8.1 考慮下列之二階非線性系統

$$\begin{cases} \dot{x_1} = 2x_2 + \theta_1 x_1 + \sin x_2 \\ \dot{x_2} = \theta_2 {x_2}^2 + x_1 + u \end{cases}$$

其中u是控制訊； θ_1和θ_2是不確定的參數，但滿足

$$|\theta_1| \le a, \qquad |\theta_2| \le b$$

本題的目的是要設計滑動控制u使得系統(1)相對於以下的滑動曲面為 Lyapunov 穩定

$$S = (1+a)x_1 + x_2$$

（a）試證明在不確定性參數θ_1、θ_2的作用下，能確保$S\dot{S} < 0$的滑動控制u為

$$u = u_{eq} - \beta(x) sgn(S)$$

其中

$$u_{eq} = -x_1 - (1+a)x_2$$

$$\beta(x) = a(1+a)|x_1| + b{x_2}^2 + b_0, \qquad b_0 > 0$$

提示：參考 8.4.3 節的證明方法，先求出$\dot{S}$的表示式(利用(1)式)，將(4)式的u代入$\dot{S}$, 再求$\dot{S}S$的表示式，利用不等式層層化簡，得到關係式$\dot{S}S \le -\eta|S|$，其中η可用常數a、b、b_0表示之。

對於非線性系統(1a)與(1b)而言，選擇滑動面$S = (1+a)x_1 + x_2$，且滑動面要滿足 Lyapunov 穩定，故先對S做一次微分：

$$\begin{aligned} \dot{S} &= (1+a)\dot{x}_1 + \dot{x}_2 \\ &= (1+a)(x_2 + \theta_1 x_1 \sin x_2) + \theta_2 x_2^2 + x_1 + u \\ &= (x_1 + (1+a)x_2) + \left((1+a)\theta_1 x_1 \sin x_2 + \theta_2 x_2^2\right) + u \end{aligned} \tag{8.1.1}$$

對於(8.1.1)式而言，由於受控體有θ_1與θ_2不確定的參數，因此無法求得 $\dot{S} = 0$ 時的控制律u。因此須將不確定項有θ_1與θ_2拆開來處理。為了滿足滑動控制律的切換邏輯，我們令 $u = u_{eq} - \beta(x)sgn(S)$，其中 u_{eq} 將抑制未含有「結構化不確定性」參數項系統帶來的影響。另一方面，$-\beta(x)sgn(S)$ 則有抑制含有結構不確定性參數θ_1、θ_2項對系統帶來的影響，最終將使得系統滿足：

$$\frac{1}{2}\frac{d}{dt}S^2 \le -\eta|S|, \eta > 0$$

由(8.1.1)式，令u_{eq}為：

$$u_{eq} = -x_1 - (1+a)x_2$$

則

$$\begin{aligned} \dot{S} &= (x_1 + (1+a)x_2) + \left((1+a)\theta_1 x_1 \sin x_2 + \theta_2 x_2^2\right) + u \\ &= \left((1+a)\theta_1 x_1 \sin x_2 + \theta_2 x_2^2\right) - \beta(x)sgn(S) \end{aligned} \tag{8.1.2}$$

且θ_1、θ_2滿足$|\theta_1| \le a$、$|\theta_2| \le b$，因此可對(8.1.2)進行估算：

$$\begin{cases} (1+a)\theta_1 x_1 \sin x_2 \le |(1+a)\theta_1 x_1 \sin x_2| \le (1+a)a|x_1| \\ \theta_2 x_2^2 \le |\theta_2 x_2^2| \le b x_2^2 \end{cases} \tag{8.1.3}$$

由題目的(6)式

$$\beta(x) = a(1+\mathrm{a})|\mathrm{x}_1| + \mathrm{bx}_2^2 + \mathrm{b}_0，\mathrm{b}_0 > 0 \tag{8.1.4}$$

由(8.1.2)至(8.1.4)式可得：

$$\begin{aligned} \frac{1}{2}\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{S}^2 &= \dot{\mathrm{S}}\mathrm{S} = \left((1+\mathrm{a})\theta_1\mathrm{x}_1\sin\mathrm{x}_2 + \theta_2\mathrm{x}_2^2\right)\mathrm{S} - \beta(\mathrm{x})\mathrm{sgn}(\mathrm{S})|S| \\ &\le \left((1+\mathrm{a})\mathrm{a}|\mathrm{x}_1| + \mathrm{bx}_2^2\right)|\mathrm{S}| - \left(a(1+\mathrm{a})|\mathrm{x}_1| + \mathrm{bx}_2^2 + \mathrm{b}_0\right)|\mathrm{S}| \\ &= -\mathrm{b}_0|\mathrm{S}| \end{aligned} \tag{8.1.5}$$

此時系統滿足

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{S}^2 \le -\mathrm{b}_0|\mathrm{S}| = -\eta|\mathrm{S}|，\eta = \mathrm{b}_0 > 0 \tag{8.1.6}$$

