Investigation Into The Lifetime of Muons

PHAS0051 Formal Report

Lucas Curtin

Aidan Bartholomew Esther Liubarski

Word Count:2734

UCL
Physics Department
UCL Physics Building Lab 3, London
January 2022

***Abstract*—Muons are born via cosmic rays colliding with the Earth's atmosphere. Positive muons decay only via the weak force, so their mean lifetime is an indication of the strength of this force. Negative muons however, can also decay via muon capture. This experiment seeks to separate the lifetimes of these charged subatomic particles by analysing their interactions with a plastic scintillator over large portions of time.**

***Index Terms*—Muons, Weak Force, Decay, Exponential Distribution, Scintillators**

I. INTRODUCTION

High energy protons known as cosmic rays interact with the air nuclei in the Earth's atmosphere and create hadronic showers as seen in Figure 1.

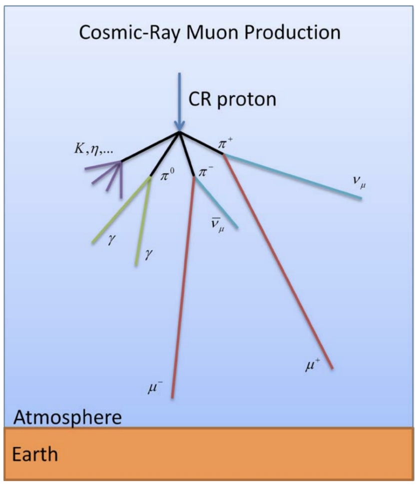

Fig. 1. Diagram of muon production from cosmic rays colliding with air molecules in the upper atmosphere[1].

These hadronic showers lead to the production of positive and negative pions. The positive pions π^+ then decay into positive muons μ^+. These positive muons will then decay as seen in (2).

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{1}$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu_\mu} \tag{2}$$

Where ν_e, ν_μ, $\bar{\nu_\mu}$ and e^+ represent an electron neutrino, a muon neutrino, a anti muon neutrino, and a poistron respectively. The positron is the positive counterpart of the electron e^-. The negative muon is also produced in an analogous decay from the negative pion, π^-, as seen in (3).

$$\pi^- \rightarrow \mu^- + \bar{\nu_\mu} \tag{3}$$

The negative muon μ^- however has two paths of decay.

$$\mu^- + p \rightarrow n + \nu_\mu \tag{4}$$

$$\mu^- \rightarrow e^- + \bar{\nu_e} + \nu_\mu \tag{5}$$

The path of decay in equation (4) is when negative muons terminate by a charge-exchange reaction with a proton in a nucleus of a substance they are moving through. However, the weak force is responsible for the decays in equations (2) and (5). Since positive muons only have one path of decay (which is via the weak force), the strength of the weak force will directly determine their mean lifetime. The negative muons, since they have two paths of decay, will have a lower mean lifetime which is dependent on the weak force and the number of protons in the material it is moving through (this is known as the Z number). The aim of this experiment was to calculate the lifetime of free muons. This lifetime can then be used to determine the strength of the fundamental weak force, which is the mechanism for free muon decay, by calculating the Fermi Coupling Constant,G_f, using equation (6)[2].

$$\frac{1}{\tau_\mu} = \frac{{G_f}^2 {m_\mu}^5}{192\pi^3}(1 + \Delta q) \tag{6}$$

Where τ_μ, m_μ, and Δq represent the positive muon lifetime, the muon rest mass and a Quantum Electrodynamics (Q.E.D.) correction respectively. The latter correction was not be considered so the $(1+\Delta q)$ term was just set to 1. This was be achieved by slowing and stopping muons in a scintillator which will cause excitations when muons enter and also when they decay inside of it. These excitations will cause photons to be emitted from the scintillator which will provide a series of signals which can be analysed to determine the time frame at which the detected muons decay. The measured muons will be produced approximately 15km above sea-level[3] so will live out a portion of their lifetimes before they reach the scintillator. Their lifetimes however follow an exponential distribution which exhibit memorylessness, where the past has no bearing on the future. The exponential distribution can be measured over any portion and will still return information about the whole distribution.

