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# 多传感器融合讲评

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# 及格:补全代码,且功能正常。

- 一些注意点
  - 作业中的lidar_localization/slam_data下的地图有些问题，大家可以使用之前课程建立的地图
  - lidar_localization/config/scan_context 中的文件有问题。需要使用上一章的文件。
  - lidar_localization/config/matching/matching.yaml 中的 scan_context_path记得改一下。默认的不对

# 及格:补全代码,且功能正常。

```
▼ /home/wangzhiyong/sensor_ws/src/lidar_localization/include/lidar_localization/models/sliding_window/factors/factor_prvag_imu_pre_integration.hpp 4
    (79, 8) // TODO: get square root of information matrix:
    (86, 8) // TODO: compute residual:
    (97, 8) // TODO: compute jacobians:
    (164, 8) // TODO: correct residual by square root of information matrix:
▼ /home/wangzhiyong/sensor_ws/src/lidar_localization/include/lidar_localization/models/sliding_window/factors/factor_prvag_map_matching_pose.hpp 4
    (53, 8) // TODO: get square root of information matrix:
    (60, 8) // TODO: compute residual:
    (68, 8) // TODO: compute jacobians:
    (83, 8) // TODO: correct residual by square root of information matrix:
▼ /home/wangzhiyong/sensor_ws/src/lidar_localization/include/lidar_localization/models/sliding_window/factors/factor_prvag_marginalization.hpp 9 item
    (48, 8) // TODO: Update H:
    (54, 8) // TODO: Update b:
    (75, 8) // TODO: Update H:
    (88, 8) // TODO: Update b:
    (111, 8) // TODO: Update H:
    (135, 10) // TODO: implement marginalization logic
    (150, 8) // TODO: shall we improve numeric stability following VIO/LIO-mapping's practice?
    (183, 8) // TODO: compute residual:
    (189, 8) // TODO: compute jacobian:
▼ /home/wangzhiyong/sensor_ws/src/lidar_localization/include/lidar_localization/models/sliding_window/factors/factor_prvag_relative_pose.hpp 4 items
    (58, 8) // TODO: get square root of information matrix:
    (65, 8) // TODO: compute residual:
    (73, 8) // TODO: compute jacobians:
    (102, 8) // TODO: correct residual by square root of information matrix:
▼ /home/wangzhiyong/sensor_ws/src/lidar_localization/include/lidar_localization/models/sliding_window/params/param_prvag.hpp 1 item
    (49, 12) // TODO: evaluate performance penalty of applying exp-exp-log transform for each update
▼ /home/wangzhiyong/sensor_ws/src/lidar_localization/src/matching/back_end/sliding_window.cpp 5 items
    (315, 12) // TODO: add init key frame
    (318, 12) // TODO: add current key frame
    (336, 12) // TODO: add constraint, GNSS position:
    (356, 12) // TODO: add constraint, lidar frontend / loop closure detection:
    (366, 16) // TODO: add constraint, IMU pre-integraion:
```

# 及格:补全代码,且功能正常。

```
/home/wangzhiyong/sensor_ws/src/lidar_localization/src/models/sliding_window/ceres_sliding_window.cpp 10 items
    (247, 12) // TODO: create new sliding window optimization problem:
    (250, 12) // TODO: a. add parameter blocks:
    (255, 16) // TODO: add parameter block:
    (263, 12) // TODO: add residual blocks:
    (311, 12) // TODO: b.2. map matching pose constraint:
    (320, 20) // TODO: add map matching factor into sliding window
    (329, 12) // TODO: b.3. relative pose constraint:
    (339, 20) // TODO: add relative pose factor into sliding window
    (348, 12) // TODO: b.4. IMU pre-integration constraint
    (358, 20) // TODO: add IMU factor into sliding window
```

# Relative Pose from Lidar Frontend

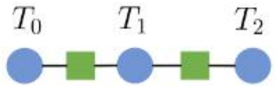
ResidualBlock

(1).Relative Pose from Lidar Frontend

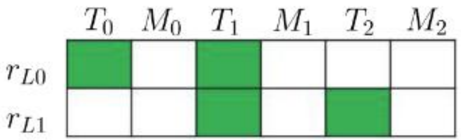
3）激光里程计相对位姿和优化变量的残差

该残差对应的因子为激光里程计因子。

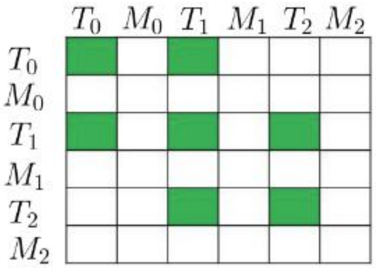
一个因子约束两个位姿，其模型如下：




残差关于优化变量的雅可比，可视化如下：




因此，对应的Hessian矩阵可视化为：



# Relative Pose from Lidar Frontend

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3.b. relative pose from lidar frontend.

