Estimation and Hypothesis Testing

Agenda

- Estimation & Hypothesis Testing Quiz
- Sampling & Sampling Distributions
- Central Limit Theorem & Assumptions
- Estimation and Confidence Intervals
- Null and Alternative Hypotheses
- Hypothesis Testing Steps
- One-tailed and Two-tailed Tests
- Type I and Type II Errors

Let's begin the discussion by answering a few questions on estimation and hypothesis testing.

Estimation & Hypothesis Testing Quiz

What does the Central Limit Theorem state about the sampling distribution of the sample mean?

A. It will always be skewed

B. It will be identical to the population distribution

C. It will approach a normal distribution as the sample size increases

D. It will be uniformly distributed irrespective of the sample size

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Sampling and Sampling Distributions

Sampling is a **technique of selecting a subset of the population** to make statistical inferences from them and estimate the characteristics of the whole population.

Needed as we have **limited resources and time**, so it's **not always possible to study the population**

We can **draw multiple samples** from a population

For each sample we can **compute a statistic** (mean, median, standard deviation)

Sampling Distribution is the **distribution of the statistic computed from all possible samples** drawn from a specific population

Central Limit Theorem

The sampling distribution of the sample means will approach normal distribution as the sample size gets bigger, no matter what the shape of the population distribution is.

As the **sample size increases**, the shape of the **distribution approaches a normal distribution**

Let's see CLT in action via a simulation

Estimation & Hypothesis Testing Quiz

Which of the following is generally considered a sufficiently large sample size for the Central Limit Theorem to hold true?

A. Greater than or equal to 10

B. Greater than or equal to 50

C. Greater than or equal to 30

D. Greater than or equal to 40

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C. Greater than or equal to 30

D. Greater than or equal to 40

Central Limit Theorem - Assumptions

Assumptions of Central Limit Theorem:

Data must be **randomly sampled**

Sample size must be **sufficiently large (≥30)**

Samples should come from the **same distribution**

Sample values must be **independent** of each other

Estimation & Hypothesis Testing Quiz

What does a confidence interval represent in statistics?

A. The range of sample values

B. The range within which the true population parameter is expected to lie

C. The variability of the sample data

D. The exact value of the population mean

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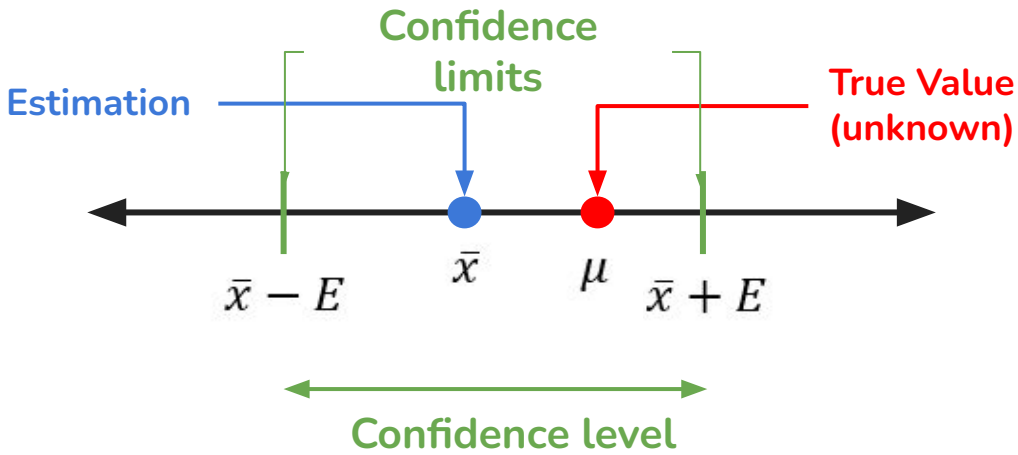
Estimation & Confidence Intervals

Estimation refers to making inference about a population parameter based on sample statistic

Confidence interval provides an interval, or a range of values, which is expected to cover the true unknown parameter

The upper and lower limits of the interval are determined using the distribution of the sample mean and a multiplier that specifies the 'confidence'



Estimation & Hypothesis Testing Quiz

A manufacturer wants to test if the average lifetime of light bulbs exceeds 1000 hours. With a sample of 40 bulbs, the mean lifetime is found to be 1020 hours and a standard deviation of 50 hours.

Which of the following represents the correct null and alternative hypotheses for this scenario?