亦即在控制律$\mathrm{u} = \mathrm{u}_{\mathrm{eq}} - \beta(\mathrm{x})\mathrm{sgn}(\mathrm{S})$的作用下，能夠在有限的時間內讓系統的軌跡達到滑動面，且假設到達的時間$\mathrm{t} = \mathrm{t}_\mathrm{f}$故可被表示成：

$$\mathrm{t}_\mathrm{f} \le \mathrm{t}_0 + \frac{|s_0|}{\eta} \tag{8.1.7}$$

(b)用 Matlab 模擬以上滑動控制律的正確性。設定$a = b = 1$，並使θ_1和θ_2在區間$[-1,1]$內任意變化，每次模擬均取不一樣的θ_1和θ_2，例如 $\theta_1 = \sin t$ 和 $\theta_2 = \cos t$，或是取成± 1之間的任意隨機亂數(利用 Matlab 的隨機亂數產生器)。用數值模擬驗證，當θ_1和θ_2在區間$[-1,1]$內任意變化時，滑動控制律(4)都可確保相平面軌跡進入滑動曲面$S = 0$，同時觀察是否有顫動現象伴隨發生。

利用 MATLAB 模擬，且令結構化不確定參數$a = b = 1$、$\mathrm{b}_0 > 0$，令$\mathrm{b}_0 = 0.03$，並選擇初始條件 x01=[1,0]，x02=[0,1]，x03=[-1,0]，x04=[0,-1]，x05=[1,1]，x06=[-1,1]，x07=[-1,1]，x08=[1,-1]，模擬結果如圖 8.1.1 至圖 8.1.3 所示。

