1 Uninformed Search

Search Problem State space S: all possible configs of domain, **init state** $s_0 \in S$, **goal states** $G \subset S$ (end states), **Operators** A (acts avail), **Path, Path cost**, c, **Soln** path $s_0 \to s_g \in G$, **Opt soln**: path with min $
Eight puzzle: States (conf of puzzle), goals (target conf), ops (swap blank with adj), path cost (# moves)
Rep state space search as graph, verts are states, edges are ops. Build **search tree** to find goal state. Search tree nodes not same as graph nodes
Data struct for search tree: **Node** (state id, parent state + op, cost of path, depth). To expand node, apply all legal ops and gen new states.

Generic search alg Init search tree with s_0. Loop: If no nodes can be expanded, fail. Else choose node to expand, if node has goal, return path, else expand node by applying each op and getting new states, adding to tree.

Uninformed (blind) search If state isn't goal, you don't know how close to goal it might be.

Uninformed Search algs Key props: **Completeness** (guarantee soln if it exists), **Optimality** (how good is soln), **space complexity**, **time complexity**.

Search complexity Branching factor b, **solution depth** d
All uninf search time complex $O(b^d)$, very general and very expensive, no knowledge.

Breadth-first search (complete w/ finite b, guaranteed shortest path if unit cost = optimal, but not if weighted graph). $O(b^d)$ space. **depth-first search** ($O(bm)$ space, easy to do recursively, more efficient than BFS if many goal paths, **not optimal**, may not complete (cycles), don't use DFS for big d), **uniform-cost search** (BFS but with **general** (weighted graph) step costs, use a pqueue, opt & comp), **depth-limited search** (DFS but stop at goal or max depth, always terms, but not complete), **iterative deepening search**(depth-lim search but increasing depth, expands nodes mult times, complete , linear mem req like DFS, but more time, optimal if unit cost, preferred for large state spaces)
Revisiting states: maintain **closed list** to store expanded nodes, good for probs with repeated states. $O(|S|)$ time and space. Sometimes re-expanding states could be better (compare old and new path cost, also sometimes domain may be too large to store all states).

What method to use? To find opt: BFS, IDS for unit cost, uniform-cost search if general cost. Large state space: DFS max length is known, IDS otherwise. Limited mem: DFS/IDS. Quickly find best soln with budget: Depth-limited search if unit cost, UCS if gen cost.

2 Informed Search

Use **heuristics** to guide search. Uninformed expand nodes based on dist from start node. Informed expands based on **distance to goal**. If we don't know exact distance, we use intuition, **heuristic**. Heuristics come from prior knowledge of prob, exact soln to relaxed vers of prob, learning from exp
Heuristic for path planning: straight-line distance between two places.
Eight puzzle: number of misplaced tiles, total Manhattan dist

Algs **Best-First Search**(greedy , expand most promising node first, close to BFS, if heuristic is 0, then same as BFS, opp of UCS(cost-so-far) vs cost-to-go, $O(b^d)$ time/space, good heuristic can make $O(bd)$, **not always complete**(loops), but complete if finite space if we check repeated, not optimal), **Heuristic Search**(Problem: best-first too greedy, doesn't account for cost so far. Soln: **Heuristic search**, greedy wrt to $f = g + h$, g is cost so far, h is heur. Use pq, add to q w/ p: $f = g + h$, end when goal popped from q. Note we cont expanding nodes after finding goal if $\exists$ unexpanded w/ lower cost than current path to goal. Not optimal , unless we put conds on heuristics), **A* Search** (Heuristic search with AH. Complete , $f(s) = g(s) + h(s) \leq g(s) + c(s,s') + h(s') = f(s')$ by consistency, so no node can be re-expanded. If soln, c must b bounded, so $A*$ will find. Opt , prove by contradiction, showing impossible that subopt goal is expanded before opt. Still worst case $O(b^d)$, but $O(bd)$ with perfect heuristic because only expand nodes on opt path. With given h, no other search alg can expand less nodes), **Iterative Deepening A***(DFS, but use f to determine order to explore children. Stop at f val instead of d. Same props as $A*$, but less mem. If we remember expansion of old nodes $\to$ SMA*)

Admissible heuristics $h^*(n)$ is shortest p from n to any goal. h is **admissible heuristic** if $h(n) \leq h^*(n) \forall n$. They are optimistic. Trivial ah: $h(n) = 0 \forall n$, get UCS. Obviously $h(g) = 0 \forall g \in G$ for AH. Usually relaxed vers of prob gives AH.

Consistency AH h is called **consistent/monotone*** if for every state s and succ s', $h(s) \leq c(s,s') + h(s')$, i.e. h gets more precise as we get closer to goal. Vers of triangle ineq. Can fix inconsistent heuristics by: $f(s') = g(s') + h(s') \to f(s') = \max[g(s') + h(s'), f(s)]$

Dominance $h_2(n) \geq h_1(n) \forall n$ and both AH, then h_2 dominates h_1, i.e. more informative.

Decomposition Break complex prob into smaller parts. Decomp and putting soln together may give up optimality. Use decomp for probs we can't solve w/o. Subsolns can be cached & reused. Need to be careful that when we chose subgoal, overall prob still has soln. **Macro-action** is sequence of actions from orig problem (e.g. make T for Rubik's)
Abstraction ignores info to speed up comp, make compact representation, map several real states to one abstract state
Consistency stronger than admissibility, cannot be cons but not admis. A* space vs ID depends on how good heur is. DFS is special case of best-first.

3 Optimization

large cont/combin state space. Can't search all possible soln. Non-uniform cost.

Traveling salesman prob Vertices+dist between pairs. Get shortest path to visit each vert once. Tour = path that satisfies goal.
Optimization prob described by **states** and **evaluation function**, note that states are candidate solutions (can be partial or wrong) here, not descrp of world . Func corresp to path cost.

Optimization Search **Constructive methods**, start from scratch, build up. **Iterative improvement**, start with soln, improve. Both involve local search.

Generic local search Start at init X_0, repeat until satisfied: Gen neighbors of X_i; eval them. Select 1 neighbor X_{i+1} to become current config.

Discrete Hill-Climbing Start with X_0, val $E(X_0)$. Repeat until sat: Gen neighbors of X_i and $E(X_i)$. Get $\max_i E(X_i)$. If max is less than $E_{\max}$ of init, then stop. Else update X_i to be new X_i and E to be new $E_{\max}$. This is a variant of best-first search, easy to prog, no memory of past req, can handle large probs. Small neighborhood = less neighbors, possibly worse soln, large neighborhood = more to eval, possibly less local optima, better soln.

Problem hill climbing can easily get stuck in plateau or local opt, to fix, use random re-starts or pick any move that leads to improvement (**randomize hill climbing**).

Simulated annealing Like hill climbing, but allows bad moves to escape local opt. Decrease size+freq of bad moves over time.
Alg: Start with X_0 and $E(X_0)$. Loop until satisfied: choose random neighbor, if val is greater than E_{max}, replace current max. If greater than current val we holding, replace current X. Else, with prob p, still replace cur val. Return max at end.
What to use for p? Constant, val that decays to 0, val that depends on how bad move is. We usually use Boltzmann distribution $p = e^{-(E - E_i)/T}$, bad $E_i \to$ small p. T is called **temperature**, usually start high then decrease to 0 over time. Can decrease T by mult by constant $0 < \alpha < 1$ at every iter. If T is high $\to$ alg is in exploratory phase, if low, exploitation phase.

If T decreases "slowly enough", optimal , but may take ∞ moves. SA better than HC when lots of local opt. HC preferred if func is smooth, not many local opt, most local opt are similar.

Parallel search Run mult separate searches (HC or SA) in parallel, keep best soln

Local beam search Like parallel search, but share info across searches. Start k searches in parallel, but keep k (**beam width**) top solns at each step.

Genetic algs **Individual**=candidate soln. Each indiv has **fitness** (quality of soln). **Population** = set of indivs. Pops change over **generations** by applying **operations**(mutation(inject random change with mutation rate = prob of mutation occur)/crossover(combine parts of indivs to make new indiv, use **crossover mask** to specify which parts taken from 1 indiv as bin string, rest taken from other)/selection) to indivs. Higher fitness = more likely to survive & reproduce. Usually represent indivs by **binary string**.

Alg : (params: fitness,threshold,p,r,m) init P with p rand indivs. Eval, get $Fitness(h) \forall h \in P$. While $\max_h Fitness(h) < threshold$: Select $(1-r)p$ members of P to put in P_s. Crossover $\frac{rp}{2}$ pairs of indivs. For each pair, produce two offspring and incl in P_s. Mutate 1 rand bit in mp rand indivs of P_s. Update $P \leftarrow P_s$. Eval fitness $\forall h \in P$. After loop return ind w/ max fit.