Residual:

$$\because T_i^{-1} T_j = \begin{bmatrix} R_i^T, & -R_i^T t_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_j, & t_j \\ 0, & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_i^T R_j, & R_i^T (t_j - t_i) \\ 0, & 1 \end{bmatrix}$$

$$\therefore r_p = R_i^T (t_j - t_i) - t_{obs}$$

# Relative Pose from Lidar Frontend

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$$r_q = \ln(R_i^T R_j R_{obs}^T)^\vee$$

Jacobian, pos:

$$\frac{\partial r_p}{\partial t_i} = -R_i^T$$

$$\frac{\partial r_p}{\partial t_j} = R_i^T$$

Jacobian, ori:

$$\frac{\partial r_q}{\partial R_i} = \lim_{\phi \to 0} \frac{\ln[\exp(-\phi^\wedge) R_i^T R_j R_{obs}^T]^\vee - \ln[R_i^T R_j R_{obs}^T]^\vee}{\phi}$$

$$= \lim_{\phi \to 0} \frac{\ln[R_i^T R_j R_{obs}^T \cdot \exp(-R_{obs} R_j^T R_i \phi)^\wedge]^\vee - \ln(R_i^T R_j R_{obs}^T)^\vee}{\phi}$$

$$= -J_r^{-}(r_q)\, \exp(r_q^\wedge).\mathrm{inverse}()$$

# Relative Pose from Lidar Frontend

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$$\frac{\partial r_q}{\partial R_j} = \lim_{\phi \to 0} \frac{\ln\left(R_i^T R_j \exp(\phi^\wedge) R_{obs}^T\right)^\vee - \ln\left(R_i^T R_j R_{obs}^T\right)^\vee}{\phi}$$

$$= \lim_{\phi \to 0} \frac{\ln\left[R_i^T R_j R_{obs}^T \cdot \exp\left((R_{obs}\phi)^\wedge\right)\right]^\vee - \ln\left(R_i^T R_j R_{obs}^T\right)^\vee}{\phi}$$

$$= J_r^{-1}(r_q) R_{obs}$$

$J_r$ should be implemented.

# Relative Pose from Lidar Frontend

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```
//
// TODO: get square root of information matrix:
//
Eigen::Matrix<double, 6, 6> sqrt_info = Eigen::LLT<Eigen::Matrix<double, 6, 6>>(
    I_
).matrixL().transpose();

//
// TODO: compute residual:
//
Eigen::Map<Eigen::Matrix<double, 6, 1>> residual(residuals);

residual.block(INDEX_P, startCol: 0, blockRows: 3, blockCols: 1) = ori_i.inverse() * (pos_j - pos_i) - pos_ij;
residual.block(INDEX_R, startCol: 0, blockRows: 3, blockCols: 1) = (ori_i.inverse()*ori_j*ori_ij.inverse()).log();

//
```

# Relative Pose from Lidar Frontend

```
// TODO: compute jacobians:
//
if ( jacobians ) {
  // compute shared intermediate results:
  const Eigen::Matrix3d R_i_inv = ori_i.inverse().matrix();
  const Eigen::Matrix3d J_r_inv = JacobianRInv( w: residual.block(INDEX_R,  startCol: 0,  blockRows: 3,  blockCols: 1));

  if ( jacobians[0] ) {
    Eigen::Map<Eigen::Matrix<double, 6, 15, Eigen::RowMajor>> jacobian_i( dataPtr: jacobians[0] );
    jacobian_i.setZero();

    jacobian_i.block<3, 3>(INDEX_P, INDEX_P) = -R_i_inv;
    jacobian_i.block<3, 3>(INDEX_R, INDEX_R) = -J_r_inv*(ori_ij*ori_j.inverse()*ori_i).matrix();

    jacobian_i = sqrt_info * jacobian_i;
  }

  if ( jacobians[1] ) {
    Eigen::Map<Eigen::Matrix<double, 6, 15, Eigen::RowMajor>> jacobian_j( dataPtr: jacobians[1]);
    jacobian_j.setZero();

    jacobian_j.block<3, 3>(INDEX_P, INDEX_P) = R_i_inv;
    jacobian_j.block<3, 3>(INDEX_R, INDEX_R) = J_r_inv*ori_ij.matrix();

