	5
15	Acc	=+((B10/60)*2*3.1416)/B11
16	Dec	=+((B10/60)*2*3.1416)/B12
17	Friction Torque	144
18	Load Torque	720
19		
20	MOTOR SPECS	
21		
22	Torque (cont)	5616
23	Torque (peak)	16 920
24	Max Velocity	4000
25	Resistance	11.9
26	Torque Constant	2880
27	Inertia	0.094
28		
29	SYSTEM DATA	
30		
31	Total Inertia	=+B8+B9+B27
32	Acc Torque	=+B31*B15+B17+B18
33	Dec Torque	=-B31*B16+B17+B18
34	Run Torque	=+B17+B18
35	RMS Torque	$=+((((B32)^2*B11)+((B34)^2*B13)+((B33)^2*B12))/(B11+B12+B13+B14))^{0.5}$
36	Acc Current	=+B32/B26
37	Dec Current	=+B33/B26
38	Run Current	=+B34/B26
39	RMS Current	=+B35/B26

Table 4.4 Results using Excel program in Table 4.3 for first motor selection Example in 4.9.1

	A	B
1		
2	**TABLE 4.4**	
3		
4	**Results for First Motor Selection**	
5		
6	SYSTEM SPECS	
7		
8	Load Inertia	5.4
9	Shaft Inertia	0.072
10	Motor Velocity (max)	3000
11	Acc Time	0.5
12	Dec Time	0.5
13	Run Time	5
14	Stop Time	5
15	Acc	628.32
16	Dec	628.32
17	Friction Torque	144
18	Load Torque	720
19		
20	MOTOR SPECS	
21		
22	Torque (cont)	5616
23	Torque (peak)	16 920
24	Max Velocity	4000
25	Resistance	11.9
26	Torque Constant	2880
27	Inertia	0.094
28		
29	SYSTEM DATA	
30		
31	Total Inertia	5.566
32	Acc Torque	4361.23
33	Dec Torque	–2633.23
34	Run Torque	864
35	RMS Torque	1232.50
36	Acc Current	1.51
37	Dec Current	–0.91
38	Run Current	0.30
39	RMS Current	0.43

4.9.2 Rotary Motion – Gearhead Drive

N is the gear ratio of the gearhead. For most applications, this ratio is greater than 1 in order to create a condition in which the motor is operating at as high an efficiency as possible.

For gearheads in which N is 10 or greater, the load inertia reflected back to the motor will often be relatively small, in which case a good initial approximation for the total inertia seen by the motor is $J_M + J_{GH}$. See Figure 4.75.

System Data:

$$J_M = \text{motor inertia (g cm s}^2\text{)}$$
$$J_L = \text{load inertia (g cm s}^2\text{)}$$
$$J_{GH} = \text{gearhead inertia (g cm s}^2\text{)}$$
$$J_S = \text{shaft inertia (g cm s}^2\text{)}$$
$$T_L = \text{load torque (g cm)}$$
$$T_F = \text{friction torque (g cm)(seen at gearhead input)}$$

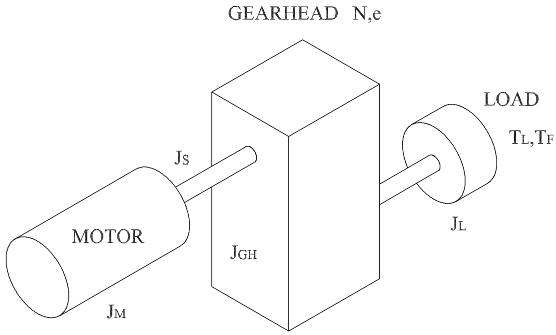

Figure 4.75 Rotary motion; gearhead coupled

Gearhead Efficiency (e): Spur or Bevel = 0.90; Worm = 0.65; Planetary = 0.95
Motion:

$$\text{Position: } \theta_M = N\theta_L \text{ (rad)}$$
$$\text{Velocity: } \theta'_M = N\theta'_L \text{ (rad s}^{-1}\text{)}$$
$$\text{Acc/Dec: } \theta''_M = N\theta''_L \text{ (rad s}^{-2}\text{)}$$

At Motor:

$$J_T \text{ (total inertia)} = J_M + J_L/N^2 + J_{GH} \tag{4.76}$$

(theoretical, assuming the gearhead efficiency is 100%)
In order to account for the actual gearhead efficiency, the torques are calculated as follows:

$$T_{ACC} = (J_M + J_{GH})\theta''_M + (J_L/N^2e)\theta''_M \pm T_L/Ne + T_F \tag{4.77}$$
$$T_{DEC} = -(J_M + J_{GH})\,\theta''_M - \left(J_L/N^2e\right)\theta''_M \pm T_L/Ne + T_F \tag{4.78}$$
$$T_{RUN} = +T_F \pm T_L/Ne \tag{4.79}$$
$$T_{SS} = \pm(T_L/Ne - T_F) \text{ if } T_L/Ne > T_F \tag{4.80}$$
$$= 0 \text{ if } T_L/Ne < T_F$$

The sign for T_L depends on whether it is bi-or unidirectional and whether it opposes or aids acceleration or deceleration. At zero velocity, if a load torque is present and it is greater than friction torque, the motor must develop a counter torque to maintain zero velocity.

Example
A system with a load inertia of 2736 g cm s^2 must be accelerated and decelerated between 0 and 300 rpm in 0.5 s. A spur gearhead with a ratio of 10:1 has been chosen so as not to exceed a maximum gearhead input speed of 3200 rpm. The gearhead has an input inertia of 0.072 g cm s^2. There is 2880 g cm of system friction torque at the load and a bidirectional load torque of 360 g cm opposing the direction of rotation. A trapezoidal profile will be used

with accelerate and decelerate times of 0.5 s, run time of 3 s and stop time of 5 s.

$$\theta'_L = 300 \text{ rpm} = 31.4 \text{ rad s}^{-1}$$
$$\theta'_M = N\theta'_L = (10)(31.4) \text{ rad s}^{-1}$$
$$\theta''_L = 31.4/0.5 = 62.8 \text{ rad s}^{-2}$$
$$\theta''_M = N\theta''_L = (10)(62.8) = 628 \text{ rad s}^{-2}$$
$$\text{Initial inertia} = J_L/N^2 + J_{GH} = 2736/10^2 + 0.072 = 27.4 \text{ g cm s}^2$$
$$\text{Initial } T_{ACC} = (27.4)(628) + \boxed{2880/10} + 360/10 = 17531 \text{ g cm}$$