Selection Survival of fittest: **Fitness proportionate**, might lead to crowding (mult copies of same soln), **tournament selection**, pick 2 rand indivs, with prob p select fitter one. **Rank selection**, sort all by fitness, prob of selection proportional to rank. **Softmax (Boltzmann) selection** $P(i) = \frac{e^{Fitness(i)/T}}{\sum_{j=1}^{p} e^{Fitness(j)/T}}$.

Elitism , best soln can die during evol, so we preserve best soln encountered. Gen algs more expensive than HC & SA.

Pros cons of gen alg Pro: Intuitive due to analogy, can be effective if tuned properly. Con: Perform dependent on encoding of problem. Many params to tweak. Low mutation rate = overcrowding. Too high = too random. Search ops: mutation, xover, select

4 Constraint Satisfaction Problems

Use constraints to narrow search space. Def: **variables** V_i that can take vals from domain D_i. **Constraints** specifying allowed combinations of values for variables. Constraints can be represented as a function or list of allowable vals. **CSP solution** is assignment of vals to vars st all constraints true. Usually want to find any soln or find that no soln exists.

Approaches **Constructive approach**, state = vals assigned so far. Use forward search to fill soln. Gen purpose, works for all CSPs. **Random approach**, start with broken complete assn of vals to vars. Fix broken constrs by re-assign vars. Use optimization.

Problem def **State** (vals assigned so far, can be partial/inconsistent). **Initial state** (all vars unassgn). **Operators** (assign val to unass var). **Goal test** = all vars assigned, no constraint false, complete and consistent assignment.

Problem is deterministic . Note that depth is limited to # of vars , can use DFS or depth-limited search.
Uninformed search for map coloring: choose unassigned var, assign a val. This is complete and optimal, but complexity is worst possible, $n!d^n$, (n vars, d vals). Branching factor v v high . Var assgnment order irrelevant, many paths equiv.

Constraint graph Nodes are vars, arcs show constraints. Can use graph struct to accelerate search. Use **inference** to reduce search space. pre-process graph to remove inconsistencies. Var is **arc-consistent** if all val in domain satisfies vars binary constraints. Network is **generalized arc-consistent** if all vals in domain of all vars are all arc-cons.
Keep applying arc-consistency until no changes to get generalized arc-cons.

Ex Map coloring: vars = countries, domains = r,g,b, constraints = adj countries cannot be same color, $C_1 \neq C_2 \ldots$
4 queens: 1 queen/col, vars = $Q_1, \ldots$, row of each queen. Domain = 1,2,3,4. Constraints: $Q_i \neq Q_j$ (not same row), $|Q_i - Q_j| \neq |i - j|$ (not same diag)

Backtracking search DFS but fix order of var assn $b = |D_i|$. If no assignment for specific var, backtrack to prev var and try diff val. Basic uninformed alg.

Forward checking Keep track of legal vals for unassigned vars. When you assign vars, look at unassigned vars connected via constraint and delete from their domain any inconsistent vals to new assgn.

Heuristics for CSP For selecting vars: **minimum-remaining vals** = choose var that is most constrained (least remain vals) more info if one branch is not satisfiable (1/2 of branches bad vs 1/100 branches bad). **Degree heuristic** = choose var that imposes most constraints on remaining vars, can use to break ties from min-remain vals heuristic. To select a val: **least-constraining val**: assign a val that rules out fewest vals for other vars (less chance of conflict in future).
Worst-case d^n. d # vals, n # vars. Tree structured constraint graph gives $O(nd^2)$. Nearly-tree structured: $O(d^c(n-c)d^2)$ using **cutset conditioning**, find vars st removing them turns graph into tree. Instantiate them all possible ways, c is size of cutset.

Local search **Iterative improvement alg** Start with broken but complete assnment of vals & vars. Allow var assgns that don't satisfy some constraints. Randomly select conflicted vars. Ops reassign var vals. **min-conflict heuristic** chooses val that violates fewest constraints. This is hill climbing.

5 Uncertainty

Actions may be **non-deterministic** or det. Problems can be **(fully) observable, partially observable**(det or nondet) or **non-observable**.

Searching under uncert Cannot determine future states in advance (i.e. depend on die roll). Soln is not path, but **contingency plan/strategy**.
Vacumm ex. Two rooms, vacuum in one of the rooms, rooms can be dirty or clean. When non-observable, need plan. What states possible after doing an action? Reason over **beliefs** (sets of states). Total # possible beliefs = power set of all states w/o empty. Less reachable beliefs though.

Conformant planning Find plan that leads to goal despite state uncertainty . Good heur, use acts that red uncertainty, red belief to 1 state, then do standard search.
Non-deterministic case: vacuum may sometimes deposit dirt instead of cleaning, sweep may sometimes clean adjacent. Make **AND-OR** search tree: **OR nodes** (agent chooses between actions), **AND nodes** (choice induced by env choice of outcome, non-det). Want subtree st all leaves are goal leaves .
Soln is subtree that specifies one act at each OR node, inclds every outcome at each AND node, has goal node at ea leaf.
Slippery vacuum, moving sometimes fails. Apply **cyclic** soln, keep trying until it works. Can be ok soln if caused by random event but not if caused by unobserved event, like broken vacuum, can't move.

Partial observability Can only sense things locally, i.e. if current room is dirty. Account for possible observations that tell us about next state. Search over belief states. Problems # of reachable beliefs can be v large (use sampling or pruning). # of states in each belief can b v large (use compact state rep, plan for each state sep)
Mastermind: belief space: pow set of all colors. Init belief set of possible col. Act space: choosing col combo. Det: code doesn't change, non-det: don't know percepts. Percepts: white/red pegs. Goal test: 4 reds, step cost $\frac{1}{\#\ tries}$

6 Game Playing

We have **perfect** vs **imperfect**(hidden info) info, and **deterministic** vs **stochastic** (chance) games.

Game playing as search 2-player, perfect, determ games. State: state of board/player turn, ops: legal moves, goal: states st W/L/D, cost: basic $(+1, 0, -1 \to W/D/L)$, complex (points won, money, ...).
Want strat(way of picking moves) to max utility(prob of win/min cost) .
We assume adversary is trying to minimize & playing optimally, bad assumption .
Define **max player** (wants to max util) & **min player** (to min util).

Minimax search Expand complete search tree until terminal states have been reached, compute util. Go back up from leaves towards cur state. At **min nodes**, backup worst val of children, at **max nodes**, backup best val, where min/max nodes correspond to min/max players. Complete (if tree finite), optimal if advers playing opt, $O(b^m)$ time, $O(bm)$ space if DFS.
Issues v expensive even w/ pruning. Req reasonable eval func. Assum both players playing opt wrt same eval func. What if non-determinism in game or don't know game well enough to make good eval func? $\to$ **random sims**.
Resource limitations Might be time restricted, can't search all nodes. Can use cutoff test (based on depth) & eval func ($v(s)$ represents "goodness" of board state, chance of winning at that pos. If features of board can be eval indep, use weighted linear func) for nodes @ cutoff. **Real-time search**. Eval func for chess can be # white queens − # black queens + # white pawns − # black pawns .... Move chosen should be same if we apply monotonic trans to eval func. Minimax cutoff: stop at some max depth, use eval func.

α-β pruning If path looks worse than what we have, discard. If best move at node cannot change , don't search further . Minimax but keeps track of best leaf val for player (α) and opponent (β), gives bounds $[\alpha, \beta] = [-\infty, \infty]$ at start. Update α at max and β at min. Pass vals up to parents as normal minimax, parents copy their bounds to children.
Pruning can greatly incr eff . Pruning does not affect final result, best moves are same as minimax , assuming opponent is optimal and eval func is good. With bad ordering, $O(b^m)$, nothing pruned. Perfect ordering: $O(b^{\frac{m}{2}})$. Usually $O(b^{\frac{3m}{4}})$ Cons: big b means depth is still too limited. Optimal only if opp is optimal. If using heuristics, opponent needs to use heuristics too.

Forward pruning (for domains with large b) only explore n best moves for our state. May lead to sub-optimal soln. Can b v eff.

Strats Make compact state rep. Using IDS for real-time. Use α-β pruning w/ eval func. Searching deeper usually more important than having good eval func. Consider diff strats for begin, mid, end. Use rand to break ties. Consider non opt opp.

Random Simulations Sim games by rand selecting moves for both players. At end, check if won or lost, keep track of initial move. After lots of sims, pick move w/ highest win rate. Using rand to gen sample for estimation called **Monte Carlo method**. Spend more search effort at **promising moves**. Can use minimax style search for few moves at top.