    jacobian_j = sqrt_info * jacobian_j;
  }
```

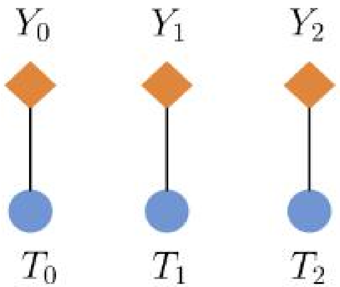
# 及格:补全代码,且功能正常

## (2).Map Matching

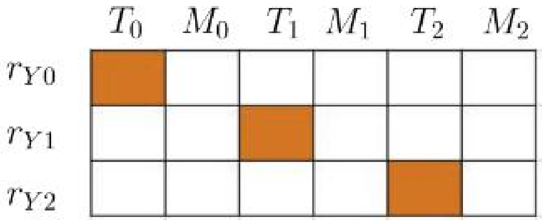
2）地图匹配位姿和优化变量的残差

该残差对应的因子为地图先验因子。

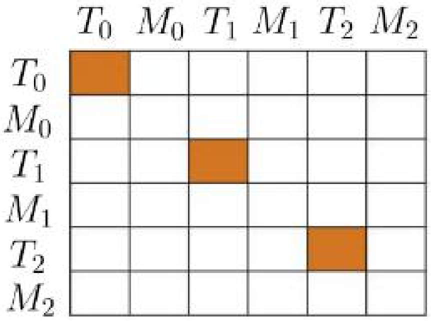
一个因子仅约束一个位姿，其模型如下：




残差关于优化变量的雅可比，可视化如下：




因此，对应的Hessian矩阵的可视化为：



# Map Matching

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3. a. pose from map matching / GNSS position

Residual:

$$r_p = t - t_{obs}$$

$$r_q = \ln(R \cdot R_{obs}^T)^\vee$$

Jacobian:

$$\frac{\partial r_p}{\partial t} = I_3$$

# Map Matching

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$$\frac{\partial r_q}{\partial R} = \lim_{\phi \to 0} \frac{\ln(R\exp\phi^\wedge R_{obs}^T)^\vee - \ln(RR_{obs}^T)^\vee}{\phi}$$

$$= \lim_{\phi \to 0} \frac{\ln(RR_{obs}^T R_{obs}\exp\phi^\wedge R_{obs}^T)^\vee - \ln(RR_{obs}^T)^\vee}{\phi}$$

$$= \lim_{\phi \to 0} \frac{\ln[RR_{obs}^T \cdot \exp(R_{obs}\phi)^\wedge]^\vee - \ln(RR_{obs}^T)^\vee}{\phi}$$

$$= \lim_{\phi \to 0} \frac{\ln(RR_{obs}^T)^\vee + J_r^{-1}R_{obs}\phi - \ln(RR_{obs}^T)^\vee}{\phi}$$

$$= J_r^{-1}(r_q)R_{obs}$$

$J_r$ should be implemented.

# Map Matching

```
//
// TODO: compute residual:
//
Eigen::Map<Eigen::Matrix<double, 6, 1>> residual(residuals);

    residual.block(INDEX_P,  startCol: 0,  blockRows: 3,  blockCols: 1) = pos - pos_prior;
    residual.block(INDEX_R,  startCol: 0,  blockRows: 3,  blockCols: 1) = (ori*ori_prior.inverse()).log();

//
// TODO: compute jacobians:
//
if ( jacobians ) {
  if ( jacobians[0] ) {
    Eigen::Map<Eigen::Matrix<double, 6, 15, Eigen::RowMajor> > jacobian_prior( dataPtr: jacobians[0] );
    jacobian_prior.setZero();

    jacobian_prior.block<3, 3>(INDEX_P, INDEX_P) = Eigen::Matrix3d::Identity();
    jacobian_prior.block<3, 3>(INDEX_R, INDEX_R) = JacobianRInv(
            w: residual.block(INDEX_R,  startCol: 0,  blockRows: 3,  blockCols: 1)) * ori_prior.matrix();