Motor Choice: A NEMA size 34 brushless servo motor with the following specs:

$$T_{CONT} = 7920 \text{ g cm} \qquad T_{PEAK} = 26\,712 \text{ g cm}$$
$$N_{MAX} = 5400 \text{ rpm} \qquad R = 4.8\ \Omega$$
$$K_T = 3526 \text{ g cm A}^{-1} \qquad J_M = 0.15 \text{ g cm s}^2$$

Gearhead Choice: a spur gearhead with the following specs:

$$T_{ACC} \text{ (output)} = 345\,600 \text{ g cm} \qquad T_{NOM} \text{ (output)} = 253\,440 \text{ g cm}$$
$$N_{IN} = 3200 \text{ rpm nom; } 6000 \text{ rpm max}$$
$$J_{GH} = 0.072 \text{ g cm s}^2 \qquad \text{eff.} = 90\%$$

$$T_{ACC} = (0.15 + 0.072)(628) + (2736/(10^2 \times 0.9))(628) + 2880/(10 \times 0.9) + \boxed{360/(10 \times 0.9)}$$
$$= 19\,590 \text{ g cm}$$
$$T_{DEC} = -(0.15 + 0.072)(628) - \left(2736/(10^2 \times 0.9)\right)(628) + 2880/(10 \times 0.9) + 360/(10 \times 0.9)$$
$$= -18\,871 \text{ g cm}$$
$$T_{RUN} = (2880 + 360)/(10 \times 0.9) = 369 \text{ g cm}$$

$$T_{ACC} = (J_M + J_{GH})\theta''_M + (J_L/N^2e)\theta''_M \pm T_L/Ne + T_F \quad (4.77)$$
$$T_{DEC} = -(J_M + J_{GH})\theta''_M - \left(J_L/N^2e\right)\theta''_M \pm T_L/Ne + T_F \quad (4.78)$$
$$T_{RUN} = +T_F \pm T_L/Ne \quad (4.79)$$
$$T_{SS} = \pm(T_L/Ne - T_F) \text{ if } T_L/Ne > T_F \quad (4.80)$$
$$= 0 \text{ if } T_L/Ne < T_F$$

Since $T_F > T_L$

$$T_{SS} = 0$$
$$T_{RMS} = \sqrt{\frac{19\,590^2(0.5) + 360^2(3) + 18\,871^2(0.5)}{0.5 + 3 + 0.5 + 5}} = 6415 \text{ gm cm}$$
$$I_{ACC} = 19\,590/3528 = 5.6 \text{ A} \qquad I_{DEC} = 18\,871/3528 = 5.3 \text{ A}$$
$$I_{RUN} = 360/3528 = 0.1 \text{ A} \qquad I_{RMS} = 6415/3528 = 1.8 \text{ A}$$

The torques and speed are within the specs of both the motor and gearhead, although the RMS torque only has an 18% safety margin with respect to the motor continuous torque rating and possibly a somewhat larger motor should be considered.

$$T_{ACC} \text{ at load} = (2736)(62.8) = 171\,820 \text{ g cm; a } 2:1 \text{ safety margin for the gearhead}$$

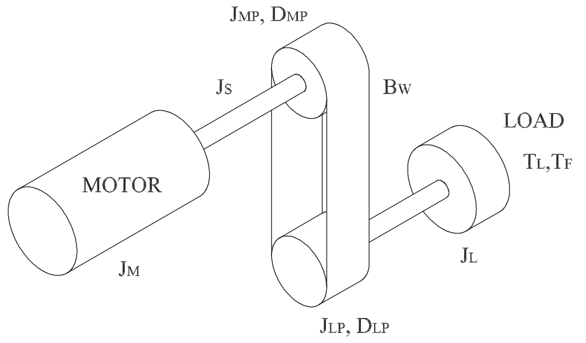

Figure 4.76 Rotary motion; belt and pulley coupled

4.9.3 Rotary Motion – Belt and Pulley Drive

The belt and pulley drive is similar to the gearhead drive in that there is a ratio between the motor and the load. In this case, N is the ratio of the load pulley diameter (D_{LP}) to the motor pulley diameter (D_{MP}).

$$\boxed{N = D_{LP}/D_{MP}}$$

In addition, the weight of the belt must be reflected to the motor as an inertia, especially if the belt is long or of a high mass material such as a chain link belt. See Figure 4.76.

System Data:

J_M = motor inertia (g cm s^2)
J_L = load inertia (g cm s^2)
J_{MP} = motor pulley inertia (g cm s^2)
J_{LP} = load pulley inertia (g cm s^2)
J_S = shaft inertia (g cm s^2)
T_L = load torque (g cm)
T_F = friction torque (g cm) (seen at motor pulley)
B_W = belt weight (g cm)

Belt weight reflected to the motor as an equivalent inertia:

$$J_B = \left(\frac{B_W}{980.6}\right)\left(\frac{D_{MP}}{2}\right)^2$$

Motion:

Position: $\theta_M = N\theta_L$ (rad)
Velocity: $\theta'_M = N\theta'_L$ (rad s^{-1})
Acc/Dec: $\theta''_M = N\theta''_L$ (rad s^{-2})

At Motor:

$$J_T(\text{total inertia}) = J_M + J_L/N^2 + J_{MP} + J_{LP}/N^2 + J_B \text{ (theoretical)} \tag{4.81}$$

$$T_{ACC} = +J_T\theta''_M + T_F \pm T_L/Ne \tag{4.82}$$

$$T_{DEC} = -J_T\theta''_M + T_F \pm T_L/Ne \tag{4.83}$$

$$T_{RUN} = +T_F \pm T_L/Ne \tag{4.84}$$

$$T_{SS} = \pm(T_L/Ne - T_F) \text{ if } T_L/Ne > T_F \tag{4.85}$$

$$= 0 \text{ if } T_L/Ne < T_F$$

The sign for T_L depends on whether it is bi- or unidirectional and whether it opposes or aids acceleration or deceleration. At zero velocity, if a load torque is present and it is greater than the friction torque, then the motor must develop a counter torque to maintain zero velocity.

