Rotational Magnetic Levitation: A Folded/SHS Framework with Practitioner Playbook

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Abstract

We provide an experiment-ready guide to rotational magnetic levitation (RML) accompanied by a concise theoretical scaffold based on Floquet averaging, stable Hamiltonian structures (SHS), and a folded symplectic normal form for vertical equilibrium. The practical portion covers hardware setup, tuning, measurement plans, and troubleshooting. The theoretical portion explains why a small vertical bias field $B_{r,z}$ selects a Reeb-like direction, how fast rotation yields a phase-locked "blue phase," and why stable levitation occurs at a fold where the averaged vertical force crosses zero with negative slope. We conclude with experiment recipes, scaling laws, and data-analysis templates that tie directly to the model.

Contents

1 Quick-Start for Practitioners

1.1 Hardware and Materials

- **Rotor magnet (Rm):** Diametrically magnetized cylinder or sphere on a high-speed motor with RPM readout.
- **Floater magnet (Fm):** Smaller permanent magnet (sphere or short cylinder) to levitate.
- **Bias magnet (Bm):** Small static magnet mounted above the rotor to supply a controllable vertical field component $B_{r,z}$. A micrometer stage is recommended.
- **Alignment:** 3-axis stages to set rotor–floater separation d and to position Bm.
- **Damping option:** Adjustable aluminum plate or viscous bath (water/glycerin) for capture assistance.
- **Sensing:** High-speed camera, optional IMU on Fm, Hall sensors, and a laser distance sensor for vertical height z.

Recommended starting geometry. Rotor radius $R_r \approx 15$–$25\,\text{mm}$; Floater radius $R_f \approx 5$–$10\,\text{mm}$; initial separation $d_0 \approx 10$–$40\,\text{mm}$; NdFeB (N42–N52).

1.2 Safety and Alignment

Use a clear shield around the rotor; verify balance to target RPM (e.g., 12–18k RPM). Fix the spin axis vertical. Verify polarities and field magnitudes with a Gaussmeter.

1.3 First-Levitation Protocol (Minimal Knobs)

1. **Dial a small vertical bias:** set $B_{r,z} \sim 1\%$–10% of the transverse field at Fm.
2. **Set separation:** start at $d \approx 25$ mm.
3. **Spin up:** sweep rotor frequency ω_r slowly from low to high.
4. **Watch for capture (blue phase):** small constant tilt θ_f, steady phase lag ϕ, stable height.
5. **Remove damping (if used):** steady levitation should persist after capture.

Tell-tales of success. Nearly constant $\theta_f \ll 1$ rad, steady ϕ, height decreases mildly as ω_r increases.

If capture fails. Increase $B_{r,z}$ and/or introduce damping (aluminum plate within 5–10 mm below Fm) to enlarge the basin of attraction.

1.4 Knobs and Qualitative Effects

- Increasing ω_r: decreases levitation height and θ_f.
- Increasing $B_{r,z}$: widens capture basin, decreases θ_f, stabilizes phase-lock.
- Increasing R_f: raises required ω_r (since $I_f \propto R_f^5$).
- Increasing remanence: weaker effect on thresholds; modestly increases height.

1.5 Minimal Measurement Set

- Levitation height $z(\omega_r)$ (or separation $d(\omega_r)$).
- Floater orientation: tilt $\theta_f(\omega_r, B_{r,z})$ and phase lag $\phi(\omega_r, B_{r,z})$.
- Capture probability vs $B_{r,z}$ (with/without damping).

1.6 Troubleshooting

- Jitter or no capture: increase $B_{r,z}$ or add damping; check rotor wobble.
- Sudden drop at high ω_r: mode transition; back off ω_r or adjust d.
- Floater spins uncontrollably: excess $B_{r,\perp}$ asymmetry; reduce rotor tilt or lower ω_r.
- Extreme sensitivity to nearby objects: likely near the fold $F_z \simeq 0$; increase $B_{r,z}$.

2 Design of Experiments (DoE)

2.1 Core Factor Sweeps

- ω_r: 6–18 krpm, 8–10 levels.
- $B_{r,z}$: 0, 0.5, 1, 2, 5 mT (measured at floater location with rotor off).
- d: 10–40 mm in steps of 2.5–5 mm.
- R_f: 5, 6, 8, 10 mm (same material).

2.2 Response Variables

Levitation height z, tilt θ_f, phase lag ϕ, capture probability, time-to-capture, mode labels (UD/Side/Mixed/U), drop frequency.

2.3 Recommended Plots

- z vs ω_r at fixed $B_{r,z}$ (expect decreasing trend).
- θ_f, ϕ vs ω_r for several $B_{r,z}$.
- Stability maps in (ω_r, d) for each $B_{r,z}$.
- Capture probability vs $B_{r,z}$ with/without damping.
- Threshold ω_r vs R_f (log–log slope $\approx 5/2$; see §D).