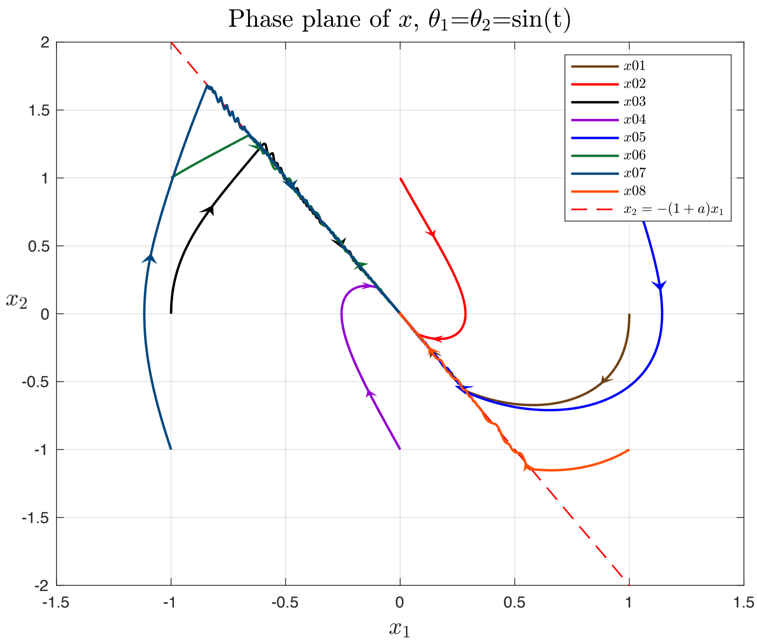

圖 8.1.1

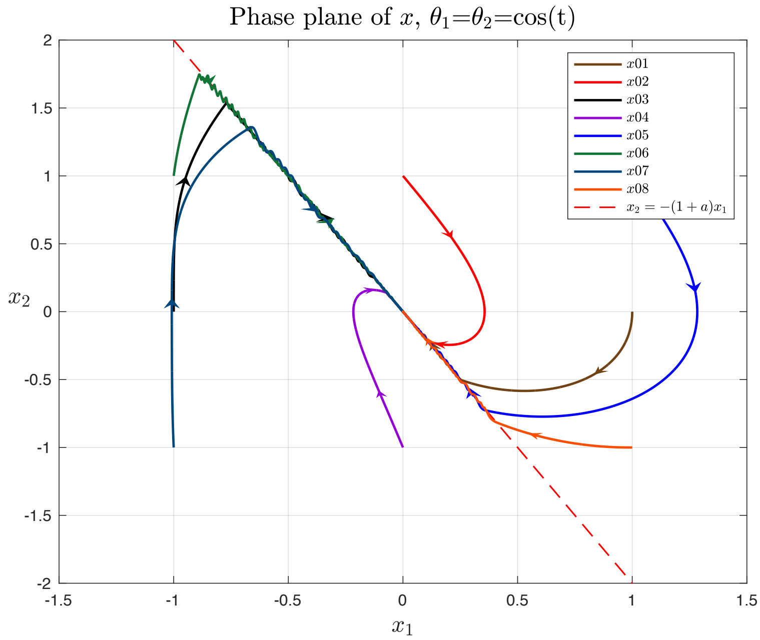

圖 8.1.2

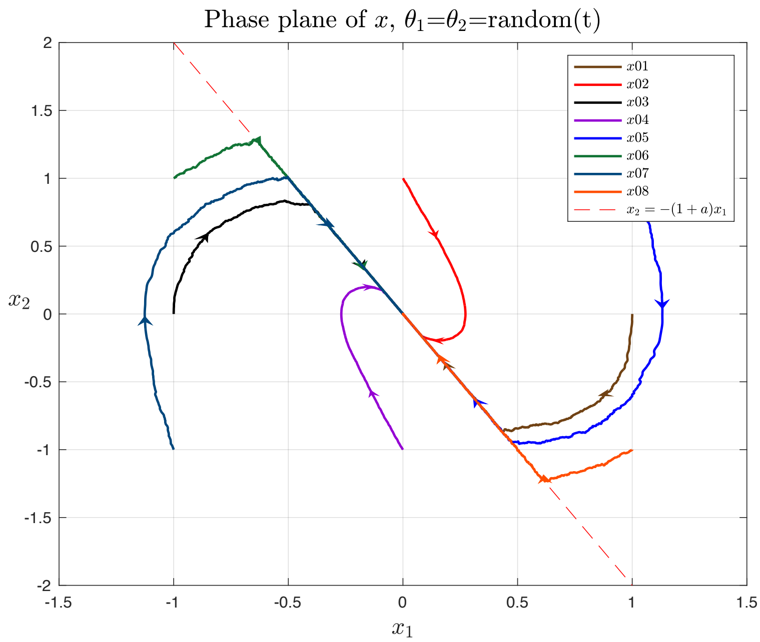

圖 8.1.3

由圖可發現儘管系統有不確定參數θ_1、θ_2，但只要兩者的上下界已知，即可透過滑動控制使系統軌跡在有限的時間內相對於滑動面為 Lyapunov 穩定，亦即$S \rightarrow 0$，但因為不確定性的存在，所以無法設計一滑動控制使系統軌跡維持在滑動面上。因此，可以透過切換控制可有效地將狀態軌跡調回滑動面，故會形成在滑動面兩邊不斷跳躍的情況，亦即系統的真實軌跡將落在$S = 0$為中心的帶狀區，而帶狀區的大小將由系統的不確定參數直接影響，這也是產生顫動效應的主因。

(c)設定$a = b = 2$，重覆上面步驟的模擬，並觀察顫動的情況有何改變。

i.

在此討論調變參數 a 對於系統的影響，固定$b = 1$，$b_0 = 0.03$，且比較$a = 1, a = 2, a = 4$的差異。相平面模擬如圖 8.1.4 所示，時域響應如圖 8.1.5~8.1.6 所示。從圖中可以看到雖然能保證系統在有限時間內進入滑動面附近的區域，然而在模擬階段可發現系統有進一步朝原點趨近的趨勢，且由於滑動面為$S = (1 + a)x_1 + x_2$，表示不同的 a 會對應到不同的滑動面。從相平面圖可以發現當 a 越大，系統顫動效應增強，然而，從時域響應可觀察出當 a 調大，系統響應收斂至原點的速度有增加的趨勢，且降低x_1的低射現象，不過會增加x_2的最大超越量。