Pulley efficiencies (e): timing belt = 0.97; chain and sprocket = 0.96

Example

A system with a motor pulley diameter of 1.27 cm and a width of 2.54 cm plus a load pulley diameter of 5.08 cm and a width of 2.54 cm is to accelerate an inertial load of 144 g cm s^2 from 0 to 600 rpm in 0.3 s, run for 5 s, decelerate to 0 rpm in 0.3 s and remain at 0 rpm for 2 s. There is 720 g cm of system friction torque at the motor shaft and a load torque of 1440 g cm in the direction of rotation. A timing belt with a weight of 227 g will be used.

$$J_{MP} = \frac{\rho\pi W R^4}{2g} = 0.0051 \text{ g cm s}^2$$

$$J_{LP} = 0.018 \text{ g cm s}^2$$

$$\theta'_L = 600 \text{ rpm} = 62.8 \text{ rad s}^{-1}$$

$$\theta'_M = N\theta'_L = (2/0.5)(62.8) = 251.3 \text{ rad s}^{-1}$$

$$\theta''_M = 251.3/0.3 = 837.8 \text{ rad s}^{-2}$$

$$J_B = (227/980.6)(1.27/2)^2 = 0.093 \text{ g cm s}^2$$

$$\text{Initial inertia} = 0.0051 + 0.093 + (1.3 + 144)/4^2 = 9.2 \text{ g cm s}^2$$

$$\text{Initial } T_{ACC} = (837.8)(9.2) + 720 - 1440/4 = 8068 \text{ g cm}$$

Motor Choice: A NEMA 23 brushless servo motor with the following specs.

$$T_{CONT} = 3816 \text{ g cm} \quad T_{PEAK} = 11\,520 \text{ g cm}$$

$$N_{MAX} = 5000 \text{ rpm} \quad R = 1.22\ \Omega$$

$$K_T = 635 \text{ g cm A}^{-1} \quad J_M = 0.053 \text{ g cm s}^2$$

$$T_{ACC} = +(837.8)\big(0.053 + 0.0051 \times 10^{-4} + 0.093\big) + (837.8)\big((1.3 + 144)/(4^2 \times 0.97)\big) + 720 - 1440/(4 \times 0.97) = +8319 \text{ g cm}$$

$$T_{DEC} = -(837.8)(0.053 + 0.0051 + 0.093) - (837.8)\left((1.3 + 144)/(4^2 \times 0.97)\right) + 720 - 1440/(4 \times 0.97) = -7621 \text{ g cm}$$

$$T_{RUN} = +720 - 1440/(4 \times 0.97) = 349 \text{ g cm}$$

$$T_{SS} = 0$$

$$T_{RMS} = \sqrt{\frac{8319^2(0.3) + 349^2(5) + 7621^2(0.3)}{0.3 + 5 + 0.3 + 2}} = 2259 \text{ g cm}$$

$$I_{ACC} = 8319/635 = 13.1 \text{ A} \quad I_{DEC} = 7621/635 = 12 \text{ A}$$

$$I_{RUN} = 349/635 = 0.55 \text{ A} \quad I_{RMS} = 2259/635 = 3.6 \text{ A}$$

4.9.4 Linear Motion – Leadscrew/Ballscrew Drive

In evaluating a screw drive all of the following must be considered:

Screw Inertia (J_S): This can be determined using the inertia formula for a cylinder. Quite often, for a high pitch screw made of steel, the screw inertia is much larger than the reflected load inertia and the reflected load inertia can be ignored for initial calculations.

Friction Force (F_F): This is the opposing force created by the friction between the load and the load bearing surface. Do not confuse the coefficient of friction (μ) with the screw efficiency (e).

Nut Preload (T_P): To eliminate backlash, the drive nut, through which the screw rotates, is sometimes preloaded. This preload creates an additional torque load on the motor.

Note: When performing screw calculations do not confuse the screw pitch (P_S) which has the units of rev/cm with the screw lead (L_S) which has the units of cm/rev. (See Figure 4.77).

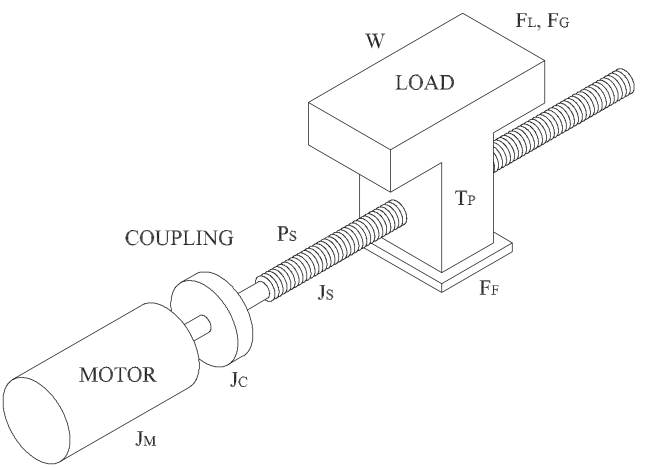

Figure 4.77 Linear motion; ballscrew drive

System Data:

$$
\begin{aligned}
J_M &= \text{motor inertia (g cm s}^2\text{)}\\
J_C &= \text{coupling inertia (g cm s}^2\text{)}\\
W &= \text{load weight (g)}\\
T_P &= \text{preload torque (g cm)}\\
F_L &= \text{load force (g)}\\
F_G &= \text{gravity force} = W \text{ (g)}\\
F_F &= \text{friction force} = \mu W \text{ (g)}\\
J_S &= \text{screw inertia (g cm s}^2\text{)}\\
T_B &= \text{bearing torque (g cm)}\\
P_S &= \text{screw pitch (rev/cm)}\\
e &= \text{screw efficiency}\\
\mu &= \text{friction coefficient}
\end{aligned}
$$

Motion:

$$
\begin{aligned}
&\text{Position: } \theta_M = (2\pi P_S)(S) \text{ (rad)}\\
&\text{Velocity: } \theta'_M = (2\pi P_S)(S') \text{ (rad s}^{-1}\text{)}\\
&\text{Acc/Dec: } \theta''_M = (2\pi P_S)(S'') \text{ (rad s}^{-2}\text{)}
\end{aligned}
$$

At Motor:

$$
\begin{aligned}
&T_R = T_P + T_B + \frac{\pm F_L + F_F \pm F_G}{2\pi P_S e} \text{(reflected torque)}\\
&J_R = \left(\frac{W}{980.6}\right)\left(\frac{1}{2\pi P_S}\right)^2\left(\frac{1}{e}\right) \text{ (load reflected as inertia)}\\
&J_T \text{(total inertia)} = J_M + J_C + J_S + J_R\\
&T_{ACC} = +J_T\theta''_M + T_R\\
&T_{DEC} = -J_T\theta''_M + T_R\\
&T_{RUN} = T_R\\
&T_{SS} = \frac{\pm(F_L - F_F) + F_G}{2\pi P_S e} - T_P
\end{aligned}
$$