Monte Carlo Tree Search Search tree + Monte Carlo sims. Select promising node in search tree using **tree policy** (mapping from states to acts). Sample possible continuations from leaves using rand **default policy** for both players (usually @ end of gm). Val of move = **avg of evals** from sampled lines.

Pick move with best avg/expected val .

Alg: Init search tree with curr state of gm. Repeat until no more comp budget: **Descent**: Choose + expand node in curr tree, use minimax or which 1 seems more promising. **Rollout**: when @ leaf, use MC sim to end of game/affordable d. **Update**: update stats for all nodes visited during descent by backpropagating. **Growth**: First state in rollout added to tree and stats initialized. Advantages vs α-β Not as pessimistic, converges to minimax soln in limit. Perf increases w/ # lines of play. Unaffected by b. Easy to parallelize.

Disadvantages May miss opt play, policy is very important.

Tree Policy How to select next move in search tree? Balance **exploitation** (node that seems promising acc to estimates & sims) & **exploration** (node hasn't received many sims, so want **more info**). Def: $Q(s,a)$: val of taking act a from state s. Win rate of node based on sims so far. $n(s,a)$ # tries taken act a from state s. $n(s)$ # times visited s.

Upper Confidence Trees $Q^{\oplus}(s,a) = Q(s,a) + c\sqrt{\frac{\log n(s)}{n(s,a)}}$, c is scaling constant. 1^{st} term is upper bound on val of taking a in s. 1^{st} term after eq is exploitation, last term is exploration (gets smaller the more you select it). To decide which action to take, calc upper bd of all children, select min/max.
Can incr calc avg, $V_{k+1} \sum_{i=1}^{k+1} R_i = \frac{k}{k+1} V_k + \frac{R_{k+1}}{k+1}$

Rapid Action-Value Estimate Assume val of move is same no matter when played. Introduces bias, but reduces variability in MC estimates. Since only the move itself matters , state space is simplified, requires less sims (don't need many sims for each indiv pos), but might oversimplify board $\to$ bad estimates. Trade-off between **model complexity** and **representational power**.

7 Logic

Need a notion of knowledge, how to represent and reason.

Knowledge representation **Perception** what is my state? **Cognition** what act should I take? State recognition requires some form of rep. Choosing right act implies some sort of **inference**.

Declarative problem solving Agent has **knowledge base** (facts in some standard lang, domain specific) and **inference engine** (with rules for deducing new facts & concl, domain independent)
Logics = **formal langs** for rep info st concl can be drawn. Defined by **syntax** which defines valid **sentences** and **semantics**, giving meaning to sentences.

Propositional logic **Propositions** = assertions about state of world/game/prob, can be true or false. Can combine with **logical connectives.Interpretation** specifies T/F for each prop sym. **Model** of set of clauses = interpretation st each clause is T. Sentence is **valid** if T in all interps (tautology); **satisfiable** if T in ≥ 1 interp ; **unsatisfiable** if F in all interps. Truth of sentence depends on interp.
KB **entails** $\alpha \iff \alpha$ is T in all worlds where KB is T. Check validity via **inference**: $KB \models \alpha \iff (KB \implies \alpha)$ is valid. Check satisfiability via inference: $KB \models \alpha \iff (KB \wedge \neg\alpha)$ unsat, proof by contra.
KB $\vdash_i \alpha \implies \alpha$ can be derived from KB by inf proced i. Want i to be **sound** (KB $\vdash_i \alpha \implies$ KB $\models \alpha$) and **complete** (KB $\models \alpha \implies$ KB $\vdash_i \alpha$)

Inference methods **Model checking**: use truth table, KB $\models \alpha$ if $KB = T \implies \alpha = T$. Sound & complete, but inefficient, needs 2^n models for n literals.

Appl of inf rules Sound gen of new sentences from old. **Proof** = seq of inf rule appl. Can use inf rules as ops in search alg. Complexity of verifying validity of sent w/ n lits = 2^n. If we only use Horn clauses we can get **poly time** infer.

Normal/Standardized forms **Conjunctive Normal Form (CNF)**: conjunctions ($\wedge$) of disjunctions ($\vee$) of literals $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$ **Disjunctive Normal Form (DNF)**: opp of CNF, $(A \wedge B) \vee (A \wedge \neg C)$ **Horn Form**: Conjunction of Horn clauses (clauses w/ ≤ 1 +ve lit) $(A \vee \neg B) \wedge (B \vee \neg C \neg D)$, often written as $B \implies A, (C \wedge D) \implies B$.

Inf rules **CNF resolution** $\frac{(\alpha \vee \beta), (\neg\beta \vee \gamma)}{(\alpha \vee \gamma)}$ (comp for prop log) **Horn Modus Ponens** $\frac{\alpha_1, \ldots, \alpha_n, (\alpha_1 \wedge \ldots \wedge \alpha_n \implies \beta)}{\beta}$ (comp for Horn KBs).**And-elim** $\frac{\alpha_1 \wedge \ldots \wedge \alpha_n}{\alpha_1, \ldots, \alpha_n}$. **Impl elim** $\frac{\alpha \implies \beta}{\neg\alpha \vee \beta}$. **De Morgan's law** $\neg(\alpha \vee \beta) \iff (\neg\alpha) \wedge (\neg\beta), \neg(\alpha \wedge \beta) \iff (\neg\alpha) \vee (\neg\beta)$ Can use rules with forward or backward search.

Forward chaining When new sent p added to KB, look for all sent that share lits with p, perform resolution and add new sent to KB and cont. data-driven, eager method, new facts inferred ASAP.
extends KB, improves understanding of world, used when focus is finding model of world.

Backward chaining When query q asked of KB: if $q \in$KB $\to$ T. Else use resolution for q with other sent in KB and cont. goal-driven, lazy reasoning method, facts only inferred as needed.
Frugal in terms of comp, KB grows less, focus on proof (usually more eff), does nothing until asked questions, used in proofs by contra.

Prop log is good because very simple, few rules. But bad because cannot express in compact way. Want to describe world in more compact and eff way and want to quantify over objs.

First-order logic Adds new elements: **predicates** to describe objects/props/relations, **quantifiers** $\forall, \exists$, **functions** to give you obj related to another obj, domain elements to domain elements, like $RightOf$ (includes **constants**). Can handle **infinite domains** w/ quantifiers.

Types of sentences **Term** (const, var, func), **atomic sentences** (predicates, equality of terms), **complex sentences** (combine atomic sentences w/ connectives)

$\forall x \forall y = \forall y \forall x, \exists x \exists y = \exists y \exists x,$ but $\forall x \exists y \neq \exists y \forall x$. $\forall x A(x) = \neg\exists\neg A(x), \exists x A(x) = \neg\forall x \neg A(x)$, $t_1 = t_2$ iff they refer to same obj in D

Truth in FOL Sent true wrt a **model** = $M = (D, I)$, D is domain of obj, I is interpretations s.t. const symbols $\to$ obj, pred sym $\to$ relations of objs, func sym $\to$ func relations of objs

Inf Algs for FOL **Propositionalize** FOL $\to$ prop log, too expensive (cept if trivial). **Search** (forward/backward chaining using generalized MP), w/ inf rules: **MP** $\frac{\alpha, \alpha \implies \beta}{\beta}$, **$\wedge$ intro (AI)** $\frac{\alpha\ \beta}{\alpha \wedge \beta}$, **Universal Elim** $\frac{\forall x \alpha}{\alpha\{x/\tau\}}$. Ops are inf rules, states are sentences, goal is to check states to see if they contain query. Problem b is huge, esp for UE. Try to find substitution that makes rule match known facts. **Unification** = pattern matching to find good candidate for UE. Sub σ unifies atom sent. p and q if $p\sigma = q\sigma$

Generalized Modus Ponens $\frac{p_1\sigma, \ldots, p_n\sigma, (p_1 \wedge \ldots \wedge p_n \implies q)}{q\sigma}$. If we use GMP w/ KB of **Horn** clauses gives single atomic sent or clause of form (conj of atom sent) $\implies$ (atom sent). All vars assumed universally quant .

GMP is complete for KBs of universally quantified Horn clauses, but incomplete for general FOL. Entailment in FOL is **semi-decidable**, can find proof if $KB \models \alpha$, but not always if $KB \not\models \alpha \to$ **Halting Problem**, don't know if proof will term.

Resolution Can **resolve** 2 clauses if they have complementary literals, one lit unifies with neg of other. Same as prop resol, except with unifications. Sound and complete inf meth for FOL. **Proof by negation**, to prove $KB \models \alpha$, prove $(KB \wedge \neg\alpha)$ unsat. Do so by expressing KB and $\neg\alpha$ are expressed in univ quant CNF. Use resolution to combine 2 clauses into 1. Continue until empty clause (**contradiction**).

