Efficient Packing of 3D-Polytopes into a Parallelepiped using an SMT-Solver*

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Abstract

This report describes an approach to the problem of polytope packing into a parallelepiped by means of an encoding into an SMT-Solver. The encoding is simple and flexible, and it can easily handle both convex and non-convex polytopes. A simple experiment with an unoptimized prototype shows that this technique may outperform heuristic-based approaches in finding the minimal height of the packing.

1 Introduction

Polytope Packing is the problem of placing a given set of polytopes into a parallelepiped of given length and width with the goal of finding the minimal possible height that avoids polytopes collision (polytopes may touch but they cannot compenetrate).

The problem has been studied for instance by Stoyan et. al in [1]. We take this paper as a reference and comparison for this report. More related work can be found in the aforementioned paper.

Our approach is a plain encoding into an SMT formula. SMT, Satisfiability Modulo Theories, is an area of research that combines efficient SAT-Solving and domain-specific decision procedures to build efficient tools that could reason about, for instance, arbitraty boolean combinations of linear arithmetic costraints. The encoding exploits the notion of Minkowski sum to formally describes concepts such as "polytope intersection".

Therefore, the approach can be summarized as follows: take a set of polytope descriptions, encode the problem into SMT, execute an SMT-Solver to find a solution (if any exists), read the solution and translate it back to coordinates that describes the polytopes placement.

2 Notation

We shall use $P_1, P_2, \ldots$ to denote three dimensional polytopes in $\mathbb{R}$

*This work dates back to 2005. We never had the chance to make it public before now.

3 Minkowski sum and difference

4 Encoding

5 Experiments

6 Conclusion

References

[1] et. al Y. Stoyan. Packing of convex polytopes into a parallelepiped. 2003.