For horizontal systems, $F_G = 0$

For vertical systems, use: $+F_G$ when accelerating “up” or decelerating “down”
$-F_G$ when accelerating “down” or accelerating “up”

Coefficient of friction (μ)		Efficiency (e)	
Steel on steel	0.580	Ball nut	0.90
Steel on steel (lub)	0.150	Ball nut (preloaded)	0.80
Aluminum on steel	0.450	Acme w/metal nut	0.40
Brass on steel	0.350	Acme w/plastic nut	0.65
Dove-tail slide	0.200		
Ball bushing	0.001		
Linear bearing	0.001		
Teflon on steel	0.400		

Example

A mechanism to be used in an injection molding machine is required to move a 45 360 g load at a speed of 12.7 cm s^{-1} for a 107 cm travel. A linear bearing slide assembly will be used with a coefficient of friction of 0.001. A 1.97 pitch lead screw, 1.27 cm diameter by 122 cm long with preloaded ball nut will drive the load. The preload will be 1440 g cm.

Acceleration and deceleration times are to be 0.2 s. Zero velocity times between motions will be 5 s minimum. The assembly will have an estimated bearing friction torque of 360 g cm. A coupling with an inertia of 0.029 g cm s^2 has been selected.

$$J_S = \frac{\rho \pi L R^4}{2g} = 0.252 \text{ g cm s}^2$$

$$J_R = (45\,360/980.6)\,(1/(2\pi \times 1.97))^2\,(1/0.8) = 0.377 \text{ g cm s}^2$$

Note how the screw inertia and the reflected load inertia are the same order of magnitude.

$$\theta'_M = (2\pi)(12.7)(1.97) = 157 \text{ rad s}^{-1} = 1500 \text{ rpm}$$

$$\theta''_M = 157/0.2 = 785 \text{ rad s}^{-2}$$

$$F_F = (0.001)(45\,360)(16) = 45.4 \text{ g}$$

$$\text{Initial inertia} = 0.029 + 0.252 + 0.377 = 0.658 \text{ g cm s}^2$$

$$\text{Initial } T_{ACC} = (0.658)(785) + 1440 + 360 + (45.4)\,(1/\,(2\pi \times 1.97 \times 0.8)) = 2321 \text{ g cm}$$

Motor Choice: A NEMA 23 brushless servo motor with the following specs:

$T_{CONT} = 3888$ g cm $\quad T_{PEAK} = 11\,592$ g cm

$N_{MAX} = 5000$ rpm $\quad R = 4.57\ \Omega$

$K_T = 1236$ gm cm/amp $\quad J_M = 0.053$ gm cm sec^2

$$T_{ACC} = +\,(0.658 + 0.053)\,(785) + 1440 + 360 + (45.4)\,(1/(2\pi \times 1.97 \times 0.8)) = +2363 \text{ g cm}$$

$$T_{DEC} = -\,(0.658 + 0.053)\,(785) + 1440 + 360 + (45.4)\,(1/(2\pi \times 1.97 \times 0.8)) = +1246 \text{ g cm}$$

$$T_{RUN} = 1805 \text{ g cm}$$

$$T_{SS} = 0$$

In order to determine T_{RMS}, the run time must be calculated from the specifications that is, total travel is 107 cm, run velocity is 12.7 cm s^{-1} and acceleration and deceleration times are 0.2 s, therefore:

$$(0.5)\,(0.2)\,(12.7)\,(2) + (12.7)\,(t_{RUN}) = 107$$

$$t_{RUN} = 8.2 \text{ s}$$

$$T_{RMS} = \sqrt{\frac{2363^2\,(0.2) + 1805^2\,(8.2) + 1246^2\,(0.2)}{0.2 + 8.2 + 0.2}} = 1809 \text{ g cm}$$

$$I_{ACC} = 2363/1236 = 2 \text{ A} \quad I_{DEC} = 1246/1236 = 1.0 \text{ A}$$

$$I_{RUN} = 1805/1236 = 1.5 \text{ A} \quad I_{RMS} = 1809/1236 = 1.5 \text{ A}$$

4.9.5 Linear Motion – Belt and Pulley Drive

Similar to the belt and pulley drive for rotary load motion, the inertias of the pulleys and the weight of the belt must be considered in determining the total motor load (see Figure 4.78).

System Data:

$$\begin{aligned}
J_M &= \text{motor inertia (g cm s}^2\text{)} \\
J_C &= \text{coupling inertia (g cm s}^2\text{)} \\
W &= \text{load weight (g)} \\
F_L &= \text{load force (g)} \\
F_G &= \text{gravity force} = W\text{(g)} \\
F_F &= \text{friction force} = \mu W\text{(g)} \\
B_W &= \text{belt weight(g)} \\
J_{PM} &= \text{motor pulley inertia (g cm s}^2\text{)}
\end{aligned}$$

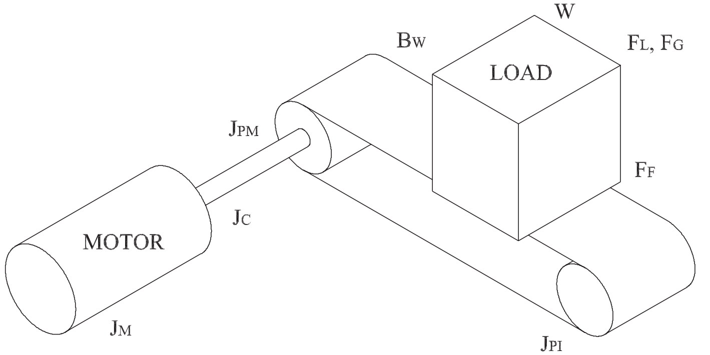

Figure 4.78 Linear motion; belt and pulley drive

$$J_{PI} = \text{idler pulley inertia (g cm s}^2\text{)}$$
$$e = \text{screw efficiency}$$
$$\mu = \text{friction coefficient}$$

Motion:

$$\text{Position: } \theta_M = \frac{S}{D_{PM}/2} \text{ (rad)}$$
$$\text{Velocity: } \theta'_M = \frac{S'}{D_{PM}/2} \text{(rad s}^{-1}\text{)}$$
$$\text{Acc/Dec: } \theta''_M = \frac{S''}{D_{PM}/2} \text{(rad s}^{-2}\text{)}$$

Belt weight reflected to motor as an equivalent inertia:

$$J_B = \left(\frac{B_W}{980.6}\right)\left(\frac{D_{PM}}{2}\right)^2\left(\frac{1}{e}\right)$$