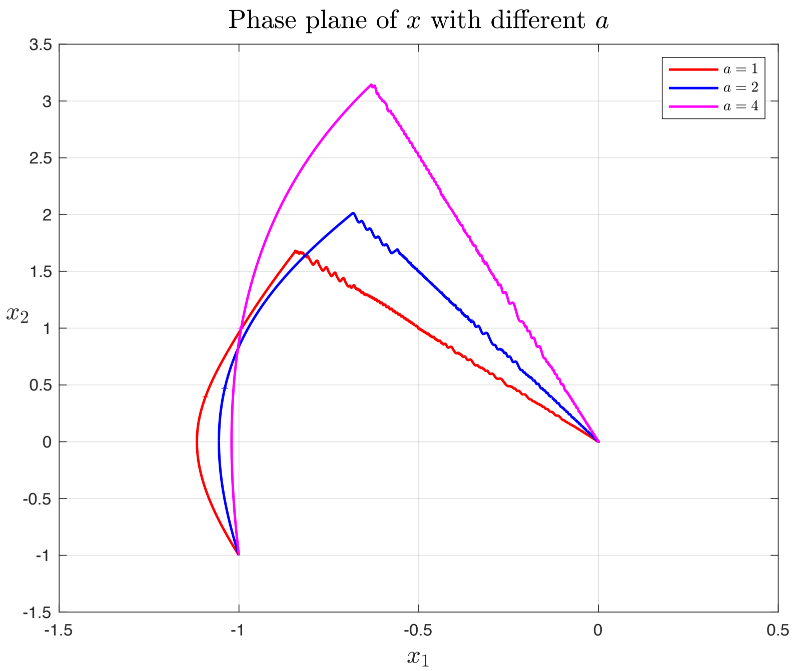

圖 8.1.4

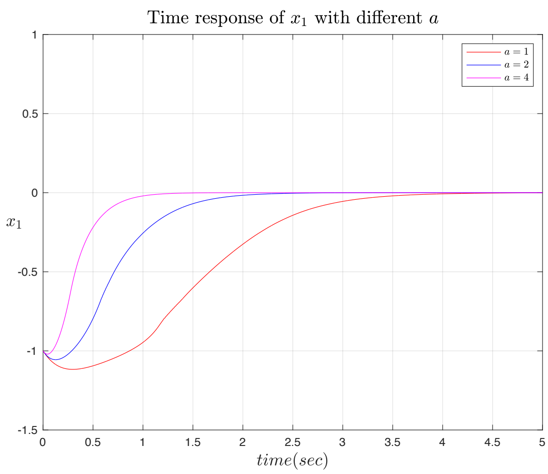

圖 8.1.5

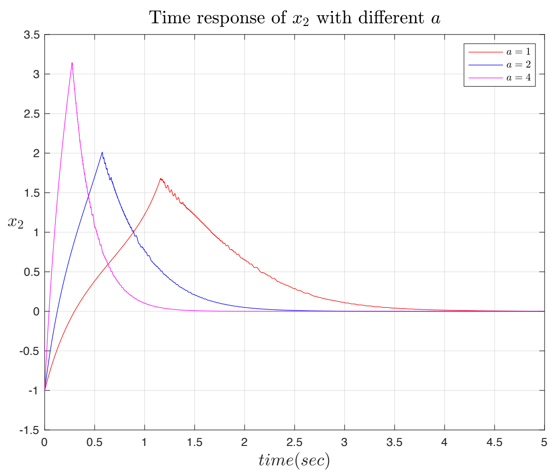

圖 8.1.6

ii.

在此討論調變參數 b 對於系統的影響，固定$a = 1$，$b_0 = 0.03$，比較 b=1、b=2、b=4 的差異。相平面模擬如圖 8.1.7 所示，時域響應如圖 8.1.8~8.1.9 所示，從圖中可以看到雖然能保證系統在有限時間內進入滑動面附近的區域，然而在模擬階段可發現系統有進一步朝原點趨近的趨勢。另一方面，由於滑動面選擇$S = (1 + a)x_1 + x_2$與參數 b 無關，因此不同的 b 將都對應到相同的滑動面上，從相平面軌跡可觀察出，當 b 調大時，將導致系統的顫動效應增加，與調整 a 相比更加明顯，從時域響應上可以看到 b 的上升雖然有加速系統收斂到原點的趨勢，在降低$x_1(t)$的低射現象同時提升$x_2(t)$的最大超越量，但在收斂速度上與放大 a 相比，調整 b 對收斂速度並沒有顯著的增加。

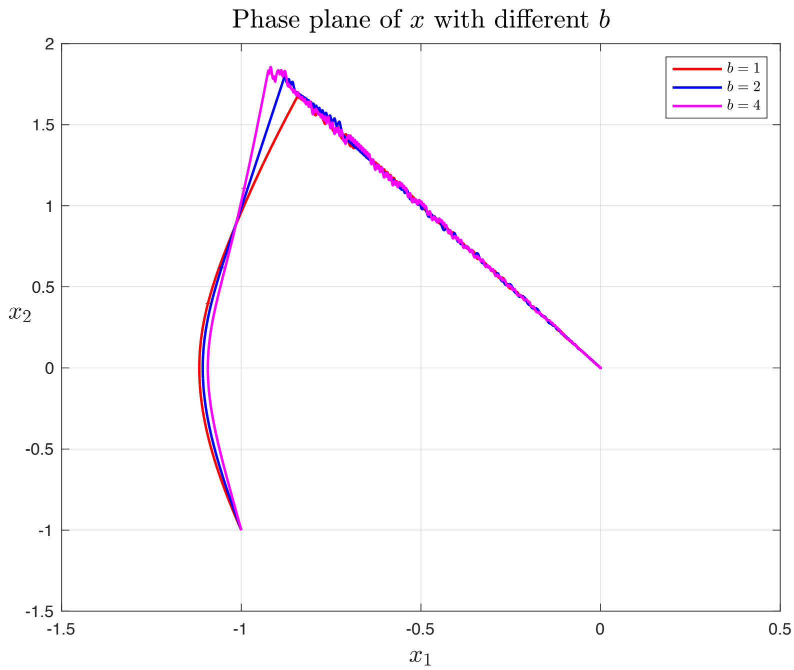

圖 8.1.7

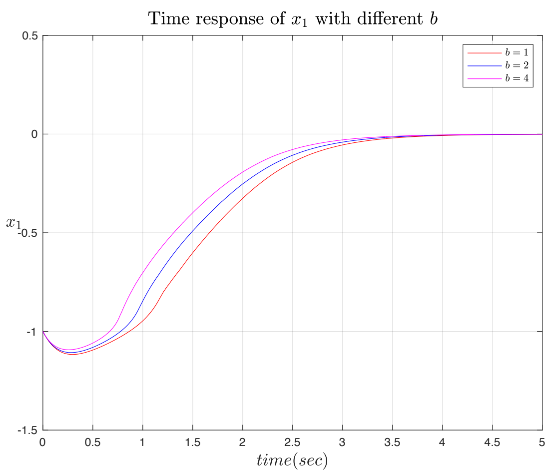

圖 8.1.8

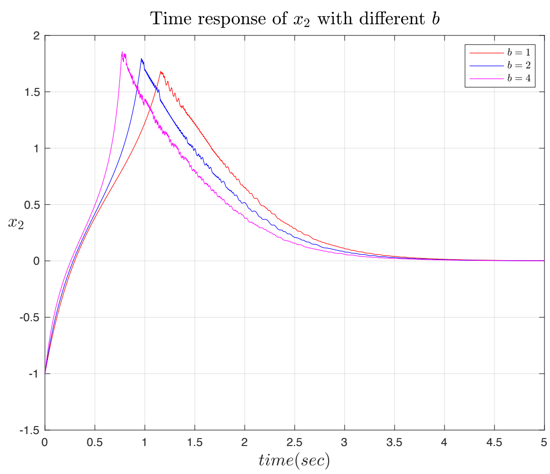

圖 8.1.9

iii.