II. METHOD

When a muon enters the scintillator, it is slowed down which causes a constant stream of photons to be released as it transfers energy. This signal of photons stops when the muon finally stops or exits the scintillator. Muons that stop in the scintillator will shortly decay after which will in turn release another photon. When a photon is released from the scintillator as seen in Figure 2, the photomultiplier tube (P.M.T.) turns it into an electrical signal. The photon hits a photocathode in the P.M.T. which causes an electron to be released. This electron is directed by the focusing electrode where it is then accelerated to a series of dynodes. The electrons are exponentially multiplied by a process of secondary emission at each dynode. The scale by which they are multiplied is controlled by increasing the potential difference at each dynode with the High Voltage (H.V.) adjust, seen in Figure 3. The signal from the P.M.T. goes through a two stage amplifier before it goes through the discriminator. The discriminator removes any signals below a certain voltage. It then goes through the F.P.G.A. (Field Programmable Gate Array) which determines the time between two signals.

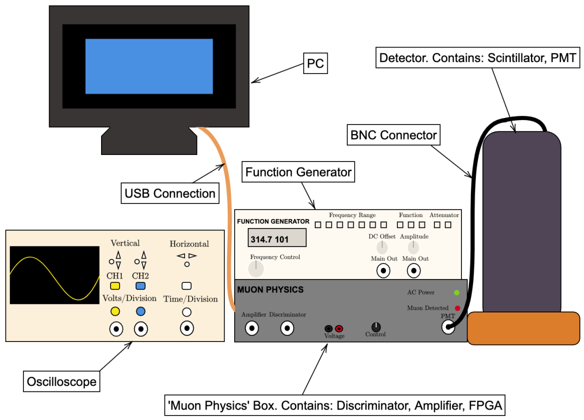

Fig. 2. Diagram of experimental setup including the detector which contains the P.M.T. and the plastic scintillator.

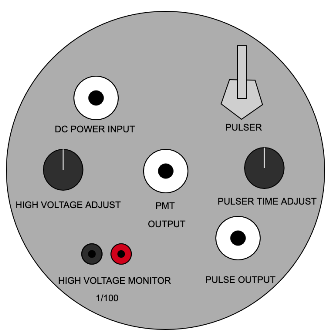

Fig. 3. Diagram of detector settings for both the P.M.T. and the L.E.D. pulser

The gain of the amplifier is not constant over all frequencies. The function generator was used to generate a signal of known voltage through the amplifier which was then analysed on the oscilloscope. As seen in Figure 4, there is a range of frequencies where the gain is constant (between 10kHz$\rightarrow$ 315 kHz). This limited the data that could be reliably used and the range of muon lifetimes that could be measured was 3.17μs$\rightarrow$ 100μs. The gain was then calculated from 3 frequency values within this range: 15,165 and 315 kHz. The gain of the amplifier was equal to 10.2. However, due to attenuation resistors inside the 'Muon Physics' box inserted between the amplifier output and the front panel connector, the gain must be multiplied by the factor $\frac{1050}{5021}$. The final amplifier gain was equal to 214.2. The FPGA was then calibrated so that it correctly outputted the timescale of the signal received from the discriminator. The discriminator output was connected to the oscilloscope and the frequency of the LED pulses was varied using the 'PULSER TIME ADJUST' seen in Figure 3. A straight line of gradient 1 between the two time intervals

TABLE I
PARAMETERS OF THE LINEAR RELATIONSHIP BETWEEN THE FPGA TIME AND THE OSCILLOSCOPE TIME.