At Motor:

$$T_R = (\pm F_L + F_F \pm F_G)\left(\frac{D_{PM}}{2}\right)^2\left(\frac{1}{e}\right) \text{ (reflected torque)} \tag{4.86}$$
$$J_R = \left(\frac{W}{980.6}\right)\left(\frac{D_{PM}}{2}\right)\left(\frac{1}{e}\right) \text{ (load reflected as inertia)} \tag{4.87}$$
$$J_T = J_M + J_{PM} + J_{PI} + J_B + J_R \text{ (total inertia)} \tag{4.88}$$
$$T_{ACC} = +J_T\theta''_M + T_R \tag{4.89}$$
$$T_{DEC} = -J_T\theta''_M + T_R \tag{4.90}$$
$$T_{RUN} = T_R \tag{4.91}$$
$$T_{SS} = (\pm(F_L - F_F) + F_G)\left(\frac{D_{PM}}{2}\right)\left(\frac{1}{e}\right) \tag{4.92}$$

For horizontal systems, $F_G = 0$

For vertical systems, use: $+F_G$ when accelerating "up" or decelerating "down"
$-F_G$ when accelerating "down" or decelerating "up"

Example

In a packaging machine, a 2268 g load is to be accelerated and decelerated between 0 and 122 cm s^{-1} in 0.4 s, with a total travel of 122 cm. A timing belt and pulley drive system is to be used, in which the pulleys are 5.1 cm in diameter, 15.2 cm long and made of steel. The belt weighs 2268 g. A lubricated dove tail slide structure, with a friction coefficient of 0.10 is to be used to support the load. The motor pulley will be integral to the motor shaft, eliminating the need for a coupling.

$$\begin{aligned}
J_{PM} &= \pi\,(7.81)\,(15.2)\left(2.54^4\right)/(2 \times 980.6) = 7.92 \text{ g cm s}^2 \\
J_{PI} &= 7.92 \text{ g cm s}^2 \\
J_B &= (2268/980.6)\left(2.54^2\right)(1/0.97) = 15.5 \text{ g cm s}^2 \\
J_R &= (22\,680/980.6)\left(2.54^2\right)(1/0.97) = 155 \text{ g cm s}^2 \\
\theta'_M &= \left(122 \text{ cm s}^{-1}\right)(\text{rev}/5.1\pi \text{ cm}) = 457 \text{ rpm} = 48 \text{ rad s}^{-1} \\
\theta''_M &= 48/0.4 = 120 \text{ rad sec}^{-2} \\
F_F &= (0.1)\,(22\,680) = 2268 \text{ g} \\
T_R &= (2268)\,(5.1/2)\,(1/0.97) = 5962 \text{ g cm} \\
\text{Initial inertia} &= 7.92 + 7.92 + 15.5 + 155 = 186 \text{ g cm s}^2 \\
\text{Initial } T_{ACC} &= (186)(120) + 5962 = 28\,282 \text{ g cm}
\end{aligned}$$

First Motor Choice: A NEMA 23 brushless servo motor with the following specs:

$$\begin{aligned}
T_{CONT} &= 11\,448 \text{ g cm} \quad T_{PEAK} = 34\,272 \text{ g cm} \\
N_{MAX} &= 5000 \text{ rpm} \quad R = 7.72\ \Omega \\
K_T &= 1691 \text{ g cm A}^{-1} \quad J_M = 0.173 \text{ g cm s}^2 \\
T_{ACC} &= +(0.173 + 7.92 + 7.92)(120) + (15.5 + 155)(120) + 5962 = +28\,344 \text{ g cm} \\
T_{DEC} &= -(0.173 + 7.92 + 7.92)(120) - (15.5 + 155)(120) + 5962 = -16\,420 \text{ g cm} \\
T_{RUN} &= 5962 \text{ g cm} \\
T_{SS} &= 0
\end{aligned}$$

In order to determine $T_{RMS,}$ the run time must be calculated from the specifications, that is, total travel is 122 cm, run velocity is 122 cm s^{-1} and acceleration and deceleration times are 0.4 s, therefore:

$$(0.5)(0.4)(122)(2) + (122)\,(t_{RUN}) = 122$$

$$t_{RUN} = 0.6 \text{ s}$$

$$T_{RMS} = \sqrt{\frac{28344^2(0.4) + 5962^2\,(0.6) + 16420^2(0.4)}{0.4 + 0.6 + 0.4}} = 17\,938 \text{ g cm}$$

This RMS torque is 50% larger than the continuous torque rating of the motor and therefore the motor is too small for this application even though the peak torque rating is larger than the acceleration and deceleration torques.

A new selection must be made based on the RMS torque requirement.

Second Motor Choice: A NEMA 34 brushless servo motor with the following specs:

$$
\begin{array}{ll}
T_{CONT} = 406 \text{ oz in} & T_{PEAK} = 1217 \text{ oz in} \\
N_{MAX} = 4500 \text{ rpm} & R = 1.7\ \Omega \\
K_T = 59.23 \text{ oz in A}^{-1} & J_M = 0.0053 \text{ oz in s}^2
\end{array}
$$

Although this selection has twice the inertia of the first choice, it is a small percentage of the total inertia and the torques will essentially remain the same as initially calculated. This motor can therefore be used for the application.

$$
\begin{array}{ll}
I_{ACC} = 28\,344/4265 = 6.7 \text{ A} & I_{DEC} = 16\,420/4265 = 3.8 \text{ A} \\
I_{RUN} = 5962/4265 = 1.4 \text{ A} & I_{RMS} = 17\,938/4265 = 4.2 \text{ A}
\end{array}
$$

4.9.6 Linear Motion – Rack and Pinion Drive

Rack and pinion systems typically have the motor + pinion stationary, with the rack and load moving; however, for some designs, especially those with long travel, it is often more expeditious to have the rack remain stationary and have the motor + pinion plus load mounted together and move (see Figure 4.79).