    jacobian_prior = sqrt_info * jacobian_prior;
  }
}
```

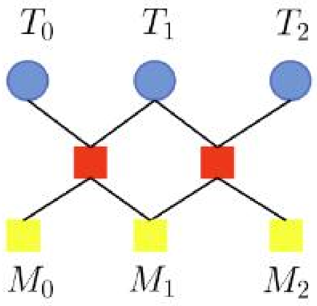
# 及格:补全代码,且功能正常

(3). IMU Pre-Integration

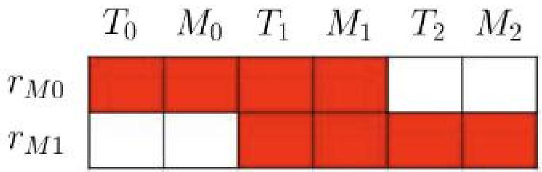
4）IMU预积分和优化变量的残差

该残差对应的因子为IMU因子。

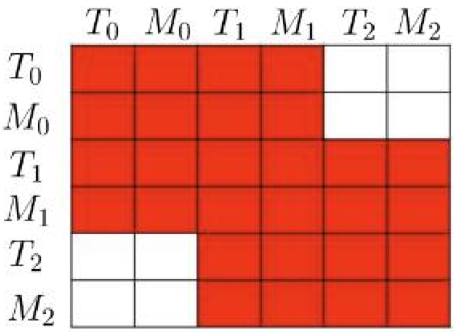
一个因子约束两个位姿，并约束两个时刻 IMU 的速度与 bias。




残差关于优化变量的雅可比，可视化如下：




因此，对应的Hessian矩阵可视化为：



# IMU Pre-Integration

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3.c IMU pre-integration:

The residual, when parameterized using so3, is as follows:

①
$$r_p = R_i^T\left(p_j - p_i - v_i T + \tfrac{1}{2} g T^2\right) - \alpha_{ij}$$

$$\propto p_i, r_i, v_i, b_{ai}, b_{gi} \;/\; p_j, b_{aj}, b_{gj}$$

$\therefore i$ $\quad \frac{\partial r_p}{\partial p_i} = -R_i^T$ $\qquad\qquad \therefore j$ $\quad \frac{\partial r_p}{\partial p_j} = R_i^T$

$$\frac{\partial r_p}{\partial r_i} = \left[R_i^T\left(p_j - p_i - v_i T + \tfrac{1}{2} g T^2\right)\right]^{\wedge} \qquad \frac{\partial r_p}{\partial r_j} = 0$$

# IMU Pre-Integration

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$$\frac{\partial r_p}{\partial v_i} = -T \cdot R_i^T \qquad \frac{\partial r_p}{\partial v_j} = 0$$

$$\frac{\partial r_p}{\partial b_{ai}} = -J.block<3,3>(P,A) \qquad \frac{\partial r_p}{\partial b_{aj}} = 0$$

$$\frac{\partial r_p}{\partial b_{gi}} = -J.block<3,3>(P,G) \qquad \frac{\partial r_p}{\partial b_{gj}} = 0$$

# IMU Pre-Integration

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②

$$r_r = \ln(R_{ij}^T R_i^T R_j)^\vee$$

$\propto \quad r_i,\ b_{gi} \qquad\qquad r_j,\ b_{gj}$

$$\therefore \underline{\underline{i}} \quad \frac{\partial r_r}{\partial r_i} = \lim_{\varphi \to 0} \frac{\ln[R_{ij}^T R_i^T R_j \exp(-R_j^T R_i \varphi)^\wedge]^\vee - r_r}{\varphi} = -J_r^{-1}(r_r) R_j^T R_i$$

$$\frac{\partial r_r}{\partial b_{gi}} = \lim_{\delta b_{gi} \to 0} \frac{\ln[\exp(-J_{b_{gi}}^q \delta b_{gi})^\wedge R_{ij}^T R_i^T R_j)]^\vee - r_r}{\delta b_{gi}}$$

$$= -J_r^{-1}(r_r)\ \exp(r_r^\wedge).\mathrm{inverse}()\ J.\mathrm{block}<3,3>(R,G)$$

$$\therefore \underline{\underline{j}} \quad \frac{\partial r_r}{\partial r_j} = \lim_{\varphi \to 0} \frac{\ln[R_{ij}^T R_i^T R_j \exp \varphi^\wedge]^\vee - r_r}{\varphi} = J_r^{-1}(r_r)$$

$$\frac{\partial r_r}{\partial b_{gj}} = 0$$

# IMU Pre-Integration



③.

$$r_v = R_i^T(v_j - v_i + gT) - \beta_{ij}$$

$\propto r_i, v_i, b_{ai}, b_{gi}$ $\quad v_j, b_{aj}, b_{gj}$

$\therefore \dot{z} \doteq$: $\dfrac{\partial r_v}{\partial r_i} = [R_i^T(v_j - v_i + gT)]^\wedge$

$\dfrac{\partial r_v}{\partial v_i} = -R_i^T$ $\quad \dfrac{\partial r_v}{\partial v_j} = R_i^T$

$\dfrac{\partial r_v}{\partial b_{ai}} = -J.block\langle 3,3\rangle(V,A)$ $\quad \dfrac{\partial r_v}{\partial b_{aj}} = 0$

$\dfrac{\partial r_v}{\partial b_{gi}} = -J.block\langle 3,3\rangle(V,G)$ $\quad \dfrac{\partial r_v}{\partial b_{gj}} = 0$

# IMU Pre-Integration

④: $r_a = b_{aj} - b_{ai}$ $\qquad$ $r_g = b_{gj} - b_{gi}$

$$\frac{\partial r_a}{\partial b_{ai}} = -I \quad \frac{\partial r_a}{\partial b_{aj}} = I \qquad \frac{\partial r_g}{\partial b_{gi}} = -I \quad \frac{\partial r_g}{\partial b_{gj}} = I$$

# IMU Pre-Integration