在此討論同時調變參數 a 和 b 對於系統的影響，固定$b_0 = 0.03$，比較 a = b = 1，a = b = 2，a = b = 4 的差異。相平面模擬如圖 8.1.10 所示，時域響應如圖 8.1.11~8.1.12 所示。由圖可知雖然能保證系統在有限時間內進入滑動面附近的區域，然而在模擬階段可發現系統有進一步朝原點趨近的趨勢。另一方面，由於滑動面選擇$S = (1 + a)x_1 + x_2$，所以改變 a 時，b 的改變可視為 a 先改變再進行疊加。由相平面軌跡發現將 a、b 調大，則顫動效應將增加，與上一部份放大 b 相比效果類似，只是由於 a 的改變會導致不同的滑動面產生，從時域響應上可以看到 a，b 的上升有加速系統收斂到原點的趨勢，一方面降低$x_1(t)$的最大超越量，另一方便更大幅度提升了$x_2(t)$的最大超越量，因此同時調 a、b 對系統性能並沒有改善。

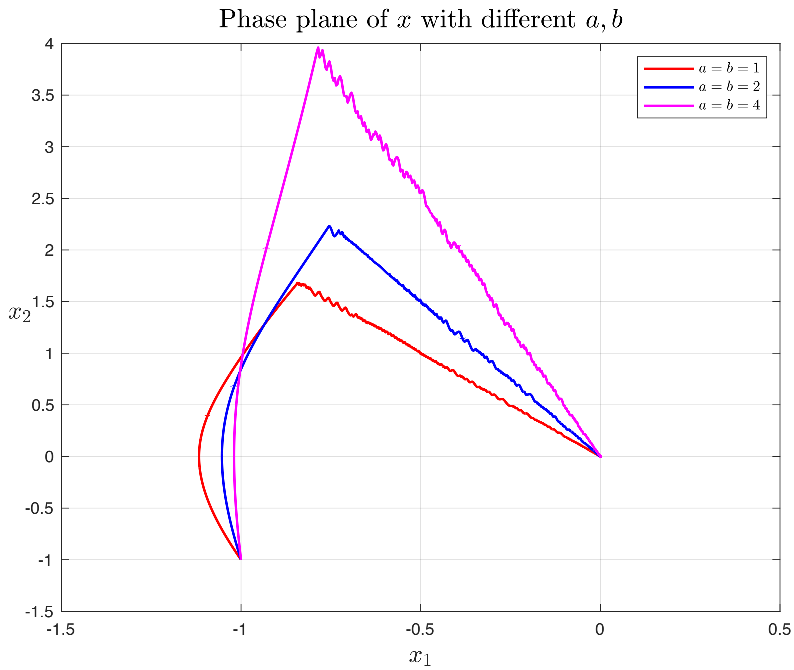

圖 8.1.10

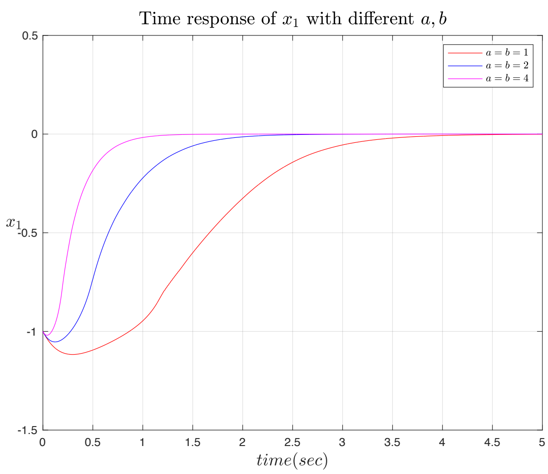

圖 8.1.11

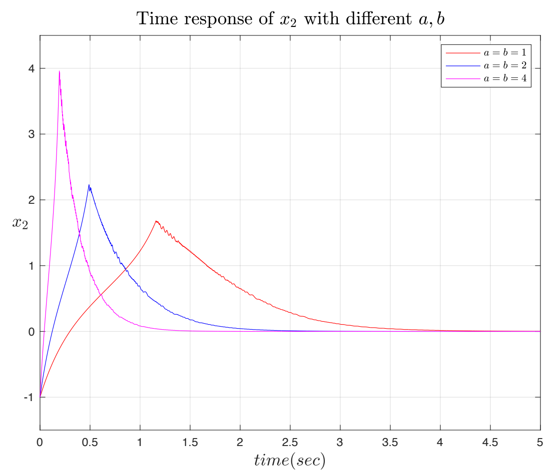

圖 8.1.12

iv.