System Data:

$$
\begin{aligned}
J_M &= \text{motor inertia (g cm s}^2) \\
J_P &= \text{pinion inertia (g cm s}^2) \\
W &= \text{load weight (g)} \\
F_L &= \text{load force (g)} \\
F_G &= \text{gravity force} = W\text{(g)}
\end{aligned}
$$

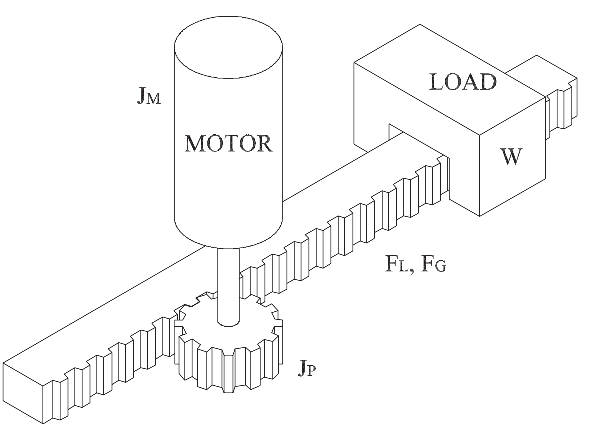

Figure 4.79 Linear motion; rack and pinion drive

$$F_F = \text{friction force} = \mu W(\text{g})$$
$$e = \text{system efficiency}$$
$$\mu = \text{friction coefficient}$$

Note: For the case in which the rack moves, the rack weight must be included in W.
For the case in which the motor moves, the motor + pinion weight must be included in W.
Motion:

$$\text{Position: } \theta_M = \frac{S}{D_P/2} \text{ (rad)}$$
$$\text{Velocity: } \theta'_M = \frac{S'}{D_P/2} \text{ (rad s}^{-1}\text{)}$$
$$\text{Acc/Dec: } \theta''_M = \frac{S''}{D_P/2} \text{ (rad s}^{-2}\text{)}$$

At Motor:

$$T_R = (\pm F_L + F_F \pm F_G)\left(\frac{D_P}{2}\right)\left(\frac{1}{e}\right) \text{ reflected torque} \tag{4.93}$$

$$J_R = \left(\frac{W}{980.6}\right)\left(\frac{D_P}{2}\right)^2\left(\frac{1}{e}\right) \text{ (load reflected as inertia)} \tag{4.94}$$

$$J_T = J_M + J_P + J_R \text{ (total inertia)} \tag{4.95}$$

$$T_{ACC} = +J_T\theta''_M + T_R \tag{4.96}$$

$$T_{DEC} = -J_T\theta''_M + T_R \tag{4.97}$$

$$T_{RUN} = T_R \tag{4.98}$$

$$T_{SS} = (\pm(F_L - F_F) + F_G)\left(\frac{D_P}{2}\right)\left(\frac{1}{e}\right) \tag{4.99}$$

For horizontal systems, $F_G = 0$

For vertical systems, use: $+F_G$ when accelerating "up" or decelerating "down"
$-F_G$ when accelerating "down" or decelerating "up"

Example

A 68 040 g table is to move a total of 396 cm at a velocity of 305 cm s^{-1}. during the constant velocity portion of the profile. The table will be mounted on roller bearing slides with a friction coefficient of 0.01. Acceleration and deceleration times will be 0.3 s. A pinion with a working diameter of 8 cm together with a rack rated at 224 532 g of dynamic thrust has been selected for preliminary consideration.

$$
\begin{aligned}
J_P &= 26 \text{ g cm s}^2 \text{ (catalog data)} \\
J_R &= \left(\frac{68\,040}{980.6}\right)(8/2)^2\,(1/0.97) = 1145 \text{ g cm s}^2 \\
\theta'_M &= \left(\frac{305 \times 2\pi}{\pi \times 8}\right) = 76.3 \text{ rad s}^{-1} = 728 \text{ rpm} \\
\theta''_M &= 76.3/0.3 = 254 \text{ rad sec}^{-2} \\
F_F &= 0.01 \times 68\,040 = 680 \text{ g} \\
T_F &= (680)\,(8/2)\,(1/0.97) = 2806 \text{ g} \\
\text{Initial inertia} &= 26 + 1145 = 1171 \text{ g cm s}^2 \\
\text{Initial } T_{ACC} &= (1171)\,(254) + 2806 = 300\,240 \text{ g cm}
\end{aligned}
$$

Motor Choice: A size 142 brushless servo motor with the following specs:

$$
\begin{aligned}
T_{CONT} &= 230\,400 \text{ g cm} \quad T_{PEAK} = 691\,200 \text{ g cm} \\
N_{MAX} &= 3000 \text{ rpm} \quad R = 0.34\ \Omega \\
K_T &= 11\,520 \text{ g cm A}^{-1} \quad J_M = 22.8 \text{ g cm s}^2 \\
T_{ACC} &= +(22.8 + 26)(254) + (1145)(254) + 2806 = 306\,031 \text{ g cm} \\
T_{DEC} &= -(22.8 + 26)(254) - (1145)(254) + 2806 = -300\,419 \text{ g cm} \\
T_{RUN} &= 2806 \text{ g cm} \\
T_{SS} &= 0
\end{aligned}
$$

In order to determine T_{RMS}, the run time must be calculated from the specifications, that is, total travel is 396 cm, run velocity is 305 cm s^{-1} and acceleration and deceleration times are 0.3 s, therefore:

$$(0.5)(0.3)(305)(2) + (305)\,(t_{RUN}) = 396$$

$$t_{RUN} = 1 \text{ s}$$

$$T_{RMS} = \sqrt{\frac{306\,031^2(0.3) + 2806^2(1) + 300\,419^2(0.3)}{0.3 + 1 + 0.3}} = 185\,707 \text{ g cm}$$

$$I_{ACC} = 306\,031/11\,520 = 26.6 \text{ A} \quad I_{DEC} = 300\,419/11\,520 = 26 \text{ A}$$

$$I_{RUN} = 2806/11\,520 = 0.25 \text{ A} \quad I_{RMS} = 185\,707/11\,520 = 16 \text{ A}$$

4.9.7 Linear Motion – Roll Feed Drive

In designing a roll feed system, four items that must be considered are:

1. The unsupported weight of the material being moved must be included with the inertia of the rollers.
2. The spool of material from which the material is being removed may have to be accelerated by the roll feed.
3. The force which is used to load the two rollers together can cause an increase in the bearing torque above the normal free wheeling bearing torque.
4. Due to mounting tolerances and material compression, this force will not be perfectly normal to the center line of the roller bearing axis, and will create an additional load torque.