```cpp
//
// TODO: compute residual:
//
Eigen::Map<Eigen::Matrix<double, 15, 1>> residual(residuals);

residual.block<3, 1>(INDEX_P, startCol: 0) = ori_i.inverse().matrix() * (pos_j - pos_i - (vel_i - 0.50 * g_ * T_) * T_) - alpha_ij;
residual.block<3, 1>(INDEX_R, startCol: 0) = (Sophus::SO3d::exp(theta_ij).inverse()*ori_i.inverse()*ori_j).log();
residual.block<3, 1>(INDEX_V, startCol: 0) = ori_i.inverse().matrix() * (vel_j - vel_i + g_ * T_) - beta_ij;
residual.block<3, 1>(INDEX_A, startCol: 0) = b_a_j - b_a_i;
residual.block<3, 1>(INDEX_G, startCol: 0) = b_g_j - b_g_i;

//
// TODO: compute jacobians:
//
if ( jacobians ) {
  // compute shared intermediate results:
  const Eigen::Matrix3d R_i_inv = ori_i.inverse().matrix();
  const Eigen::Matrix3d J_r_inv = JacobianRInv( w: residual.block(INDEX_R, startCol: 0, blockRows: 3, blockCols: 1));

  if ( jacobians[0] ) {
```

# IMU Pre-Integration

```
// a. residual, position:
jacobian_i.block<3, 3>(INDEX_P, INDEX_P) = -R_i_inv;
jacobian_i.block<3, 3>(INDEX_P, INDEX_R) = Sophus::SO3d::hat(
    omega: ori_i.inverse() * (pos_j - pos_i - (vel_i - 0.50 * g_ * T_) * T_)
);
jacobian_i.block<3, 3>(INDEX_P, INDEX_V) = -T_ * R_i_inv;
jacobian_i.block<3, 3>(INDEX_P, INDEX_A) = -J_.block<3,3>(INDEX_P, INDEX_A);
jacobian_i.block<3, 3>(INDEX_P, INDEX_G) = -J_.block<3,3>(INDEX_P, INDEX_G);

// b. residual, orientation:
jacobian_i.block<3, 3>(INDEX_R, INDEX_R) = -J_r_inv * (ori_j.inverse() * ori_i).matrix();
jacobian_i.block<3, 3>(INDEX_R, INDEX_G) = -J_r_inv*(
    Sophus::SO3d::exp( omega: residual.block<3, 1>(INDEX_R,  startCol: 0))
).matrix().inverse()*J_.block<3,3>(INDEX_R, INDEX_G);

// c. residual, velocity:
jacobian_i.block<3, 3>(INDEX_V, INDEX_R) = Sophus::SO3d::hat(
    omega: ori_i.inverse() * (vel_j - vel_i + g_ * T_)
);
jacobian_i.block<3, 3>(INDEX_V, INDEX_V) = -R_i_inv;
jacobian_i.block<3, 3>(INDEX_V, INDEX_A) = -J_.block<3,3>(INDEX_V, INDEX_A);
jacobian_i.block<3, 3>(INDEX_V, INDEX_G) = -J_.block<3,3>(INDEX_V, INDEX_G);

// d. residual, bias accel:
jacobian_i.block<3, 3>(INDEX_A, INDEX_A) = -Eigen::Matrix3d::Identity();

// d. residual, bias accel:
jacobian_i.block<3, 3>(INDEX_G, INDEX_G) = -Eigen::Matrix3d::Identity();

jacobian_i = sqrt_info * jacobian_i;
```

# IMU Pre-Integration

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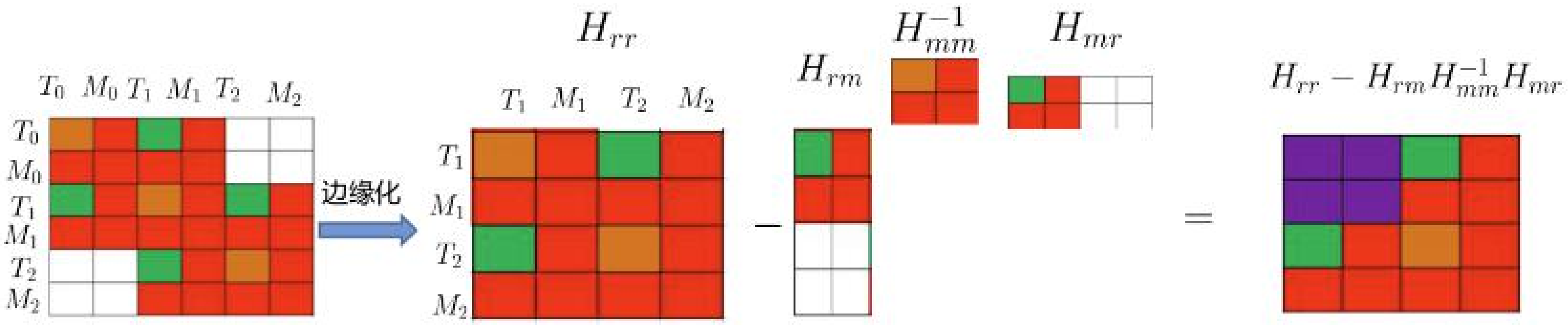
```
    if ( jacobians[1] ) {
      Eigen::Map<Eigen::Matrix<double, 15, 15, Eigen::RowMajor>> jacobian_j( dataPtr: jacobians[1]);
      jacobian_j.setZero();