A. $H_0: \mu = 1020, H_a: \mu \neq 1020$

B. $H_0: \mu = 1000, H_a: \mu \neq 1000$

C. $H_0: \mu \leq 1000, H_a: \mu > 1000$

D. $H_0: \mu \leq 1020, H_a: \mu > 1020$

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Hypothesis Testing - Introduction

Question of Interest

Has the new online Ad increased the conversion rates for an E-commerce website?

Hypotheses about the population parameter(s)

Null Hypothesis (H_0)
The status quo

The new Ad has not increased the conversion rate.

Alternative Hypothesis (H_a)
The research hypothesis

The new Ad has increased the conversion rate.

Hypothesis Formulation

Let μ denote the population mean; the hypothesized value is 1000 hours; the sample mean is 1020 hours

We want to formulate the null and alternative hypotheses for testing if the **average lifetime of light bulbs exceeds the hypothesized value** (1000 hours)

The null hypothesis will represent the **status quo**, i.e., the **average lifetime does not exceed 1000 hours**

$$H0: \mu \leq 1000$$

The alternative hypothesis will represent the **research hypothesis**, i.e., the **average lifetime exceeds 1000 hours**

$$Ha: \mu > 1000$$

Estimation & Hypothesis Testing Quiz

Which of the following option represents the correct sequence for hypothesis testing?

I. Formulate H0 and Ha
II. Set Level of Significance, α
III. Select Appropriate Test
IV. Reject or Do Not Reject H0
V. Compare with α
VI. Determine p-value

A. I, II, III, IV, V, VI

B. I, II, III, IV, VI, V

C. I, II, IV, V, III, VI

D. I, III, II, VI, V, IV

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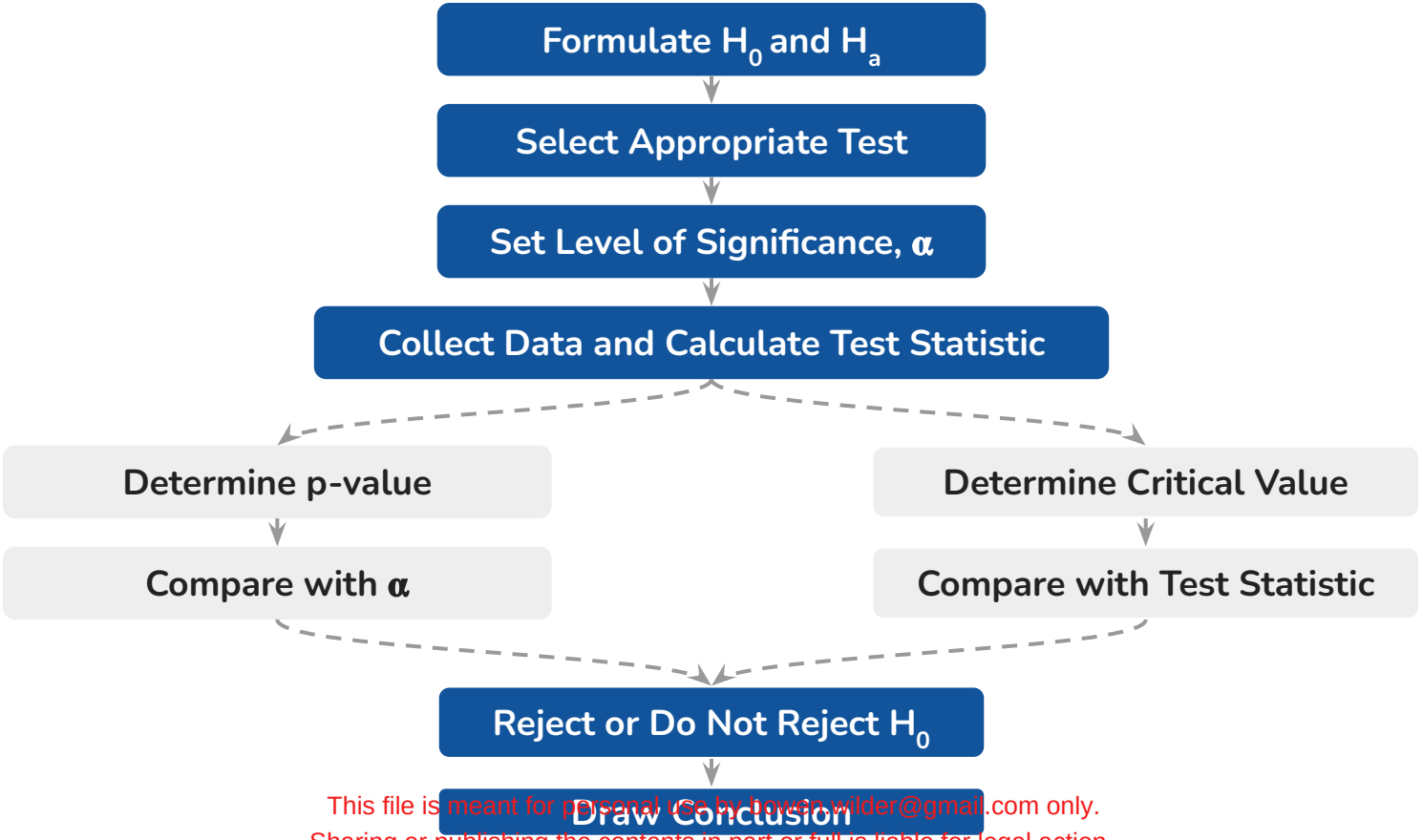
Hypothesis Testing - Steps