Convert KB to CNF $P \implies Q \equiv \neg P \vee Q$. Move $\neg$ inwards: $\neg\forall x P \equiv \exists x \neg P$. Standardize vars apart, i.e. $\forall x \exists x \to \forall x \exists y$. Move quantifiers to left. Eliminate existential quantifiers by **skolemization** ($\exists x Rich(x) \equiv Rich(G1)$, $G1$ is a new **Skolem constant**. When $\exists$ inside $\forall$: $\forall x f(x) \implies \exists y g(y) \wedge l(x,y) \equiv \forall x f(x) \implies g(H(x)) \wedge l(x, H(x)))$, $H(x)$ is a **Skolem function**. Drop universal quants, distribute over $\vee \to (P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

Resolution Strats **Unit res** prefer to do res if one clause is literal $\to$ shorter sentences. **Set of support** identify (hopefully small) subset of KB that every res will take clause from to resolve with another sent, add to set of support , can make inf (incomp). **Input resolution** always combine sent from query or Kb with another sent, not complete in general, doesn't use new sentences.

Pros KB-systems Expressible/human readable, simple inf proc, easy to change, easy to explain, machine readable, parallel.

Cons Can b hard to express, undesirable interactions among rules, non-transparent behavior, hard to debug, slow, where does KB come from?

Planning Coll of act for some task. Is a search problem, but more structured. In search, states and actions are atomic, in planning, states and goals are logical sent , actions are preconditions+outcomes.

STRIPS (Stanford Research Institute Planning System) **Domain**: typed objs as props. **States** as first-order preds over obj(repr as conj ($\wedge$) of preds), **closed-world assumption** (not stated = false, only obj in world are defined). **Operators** = **preconditions** (when can use act, rep as conj), **effects** = what happens after (rep as conj). **Ex.** S: $In(robot, room) \wedge Closed(door)$, G: $In(robot, r) \wedge In(Charger, r)$, OP: $Go(x,y)$, precond: $At(robot, x) \wedge Path(x,y)$, postcond/effects: $At(robot, y) \wedge \neg At(robot, x)$. **Action schema** defines name, prec, effects for each op. Effects: **Add-list** list of props that become T after act. **Delete-list** list of props that become F after
Semantically we have: If precond = F, do nothing, act cannot be applied. Else if T, delete items on delete-list, add items on add-list, order of ops important! . Can rep STRIPS state transitions as a tree. Pros restricted, inf more efficient. All ops = deletion+Addition of props of KB. Cons Assumes small # props change per act (else ops hard to def and reasoning expens). Limited lang (everything = conj, not applc to all domains). STRIPS is sound , not complete (no backtracking), not optimal (no guarantee on shortest plan), expensive

Planning approaches **State-space planning** use search w/ states and ops, **plan-space planning** work at level of plans (won't talk)
Progression (forward) planning match preconds. Det all ops applic from start. Ground ops, replace vars with constants. Choose op to apply, det new content of KB. Repeat until goal.

Regression (backwards) planning match effects. Pick acts that satisfy some goal props. Make new goal w/ preconds of these acts + unsolved goal props. Repeat until goal set satisfied by start.
SatPlan (satisfiability) Plan prob $\to$ gen all possible literals at all time slices. Solve humongous SAT prob. Optimal and complete, but NP-hard, PSpace if we allow plan duration to vary. **Heuristic-search planning** Don't use domain heuristics, use heuristics based on planning prob itself. Simple heur: ignore delete lists. **GraphPlan** make graph encoding constraints on possible plans. If $\exists$ valid plan, will be part of this graph, so search only this graph.

Planning Problems **Incomplete info** (unknown preds), disjunctive effects, things might cause more than we think. **Incorr info** cur state incorr, unanticipated outcomes $\to$ failure. **Qualification prob** Can never finish listing all req prec and cond outcomes of acts.
Solns? **Conditional (contingency) planning** plan w/ observation acts to get info (i.e. check tire, if intact ...), sub-plans made for each contingency. Expensive, plans for unlikely cases. **Monitoring/Replanning** Assume normal states+outcomes, check prog during exec, replay if needed.

8 Uncertainty with Probabilities

Dealing with uncertainty: Implicitly: ignore as much as poss, mk procedures robust to uncertainty. What we've seen so far. Explicitly: build model of world **describing uncertainty**, reason about effect of acts given model.
How to represent? Logic has 2 probs , falsehood (implications are 100%), leads to weak conclusions (check tire after each act, very expensive inf). Logic makes assumptions unless contrad by evidence, reasoning under uncertainty considers all possible states, but all states equally likely. Use probability !

Bayesian Probability Use probs to descr world and uncertainties. **Beliefs** relate logical props to knowledge, they are subjective . Probs **change** with new evidence. **Prior (uncond) beliefs** = belief before new evid

Probalistic Models World is a set of RV $\Omega = \{X_1, \ldots, X_n\}$. Div by mutually excl **events/states**.Prob model allows computation of **any event** in world. **Joint prob dist fun** assigns non neg weight to ea event (sums to 1).

Inf using joint dist **Uncond prob** of any prob = $\sum$ of entries from full joint dist (marginalization)

	H	$\overline{H}$
R	0.05	0.10

$P(H) = P(H,R) +$

$P(H,\bar{R}) = 0.65$
$P(A|B) = \frac{P(A\cap B)}{P(B)}$

Bayes Rule for inf Want to form hyp based on obs vars. $P(H|e) = \frac{P(e|H)P(H)}{P(e)}$, $P(H|e)$ **posterior prob**, $P(H)$ **prior prob**, $P(e|H)$ **likelihood**, $P(e) = P(e|H)P(H) + P(e|\bar{H})P(\bar{H})$ **normalizing constant**. Update prior w/ data(likelihood) to get post

Computing cond probs Often want **posterior joint dist** of **query vars** given vals for **evidence vars**. "Sum out" hidden vars $P(Y,e) = \sum_z P(Y,e,z)$, often too large. If **independent vars**, $P(x_1,\dots,x_n) = \prod_{i=1}^n P(x_i)$, only need n # instead of 2^n. Indep is too strong of a cond, use **cond indep** $P(x|y,z) = P(x|z)\forall x,y,z \implies x \perp y|z$

Chain Rule $P(A_n,\dots,A_1) = P(A_n|A_{n-1},\dots,A_1)\cdot P(A_{n-1},\dots,A_1)$

Naïve Bayes Model Assume symptoms indep given disease. $P(D,s_1,\dots,s_n) = P(D)P(s_1|D)\dots P(s_n|D)$. $P(D,s_1,\dots,s_n)$ joint dist has $2^{n+1}-1$ vars w/o Naïve (can use compl for last one). Assuming Naïve Bayes, we get $2n+1$ vars, 1 for each $P(s_i|D)$ and 1 for $P(D)$. Use $P(D|s_1,\dots,s_n)$ to diag patient

9 Bayesian Networks

Represent cond ind using graph structure + params. Each node has cond prob dist (**CPD**) for var given parents. Joint dist = prod of all CPD Is **directed graph** s.t., 1 node for each var in prob. Directed edges → direct influence. No no dir cycles. Each node has cond prob dist $P(X_i|parents(X_i))$. Joint prob dist = $P(x_1,\dots,x_n) = \prod_{i=1}^n P(x_i|parents(x_i))$. Each node has $2^{\#parents}$ params, 1 cond for par=T, 1 for F

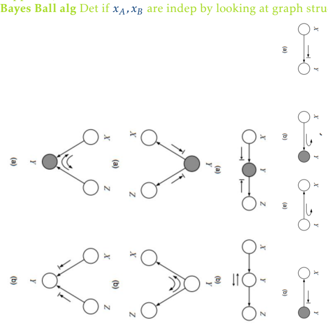
Types of queries **Uncond** $P(Y)$, **Cond** $P(Y|Z=z)$, **Maximum a posteriori (MAP)** $MAP(Y|Z=z) = arg\max_y P(Y=y|Z=z)$

Marginalization Given Bayes net, to marginalize the unconditional prob, sum enumeration through all possible vals of other events.

Inference in BNs Given Bayes net & RV X, deciding $P(X=x)>0$ is NP-hard. No inf proc efficient for all networks. But for **some family** of networks, inf can be efficient, i.e. Naïve Bayes. Allows incremental updating of beliefs when new evid obtained. (Add as new factor in prod) Full joint prob can be decomposed into prod of prob and cond probs in BN
To get desired expr might need to marginalize over many vars (big tree)
Inf poly time for tree-structured, NP-comp in worst. Best exact inf alg converts net to tree then does exact inf. For large nets, approx inf meth better.