      // a. residual, position:
      jacobian_j.block<3, 3>(INDEX_P, INDEX_P) = R_i_inv;

      // b. residual, orientation:
      jacobian_j.block<3, 3>(INDEX_R, INDEX_R) = J_r_inv;

      // c. residual, velocity:
      jacobian_j.block<3, 3>(INDEX_V, INDEX_V) = R_i_inv;

      // d. residual, bias accel:
      jacobian_j.block<3, 3>(INDEX_A, INDEX_A) = Eigen::Matrix3d::Identity();

      // d. residual, bias accel:
      jacobian_j.block<3, 3>(INDEX_G, INDEX_G) = Eigen::Matrix3d::Identity();

      jacobian_j = sqrt_info * jacobian_j;
    }
  }
```

# 及格:补全代码,且功能正常

Sliding Window Marginalization

1）移除老的帧

上述过程，通过可视化可以表示为



# Sliding Window Marginalization

Marginalization的实现理念参考关键计算步骤的推导

4. In order to adapt Marginalization to Ceres solver

a. When the sliding window has been filled, start to create Marginalization factor for next optimization

# Sliding Window Marginalization



b. In order to fit into Ceres solver marginalization has to be implemented as follows:

$$\because \begin{cases} H_{rr} - H_{rm} H_{mm}^{-1} H_{mr} \doteq H = J^T J \\ b_r - H_{rm} H_{mm}^{-1} b_m \doteq b = -J^T r \end{cases}$$

$\because H = V \Lambda V^T$

$\therefore J = \sqrt{\Lambda}\, V^T$ → Marg. Res. Block. Jacobians

$\therefore r = J^{-T} b = \sqrt{\Lambda}^{-1} V^T b$ → Marg. Res. Block. Residuals.

# Sliding Window Marginalization



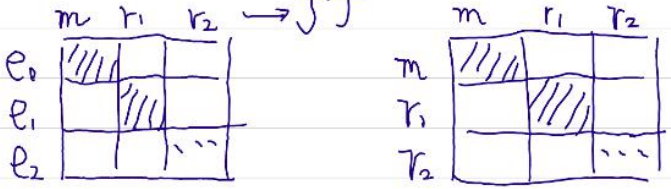
c. Add the Marg. Factor directly to next Ceres problem.

Marginalization Residual Block Building, PRVAG in $\underline{\underline{SO3}}$

for the to-be-marginalized param block m and its next param block $r_1$

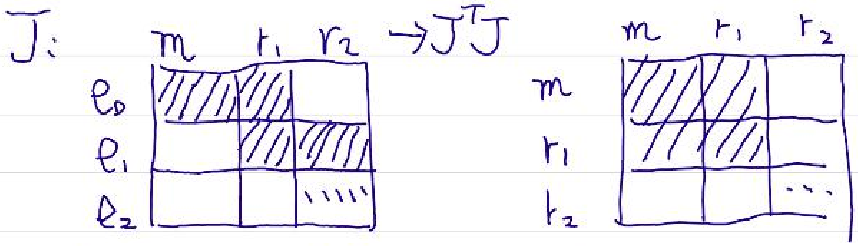
a. Map Matching:

$J: \rightarrow J^T J$



# Sliding Window Marginalization

b. Relative Pose & IMU Pre Integration




∴ The marginalization res. block is only relevant to param block $\underline{\underline{r_1}}$

and $H_{rr}$, $H_{rm}$, $H_{mm}$, $H_{mr}$, $b_r$, $b_m$

should be computed using res. block $\begin{cases} \text{MapMatching}(m) \\ \text{RelativePose}(m, r_1) \\ \text{IMU}(m, r_1) \end{cases}$

# Sliding Window Marginalization



$$\frac{1}{2}(r+J\Delta x)^T(r+J\Delta x) = \frac{1}{2}\Delta x^T J^T J \Delta x + r^T J \Delta x + r^T r$$

$$J^T J \Delta x + J^T r = 0$$

$$J^T J \Delta x = -J^T r$$

# Sliding Window Marginalization

```
//
// TODO: Update H:
//
// a. H_mm:
H_.block<15, 15>(INDEX_M, INDEX_M) += J_m.transpose() * J_m;

//
// TODO: Update b:
//
// a. b_m:
b_.block<15,  1>(INDEX_M,        startCol: 0) += J_m.transpose() * residuals;
```

```
//
// TODO: Update H:
//
// a. H_mm:
H_.block<15, 15>(INDEX_M, INDEX_M) += J_m.transpose() * J_m;
// b. H_mr:
H_.block<15, 15>(INDEX_M, INDEX_R) += J_m.transpose() * J_r;
// c. H_rm:
H_.block<15, 15>(INDEX_R, INDEX_M) += J_r.transpose() * J_m;
// d. H_rr:
H_.block<15, 15>(INDEX_R, INDEX_R) += J_r.transpose() * J_r;