Estimation & Hypothesis Testing Quiz

A pharmaceutical company is conducting a clinical trial to evaluate the effectiveness of a new drug for treating high blood pressure. The company wants to determine whether the drug has a significant impact on systolic blood pressure levels. Based on the alternative hypotheses below, which type of test should be used?

A: The new drug increases systolic blood pressure levels compared to the placebo.
B: The new drug has a different effect on systolic blood pressure levels compared to the placebo.
C: The new drug decreases systolic blood pressure levels compared to the placebo.

A. A: One-tailed test, B: One-tailed test, C: Two-tailed test

B. A: Two-tailed test, B: Two-tailed test, C: One-tailed test

C. A: Two-tailed test, B: One-tailed test, C: Two-tailed test

D. A: One-tailed test, B: Two-tailed test, C: One-tailed test

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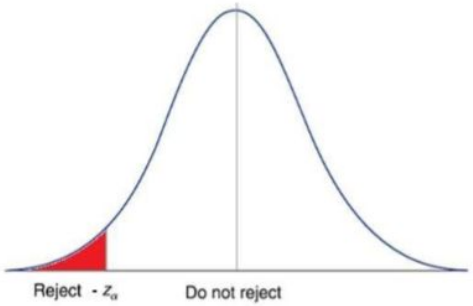
B. A: Two-tailed test, B: Two-tailed test, C: One-tailed test

C. A: Two-tailed test, B: One-tailed test, C: Two-tailed test

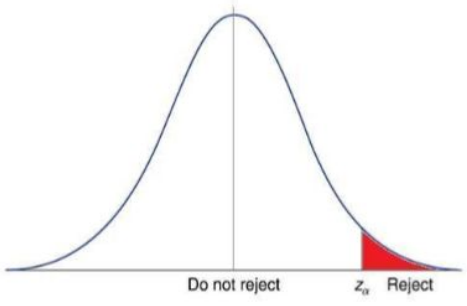
D. A: One-tailed test, B: Two-tailed test, C: One-tailed test

One-tailed vs Two-tailed Tests



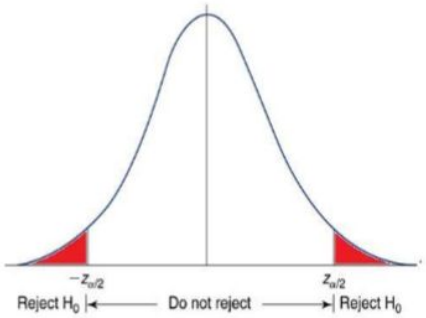
- Lower tail test.
- $H_1: \mu < \ldots\ldots$

Reject H0 if the value of test statistic is too small



- Upper tail test.
- $H_1: \mu > \ldots\ldots$

Reject H0 if the value of test statistic is too large



- Two tail test.
- $H_1: \mu \neq \ldots\ldots$

Reject H0 if the value of test statistic is either too small or too large

Estimation & Hypothesis Testing Quiz

Which of the following statements correspond to Type-I and Type-II errors?

A. A researcher is conducting a hypothesis test to check if a new drug significantly reduces cholesterol levels in patients, and concludes that it does reduce cholesterol levels when actually it has no effect.

B. A company is conducting a hypothesis test to check if a new manufacturing process increases the lifespan of light bulbs, and it fails to conclude that the lifespan increases when it actually does.

