MF 728: Fixed Income

Professors Eugene Sorets and Chris Kelliher — Spring 2024

Problem set # 2

Due: Thurssday, February 29, by 11pm.

1. **CDS Pricing** Assume that the risk-free interest rate is 3% for all maturities and suppose that the CDS spreads for contracts that are starting today are given by the table below. Also, assume that the expected recovery $R = 40\%$.

Maturity	premium (bps)
1y	100
2y	110
3y	120
5y	140

   (a) Assume that hazard rate is piecewise-constant and bootstrap the CDS survival curve using the information above.

   (b) What would be a fair spread for a 4y CDS, that starts today.

   (c) If you had bought a 5y CDS exactly one year ago with the contractual spread of 80bps, what would you charge me to buy it off you today.

   (d) Working with the 4y CDS in (1.b), compute the dv01 of the CDS with respect to the CDS curve in Table 1.

2. **Caplet Pricing in Different Models:** Consider a 1Y expiry 1.25% strike put option on a 1Y3m forward rate, i.e., a forward rate with 3m tenor that starts 1Y from now. Recall that in the T-Forward measure the pricing equation for a caplet put option can be written as:

$$P(K) = \delta D(0, T + \delta)\mathbb{E}\left[(K - F_T)^+\right] \tag{1}$$

You may assume $\delta = 0.25$.

   (a) Suppose you conducted a bootstrapping algorithm that identified the constant instantaneous forward rate over the entire period to be 1.25%. Calculate the discount factor needed to obtain the caplet price.

   (b) Assume that the 1Y3m forward rate follows a log-normal distribution:

   $$dF_t = \sigma F_t dW_t \tag{2}$$

   and that $\sigma = 0.15$. You may assume a constant instantaneous forward rate of 1.25% when calculating the current forward rate, F_0. Calculate the price of the option on the 1Y3m forward rate by adapting the Black-Scholes formula.

   (c) Consider the Bachelier or Normal model:

   $$dF_t = \sigma_n dW_t \tag{3}$$

   Calculate what you believe to be the σ_n that will make the Normal model best approximate the Log-Normal model above.

   (d) Using this σ_n, calculate the price of the put option defined above (1) under the Bachelier model. Compare to the Black-Scholes model price. Are they similar? Why or why not? Which one is higher? Why?

3. **Stripping Caplet Volatilities:**

(a) Consider the following table of two-year at-the-money cap volatilities under the Black's model:

Start	Length	Black Vol
1Y	2Y	0.15
2Y	2Y	0.2
3Y	2Y	0.225
4Y	2Y	0.225
5Y	2Y	0.25

Assume that caplets are paid quarterly on three-month forward rates each with $\delta = 0.25$. Further assume a flat interest rate curve with 1% instantaneous forward rates along the entire curve. Calculate the price of each cap using Black's model.

(b) Extract the Black-Scholes at-the-money implied volatilities for each caplet from the provided set of cap volatilities. Comment on the shape of the caplet vs. cap volatilities.