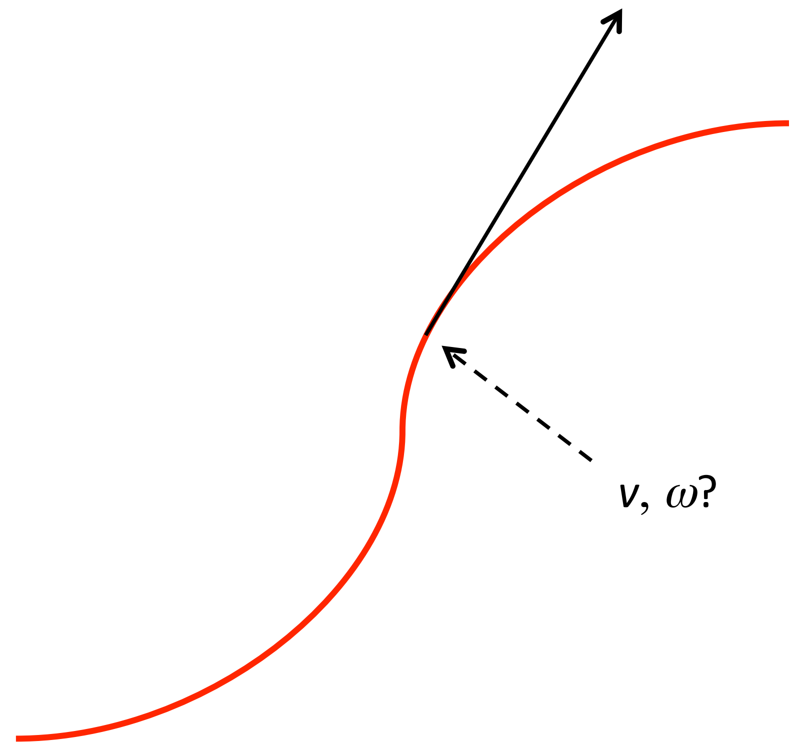
Kinematics
Moving Along an Arbitrary Path

Kinematics
Moving Along an Arbitrary Path

For a differential navigation robot to follow an arbitrary path, linear speed *v* and angular speed ω have to be assigned and controlled for all points in the curve at all times.



Kinematics
Example - Parabola

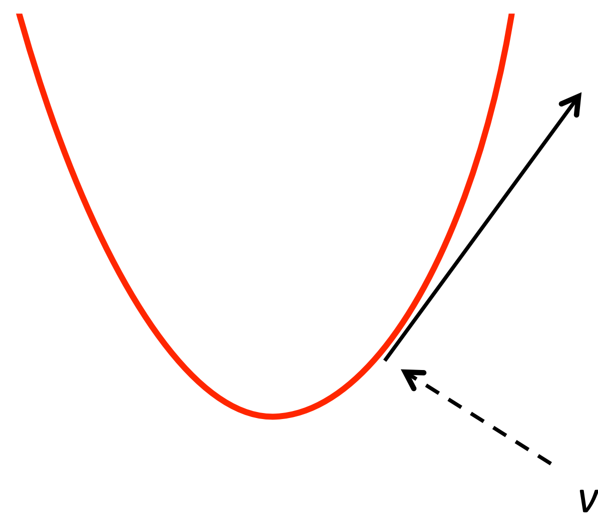
or example, we want the robot to move ong a parabola $y=x^2$

ow do we calculate v and ω?

e can make the robot move along the arabola at different speeds v, e.g., the robot ay travel at v=1, v=2, etc.

ow does ω vary according to the different lues of v?

e present the concept of a curvature efined by variable k and show that $\omega = k \cdot v$



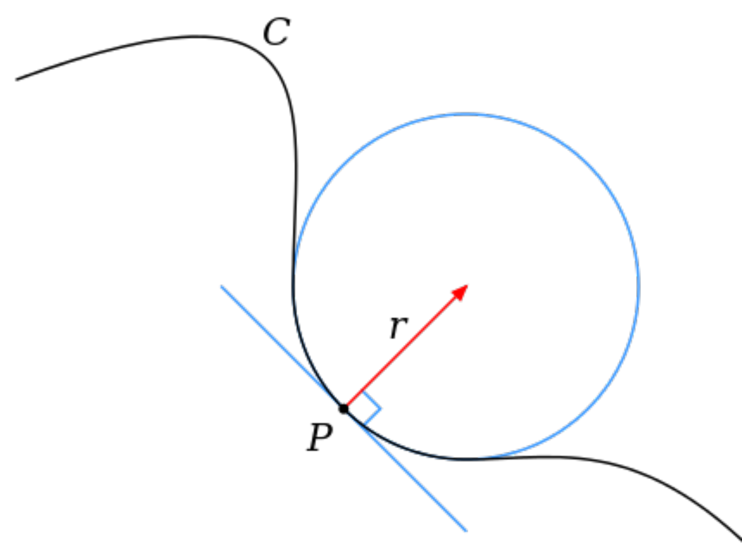
Kinematics
Curvature and Radius of a Curve