2.4 Example DoE Table

Factor	Levels	Units	Notes	Role
ω_r	6–18k (10)	rpm	evenly spaced	primary sweep
$B_{r,z}$	0, 0.5, 1, 2, 5	mT	measured w/ rotor off	bias control
d	10:5:40	mm	fixed by stage	geometry
R_f	5, 6, 8, 10	mm	same grade NdFeB	inertia scaling

Table 1: Suggested factors and levels for initial DoE.

3 Theory to Practice: Intuition, Formulas, Predictions

3.1 One-Paragraph Intuition

A fast, nearly transverse rotating field from the rotor produces a periodic torque on the floater. A small *static* vertical field $B_{r,z}$ selects a preferred axis (the "Reeb" direction). Above a critical speed the floater phase-locks: its tilt θ_f and phase lag ϕ become approximately constant while it precesses at the drive rate. In this locked regime the time-averaged vertical force F_z is non-monotone in distance and crosses zero with *negative slope*, producing a restoring force (stable levitation). Damping helps capture but is not required to maintain the locked state.

3.2 Working Formulas in the Blue Phase

Let I_f be the floater moment of inertia, m_f its magnetic moment, and $B_{r,\perp}$ the transverse amplitude of the rotor field at the floater. In the locked steady state,

$$\phi \approx \frac{\zeta_{\mathrm{rot}}\, \omega_r}{I_f \omega_r^2 - m_f B_{r,z}}, \qquad \theta_f \approx \frac{m_f B_{r,\perp}}{I_f \omega_r^2 - m_f B_{r,z}}. \tag{1}$$

Implications. Increasing ω_r or $B_{r,z}$ decreases θ_f and ϕ. Finite I_f is essential (drag alone cannot replace it). For a sphere of radius R_f, $I_f \propto R_f^5$, so thresholds scale strongly with size.

3.3 Folded Normal Form for Vertical Equilibrium

Let $F_z(d; \omega_r, B_{r,z}, \ldots)$ denote the time-averaged vertical force. Near a stable levitation distance $d_\star$,

$$F_z(d) \approx K\,(d - d_\star) + \cdots\,, \qquad K < 0. \tag{2}$$

The *fold set* is $\Sigma = \{F_z = 0\}$. Stability corresponds to crossing with negative slope; practically, a gentle tap yields return rather than escape.

3.4 SHS Picture & Reeb-Locked Orbit

On a fixed energy shell $\mathcal{M}^3 = \{H_{\text{eff}} = E\}$, there exists a 1-form λ (contact-like) such that $\lambda \wedge d\lambda > 0$ and the restricted dynamics are conformal to the Reeb field R of λ. The small $B_{r,z}$ provides coorientation and sets the Reeb direction (precession). The blue phase corresponds to a closed Reeb-type orbit whose linearization has one stable direction (vertical) and a neutral direction along the orbit.

3.5 Practitioner Predictions (Immediately Testable)

P1. $\theta_f(\omega_r, B_{r,z})$ and $\phi(\omega_r, B_{r,z})$ follow Eq. (1); fits estimate I_f and effective field coefficients.

P2. $z(\omega_r)$ strictly decreases at fixed $B_{r,z}$, with inflections near mode transitions.

P3. Hysteresis/monodromy under slow loops in $(\omega_r, B_{r,z})$ encircling Σ: height and phase do not return identically.

P4. Capture probability increases with $B_{r,z}$ and with damping during approach; steady state persists after damping is removed.

P5. Nano-regime crossover: intrinsic spin angular momentum can substitute for drive-induced twist, enabling stabilization in static fields.

4 Experiment Recipes

4.1 Mapping the Reeb Direction (Bias-Sweep)

Fix d. For each $B_{r,z}$, sweep ω_r upward; record θ_f, ϕ, z. *Expected:* decreasing θ_f, ϕ with both ω_r and $B_{r,z}$; widening capture basin with larger $B_{r,z}$.

4.2 Height–Frequency Curves (Core Plot)

For multiple $B_{r,z}$, measure $z(\omega_r)$. *Expected:* monotone decrease; inflection indicates proximity to a mode transition.

4.3 Basin Mapping with Damping

Place an aluminum plate at gaps 2–20 mm. From randomized initial conditions, measure capture probability into the blue phase. *Expected:* strong improvement at small gaps; once captured, levitation persists after removing the plate.

4.4 Size Scaling Test

Repeat the core plot for $R_f = 5, 6, 8, 10$ mm. Estimate minimum ω_r for robust capture. *Expected:* $\omega_{r,\min} \propto R_f^{5/2}$ (holding geometry constant).

4.5 Hysteresis/Monodromy Loop

Perform slow parameter loops: increase ω_r while decreasing $B_{r,z}$, then reverse; track z, ϕ. *Expected:* loop area > 0 in $(\omega_r, B_{r,z})$ plane (contact-geometric phase).