Parameters	Gradient	Y-intercept, s
Value	(1.001 ± 0.003)	$(0.3 \pm 0.4) \times 10^{-7}$

is necessary to accept that the values produced by the FPGA are correct. The y-intercept of this line should equal 0 if there is no delay between the two signals. There was found to be a linear relationship for time intervals below 20 µs. This was because the FPGA takes 20 µs to update between two signals. This overclocking time was chosen to reduce the number of coincidental muons passing through the detector that would be registered as one muon entering and then decaying. The values above 20 µs were then removed to test the linearity between the FPGA time interval and the oscilloscope time interval. This was determined by using the python library Numpy and its polyfit function. This was plotted as shown in Figure 5. The parameters of the line can be seen in Table I. The gradient of the line satisfied the 1:1 relationship between the two time intervals. The y-intercept was also negligibly small and was taken as 0, so there was no significant delay between the two signals.

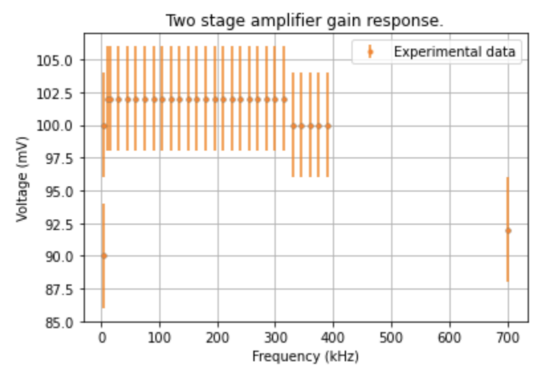

Fig. 4. Plot of the voltage response from the oscilloscope of the amplified signal over a range of frequencies

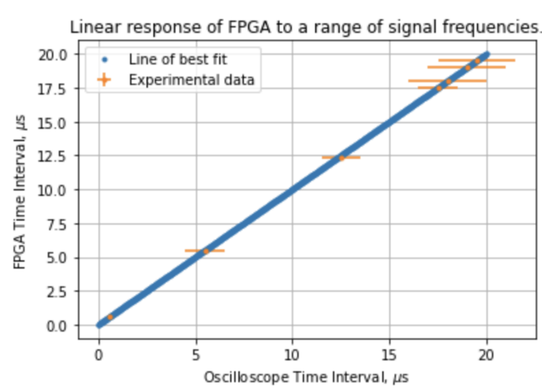

Fig. 5. FPGA linearity

TABLE II
CALIBRATION VALUES FOR THE H.V. ADJUST AND DISCRIMINATOR FOR THE CIRCUIT.

Settings	H.V. Adjust, V	Discriminator, V
Value	0.54	0.6

The discriminator value was also calibrated so that it was large enough to tune out noise generated by fluctuations in the circuit but small enough that it didn't tune out the amplified signals. An expected intensity flux for vertical muons was used to determine the values: $I = 1\text{cm}^{-2}\text{min}^{-1}$[4]. This is a lower-end estimate of the number of muons entering the detector per second. One should expect a higher number of detections due to muons that have entered into the scintillator from any horizontal angle. These horizontal muons however are less likely to decay inside the scintillator because they will pass through less of the scintillator so will more frequently just pass straight through without decaying. The discriminator was set so the rate of detections was lower than the expected value of vertical muon detections per second. This meant less data would be gathered, but the data that was collected was certainly from excitations of the scintillator. The H.V. adjust seen in Figure 3 was increased so the signal strength from the muon detections did not fall under the discriminator value. Noise due to electrical fluctuations in the circuit (not affected by the H.V. adjust) was removed by a high discriminator value. The discriminator was set to the maximum value, and the H.V. adjust was varied to find a region where the rate of incident muons was less than the expected value calculated above. This ensured that all muons detected by the equipment were caused by scintillation excitations and not noise from the circuit. The voltage values for the discriminator and the H.V. adjust were determined by connecting a multimeter to the respective positive and negative voltage terminals as seen in Figure 2 and Figure 3.