See Figure 4.80.
System Data:

$$
\begin{aligned}
J_M &= \text{motor inertia (g cm s}^2) \\
J_{RM} &= \text{motor roller inertia (g cm s}^2) \\
J_{RP} &= \text{pinch roller inertia (g cm s}^2) \\
W &= \text{load weight (g)} \\
F_L &= \text{load force (g)} \\
F_F &= \text{friction force (g)} \\
T_P &= \text{pressure torque (g cm)} \\
T_B &= \text{bearing torque (g cm)} \\
D_{RM} &= \text{motor roller diameter (cm)} \\
J_{SUPPLY} &= \text{supply reel inertia (g cm s}^2)
\end{aligned}
$$

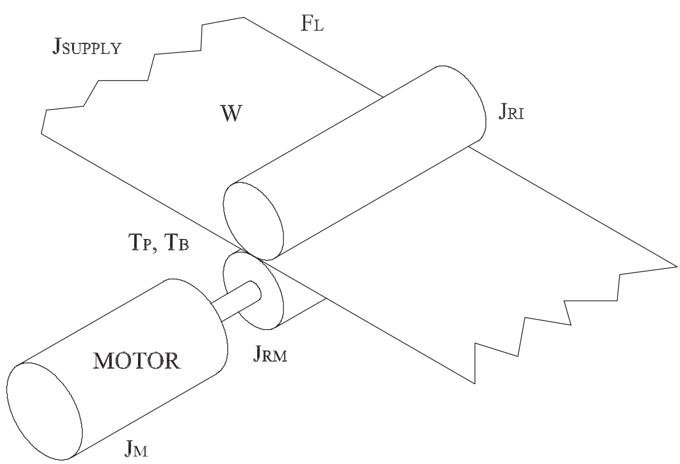

Figure 4.80 Linear motion; roll feed drive

Material weight reflected to motor as an equivalent inertia:

$$J_W = \left(\frac{W}{980.6}\right)\left(\frac{D_{RM}}{2}\right)\left(\frac{1}{e}\right)$$

Motion:

$$\text{Position: } \theta_M = \frac{S}{D_{RM}/2} \text{ (rad)}$$

$$\text{Velocity: } \theta'_M = \frac{S'}{D_{RM}/2} \text{ (rad s}^{-1}\text{)}$$

$$\text{Acc/Dec: } \theta''_M = \frac{S''}{D_{RM}/2} \text{ (rad s}^{-2}\text{)}$$

At Motor:

$$T_R = (F_L + F_F)\left(\frac{D_{RM}}{2}\right)\left(\frac{1}{e}\right) + T_P + T_B \tag{4.100}$$

$$J_T = J_M + J_{RM} + J_{RP} + J_W + J_{SUPPLY} \text{ (total inertia)} \tag{4.101}$$

$$T_{ACC} = +J_T\theta''_M + T_R \tag{4.102}$$

$$T_{DEC} = -J_T\theta''_M + T_R \tag{4.103}$$

$$T_{\mathrm{RUN}} = T_R \tag{4.104}$$

$$T_{SS} = (F_L - F_F)\left(\frac{D_{RM}}{2}\right)\left(\frac{1}{e}\right) + T_P - T_B \tag{4.105}$$

Example

A roll feed system is to be used to uncoil 30 480 cm feet of a thin film, 17.8 cm wide from a 15.2 cm diameter full reel weighing 2268 g at 61 cm s^{-1}. A hysteresis brake will be back driven to create 907 g of tension in the film. The capstan will be a rubber-coated aluminum cylinder 5.1 cm in diameter and 20.3 cm long. The pinch roll will be an aluminum cylinder 2.54 cm in diameter and 20.3 cm long. A force of 2268 g will be used to press the pinch roller against the capstan. It is anticipated that the 2268 g force will be off center no more than 0.318 cm. Total bearing torque is expected to be 432 g cm maximum. 30.5 cm of the film will be suspended between the supply roll and the roll feed system

$$J_{RM} = \frac{\pi(2.66)(20.3)(5.1/2)^4}{2 \times 980.6} = 3.6 \text{ g cm s}^2$$

$$J_{RP} = \frac{\pi(2.66)(20.3)(2.54/2)^4}{2 \times 980.6} = 0.22 \text{ g cm s}^2$$

$$J_{SUPPLY\ REEL} = (2268/2 \times 980.6)\left(7.62^2\right) = 67.1 \text{ g cm s}^2$$

$$T_P = (2268)(0.318) = 721 \text{ g cm}$$

$$T_B = 432 \text{ g cm}$$

$$
\begin{aligned}
F_L &= 907 \text{ g} \\
F_F &= 0 \text{(the film is suspended between the supply reel and the roll feed)} \\
T_R &= (907)\,(5.1/2) + 721 + 432 = 3466 \text{ g cm} \\
W &= (30.5/30\,480)(2268) = 2.3 \text{ g (weight of the 30.5 cm of film in suspension)} \\
J_W &= (2.3/980.6)\,(5.1/2)^2 = 0.015 \text{ g cm s}^2 \text{ (can be neglected)}
\end{aligned}
$$

Assume accelerate time = 1 s

$$
\begin{aligned}
S' &= 61 \text{ cm s}^{-1} \\
S'' &= 61 \text{ cm s}^{-2} \\
\theta'_M &= \left(61 \text{ cm s}^{-1}\right)(1 \text{ rev}/\pi \times 5.1 \text{ cm}) = 3.81 \text{ rev s}^{-1} = 24 \text{ rad s}^{-1} = 230 \text{ rpm} \\
\theta''_M &= 24 \text{ rad s}^{-2} \\
\text{Initial inertia} &= 3.6 + 0.22 + 67.1 = 71 \text{ g cm s}^2 \\
\text{Initial } T_{ACC} &= (71)(24) + 3466 = 5179 \text{ g cm}
\end{aligned}
$$

First Motor Choice: A size 16 brushless servo motor with the following specs:

$$
\begin{aligned}
T_{CONT} &= 4176 \text{ g cm} \quad T_{PEAK} = 12\,456 \text{ g cm} \\
N_{MAX} &= 5000 \text{ rpm} \quad R = 4.65\ \Omega \\
K_T &= 1574 \text{ g cm A}^{-1} \quad J_M = 0.036 \text{ g cm s}^2 \\
T_{ACC} &= +\,(0.036 + 3.6 + 0.22 + 67.1)\,(24) + 3466 = 5169 \text{ g cm} \\
T_{DEC} &= -\,(0.036 + 3.6 + 0.22 + 67.1)\,(24) + 3466 = 1763 \text{ g cm} \\
T_{RUN} &= 347 \text{ g cm} \\
T_{SS} &= 0
\end{aligned}
$$

T_{RMS} need not be calculated since it will take 8.3 min to unload the complete reel and therefore $T_{RMS} \approx T_{RUN}$

Note: The motor always supplies a "+" torque; during deceleration, the hysteresis brake, working against the motor, will cause the system to decelerate.