Variable elim Rearrange sums s.t. less vars in one sum: $P(T,L=1) = \sum_{a,s,f} P(s|f)P(f)P(a|f,T)P(L=1|a)P(T) = \sum_{a,f} P(f)P(a|f,T)P(L=1|a)P(T)\sum_s P(s|f)$. Replace $\sum_s P(s|f) = m_s(f)$, where $m_s(f) = p(s|f)$. Repeat with a and f. $O(2^n)\to O(2^k n)$ factors. k is max # vars a var's dist is dep on. Steps:
Impose var ordering, **query var** last. **Init** active factor list w/ cond prob dist in BN. Add evidence potentials to active fl ∀ ev vars E. For $i=1:n$, take next var X_i from order, take all factors w/ X_i as arg off active fl, mult them, sum over all vals of X_i, creating m_{X_i} & put on afl
$n = \#vars, m = max\#vals$ of a var, $k = $ max # vals var can take. $O(n)$ mults to make entry in a factor. Factor can have $O(m^k)$ entries. Important to have small factors, but size depends on ordering of vars.
Choosing optimal order is NP-complete.
X_i is **evidence var** with observed val x_i. **Evidence potential** $\delta(x_j,x_i)=1 \iff x_j = x_i \to P(s|F=1) = \sum_f P(s|f)\delta(f,1)$ (cond to sum)

DAGs & Indep If we have $P(F|T)=P(F)$, then F and T are indep. Called **d-separation** (directed). Absence of links $\implies$ some cond ind relations. Read off graph, faster inf, **skip vars** that are cond indep. If P & R not directly conn but info about P gives info about R then P **must give info about vars along path** P to R.
3 types of conn **Indirect:** $X\to Y\to Z$. $P(Z|X,Y) = P(Z|Y)$, so Z is indep of X when val of Y known (blocked), else open if Y unknown. **Common cause** $X\leftarrow Y\to Z$, $P(Z|X,Y)=P(Z|Y)$ if Y known. Same res as indir. **V-structure** $X\to Y\leftarrow Z$, when given Y, X is not indep of Z, but if we don't know Y, they are indep. Called **explaining away**, competing explanations. Know Y, knowing X changes belief in Z and vice versa (if X didn't happen, Z prob happened)
Bayes Ball alg Det if x_A, x_B are indep by looking at graph struct.



10 Learning

Want to use Bayes nets w/ real life data to get vals. **Build models** of world from data

Learning in Bayes Nets Given data in form of instances, yes/no ans for all params of model. **Parameter estimation** w/ complete data: given Bayes struct G, choice of rep for CPDs ($P(X_i|Par(X_i))$). Goal to learn CPD of each node. **Assume all vars bin**. Assume instances $x_i,\dots,x_m$ are iid (**important**).
Learning prob find set θ of params s.t. data can be summarized by $P(x_j|\theta)$, θ depends on prob dist, i.e. just params of prob dist. More **samples** the more you can trust data.

Goodness of param set Depends on likelihood to gen obs data. D is data set. **Likelihood** of θ given D is: $L(\theta|D) = P(D|\theta) = P(x_1,\dots,x_m)$ if iid $= \prod_{i=1}^m P(x_i|\theta)$. **Sufficient statistic** of data is func of data that summarizes enough info to compute likelihood, i.e. $s(D) = s(D') \implies L(\theta|D) = L(\theta|D')$

Maximum Likelihood Estimation (MLE) Choose params to **maximize** likelihood. Instead of maxing prods, we can take logs and max sum, since log is monotonic. $\log L(\theta|D) = \sum_{i=1}^m \log P(x_i|\theta)$. **Derive** & set to 0 to get max. For bern: $L(\theta|D) = \theta^{N(H)}(1-\theta)^{N(T)}, \log L(\theta|D) = N(H)\log\theta + N(T)\log(1-\theta)$ derive $\to \theta = \frac{N(H)}{N(H)+N(T)}$. With k outcomes, $\theta = \frac{N_i}{N}$ Ex. $P(L|A) = \frac{P(L,A)}{P(A)} = \frac{\#l=1,a=1}{\#a=1}$

Param est in Bayes Net Given instances, we are trying to estimate prob of each node.
Problem zero probs. For probs w/ lots of vars, possible not all possible vars are seen in data, esp if rare. If val not seen, MLE is 0. To get around this, use **Laplace smoothing** for coin toss, instead of $\theta = \frac{N(H)}{N(H)+N(T)}$ use $\frac{N(H)+1}{N(H)+N(T)+2}$.
If not Bernoulli, change +2 to $+k$ in den and +1 for each outcome in num. Bad to use Laplace if lots of params, will need a lot of data.
To get MLEs of Berns in Bayes net given table, if uncond, get proportion of true over total data. If cond, look at rows where conditions are true (or false if we want given not C) and get proportion.

Missing vals Given data, some yes/no ans are missing/undefined.
Problem val missing might indicate val, i.e. no x-ray could mean no bone problems. Can we use MLE? Can throw out samples missing data, but biasing and bad. Otherwise we can **consider both vals** of missing val, get diff likelihood for both. Overall likelihood combines both. Complicated given many missing vals!
Can no longer est params locally & indep like complete data, we now have many local max (maxing likelihood is now non-linear opt, w/ complete we have unique max) and no closed form soln (complete has closed w/ assumptions)
Assume prob of X_i missing is indep its own val given observed data.
2 solns **Gradient ascent** Use hill climbing to search through params, follow gradient of likelihood. Good bc flexible for CPD forms, easy to comp gradient, closely related to other learning, but bad since soln needs to be in space of legal params to ensure we get prob dists, sensitive to params/learning rate, **slow**
Expectation Maximization use current params to mk local approx of likelihood that is nice. **Assume** underlying dist.

EM **Init** start with initial params (either randomize or est from complete data instances). **Repeat** E-step, complete data by assign vals to missing based on curr params(use probs to get expected prob of missing = x). M-step, compute MLE. **Convergence** No change in E & M step inbetween 2 consec iter
Hard EM for each missing data, assign val most likely (easier), **Soft EM** for missing, put weight on each val = prob, use weights as counts (more common). **Comparison** soft does not commit to vals for missing items. Considers all vals, better. But soft requires comp cond probs, hard only requires comp most probable
Properties Likelihood guarant to improve/stay same(stop) w/ each iter. Guar to conv to local opt of likelihood, need local search, rand restarts/diff init params.

Unsupervised Learning What if var is always missing? Can still est MLE. Relying on dist. Using Gaussian: $\frac{1}{\sigma\sqrt{2\pi}}e^{-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2}$

K-means clustering Cluster instances into K distinct classes. Need to know K in adv, assum dist for each class. **K-means alg**: 1.Ask how many clusters, 2.guess k centers $\{\mu_1,\dots,\mu_k\}$, assm σ^2 known, 3.asgn each data pt to closest μ, 4.each center finds centroid of its pts (i.e. mv center). Repeat 3-4. Essentially maxs likelihood of data.
User must specify val of K, not always poss, can try for diff vals of K, other meths try to learn K. Stop when label of data points/cluster centers stop changing a lot. **K-means is just hard EM with Gaussian**, can get local opt/diff results, run mult times. There is also soft EM vers of K-means, center calc with weights.
$O(nkd)$, $k = \#centers, n = \#data, d = dim\ data$. Local opt. rand restarts to get better local opt, alternatively, choose init centers, place μ_1 on top of random data pt, μ_2 on pt furthest from μ_1, μ_3 on pt furthest from μ_1 & μ_2

Supervised learning Machine learning. **General learning prob**: Labeled examples $\langle x_1,x_2,\dots,x_3,y\rangle$, x_j are **input vars/features/attribs**, y **desired output/output vars/targets**. Want learn func $h: X_1\times X_2\times\dots X_n \to Y$, maps input to output. Often classification prob, learn func for categorical out. Else regression, out is **continuous**. **Data set** = training examples/instances. Training ex $i = <x_{i,1},\dots,x_{i,n},y_i>$, n attribs. x_i is col vec with $x_{i,1},\dots,x_{i,n}$. Training set has m ex. $X = X_1\times\dots\times X_n$ = space of input vals, Y = space of output vals.
Dataset $D = X\times Y$, find func $h: X\to Y$ s.t. $h(x)$ good pred for y, h = **hypothesis**
Steps decide input-output pairs, how to encode in/out, class of hypotheses, error func, alg to search

Linear Hypothesis Y is linear func of X: $f_W(x) = \sum_{j=0:m} w_j x_j$, $x_0 = 1$(no x, this is intercept), w_j are params/weights, $m = dim$ of obs space/# features

Least-Squares Min $\sum_{i=1}^n (y_i - w^T x_i)^2$, use calc to get soln or gradient descent if not possible. $f_W(X) = Xw, Err(w) = (Y-Xw)^T(Y-Xw)$, minimize w/ $\frac{\partial Err(w)}{\partial w} = -2X^T(Y-Xw)$, m eq (set to 0) w/ m unknowns. $X^T(Y-Xw) = 0$ $\hat{w} = (X^TX)^{-1}X^TY$, $\hat{Y} = X\hat{w}$. To pred new data, $Y' = X'\hat{w}$ Cost: nmp ops for mat mult, m^3 for inv, polytime $O(n^3)$.