urvature

- The curvature (k) of a curve (C) at a given point (P) is a measurement of how much the curve is "bending" at that point, i.e., it is the rate at which the direction (angle) of the tangent changes per distance unit.

dius

- The radius (r) of a curve (C) at a given point (P) is the radius (r) of the circle that would "best approximate" the curve at the given point.



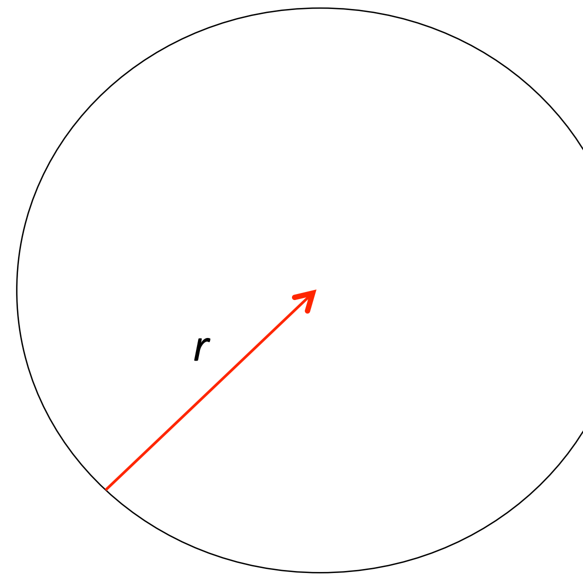
Kinematics
Circle Example and Relationship between *k* and *r*

cle

- The circle that best approximates the curvature at any given point is the circle itself. Thus, the radius of the curve at a given point is the radius (r) of the circle itself.
- Since the circle is perfectly symmetrical, the rate at which the direction changes is the same for every point in the circle.
- Since the direction changes 2π radians over a distance of $2\pi r$, we have:

$$k = 2\pi / 2\pi r = 1/r$$

- Here we can see that curvature (k) and radius (r) are equivalent inverse measurements.



Kinematics
Straight Line Example

traight Line

For a straight line, given a circle of finite radius, we can always find another circle of bigger radius that better approximates a straight line at a given point *P*.

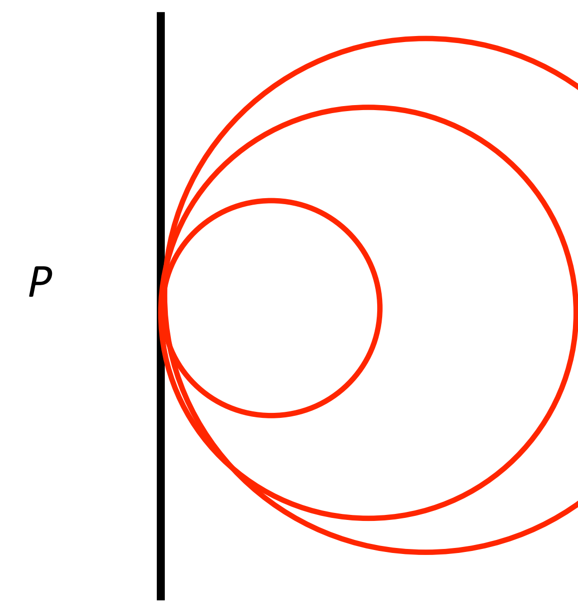
The direction of movement along the curve does not change, thus based on the curvature definition:

$$k = 0$$

The circle that "best approximates" a straight line has an infinite radius given by:

$$r = \infty$$

Thus, we have the relationship $k = 1/r$

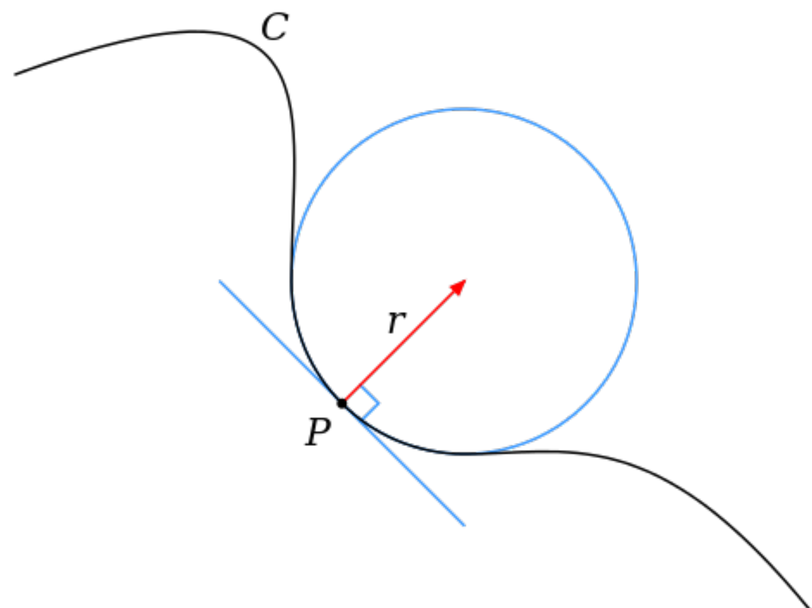


Kinematics
Semi-formal definition of curvature and radius

iven a curve C, let v and ω and be
e linear and angular speeds at
ordinate $P=(x, y)$, respectively.
hus we define:

$r = v/\omega$

$k = \omega / v$



Kinematics
Observations of the semi-formal definition

Note that in the previous definition $k=1/r$ and $\omega=kv$.
The definition of curvature is thus given by:

- ω - the rate of change of the angle per unit of time.
- v - the rate of change in distance per unit of time.
- $k=\omega/v$ - the rate of change of the angle per unit of distance.

We can move along a curve with different pairs (v, ω), where this ratio is a constant that depends only on the path shape.

Kinematics
Calculating *k* for any shape of the form *f(x,y)=c*

- $k = -\frac{sign\left(\dot{x}\frac{\partial f}{\partial y}\right)}{\left\|\nabla f\right\|^3} \nabla f^{\perp} \cdot Hf \; (\nabla f^{\perp})^T$
- In the equation above:
  - *sign*($\dot{x}$)= $\begin{cases} -1 & \text{if the motion tangent is changing, e.g. towards the negative } x-\text{axis} \\ 0 & \text{if the motion tangent is not changing, e.g. in the negative or positive } x-\text{axis} \\ +1 & \text{if themotion tangent is changing, e.g. in the positive } x-\text{axis} \end{cases}$
  - $\left\|[a \quad b]\right\| = \sqrt{a^2+b^2}$ is the norm
  - $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$ is the gradient of f, $f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$
  - $\nabla f^{\perp} = \left[-\frac{\partial f}{\partial y}, \frac{\partial f}{\partial x}\right]$ (note that $\perp$ means perpendicular)
  - $Hf = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$ is the Hessian matrix of f

Kinematics
Example curvature of the parabola

- $y = x^2$
- $f(x, y) = y - x^2$
- $\nabla f = [-2x \quad 1]$
- $\nabla f^{\perp} = [-1 \quad -2x\,]$
- $Hf = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$
- $\nabla f^{\perp} \mathrm{Hf}\, \nabla f^{\perp^T} = [-1 \quad -2x] \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -2x \end{bmatrix} = -2$
- $\left|\left|\nabla f\right|\right| = \sqrt{1 + 4x^2}$
- $k = -\frac{sign(\,\dot{x} f_y)}{\left|\left|\nabla f\right|\right|^3} \nabla f^{\perp} \cdot Hf\; (\nabla f^{\perp})^T = \frac{sign(\dot{x})}{\sqrt{1+4x^2}^3} \cdot 2$

Kinematics
Example curvature of the parabola

$\frac{sign(\dot{x})}{\sqrt{1+4x^2}^3} \cdot 2$

, suppose we are in the parabola at position $(\sqrt{2}, 2)$.

Q: If we want to travel at 3 inches per second towards the positive x axis direction, what value should we assign to ω?

- Answer:
  - Since we are travelling towards positive x axis we have sg$(\dot{x})$ =1
  - $\omega = k \cdot v = \frac{1}{\sqrt{1+4 \cdot \sqrt{2}^2}^3} \cdot 2 \cdot 3 = \frac{2}{9}$

Q: Assuming a real robot with differential navigation, how do we know what is the maximum linear speed at which the robot can move at point $(\sqrt{2}, 2)$?