//
// TODO: Update b:
//
// a. b_m:
b_.block<15,  1>(INDEX_M,        startCol: 0) += J_m.transpose() * residuals;
// a. b_r:
b_.block<15,  1>(INDEX_R,        startCol: 0) += J_r.transpose() * residuals;
```

# Sliding Window Marginalization



```
//
// TODO: Update H:
//
// a. H_mm:
H_.block<15, 15>(INDEX_M, INDEX_M) += J_m.transpose() * J_m;
// b. H_mr:
H_.block<15, 15>(INDEX_M, INDEX_R) += J_m.transpose() * J_r;
// c. H_rm:
H_.block<15, 15>(INDEX_R, INDEX_M) += J_r.transpose() * J_m;
// d. H_rr:
H_.block<15, 15>(INDEX_R, INDEX_R) += J_r.transpose() * J_r;


//
// Update b:
//
// a. b_m:
b_.block<15,  1>(INDEX_M,      startCol: 0) += J_m.transpose() * residuals;
// a. b_r:
b_.block<15,  1>(INDEX_R,      startCol: 0) += J_r.transpose() * residuals;
```

# Sliding Window Marginalization

```
  // TODO: implement marginalization logic
// save x_m_0:
Eigen::Map<const Eigen::Matrix<double, 15, 1>> x_0(raw_param_r_0);
x_0_ = x_0;
// marginalize:
const Eigen::MatrixXd &H_mm = H_.block<15, 15>(INDEX_M, INDEX_M);
const Eigen::MatrixXd &H_mr = H_.block<15, 15>(INDEX_M, INDEX_R);
const Eigen::MatrixXd &H_rm = H_.block<15, 15>(INDEX_R, INDEX_M);
const Eigen::MatrixXd &H_rr = H_.block<15, 15>(INDEX_R, INDEX_R);

const Eigen::VectorXd &b_m = b_.block<15, 1>(INDEX_M,  startCol: 0);
const Eigen::VectorXd &b_r = b_.block<15, 1>(INDEX_R,  startCol: 0);

Eigen::MatrixXd H_mm_inv = H_mm.inverse();
Eigen::MatrixXd H_marginalized = H_rr - H_rm * H_mm_inv * H_mr;
Eigen::MatrixXd b_marginalized = b_r - H_rm * H_mm_inv * b_m;
// solve linearized residual & Jacobian:
Eigen::SelfAdjointEigenSolver<Eigen::MatrixXd> saes(H_marginalized);
Eigen::VectorXd S = Eigen::VectorXd(
   x: (saes.eigenvalues().array() > 1.0e-5).select(saes.eigenvalues().array(),  elseScalar: 0)
);
Eigen::VectorXd S_inv = Eigen::VectorXd(
   x: (saes.eigenvalues().array() > 1.0e-5).select(saes.eigenvalues().array().inverse(),  elseScalar: 0)
);

Eigen::VectorXd S_sqrt = S.cwiseSqrt();
Eigen::VectorXd S_inv_sqrt = S_inv.cwiseSqrt();

J_ = S_sqrt.asDiagonal() * saes.eigenvectors().transpose();
e_ = S_inv_sqrt.asDiagonal() * saes.eigenvectors().transpose() * b_marginalized;
```

# Sliding Window Marginalization

```
//
// parse parameters:
//
Eigen::Map<const Eigen::Matrix<double, 15, 1>> x( dataPtr: parameters[0]);
Eigen::VectorXd dx = x - x_0_;

//
// TODO: compute residual:
//
Eigen::Map<Eigen::Matrix<double, 15, 1>> residual(residuals);
residual = e_ + J_ * dx;

//
// TODO: compute jacobian:
//
if ( jacobians ) {
  if ( jacobians[0] ) {
    Eigen::Map<Eigen::Matrix<double, 15, 15, Eigen::RowMajor> > jacobian_marginalization( dataPtr: jacobians[0] );
    jacobian_marginalization.setZero();

    jacobian_marginalization = J_;
  }
}
```

# sliding window optimization problem

```
// TODO: create new sliding window optimization problem:
ceres::Problem problem;

// TODO: a. add parameter blocks:
for ( int i = 1; i <= kWindowSize + 1; ++i) {
    auto &target_key_frame = optimized_key_frames_.at( n: N - i);

    ceres::LocalParameterization *local_parameterization = new sliding_window::ParamPRVAG();
    // TODO: add parameter block:
    problem.AddParameterBlock(target_key_frame.prvag,  size: 15, local_parameterization);

    if ( target_key_frame.fixed ) {
        problem.SetParameterBlockConstant(target_key_frame.prvag);
    }
}
```

# sliding window optimization problem



