Question 1

A competitive equilibirum is a set of prices, $\{w_t, r_t\}_{t=0}^{\infty}$ and allocations $\{c_t, k_{t+1}, l_t\}_{t=0}^{\infty}$ such that

1. Given prices, $\{w_t, r_t\}_{t=0}^{\infty}$, the allocation, $\{c_t, k_{t+1}, l_t\}_{t=0}^{\infty}$, solves

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$\text{s.t. } c_t + k_{t+1} \leq r_t k_t + w_t l_t, \forall t \tag{2}$$

$$c_t, k_t, l_t \geq 0, k_0 \text{ given.} \tag{3}$$

2. Given prices, $\{w_t, r_t\}_{t=0}^{\infty}$, the allocation $\{k_{t+1}, l_t\}_{t=0}^{\infty}$, solves

$$\max_{l_t, k_t} \sum_{t=0}^{\infty} (y_t - w_t l_t - r_t k_t) \tag{4}$$

$$\text{s.t. } y_t = F(k_t, l_t) \tag{5}$$

$$y_t, k_t, l_t \geq 0 \tag{6}$$

3. Markets clear: $l_t = 1$ and $c_t + k_{t+1} = F(k_t, 1)$

Question 2

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{7}$$

$$\text{s.t. } k_{t+1} = F(k_t, 1) - c_t, \tag{8}$$

$$k_t \geq 0, c_t \geq 0, k_0 \text{ given,} \tag{9}$$

Question 3

As leisure is not in the utility function, consumers allocate all time to work, $l_t = 1$, for the competive equilibrium and the social planners problem.

Competitive Equilibrium:

1. Firms problem: Sub in production constraint and take the FOC with respect to capital in period t to obtain

$$r_t = F_k(k_t, 1) \tag{10}$$

2. Consumers problem: Sub in the constraint for consumption and take FOC with respect to capital in period t to obtain

$$u_c(c_t) = \beta u(c_{t+1}) r_t \tag{11}$$

   Now sub in (10) for r_t to obtain

$$u_c(c_t) = \beta u(c_{t+1}) F_k(k_t, 1) \tag{12}$$

Social Planners Problem: Sub the constraint in and take the FOC with respect to capital to directly obtain

$$u_c(c_t) = \beta u(c_{t+1}) F_k(k_t, 1), \tag{13}$$

which is the same equilibrium condition as the competitive case, hence the allocatuions for $\{c, k, l\}$ are the same in both cases. (maybe write only in terms of k, then mention budget/ feasiblitly implies consumption is same in both cases)

1 Question 4

The planners dynamic programming problem is,

$$V(k_t) = \max_{k_{t+1}} u(F(k_t, 1) - k_{t+1}) + \beta V(k_{t+1}) \tag{14}$$

$$\text{s.t. } k_t \geq 0, F(k_t, 1) - k_{t+1} \geq 0, k_0 \text{ given}, \tag{15}$$

2 Question 5

$$V(k_t) = \max_{k_{t+1}} log(z k_t^{\alpha} - k_{t+1}) + \beta V(k_{t+1}) \tag{16}$$

$$\text{s.t. } k_t \geq 0, z k^{\alpha} - k_{t+1} \geq 0, k_0 \text{ given}, \tag{17}$$

Take FOC w.r.t k_{t+1} to obtain

$$d \tag{18}$$