A. Both A and B are Type-I Errors

B. B is Type-I Error and A is Type-II Error

C. A is Type-I Error and B is Type-II Error

D. Both A and B are Type-II Errors

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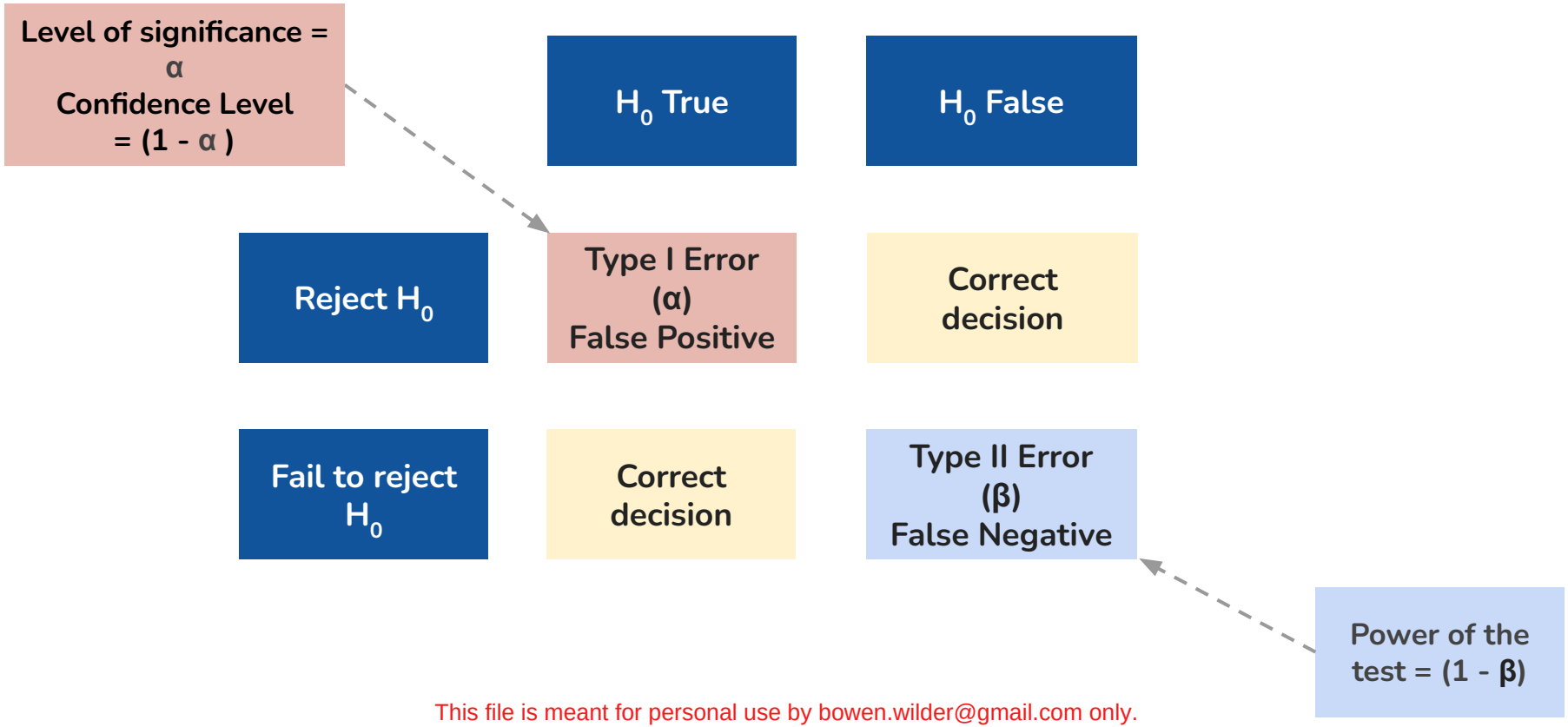
Type-I and Type-II Errors



Type-I and Type-II Errors

Scenario A: A researcher is conducting a hypothesis test to check if a **new drug significantly reduces cholesterol levels in patients**

H0: The new drug has no effect on cholesterol levels in patients

Ha: The new drug significantly reduces cholesterol levels in patients

The researcher concludes that it does reduce cholesterol levels, i.e., he **rejects the null hypothesis**

But the new drug actually has no effect, i.e., the **null hypothesis was true**

Rejected null hypothesis when it was true => Type-I Error

Type-I and Type-II Errors

Scenario B: A company is conducting a hypothesis test to check if a **new manufacturing process increases the lifespan of light bulbs**

H0: The new manufacturing process has no effect on the lifespan of light bulbs.

Ha: The new manufacturing process increases the lifespan of light bulbs.

The company fails to conclude that the new process increases the lifespan of the light bulbs, i.e., company does **not reject the null hypothesis**

But the new manufacturing process actually has the desired effect, i.e., the **null hypothesis was false**

Did not reject null hypothesis when it was false => Type-II Error

Great Learning

Happy Learning !