III. PROVISIONAL RESULTS, ANALYSIS AND DISCUSSION

There were three separate occasions of data taking for this experiment. They lasted 255, 23, and 21 hours. They were all ran with the same discriminator and H.V. combination values from Table II. This allowed for the data from all of the different sessions to be combined into one large data set. This data set was placed into bins and plotted as a histogram as seen in Figure 6. For the maximum number of data points, the number of bins should be maximised thus decreasing the size of each bin. This could be done until the theoretical limit of each bin width: the resolution of the FPGA, $20\,\text{ns}$. However, to approximate the error on each bin using the Central Limit Theorem - the frequency in each bin must be more than equal to 5. The number of bins were altered by hand until the minimum frequency in any one bin was equal to 5. The error in each bin was then estimated by using $\sqrt{N}$ [5, p. 30] where N represents the frequency in the bin. The bins could have been combined so that they'd satisfy this requirement however all the bins were kept at equal width to aid with the following noise calculation. The decay of a single muon is a Poisson process and the distribution of the muon lifetimes can be modelled as an exponential distribution as seen in (7)[[6]].

$$f(t) = \lambda e^{-\lambda t} \tag{7}$$

Where λ is the reciprocal of the mean muon lifetime, t_{μ}. By analysing equation (7), it would be expected that the proportion of muons decaying in their first lifetime to be $\frac{1}{e}$. This is known as the e-folding time and is constant for the exponential distribution. Again, this is why the mean muon lifetimes can be determined despite them living out a portion of their lives outside of the scintillator. After each time a muon lifetime passes, it'd be expected for the proportion of muons surviving to keep decreasing by a factor of $\frac{1}{e}$. This can be extended to multiple lifetimes. After 5 lifetimes it is expected that there would be $\frac{1}{e}^5 \approx 6.7 \times 10^{-3}$ of muons would survive or in other words: over 99% of the muons to have decayed. There was however a significant presence in the later bins after 5 lifetimes when there should have been close to no muons. This was likely because these bins were largely comprised of the noise that was found in the data. The average of these bins was used to find the mean offset of noise in the data. All the bins were made with equal width so the offset found in one bin could then be subtracted from the others. The sources of this noise could include:

- Thermal fluctuations in the scintillator, causing occasional photons to be released randomly.
- Light leaks in the equipment setup.
- Spurious coincidences from possible radioactive particles in the environment.
- Coincidental muons.

It was assumed due to the random nature of these errors that they are equally distributed across the bins. The vertical offset due to noise was found to be 7.2 ± 1.2. This was removed from the frequency of each bin before plotting and the error combined with the previous error found with the C.L.T. approximation.

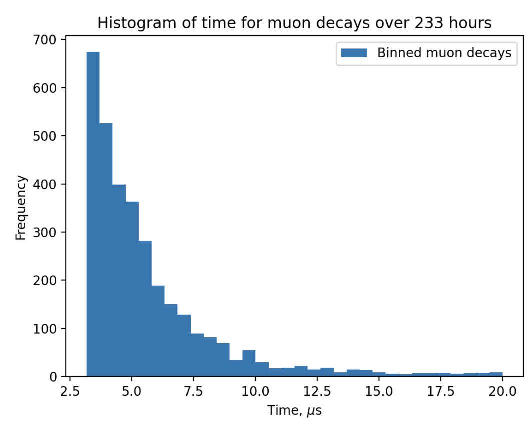

Fig. 6. Histogram of muon decays before noise correction from later bin analysis

TABLE III
FITTING PARAMETERS OF MODEL ON HISTOGRAM.