Polynomial fits $y = w_0 + w_1x + w_2x^2$, don't change method, just more inputs, x^2 is a new feature, still linear in weights. High deg bad, **overfitting**, hyp explains too perfectly, does not **generalize** well. Use **validation set** (from data) to est true error to pick best deg

Connectionist models Comp architecture similar to brain could dup some abilities. **Artificial Neural Networks**. Many neuron-like threshold switching units. Weighted intercon among units. Parallel. Tune weights automat
Sigmoid func $\sigma(z) = \frac{1}{1+e^{-z}}$, squash to 1, which neurons are on. $\frac{d\sigma(z)}{dz} = \sigma(z)(1-\sigma(z))$. Neurons w/ sigmoid, input layer, output layer, hidden layers.
Feed-forward neural nets w_{ji} = weight on conn $i\to j$. $x_{j0} = 1\forall j$ $o_j = \sigma(w_j\cdot x_j)$ output of unit j, w_j vec of weights entering j, x_j vec of inp to j, $x_{ji} = o_i$. To compute w/ NN, go through network from inputs to output neurons passing signals through activation funcs. Learn good weights. Need obj func (mean squared err), learning alg, popular is **gradient descent**. $w^{k+1} = w^k - \alpha_k\nabla f(w^k)$, $\alpha_k > 0$ step size/learning rate for iter k, $f(w^1) > f(w^2) > \dots$, $lim_{k\to\infty} w^k = w$ local opt
Overfitting (high valid err vs training err) comes from too many weights, train too long, weights too extreme

11 Temporal Inference

Model change over time. X_t **unobservable** vars @ time t. E_t **observable** ev vars @ t. Assume discrete time, same struct @ ea timestep. $X_{t:t+k} = X_t,\dots,X_{t+k}$.
Change over time = **transition model** $P(X_t|X_{0:t-1})$ (prob of being at $X=x$ at time t given all other states from 0 to $t-1$), **sensor model** $P(E_t|X_{0:t},E_{0:t-1})$
Markov Processes/Chains Assumption X_t depends on bounded subset of $X_{0:t-1}$. **First-order Markov Process** $P(X_t|X_{0:t-1}) = P(X_t|X_{t-1})$. **Second-order** $P(X_t|X_{0:t-1}) = P(X_t|X_{t-2},X_{t-1})$. **Stationary process assump** Trans + sensor models fixed ∀ time steps, same cond prob dists $P(X_t|X_{t-1}) = P(X_{t+1}|X_t)$

Dynamic Bayesian Network Bayes Net whose structure is copied over time, dependencies between vars in neighbor time steps
Inf tasks **Filtering** $P(X_t|e_{1:t})$, **Prediction** $P(X_{t+k}|e_{1:t}), k>0$, **Smoothing** $P(X_k|e_{1:t}), k<t$, **Most likely expl** $argmax_{X_{1:t}} P(X_{1:t}|e_{1:t})$. 1st 3 tasks can use forward alg, backward alg. Most likely can use Viterbi alg

Hidden Markov Model 1 state + 1 sensor var / time step. $P(X,E) = P(X_1)\prod_{t=1}^{T-1} P(X_{t+1}|X_t)\prod_{t=1}^T P(E_t|X_t)$ where terms after eq are init state prob, state transition prob, emission prob. N possible states, W possible obs. N init dist for X_1, N^2 trans dist, NW emission dist. Use MLE. Init probs $\pi_1,\dots,\pi_N$ for X_1. Trans probs for $X_t\to X_{t+1}, A = \{a_{ij}\}, i,j\in[1,N]$. Emission probs $X_t\to E_t, B = \{b_i(w_k), i\in[1,N], k\in[1,W]\}$

Forward alg Efficiently marginalize all state seq using dyn prog $P(E|\theta) = \sum_X P(E,X|\theta)$. **Trellis** of possible state seq, col head is E_i, row head is X_i. Entries are $\alpha_i(t) = P(E_{1:t,X_t=i|\theta})$. First get $\alpha_j(1) = \pi_j b_j(E_1)$ to fill first col, then use prev col to fill next one w/ recur $\alpha_j(t) = \sum_{i=1}^N \alpha_i(t-1)a_{ij}b_jE(t)$. Sum last col $P(E|\theta) = \sum_{j=1}^N \alpha_j(T)$. $O(N^2T)$ for overall likelihood of seq. **filtering**, rewrite goal as $P(X_t = i|E_{1:t},\theta) = \frac{P(X_t=i,E_{1:t}|\theta)}{P(E_{1:t}|\theta)}$, nume is $\alpha_i(t)$, den is from alg.
Prediction Do filtering for time 1 to t for $P(X_t|E_{1:t})$ then for $m = 0:k-1$ $P(X_{t+m+1}|E_{1:t}) = \sum_j P(X_{t+m+1}|X_{t+m} = j)P(X_{t+m} = j|E_{1:t})$ save computations $P(X_{t+m+1}|E_{1:t})$ for next iter

Backward alg New trellis with cells $\beta_i(t) = P(E_{t+1:T}|X_t = i,\theta)$. Prob of all prev emissions given curr state is i. Excludes prob of curr emis. Start at T, $B_i(T) = 1$. Then $\beta_i(t) = \sum_{j=1}^N a_{ij}b_j(E_{t+1})\beta_j(t+1)$, $P(E|\theta) = \sum_{i=1}^N \pi_i b_i(E_1)\beta_i(1)$, $O(N^2T)$. Use for **smoothing**, $\alpha_i(k) = P(E_{1:k},X_k = i|\theta), \beta_i(k) = P(E_{k+1:t}|X_k = i,\theta) \implies \alpha_i(k)\beta_i(k) = P(E_{1:t},X_k = i|\theta)$, compute $P(X_k|E_{1:t})$ usind cond prob and $P(E_{1:t})$ (from forward alg)

Work in Log Domain Lots of mult, avoid underflow $\log\prod_{i=1}^n p_i = \sum_{i=1}^n \log p_i = \sum_{i=1}^n a_i \implies \log\sum_{i=1}^n p_i = \log\sum_{i=1}^n e^{a_i} = \log e^b\sum_{i=1}^n e^{a_i-b} = b + \log\sum_{i=1}^n e^{a_i-b}$ where $b = \max a_i$.
Sequence labelling Viterbi, use forward alg but replace sum with max. $X^* = arg\max_X P(X,E|\theta) = arg\max_X P(X|E,\theta)$, $\delta_i(t) = \max_{X_{1:t-1}} P(X_{1:t-1},E_{1:t},X_t = i|\theta)$. Alg: $\delta_j(1) = \pi_j b_j(E_1)$, then $\delta_j(t) = \max_i \delta_i(t-1)a_{ij}b_j(E_t)$. Take $\max_i \delta_i(T)$ $O(N^2T)$ **Backpointers** keep track of where ea max entry is, work backwards to get best label seq

Unsupervised TI No state seqs, guess them, init params rand, use Viterbi EM. Pred cur state seq using cur model w/ Viterbi, then update cur params w/ cur pred & repeat

Baum-Welch/Forward-backward alg EM to HMMs. **E**: expected cts for hidden structs w/ θ^k, **M**: Get θ^{k+1} to max likel
Call req distribution for soft em in Baum as **responsibilities**, $\gamma_i(t) = P(X_t = i|E,\theta^k) = \frac{P(X_t=i,E|\theta^k)}{P(E|\theta^k)} = \frac{\alpha_i(t)\beta_i(t)}{P(E|\theta^k)}$ prob of being in state i @ t given obs seq under cur model, $\xi_{ij}(t) = P(X_t = i, X_{t+1} = j|E,\theta^k) = \frac{P(X_t=i,X_{t+1}=j,E|\theta^k)}{P(E|\theta^k)} = \frac{\alpha_i(t)a_{ij}b_j(E_{t+1})\beta_j(t+1)}{P(E|\theta^k)}$ prob of trans from i @ t to j @ $t+1$
Soft MLE updates: $\pi_i^{k+1} = \gamma_i(1), a_{ij}^{k+1} = \frac{\sum_{t=1}^{T-1}\xi_{ji}(t)}{\sum_{t=1}^{T-1}\gamma_i(t)}, b_i^{k+1}(e_k) = \frac{\sum_{t=1}^T \gamma_i(t)|E_t=e_k}{\sum_{t=1}^T \gamma_i(t)}$
Stop EM when training set likeli stops improv or prediction perf on held-out development or validation set stops improv