```
// TODO: add residual blocks:
// b.1. marginalization constraint:
if (
    !residual_blocks_.map_matching_pose.empty() &&
    !residual_blocks_.relative_pose.empty() &&
    !residual_blocks_.imu_pre_integration.empty()
) {
    auto &key_frame_m = optimized_key_frames_.at( n: N - kWindowSize - 1);
    auto &key_frame_r = optimized_key_frames_.at( n: N - kWindowSize - 0);

    const ceres::CostFunction *factor_map_matching_pose = GetResMapMatchingPose(
        res_map_matching_pose: residual_blocks_.map_matching_pose.front()
    );
    const ceres::CostFunction *factor_relative_pose = GetResRelativePose(
        res_relative_pose: residual_blocks_.relative_pose.front()
    );
    const ceres::CostFunction *factor_imu_pre_integration = GetResIMUPreIntegration(
        res_imu_pre_integration: residual_blocks_.imu_pre_integration.front()
    );

    sliding_window::FactorPRVAGMarginalization *factor_marginalization = new sliding_window::FactorPRVAGMarginalization();

    factor_marginalization->SetResMapMatchingPose(
        factor_map_matching_pose,
        parameter_blocks: std::vector<double *>{key_frame_m.prvag}
```

# sliding window optimization problem



```
        factor_marginalization->SetResRelativePose(
            factor_relative_pose,
            parameter_blocks: std::vector<double *>{key_frame_m.prvag, key_frame_r.prvag}
        );
        factor_marginalization->SetResIMUPreIntegration(
            factor_imu_pre_integration,
            parameter_blocks: std::vector<double *>{key_frame_m.prvag, key_frame_r.prvag}
        );
        factor_marginalization->Marginalize(key_frame_r.prvag);

        // add marginalization factor into sliding window
        problem.AddResidualBlock(
            factor_marginalization,
            loss_function: NULL,
            key_frame_r.prvag
        );

        residual_blocks_.map_matching_pose.pop_front();
        residual_blocks_.relative_pose.pop_front();
        residual_blocks_.imu_pre_integration.pop_front();
    }
```

# sliding window optimization problem

```
// TODO: b.2. map matching pose constraint:
if ( !residual_blocks_.map_matching_pose.empty() ) {
    for ( const auto &residual_map_matching_pose: residual_blocks_.map_matching_pose ) {
        auto &key_frame = optimized_key_frames_.at(residual_map_matching_pose.param_index);

        sliding_window::FactorPRVAGMapMatchingPose *factor_map_matching_pose = GetResMapMatchingPose(
            residual_map_matching_pose
        );

        // TODO: add map matching factor into sliding window
        problem.AddResidualBlock(
            factor_map_matching_pose,
            loss_function: NULL,
            key_frame.prvag
        );
    }
}

// TODO: b.3. relative pose constraint:
```

# sliding window optimization problem

```
// TODO: b.3. relative pose constraint:
if ( !residual_blocks_.relative_pose.empty() ) {
    for ( const auto &residual_relative_pose: residual_blocks_.relative_pose ) {
        auto &key_frame_i = optimized_key_frames_.at(residual_relative_pose.param_index_i);
        auto &key_frame_j = optimized_key_frames_.at(residual_relative_pose.param_index_j);

        sliding_window::FactorPRVAGRelativePose *factor_relative_pose = GetResRelativePose(
            residual_relative_pose
        );

        // TODO: add relative pose factor into sliding window
        problem.AddResidualBlock(
            factor_relative_pose,
            loss_function: NULL,
            key_frame_i.prvag, key_frame_j.prvag
        );
    }
}
```

# sliding window optimization problem

```
// TODO: b.4. IMU pre-integration constraint
if ( !residual_blocks_.imu_pre_integration.empty() ) {
    for ( const auto &residual_imu_pre_integration: residual_blocks_.imu_pre_integration ) {
        auto &key_frame_i = optimized_key_frames_.at(residual_imu_pre_integration.param_index_i);
        auto &key_frame_j = optimized_key_frames_.at(residual_imu_pre_integration.param_index_j);

        sliding_window::FactorPRVAGIMUPreIntegration *factor_imu_pre_integration = GetResIMUPreIntegration(
            residual_imu_pre_integration
        );

        // TODO: add IMU factor into sliding window
        problem.AddResidualBlock(
            factor_imu_pre_integration,
            loss_function: NULL,
            key_frame_i.prvag, key_frame_j.prvag
        );
    }
}

// solve:
```

# 及格:补全代码,且功能正常



```
$ evo_ape kitti ground_truth.txt optimized.txt -r full --plot --plot_mode xyz
APE w.r.t. full transformation (unit-less)
(not aligned)

       max      8.320945
      mean      5.080542
    median      5.169981
       min      0.000002
      rmse      5.308574
       sse      127575.201662
```

# 感谢各位聆听！

Thanks for Listening