Parameters	A	$\tau+$
Value	$(1.468 \pm 0.005) \times 10^3$	$(2.22 \pm 0.06)\mu s$

A fitting of the histogram to this exponential function would return a λ of a combination of the lifetime of the positive and negative muons which have different mean lifetimes. As mentioned previously, this lowering in mean lifetime is due to the muon capture of the negative muons depending on the material they are moving through. The rate at which the muons are captured is proportional to Z^4[7] where Z is the number of protons of the atoms of the material. The scintillator is made of plastic, which is largely made up of CH2 chains so the mean lifetime of negative muons in carbon can be used as an approximation of their mean lifetimes in the scintillator. The negative mean lifetimes of negative muons in carbon is (2.043 ± 0.003) µs[8]. There is also not an equal amount of positive and negative muons reaching the ground from the atmosphere. The charge ratio of atmospheric muons is (1.2766 ± 0.0064) and as there are less negative muons with their lower mean lifetimes therefore reaching the scintillator, the mean lifetime of the two is decreased by a smaller amount. The decay distributions of the positive and negative muons were then combined as seen in equation (8)[9].

$$f(t) = A(1.2766e^{-t/\tau+} + e^{-t/2.043}) \tag{8}$$

Where A represents a scaling constant for the distribution. The frequency for each bin center was then modelled as the exponential decay function in (8) using the curve fit function from the python library Scipy. These values were found to be:

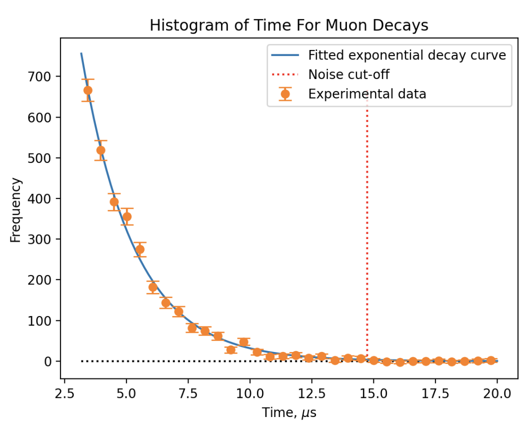

Fig. 7. Exponential fit to bin centers and corresponding frequencies of muon decay histogram

This model was plotted over the histogram bin centers as seen in Figure 7 after the noise was removed. The red dotted line indicates the point ($\approx$ 5.2 muon lifetimes) where any muon decays in the bins measured after that were taken to be made up of noise. The χ^2 statistic was then be calculated to see how the observed experimental data compares with the expected values by using equation (9).

TABLE IV
GOODNESS OF FIT STATISTICS OF EXPONENTIAL MODEL.

Statistic	*Value*
χ^2	27.76
χ^2_ν	0.93
p-value	0.58
ν	30

TABLE V
COMPARISON OF VALUES OF FERMI'S COUPLING CONSTANT, $\frac{G_f}{(\hbar c)^3}$.

Calculated value	$(1.16 \pm 0.02) \times 10^{-5}\,\mathrm{GeV}^2$
N.I.S.T. value	$(1.166\,378\,7 \pm 0.000\,000\,6) \times 10^{-5}\,\mathrm{GeV}^2$

$$\chi^2 = \sum_i (\frac{(O_i - E_i)}{\Delta O_i})^2 \tag{9}$$

Where O represents the observed values, E represents the expected values and ΔO represents the error in the observed values. The expected values for each bin were calculated by inputting the time value at the centre of the bin into (8).The p-value for this χ^2 statistic was be calculated using the Scipy. Stats python library's survival function for the χ^2 distribution. This value represents the probability of finding the χ^2 statistic at a value equal or larger than the one calculated, where a probability ≈ 0.5[5, p. 107] suggests a good fit. The reduced χ^2_ν statistic was also calculated to see how good the fit is with reference to the number of fitting parameters by using equation (10).

$$\chi^2_\nu = \frac{\chi^2}{\nu} \tag{10}$$

Where ν is the degrees of freedom which is the number of bins (32 after combination) minus the number of fit parameters. These values are shown in table IV. The produced χ^2 was close to the number of degrees of freedom which is expected for a good fitted model[5, p. 106]. The produced χ^2_ν was ≈ 1 which again suggests the fit is reasonable[5, p. 107]. The p-value for the χ^2 statistic is suitably close to 0.5 which is another indication of a good fit.