The motor will be operating far below its rated speed and, therefore, inefficiently.

A better approach would be to use a smaller motor combined with a 10:1 gearhead as follows.

Second motor choice: A size 16 brushless servo motor with the following specs:

$$
\begin{aligned}
T_{CONT} &= 1512 \text{ g cm} \quad T_{PEAK} = 4608 \text{ g cm} \\
N_{MAX} &= 5000 \text{ rpm} \quad R = 4.31\ \Omega \\
K_T &= 621 \text{ g cm A}^{-1} \quad J_M = 0.013 \text{ g cm s}^2
\end{aligned}
$$

Plus a matching size 16 gearhead with a 10:1 ratio

$$\begin{aligned}
\theta'_M &= 240 \text{ rad s}^{-1} = 2300 \text{ rpm} \\
\theta''_M &= 240 \text{ rad s}^{-2} \\
T_{ACC} &= +(0.013 + (3.6 + 0.22 + 67.1)(1/100))(240) + 3466/10 = +520 \text{ g cm} \\
T_{DEC} &= -(0.013 + (3.6 + 0.22 + 67.1)(1/100))(240) + 3466/10 = +173 \text{ g cm} \\
T_{RUN} &= 347 \text{ g cm} \\
T_{SS} &= 0 \\
I_{ACC} &= 520/621 = 0.84 \text{ A} \quad I_{DEC} = 173/621 = 0.3 \text{ A} \\
I_{RUN} &= 347/621 = 0.6 \text{ A} \quad I_{RMS} = 0.6 \text{ A}
\end{aligned}$$

4.9.8 Linear Motion – Linear Motor Drive

The latest linear motion mechanism, made practical and economically feasible by the advent of brushless motor technology, is the linear motor. It is essentially the linear dual of the rotary brushless motor and creates linear motion directly without the need for motion conversion mechanisms. As such, the basic equations are simply those determining the necessary forces required for the four parts of the profile. See Figure 4.81.

System Data:

$$\begin{aligned}
W &= \text{load weight (g)} \\
W_M &= \text{motor weight (g)} \\
F_L &= \text{load force (g)}
\end{aligned}$$

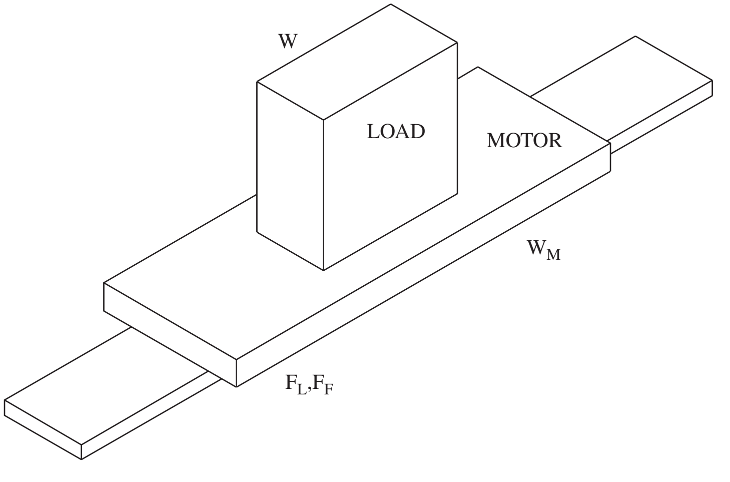

Figure 4.81 Linear motion; linear motor drive

$$F_F = \text{friction force (g)}$$
$$F_{MA} = \text{magnetic attraction force (iron core motors) (g)}$$
$$\mu = \text{friction coefficient}$$

At Motor:

$$F_F = \mu(W + W_M + F_{MA}) \tag{4.106}$$

$$F_{ACC} = +\left(\frac{W + W_M}{980.6}\right) S'' \pm F_L + F_F \tag{4.107}$$

$$F_{DEC} = -\left(\frac{W + W_M}{980.6}\right) S'' \pm F_L + F_F \tag{4.108}$$

$$F_{RUN} = \pm F_L + F_F \tag{4.109}$$

$$F_{SS} = \pm(F_L - F_F) \text{ if } F_L > F_F \tag{4.110}$$
$$= 0 \text{ if } F_L < F_F$$

Example

A 22 680 g load is to be oscillated back and forth over a 91 cm stroke with no dwell time.

Acceleration and deceleration are to be in 0.17 to and from a run velocity of 191 cm s^{-1} in a trapezoidal profile.

For initial calculations, assume that F_{MA} equals 226 800.

$$\text{Initial } F_F = (22\,680 + 226\,800)(0.03) = 7484 \text{ g}$$
$$S'' = 191/0.17 = 1124 \text{ cm s}^{-2}$$
$$\text{Initial } F_{ACC} = (22\,680/980.6)\,(1124) + 7484 = 33\,481 \text{ g}$$

Select a linear motor with the following specs:

$$F_{CONT} = 38\,102 \text{ g} \quad F_{PEAK} = 139\,255 \text{ g}$$
$$R = 4.1\ \Omega \quad W_M = 4536 \text{ g}$$
$$F_C = 4854 \text{ g A}^{-1} \quad F_{MA} = 362\,880 \text{ g}$$
$$F_F = (22\,680 + 4536 + 362\,880)\,(0.03) = 11\,703 \text{ g}$$
$$F_{ACC} = +\left(\frac{22\,680 + 4536}{980.6}\right)(1124) + 11\,703 = +42\,900 \text{ g}$$
$$F_{DEC} = -\left(\frac{22\,680 + 4536}{980.6}\right)(1124) + 11\,703 = -19\,493 \text{ g}$$
$$F_{RUN} = 11\,703 \text{ g}$$

Calculate the run time in order to calculate F_{RMS}

$$(0.5)(0.17)(191)(2) + (191)(t_{RUN}) = 91$$

$$t_{RUN} = 0.3\ \text{s}$$

$$F_{RMS} = \sqrt{\frac{42\,900^2(0.17) + 11\,703^2(0.3) + 19\,493^2(0.17)}{0.17 + 0.3 + 0.17}} = 25\,573\ \text{g}$$

$$I_{ACC} = 42\,900/4854 = 8.8\ \text{A} \quad I_{DEC} = 19\,493/4854 = 4\ \text{A}$$

$$I_{RUN} = 11\,703/4854 = 2.4\ \text{A} \quad I_{RMS} = 25\,573/4854 = 5.3\ \text{A}$$

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