Kalman Filtering HMMs with **continuous** state space
Linear Gaussian trans: Assumed by Kalman, $P(X_{t+\Delta} = x_{t+\Delta}|X_t = x_t, X_t' = x_t') = N(x_t + x_t'\Delta,\sigma^2)(x_{t+\Delta})$. i.e. next pos is linear func of current pos + **Gaussian noise**
One-step pred $P(X_{t+1}|e_{1:t}) = \int_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})dx_t$
Filtering $P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$
Limitations Cant be used if nonlinear trans model. Assume belief state = Gaussian

Particle filtering Use samples of poss states to approx belief. Pops of samples (particles) track high-likelihood. Replicate particles proportional to likelihood for e_t.
Assume belief @ t $N(x_t|e_{1:t})/N = P(x_t|e_{1:t})$. Propagate: $N(x_{t+1}|e_{1:t}) = \sum_{X_t} P(x_{t+1}|x_t)N(x_t|e_{1:t})$. Weight by likeli $W(x_{t+1}|e_{1:t+1}) = P(e_{t+1}|x_{t+1})N(x_{t+1}|e_{1:t})$. Resample: $N(x_{t+1}|e_{1:t+1})/N = \alpha W(x_{t+1}|e_{1:t+1}) = P(x_{t+1}|e_{1:t+1})$
Easy for contin space, easy to implement, good for many dist, increase # particles for precision (Time/space complex grow linearly w/ incr), hard to track rare

Speech Recognition Probs: Noise, ambiguities, co-articulation, intonation, accents, time. $P(Words|Signal) = \eta P(Signal|Words)P(Words)$, where $Signal|Words$ is acoustic HMM (words are hidden state vars, signal is ev vars) and $Words$ is lang mod. English has 50 phones (distinct speech sounds), make word pronunciation model, each state has onset/mid/end state for each phone. HMM: Training via Baum-Welch + cont speech signals. Recognition: use Viterbi for most likely seq of states -> words. MLE: $P(w_i) = \frac{\#w_i}{N}, P(w_i|w_{i-1}) = \frac{\#(w_{i-1},w_i)}{\#w_{i-1}}$

12 Utility Theory

Decision theory = prob theory + util theory. What an agent wants. Need to be **actors**, choose actions in rational way. Actions have consequences, want to max utility of acts.

Consequences of an act are **payoffs/rewards**. A rational method = eval benefit of consq & weigh by prob. To compare need: **set of consequences** $C_a = \{c_1,\dots,c_m\}$, **prob dist over cons** $P_a(c_i), \sum_i P_a(c_i) = 1$. A pair $L_a = (C_a,P_a) = [A,p;B,(1-p)]$ = **lottery**. Pref: $A > B$ (A pref to B), $A\sim B$ (indiff), $A \gtrsim$ (B not pref to A)

Axioms of util theory **Orderability**: Linearity $(A>B)\vee(B>A)\vee(A\sim B)$. Transitivity $(A>B)\wedge(B>C) \implies (A>C)$. **Continuity** If $A>B>C,\exists$ lottery L w/ A, C equiv to getting B, $\exists p, L = [p,A;(1-p)C]\sim B$. **Substitutability** Add same prize w/ same prob to two equiv lotteries, doesnt change pref. **Monotonicity** If 2 lots have same prize, 1 giving best prize often pref. **Reduction of compound lotteries** 2 consecut lots → 1 equiv lot
If we neglect an axiom, can show not rational

Utilities map outcomes/states to vals, func non-unique. Behaviour invariant wrt additive linear trans. W/ deterministic prizes only, only ordinal util (total order on prizes) matter. Util don't need to obey same things as expec vals. Util models should capture risk attitude, **risk neutral** (util = expec), **risk averse** (util < expec, rate of change slower) **risk seeking** (util > expec, faster rate of change).
Decision-theory **normative**, describ how rational agents should act. Ppl violate axioms of util th. Ppl mk diff decision based on how choices are portrayed

MEU Choose act → maximize expected util $\exists U$ s.t. $A \gtrsim B \iff U(A)\geq U(B), U([p_1,C_1;\dots;p_n,c_n]) = \sum_i p_i U(C_i)$. **Util func** $U(x)$, **Expec Util** $EU(a|x) = \sum P(Eff(a)|x)U(Eff(a))$, **MEU** $\max_a EU(a|x)$, **Optimal Action** $arg,\max_a EU(a|x)$, **Policy** $\pi(x): X\to A$, pick action in all states

Decision Graphs RVs are oval nodes, decisions/acts = rectangles (no incoming arcs), utils = diamonds (no out-going arcs). Compute opt act as **inference**

Information gathering Env w/ hidden info, agent can choose to do info-gath acts. Is it worth getting new info? **Value of info** specs util of evid that can be obtained. W/o risk, expec of info = expec of best act w/ info − expec val of best act w/o info

Val of Perfect info (VPI) Hv evid E, best act a^* w/ outcomes c_i. $EU(a^*|E) = \max_a U(A) = \max_a\sum_i U(c_i)P(c_i|E,a)$. If we know $X = x$ then choose a_x^*: $EU(a_x^*|E,X=x) = \max_a U(A) = \max_a\sum_i U(c_i)P(c_i|E,a,X=x)$. $VPI_E(X) = \sum_x P(X=x|E)EU(a_x^*|E,X=x) - EU(a^*|E)$
Props: **Non-negative** $\forall X, EVPI_E(X)\geq$, new info can't hurt. **Non-additive** $VPI_E(X,X) \neq 2VPI_E(X)$, same info 2x doesn't benefit. **Order-indep** $VPI_E(X,Y) = VPI_E(X) + VPI_{E,X}(Y) = VPI_E(Y) + VPI_{E,Y}(X)$
Don't always want MEU, env may change/not fully known. **Random choice models** choose act w/ highest expec util, but keep non-zero probs for others (like Monte Carlo). **Minimizing regret** minimize loss between current behavior and some standard behavior. **Preference Elicitation** finding ppl's pref → utils.

Learning utils from data

Bandit problems Model of decision making under uncertainty, **bandit** = name of a slot mach. Env is unknown, don't know lottery, want to learn expect util of band. **k-armed bandit**, collection of k actions, each w/ lottery. Each choice = **play**. After each play a_t, mach gives reward r_t from distrib assoc w/ a_t. Val of act a given by expec util $Q^*(a) = E[r|a]$. Want to choose arms to max rew in long run. **Est val of act** $Q_n(a) = \frac{r_1+r_2+\dots+r_n}{n}$. By law of large numbers, $\lim_{n\to\infty} Q_n(a) = Q^*(a)$. To incrementally est: $Q_{n+1}(a) = Q_n(a) + \left(\frac{r_{n+1}-Q_n(a)}{n+1}\right)$

Exploration-exploitation trade-off **explore** to find what is best (need some random), **exploit** knowledge, greedy $a_t^* = argmax_a Q_t(a)$. Prob may be non-stationary, can't use sample avg (prob changes over time). emph most recent rewards. Use constant step size $\alpha\in(0,1), Q_{n+1}(a) = Q_n(a) + \alpha(r_{n+1} - Q_n(a)), Q_{n+1}(a) = (1-\alpha)^n Q_0(a) + \sum_{i=1:n+1}\alpha(1-\alpha)^{n-i-1}r_i$. If stationary, reduce explor over time, if not, can never stop explor.

Strats **Greedy** pick best arm a w/ best est of $Q_n(a)$. ε**-greedy** $\varepsilon\in(0,1)$ (small). On ea play, prob ε explr random arm, $1-\varepsilon$ explt best est. Can make ε depend on time. v simple, but leads discontinuities. $\varepsilon = 0$ =greedy, ε low, conv slow, ε high, big var. **Softmax** act probs func of cur vals $Q_t(a)$. At time t, choose a with $P(a_t) = e^{Q_t(a)/\tau}$, norm prob s.t sum is 1, τ is temp, if τ high, favors explor, if low exploit. **Optimistic initialization** Assume good if no knowl. Initi act vals higher than possible $Q_0(a) = \infty$, always pick best, random tie break. **Upper-confidence bounds** conf intervals $\sigma = \sqrt{E[(X-\mu)^2]}$, to explore, add σ to $Q_t(a)$. Pick a via $Q(a) + \sqrt{\frac{2\log n}{n(a)}}$, n is total # acts picked, $n(a)$ # a was picked
All algs conv in limit to correct vals given stationary. UCB fastest conv wrt regret. Simpler strats perform better in practice if finite training

Contextual bandits Add vector of measurements $\vec{s}$ s.t. there is state. Act val depends on $Q(\vec{s},a) = w_a^T\vec{s}$

13 Markov Decision Processes

Sequential decision-making, 1 decision affects next 1.

