Distribution	Probability Function	Mean	Variance	MGF
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y}$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	No closed form
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}$	λ	λ	$e^{\lambda(e^t-1)}$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$
Uniform	$f(y) = \frac{1}{\theta_2-\theta_1}$	$\frac{\theta_1+\theta_2}{2}$	$\frac{(\theta_2-\theta_1)^2}{12}$	$\frac{e^{t\theta_2}-e^{t\theta_1}}{t(\theta_2-\theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2}$	μ	σ^2	$e^{\mu t + \frac{t^2\sigma^2}{2}}$
Exponential	$f(y) = \frac{1}{\beta} e^{-\frac{y}{\beta}}$	β	β^2	$(1-\beta t)^{-1}$
Gamma	$f(y) = \left(\frac{1}{\Gamma(\alpha)\beta^\alpha}\right) y^{\alpha-1} e^{-\frac{y}{\beta}}$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{y^{\frac{\nu}{2}-1} e^{-\frac{y}{2}}}{2^{\frac{\nu}{2}}\Gamma(\frac{\nu}{2})}$	ν	2ν	$(1-2t)^{-\frac{\nu}{2}}$
Beta	$f(y) = \left(\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right) y^{\alpha-1}(1-y)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	No closed form

MATH324 Formula Sheet

Julian Lore

Bias

$$Bias(\hat{\theta}) = E[\hat{\theta}] - \theta$$

$$\begin{aligned} MSE(\hat{\theta}) &= E[(\hat{\theta}-\theta)^2] \\ &= V(\hat{\theta}) + [Bias(\hat{\theta})]^2 \end{aligned}$$

Common Point Estimators

Target Parameter θ	Sample Size(s)	Point Estimator $\hat{\theta}$	$E(\hat{\theta})$	Standard Error $\sigma_{\hat{\theta}}$
μ	n	$\overline{Y}$	μ	$\frac{\sigma}{\sqrt{n}}$
p	n	$\hat{p} = \frac{Y}{n}$	p	$\sqrt{\frac{pq}{n}}$
$\mu_1 - \mu_2$	n_1, n_2	$\overline{Y}_1 - \overline{Y}_2$	$\mu_1 - \mu_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
$p_1 - p_2$	n_1, n_2	$\hat{p}_1 - \hat{p}_2$	$p_1 - p_2$	$\sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}}$

Confidence Intervals

$$P(\hat{\theta}_L \le \theta \le \hat{\theta}_U) = 1 - \alpha$$

$$P\left[\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-(\frac{\alpha}{2})}}\right] = 1 - \alpha$$

Large Sample

$$Z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}} \sim N(0,1)$$

$z_{\alpha/2}\sigma_{\hat{\theta}} = B$

$\hat{\theta}_n - z_\alpha \sigma_{\hat{\theta}}$

$\hat{\theta}_n + z_\alpha \sigma_{\hat{\theta}}$

$\hat{\theta}_n \pm z_{\alpha/2} \sigma_{\hat{\theta}}$

Small Sample

$$T_{n-1} = \frac{\overline{Y} - \mu}{\frac{S}{\sqrt{n}}}$$

$P(-t_{\frac{\alpha}{2}} \le T \le t_{\frac{\alpha}{2}}) = 1 - \alpha$

$\overline{Y} \pm t_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right), \nu = n - 1$

$\left(\overline{Y_1} - \overline{Y_2} \pm t_{\alpha/2} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right), \nu = n_1 + n_2 - 3$

Efficiency

$$eff(\hat{\theta}_1, \hat{\theta}_2) = \frac{V(\hat{\theta}_2)}{V(\hat{\theta}_1)}$$

Misc

$$\begin{aligned}
F_{Y_{(n)}} &= [F(Y)]^n \\
F_{Y_{(1)}} &= [1 - F(Y)]^n \\
\overline{X} &= \frac{1}{n}\sum_{i=1}^n X_i \\
S^2 &= \frac{\sum_{i=1}^n (Y_i - \overline{Y})^2}{n-1} \\
V(S^2) &= \frac{2\sigma^4}{n-1} \\
S_P^2 &= \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} \\
P(g(x) \ge \lambda) &\le \frac{E[g(x)]}{\lambda}, \lambda > 0 \\
P(|X - \mu| \ge k\sigma) &\le \frac{1}{k^2} \\
Var(X) &= E[(X-\mu)^2] \\
&= E[X^2] - (E[X])^2 \\
z_{0.025} &= 1.96 \\
\Gamma(z) &= \int_0^\infty x^{z-1} e^{-x}\, dx \\
E[E(\hat{\theta}|U)] &= E[\hat{\theta}] \\
V(\hat{\theta}) &= V[E(\hat{\theta}|U)] + E[V(\hat{\theta}|U)]
\end{aligned}$$