IV. SUMMARY OF FINDINGS

The calculated mean lifetime of the free decay of muons, $\tau_+ = (2.22 \pm 0.06)\mu s$, does agree with accepted values: $\tau_+ = (2.1969811 \pm 0.0000022)\mu s$[10]. With this mean lifetime for free muon decay, Fermi's coupling constant can be calculated with equation (6). The physical constants used in the equation and their associated uncertainties were from National Institute of Standards and Technology[10]. The calculated value also agrees with accepted values also from N.I.S.T as seen in table V.

V. CONCLUSION

The calculated mean lifetime of free decay of muons was within the accepted range however still higher than expected. This is possibly due an incorrect ground charge ratio between the positive and negative muons. This experiment was completed on the second floor of a building with multiple floors above it, so it is possible that more negative muons would have been captured before reaching the scintillator. A higher proportion of positive muons decaying in the scintillator would mean a higher mean lifetime between the two, possibly explaining the higher value for the mean lifetime of the positive muons found. The lifetime of negative muons in carbon being used as an approximation could also lead to a different result. As previously mentioned, plastic is largely made up of CH2 chains rather than being a pure carbon based material so could have an effective Z number closer to 5 than 6 as was used previously. To have a better understanding of the sources of noise for this experiment, future experiments could utilise multiple scintillators. Three scintillators vertically stacked on top of each other could allow for distinguishing between coincidental and muons decaying. If a muon was detected to pass through all three, then it would be known it did not decay in the second scintillator and if one was detected in only the first two then it's likely that it decayed in the second scintillator. If this experiment was to be repeated, a lower H.V. value should be used. The max discriminator value was used during this experiment and so a large H.V. value had to be used but this meant the P.M.T. would be more sensitive to possible radioactive particles passing through it. A lower discriminator value should be chosen that still eliminates the electrical noise so a lower H.V. value can then be used. The experiment should also be ran for longer periods of time as to allow for smaller equal bin widths that would still have a large enough frequency for the C.L.T. approximation but would also provide more fitting points for the model.

REFERENCES

[1] J. Perry, *Advanced applications of cosmic-ray muon radiography*. ProQuest Dissertations Publishing, 2013.

[2] T. van Ritbergen and R. G. Stuart, "On the precise determination of the fermi coupling constant from the muon lifetime," *Nuclear Physics B*, vol. 564, no. 3, pp. 343–390, 2000.

[3] Berkeley Lab, "Cosmic ray muons." http://www2.lbl.gov/abc/cosmic/SKliewer/CosmicRays/Muons.htm,(accessed: 21.11.2021).

[4] P. D. Group, "Cosmic rays," 2021. https://pdg.lbl.gov/2020/reviews/rpp2020-rev-cosmic-rays.pdf,(accessed: 21.11.2021).

[5] I. Hughes and T. Hase, *Measurements and their uncertainties: a practical guide to modern error analysis*. OUP Oxford, 2010.

[6] School of Physics and Astronomy, Queen Mary University of London, "Scintillator cosmic ray experiments into atmospheric muons." Accessed: 1-1-2022.

[7] P. Kammel and K. Kubodera, "Precision muon capture," *Annual Review of Nuclear and Particle Science*, vol. 60, no. 1, pp. 327–353, 2010.

[8] R. Reiter, T. Romanowski, R. Sutton, and B. Chidley, "Precise measurements of the mean lives of $\mu+$ and $\mu-$ mesons in carbon," *Physical review letters*, vol. 5, no. 1, pp. 22–23, 1960.

[9] L. Liu, "The speed and lifetime of cosmic ray muons." Accessed: 1-1-2022.

[10] P. D. Group, P. A. Zyla, and et al., "Review of Particle Physics," *Progress of Theoretical and Experimental Physics*, vol. 2020, 08 2020. 083C01.