平面顯示技術導論 HW #1

B02901178 江誠敏

May 3, 2016

(a) Calculate the (x, y) location of the light source D65 (spectrum is listed with the attached excel file) by finding and normalizing the tristimulus values for a color with a spectral power distribution.

利用題目給的資料計算，以下用 python 做爲主要的計算工具。

首先將 .xls 檔讀入，並把 Sheet1 中的資料讀出。

```
In [1]: from pyexcel_xls import get_data
        data = get_data('data.xls')['Sheet1']
```

觀察一下各行的資訊。

```
In [2]: print(data[4])

Out[2]: ['Wavelength (nm)', 'xbar', 'ybar', 'zbar', '', 'wavelength(nm)', 'intensity'
```

之後把數值資料取出。

```
In [3]: rdata = data[5:]
```

接著就把 XYZ tristimulus 的值計算出來，利用

$$X_{\text{tot}} = \sum_{\omega} \bar{x}(\omega) \cdot I(\omega)$$

其中 $\bar{x}$ 是各個波長下的 color matching function，I 則是 D65 各個波長的光強度。

```
In [4]: _, X, Y, Z, _, _, I = zip(*rdata)
        N = len(X)

In [5]: xtot, ytot, ztot = 0, 0, 0

In [6]: for i in range(N):
            xtot += X[i] * I[i]
            ytot += Y[i] * I[i]
            ztot += Z[i] * I[i]
```

最後得出 $X_{\text{tot}}, Y_{\text{tot}}, Z_{\text{tot}}$

```
In [7]: 'xtot, ytot, ztot = %.6f, %.6f, %.6f' % (xtot, ytot, ztot)

Out[7]: 'xtot, ytot, ztot = 2008.773018, 2113.459438, 2301.492113'
```

接著 x, y 可由

$$x = \frac{X_{\text{tot}}}{X_{\text{tot}} + Y_{\text{tot}} + Z_{\text{tot}}}, \quad y = \frac{Y_{\text{tot}}}{X_{\text{tot}} + Y_{\text{tot}} + Z_{\text{tot}}}$$

得到

```
In [8]: sm = xtot + ytot + ztot

In [9]: x, y = xtot/sm, ytot/sm

In [10]: 'x = %.6f, y = %.6f' % (x, y)

Out[10]: 'x = 0.312712, y = 0.329008'
```

因此

$$x \approx 0.312712, \quad y \approx 0.329008.$$

和 Wiki [1] 上查到的似乎差不多。

(b) **We would like to superpose two monochromatic (meaning single wavelength) sources in order to obtain ideal white as a result. One of the source' s beam corresponds to a wavelength 480 nm. Using the excel sheet as attached, what is the wavelength of the other source?**

一樣先讀入資料，頡取有用的部分。

```
In [1]: from pyexcel_xls import get_data
        from math import sqrt
        data = get_data('data.xls')['Sheet1']
        rdata = data[5:]
        wlmap = {int(r[0]): (*r[1:4],) for r in rdata}
```

之後把所有波長的 x, y value 對照表建出。

```
In [2]: xymap = {x: (y[0]/sum(y), y[1]/sum(y)) for x, y in wlmap.items()}
```

在 CIE-1931 中，Ideal white 對應到的 x, y 作標為 $\mathbf{w} = (1/3, 1/3)$。假設兩個光源在 x, y 作標上分別在點 $\mathbf{v}_{480}$ 和點 $\mathbf{v}_\lambda$，如果他們可以混出白色的光，則 $\mathbf{v}_{480}, \mathbf{w}, \mathbf{v}_\lambda$ 共線，而且 $\mathbf{w}$ 點還要在中間，此條件等價於 $\theta = \measuredangle \mathbf{v}_{480} \mathbf{w} \mathbf{v}_\lambda = 180^\circ$，也就是

$$\frac{(\mathbf{v}_{480} - \mathbf{w}) \cdot (\mathbf{v}_\lambda - \mathbf{w})}{\|\mathbf{v}_{480} - \mathbf{w}\| \, \|\mathbf{v}_\lambda - \mathbf{w}\|} = -1$$

[1] https://en.wikipedia.org/wiki/Illuminant_D65#Definition

當然實際上不可能如此精準，因此我們找使上式最靠近 -1 的波長 λ。

```
In [3]: W = (1/3, 1/3)
        v480 = xymap[480]
```

```
In [4]: def angle(wl):
            v1 = tuple(a-b for a, b in zip(v480, W))
            s1 = sqrt(v1[0]**2 + v1[1]**2)
            v2 = tuple(a-b for a, b in zip(xymap[wl], W))
            s2 = sqrt(v2[0]**2 + v2[1]**2)
            dot = v1[0]*v2[0] + v1[1]*v2[1]
            return dot / s1 / s2
```

```
In [5]: bs, bwl = 1.0, 480
```

```
In [6]: for f in wlmap:
            t = angle(f)
            if t < bs:
                bs, bwl = t, f
```

```
In [7]: 'best wave length = %d, cos(theta) = %.6f' % (bwl, bs)

Out[7]: 'best wave length = 580, cos(theta) = -0.999879'
```

最後我們得到最佳的波長爲 $\lambda = 580\,\mathrm{nm}$，此時 $\cos(\theta) \approx -0.999879$ ，算是非常接近的了!