

Problem set # 3

Due: Thursday, March 28th, by 8pm.

1. Swaption Pricing and Risk Management under the SABR Model:

Consider the following table of **normal swaption volatilities and corresponding par swap rates**:

Expiry	Tenor	F_0	ATM - 50	ATM - 25	ATM - 5	ATM + 5	ATM + 25	ATM + 50
1Y	5Y	117.45	58.31	51.51	49.28	48.74	41.46	37.33
2Y	5Y	120.60	51.72	46.87	43.09	42.63	38.23	34.55
3Y	5Y	133.03	46.29	44.48	43.61	39.36	35.95	32.55
4Y	5Y	152.05	45.72	41.80	38.92	38.19	34.41	31.15
5Y	5Y	171.85	44.92	40.61	37.69	36.94	33.36	30.21

NOTE: All numbers in the table are reported in bps.

- Calculate the **constant instantaneous forward rate for each swap that will lead to the par swap rates listed**. You may use a different instantaneous forward rate for each swap and are not required to go through an entire bootstrapping exercise.
- Using the rates obtained above, calculate the current annuity value for each swap in the above table.
- Calculate a table of premiums for each swaption in the table using the Bachelier pricing formula and the annuities computed above.
- For each option expiry, find the set of SABR parameters that best matches the quoted normal volatilities. Utilize the asymptotic approximation formula to calculate the normal volatility for a given set of SABR parameters and look for a solution that minimizes the distance between market and model volatilities.
- Comment on the relationship of the calibrated parameters as a function of expiry.
- Using these calibrated SABR parameters, calculate the price and normal volatility of swaptions with strikes equal to ATM - 75 and ATM + 75.
- Calculate the equivalent Black volatilities for each option in the table above.
- Calculate the delta of each options under Black's model.

- (i) Estimate a SABR smile adjusted delta for each option by calculating the expected implied shift in the volatility, σ_0 for a given shift in F_0 . Use this to create a shift of F_0 and σ_0 and use this shift to approximate delta. Compare the delta you obtain using this methodology to the delta you obtained via Black's model. Comment on any differences you observe.