5 Data Analysis Checklist

- Fit θ_f and ϕ to Eq. (1) to extract I_f and field coefficients.
- Segment time-series into modes (UD/Side/Mixed/U) using spectral features of $z(t), \theta_f(t), \phi(t)$.
- Construct stability regions in (ω_r, d) for each $B_{r,z}$.
- Quantify capture probability vs $B_{r,z}$ and plate gap; fit a logistic curve.

6 Extensions and Variations

- Non-axisymmetric rotors: tilt magnetization by α to control $B_{r,z}$ without a separate bias magnet.
- Multi-rotor lattices: levitate arrays and probe synchronization/collective phases.
- Fluid media: vary viscosity to map capture-time vs damping while holding steady-state height.
- Sensing upgrade: 3D Hall-sensor ring to reconstruct $\mathbf{B}$ and test averaging model.

A Floquet/Kapitza Averaging Sketch

Let the rotor field be

$$\mathbf{B}_r(t) = B_{r,\perp}\,(\cos\omega_r t\,\hat{\mathbf{x}} + \sin\omega_r t\,\hat{\mathbf{y}}) + B_{r,z}\,\hat{\mathbf{z}}. \tag{3}$$

The floater dipole $\mathbf{m}_f = m_f \hat{m}_f$ obeys rigid-body dynamics with torque $\boldsymbol{\tau} = \mathbf{m}_f \times \mathbf{B}_r$ and optional rotational drag $-\zeta_{\text{rot}}\boldsymbol{\omega}_f$. Averaging over $2\pi/\omega_r$ yields an effective correction of order $B_{r,\perp}^2/\omega_r^2$ and a gyroscopic term $\propto I_f\omega_r$. In the locked regime, $\hat{m}_f$ maintains constant θ_f, ϕ in the drive frame, leading to Eq. (1).

B SHS on the Energy Shell

Let H_{eff} denote the averaged Hamiltonian on $T^*(\mathbb{S}^2 \times \mathbb{R})$. On a fixed energy level $\mathcal{M}^3 = \{H_{\text{eff}} = E\}$, choose a 1-form λ such that $\lambda \wedge d\lambda > 0$ and the Hamiltonian vector field restricts to be conformal to the Reeb field R of λ. The small but nonzero $B_{r,z}$ supplies coorientation and selects the Reeb direction (precession axis). The locked blue phase is a closed Reeb-type orbit with one stable transverse direction (vertical).

C Folded Symplectic Normal Form for Vertical Motion

In reduced vertical coordinates (d, p_d), the averaged 2-form may flip sign across the fold set $\Sigma = \{F_z(d) = 0\}$. A local model is

$$\omega = \omega_0 + s\ \mathrm{d}s \wedge \lambda, \tag{4}$$

where s is the signed distance to Σ and λ is the contact 1-form inherited from the locked precession. Stability at $d_\star$ corresponds to $\partial_d F_z|_{d_\star} < 0$.

D Scaling Relations

For a solid sphere of radius R_f and density ρ,

$$I_f = \frac{8\pi}{15}\rho R_f^5. \tag{5}$$

Balancing $I_f \omega_r^2 \sim m_f B_{r,z}$ with $m_f \propto R_f^3$ suggests

$$\omega_{r,\min} \propto R_f^{5/2}, \tag{6}$$

if the field geometry is approximately size-invariant. The tilt obeys $\theta_f \propto B_{r,\perp}/(I_f\omega_r^2 - m_f B_{r,z})$, and increasing ω_r typically reduces the equilibrium distance $d_\star$.

E Bill of Materials (Sketch) & CAD Notes

BLDC with encoder; 3D-printed chuck with set screws; acrylic shield; micrometer stages for d and Bm; Gaussmeter; aluminum plate on a linear stage. Provide slotted mounts to tune lateral offset and Bm height precisely.

F Analysis Notebook Outline

Inputs: RPM vs time, $z(t)$, $\theta_f(t)$, $\phi(t)$, $B_{r,z}$, d, R_f.
Processing: smoothing, phase extraction, mode classification (spectral clustering), locked-state parameter estimation.
Fits: rational θ_f, ϕ models (Eq. (1)); logistic for capture vs bias/damping; $\omega_{r,\min}(R_f)$ slope.
Outputs: stability maps, scaling plots, residuals, parameter tables.

One-Page Executive Summary

Add a small vertical bias $B_{r,z}$. Spin up slowly. Seek a small, constant θ_f and steady ϕ. Height drops as ω_r rises. Use an aluminum plate only for capture. Measure $z(\omega_r)$, $\theta_f(\omega_r, B_{r,z})$, capture probability vs $B_{r,z}$. Fit the rational formulas to extract inertial/field scales. Stable levitation sits on the fold $F_z = 0$; negative slope means you're safe. The SHS/Reeb picture explains stability of a non-minimum energy configuration once phase-locked.