最後一個部分討論調變參數b_0對於系統的影響，固定 $a = b = 1$，比較 $b_0 = 0.03$，$b_0 = 0.2$，$b_0 = 2$ 的差異。相平面模擬如圖 8.1.13 所示，時域響應分析如圖 8.1.14~8.1.15 所示。相平面模擬如圖 8.1.13 所示，時域響應如 8.1.14~8.1.15 所示，從圖中可以看到雖然能保證系統在有限時間內進入滑動面附近之區域，然而在模擬階段可發現系統有欲朝原點趨近的趨勢，另外，由於滑動面為 $S = (1 + a)x_1 + x_2$，調變b_0不影響滑動面的建構。

值得注意的是，調整b_0將決定系統軌跡第一次觸碰到滑動面的粗估有限時間，因此可透過b_0來調整系統的暫態特性，但從時域響應中發現，調整b_0對系統的性能改善並沒有很明顯，在b_0放大百倍後僅收斂時間略下降。

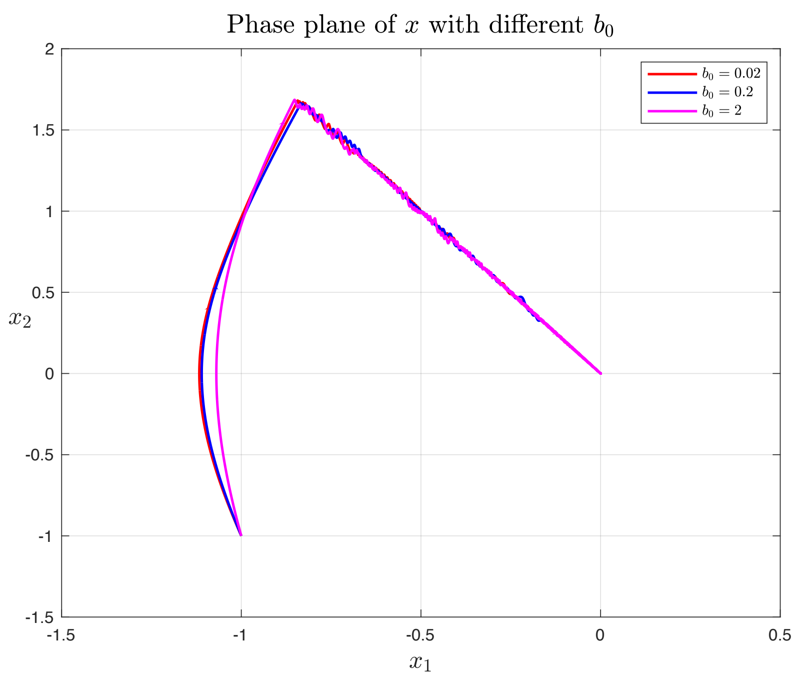

圖 8.1.13

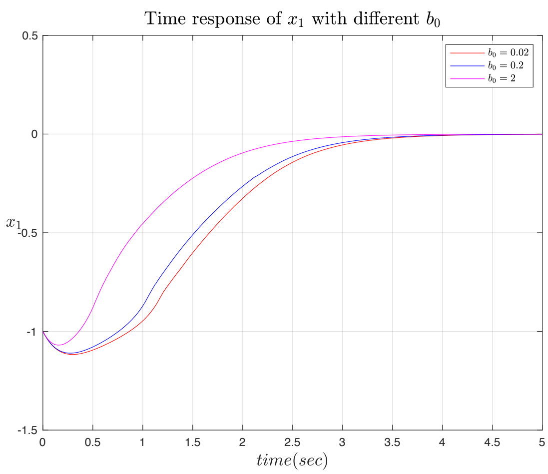

圖 8.1.14

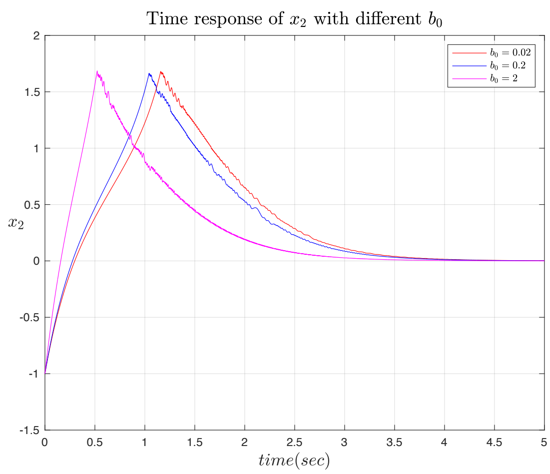

圖 8.1.15

MATLAB

圖 8.1.1～圖 8.1.3

```
clear all ; close all ; clc ;
t = [0:0.001:10] ;
%%%%%%%%%%%%%%%%%%%%
%initial conditions
x01=[1,0];x02=[0,1];x03=[-1,0];x04=[0,-1];x05=[1,1];x06=[-1,1];x07=[-1,-
1];x08=[1,-1];
x0=[x01,x02,x03,x04,x05,x06,x07,x08] ;

% Theta1=Theta2=sin(t)
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1)
s=plot([-5 5],[5*(1+1) -5*(1+1)],'color','r','LineStyle','--
','LineWidth',1);hold on;
[t,x] = ode45(@ode1,t,x01) ;
p(1) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 112,66,20 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x02) ;
p(2) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 1,0,0 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x03) ;
p(3) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,0,0 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x04) ;
p(4) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 148,0,211 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x05) ;
p(5) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,0,1 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x06) ;
p(6) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 18,116,54 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x07) ;
```

```
p(7) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,71,125 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode1,t,x08) ;
p(8) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 255,77,0 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;

xlim([-1.5 1.5]); ylim([-2 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$, $\theta_1 $=$\theta_2
$=sin(t)'},'Fontsize',16,'Interpreter','latex') ;
legend([p(1),p(2),p(3),p(4),p(5),p(6),p(7),p(8),s],{'$x01$','$x02$','$x03$','$x
04$','$x05$','$x06$','$x07$','$x08$','$x_2=-(1+a)x_1
$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Theta1=Theta2=cos(t)
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(2)
s=plot([-5 5],[5*(1+1) -5*(1+1)],'color','r','LineStyle','--
','LineWidth',1);hold on;
[t,x] = ode45(@ode2,t,x01) ;
p(1) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 112,66,20 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x02) ;
p(2) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 1,0,0 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x03) ;
p(3) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,0,0 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x04) ;
p(4) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 148,0,211 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x05) ;
```

```
p(5) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,0,1 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x06) ;
p(6) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 18,116,54 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x07) ;
p(7) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,71,125 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x08) ;
p(8) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 255,77,0 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;

xlim([-1.5 1.5]); ylim([-2 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$, $\theta_1 $=$\theta_2
$=cos(t)'},'Fontsize',16,'Interpreter','latex') ;
legend([p(1),p(2),p(3),p(4),p(5),p(6),p(7),p(8),s],{'$x01$','$x02$','$x03$','$x
04$','$x05$','$x06$','$x07$','$x08$','$x_2=-(1+a)x_1
$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Theta1=Theta2=random(t)
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(3)
s=plot([-5 5],[5*(1+1) -5*(1+1)],'color','r','LineStyle','--
','LineWidth',0.5);hold on;
[t,x] = ode45(@ode3,t,x01) ;
p(1) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 112,66,20 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x02) ;
p(2) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 1,0,0 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x03) ;
```

```
p(3) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,0,0 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x04) ;
p(4) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 148,0,211 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x05) ;
p(5) =
arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,0,1 ],'LineWidth',1.5,'scale',0.
1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x06) ;
p(6) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 18,116,54 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x07) ;
p(7) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 0,71,125 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x08) ;
p(8) = arrowPlot(x(:,1),x(:,2),'number',2,'color',[ 255,77,0 ] /
255,'LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;

xlim([-1.5 1.5]); ylim([-2 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$, $\theta_1 $=$\theta_2
$=random(t)'},'Fontsize',16,'Interpreter','latex') ;
legend([p(1),p(2),p(3),p(4),p(5),p(6),p(7),p(8),s],{'$x01$','$x02$','$x03$','$x
04$','$x05$','$x06$','$x07$','$x08$','$x_2=-(1+a)x_1
$'},'Interpreter','latex') ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% Nonlinear system model
%%%%%%%%%%%%%%%%%%%%%%%%%%%
function y=ode1(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
```

```
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode2(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+cos(t).*x(1)*sin(x(2));
y(2)=cos(t).*x(2)^2+x(1)+u;
end

function y=ode3(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+random('Uniform',-1,1).*x(1)*sin(x(2));
y(2)=random('Uniform',-1,1).*x(2)^2+x(1)+u;
end
```

圖 8.1.4～圖 8.1.6

```
clear all ; close all ; clc ;
t = [0:0.001:10] ; %time interval tspan1=linspace(0,10,100001) ;
%%%%%%%%%%%%%%%%%%%%
%initial conditions
x01=[-1,-1];
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(4)
[t,x] = ode45(@ode1,t,x01) ;
h1 = arrowPlot(x(:,1),x(:,2),'number',2,'color','r','LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = arrowPlot(x(:,1),x(:,2),'number',2,'color','b','LineWidth',1.5,'scale',0.1,'ratio','equal'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
```

```
h3 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','m','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
xlim([-1.5 0.5]); ylim([-1.5 3.5]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$ with different $ a
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$a = 1$','$a = 2$','$a = 4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(5)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,1),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,1),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,1),'color','m','LineStyle','-'); hold on;
xlim([0 5]); ylim([-1.5 1]); grid on ; axis normal ;
ylabel({'$x_1 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_1$ with different $ a
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$a = 1$','$a = 2$','$a = 4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(6)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,2),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,2),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,2),'color','m','LineStyle','-'); hold on;
xlim([0 5]); ylim([-1.5 3.5]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
```

```
title({'Time response of $x_2$ with different $ a
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$a = 1$','$a = 2$','$a = 4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%% Nonlinear system model
%%%%%%%%%%%%%%%%%%%%%%%%%%%
function y=ode1(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode2(t,x)
a=2;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode3(t,x)
a=4;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end
```

圖 8.1.7～圖 8.1.9

```
clear all ; close all ; clc ;
t = [0:0.001:10] ; %time interval tspan1=linspace(0,10,100001) ;
%%%%%%%%%%%%%%%%%%%%
```

```
%initial conditions
x01=[-1,-1];
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(7)
[t,x] = ode45(@ode1,t,x01) ;
h1 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','r','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','b','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','m','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
xlim([-1.5 0.5]);ylim([-1.5 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$ with different $ b
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$b = 1$','$b = 2$','$b = 4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(8)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,1),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,1),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,1),'color','m','LineStyle','-'); hold on;
xlim([0 5]);ylim([-1.5 0.5]); grid on ; axis normal ;
ylabel({'$x_1 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_1$ with different $ b
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$b = 1$','$b = 2$','$b = 4$'},'Interpreter','latex') ;
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(9)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,2),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,2),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,2),'color','m','LineStyle','-'); hold on;
xlim([0 5]);ylim([-1.5 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_2$ with different $ b
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$b = 1$','$b = 2$','$b = 4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%% Nonlinear system model
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function y=ode1(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode2(t,x)
a=1;b=2;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode3(t,x)
```

```
a=1;b=4;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end
```

圖 8.1.10～圖 8.1.12

```
clear all ; close all ; clc ;
t = [0:0.001:10] ; %time interval tspan1=linspace(0,10,100001) ;
%%%%%%%%%%%%%%%%%%%%
%initial conditions
x01=[-1,-1];
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(10)
[t,x] = ode45(@ode1,t,x01) ;
h1 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','r','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','b','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','m','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
xlim([-1.5 0.5]);ylim([-1.5 4]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$ with different $ a, b
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$a = b = 1$','$a = b = 2$','$a = b =
4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
figure(11)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,1),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,1),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,1),'color','m','LineStyle','-'); hold on;
xlim([0 5]);ylim([-1.5 0.5]); grid on ; axis normal ;
ylabel({'$x_1 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_1$ with different $ a, b
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$a = b = 1$','$a = b = 2$','$a = b =
4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(12)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,2),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,2),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,2),'color','m','LineStyle','-'); hold on;
xlim([0 5]);ylim([-1.5 4.5]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_2$ with different $ a, b
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$a = b = 1$','$a = b = 2$','$a = b =
4$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%% Nonlinear system model
%%%%%%%%%%%%%%%%%%%%%%%%%%%
function y=ode1(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
```

```
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode2(t,x)
a=2;b=2;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode3(t,x)
a=4;b=4;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end
```

圖 8.1.13～圖 8.1.15

```
clear all ; close all ; clc ;
t = [0:0.001:10] ; %time interval tspan1=linspace(0,10,100001) ;
%%%%%%%%%%%%%%%%%%%%
%initial conditions
x01=[-1,-1];
%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(13)
[t,x] = ode45(@ode1,t,x01) ;
h1 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','r','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
```

```
h2 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','b','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 =
arrowPlot(x(:,1),x(:,2),'number',2,'color','m','LineWidth',1.5,'scale',0.1,'rat
io','equal'); hold on;
xlim([-1.5 0.5]);ylim([-1.5 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$x_1 $'},'Fontsize',15,'Interpreter','latex');
title({'Phase plane of $x$ with different $ b_0
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$b_0 = 0.02$','$b_0 = 0.2$','$b_0 =
2$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(14)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,1),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,1),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
h3 = plot(t,x(:,1),'color','m','LineStyle','-'); hold on;
xlim([0 5]);ylim([-1.5 0.5]); grid on ; axis normal ;
ylabel({'$x_1 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_1$ with different $ b_0
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$b_0 = 0.02$','$b_0 = 0.2$','$b_0 =
2$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(15)
[t,x] = ode45(@ode1,t,x01) ;
h1 = plot(t,x(:,2),'color','r','LineStyle','-'); hold on;
[t,x] = ode45(@ode2,t,x01) ;
h2 = plot(t,x(:,2),'color','b','LineStyle','-'); hold on;
[t,x] = ode45(@ode3,t,x01) ;
```

```matlab
h3 = plot(t,x(:,2),'color','m','LineStyle','-'); hold on;
xlim([0 5]);ylim([-1.5 2]); grid on ; axis normal ;
ylabel({'$x_2 $'},'Fontsize',15 ,'Rotation',0,'Interpreter','latex');
xlabel({'$ time(sec) $'},'Fontsize',15,'Interpreter','latex');
title({'Time response of $x_2$ with different $ b_0
$'},'Fontsize',16,'Interpreter','latex') ;
legend([h1,h2,h3],{'$b_0 = 0.02$','$b_0 = 0.2$','$b_0 =
2$'},'Interpreter','latex') ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%% Nonlinear system model
%%%%%%%%%%%%%%%%%%%%%%%%%%%
function y=ode1(t,x)
a=1;b=1;b0=0.03;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode2(t,x)
a=1;b=1;b0=0.3;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end

function y=ode3(t,x)
a=1;b=1;b0=3;
S=(1+a)*x(1)+x(2);
u=-x(1)-(1+a)*x(2)-(a*(1+a)*abs(x(1))+b*x(2)^2+b0)*sign(S);
y=zeros(2,1);
y(1)=x(2)+sin(t).*x(1)*sin(x(2));
y(2)=sin(t).*x(2)^2+x(1)+u;
end
```