Markov Chain Set of states S, trans prob $T(s,s') = P(s_{t+1} = s'|s_t = s)$, initial state dist $P_0(s) = P(s_0 = s)$. Hidden Markov models = Markov chains of latent vars w/ obs.

Sequential decision-making At each t, agent in s_t. Chooses act a_t, receives reward R_{t+1} and can obs s_{t+1}, markov chains w/ acts + rew

MDPs States S, acts A, trans model $T(s,a,s') = P(s_{t+1} = s'|s_t = s, a_t = a)$, prob $s\to s'$ under a. Reward func $R(s,a)$, discount factor $\gamma\in[0,1]$, usually close to 1, future rew less than current rew, $1-\gamma$ chance agent dies at ea time step or inflation. **Planning** want to max long-term utility aka **return**. Long-term util: **episodic tasks** $U_t = R_t + R_{t+1} + \dots R_T$, T is time when terminal state. **Continuing tasks** may go on forever, $U_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \dots = \sum_{k=0}^\infty \gamma^j R_{t+k}, \gamma < 1$. **Policies** how agent should act. **Deterministic pol** $\pi(s) = a$, **stochastic policy** $\pi(s,a) = P(a_t = a|s_t = s)$. Fix policy, MDP → Markov chain w/ rew

State val func Val func of pol π is $V^\pi: S\to\mathbb{R}$. Val of s under π is expect return if agent starts from s and picks act accord to π. $V^\pi(s) = E_\pi[U_t|s_t = s]$. **Policy iteration** alg to find opt pol by computing state val funcs. 1. Start with init pol π_0. 2. Alt between: **pol eval** compute val of each s, $V^{\pi_i}(s)$. **Pol improvement** improve cur pol to get π_{i+1}

Bellman's eq $V^\pi(s) = E_\pi[U_t|s_t = s] = E_\pi[R_t + \gamma U_{t+1}|s_t = s]$ Determ pol: $V^\pi(s) = R(s,\pi(s)) + \gamma\sum_{s'\in S} T(s,\pi(s),s')V^\pi(s')$, stoch pol: $V^{\pi(s)} = \sum_{a\in A}\pi(s,a)(R(s,a) + \gamma\sum_{s'\in S} T(s,a,s')V^\pi(s'))$ In matrix form: $V^\pi = R^\pi + \gamma T^\pi V^\pi$, V^π is vector w/ val of ea state under π, R^π is vec w/ $R(s,\pi(s))$, T^π is matrix w/ $T(s,\pi(s),s')$, can sometimes solve explicitly: $V^\pi = (I-\gamma T^\pi)^{-1}R^\pi$

Iterative pol eval Start with init guess V_0, for ea iter k, update val func for all s, $V_{k+1}(s) = R(s,\pi(s)) + \gamma\sum_{s'\in S} T(s,\pi(s),s')V_k(s')$. Stop when converges (max change between 2 iter smaller than threshold).

Searching for good pol $\pi > \pi'$ if $V^\pi(s)\geq V^{\pi'}(s)\forall s\in S$. Use local search. To improve pol: $R(s,a^*) + \gamma\sum_{s'\in S} T(s,a^*,s')V^\pi(s') > \sum_{a\in A}\pi(s,a)(R(s,a) + \gamma\sum_{s\in S} T(s,a,s')V^\pi(s')) = V^\pi(s)$. Set $\pi(s,a^*) = 1$ to increase val of s. **Pol iter** $\pi'(s) = argmax_{a\in A} R(s,a) + \gamma\sum_{s'\in S} T(s,a,s')V^\pi(s')$. Alg: start w/ init pol π_0 (rand), repeat: comp V^π w/ pol eval $O(S^3)$, comp π' greedy wrt V^π $O(S^2A)$, terminate when $V^\pi = V^{\pi'}$ (at most $|A|^{|S|}$ iter)

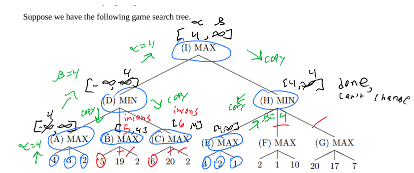
Generalized pol iter combine pol eval + pol improv, can update val of 1 state and immediately improv to save time.
opt val func $V^*(s) = \max_\pi V^\pi(s)$, best val at any state. unique for finite MDP. **Opt pol** π^* achieves opt val func, may not be unique, can have inf, ex. stochastic
Bellman Optimality Eq for V^* $V^*(s) = \max_{a\in A} E[R_t + \gamma V^*(S_{t+1})|s_t = s, a_t = a] = \max_{a\in A} R(s,a) + \gamma\sum_{s'\in S} T(s,a,s')V^*(s')$. Then we can comp $\pi^*(s) = argmax_{a\in A} R(s,a) + \gamma\sum_{s'\in S} T(s,a,s')V^*(s')$

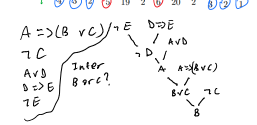
Value Iter Alg Iter on val instead of pol. Start with init approx V_0, for each iter $V_k(s) = \max_{a\in A} R(s,a) + \gamma\sum_{s'\in S} T(s,a,s')V_{k-1}(s')$. Stop when max val change is below threshold. Converges in lim to V^*. VI and PI conv to same, VI much cheaper but may take more iter. $O(S^2A)$ per iter, $O(SA)$ if few succ states, polynomial iter in $\frac{1}{1-\gamma}$
MDP limitations: get opt pol is poly in # states but # states usually huge. Dyn prog can solve up to 10^7 states. VI and PI assume model is known (trans+rew), soln: **reinforcement learning**. State sometimes not observable, soln: **POMDPs** not discussed

Function approx For large prob or continuous state space: $V_w(s)$ approxs true val func, opt params w. Pop soln is to use **linear func** $V_w(s) = \sum_m w_m\phi_m(s)$, $\phi_m(s)$ are basis features w/ info about state like eval in games. w_m are weights
Train weights from data: $MDP\to$ sample states $\to$ VI (gives x) $\to FA$ (gives y) $\to V_\phi^*$ (gives w)
$MSE: E_w = \sum_i[y_i - \sum_m w_m\phi_m(x_i)]^2$
soln: $w_m = (\sum_i\phi_m(x_i)\phi_m(x_i)^T)^{-1}(\sum_i\phi_m(x_i)y_i)$, $w_m(t+1) = w_m(t) - \alpha_i\sum_i\nabla_w(E_w^i)E_w^i$.
Good basis set is small & fits val func well

Reinforcement learning Do acts, get rewards. **Model-based**: Have policy for getting data (ex. rand explor). Observe many trans in env. Learn approx model for $R(s,a), T(s,a,s')$, use max likelihood for probs, supervised learn for rew. **Monte-Carlo**: Gen several trajectories according to some π, compute $V^\pi(s)$ via avg of observed returns after s, keep running avg $V_{n+1}(s) = V_n(s) + \frac{1}{n+1}(U_{n+1}(s) - V_n(s))$. Avoid keeping track of # visited n, use learning rate $V(s_t) = V(s_t) + \alpha(U(s_t) - V(s_t))$. **Temporal-Difference** estimate return, don't need $U(s_t)$. $V(s_t) = V(s_t) + \alpha(r_t + \gamma V(s_{t+1}) - V(s_t))$, dynamic prog like poli iter. A lot more updates. Hybrid dyn prog + monte carl. Alg: init $V(s) = 0\forall s$, repeat # wanted: pick start s. Repeat for each t: choose a from π and s. Do a, get r and s'. Comp TD error $\delta = r + \gamma V(s') - V(s)$. Update $V(s) = V(s) + \alpha_s\delta$. $s = s'$. If s' not terminal, repeat another time step t
No model req, only needs exp. on-line incremental learn, learn before final outcome, less mem+peak comp, faster conv than MC. Eligibility traces, shout δ_t backwards, with temporal distance $\gamma\lambda, \lambda\in[0,1]$. TD-learning used to get vals for given π. Explore vs exploit, can use ε-greedy.
Can correct function approximator using update rule $r_{t+1} + \gamma V(s_{t+1})$. **Online Gradient Descent TD** Init weight vector of func approx $\vec{w}$. Pick start s. Repeat for ea t: choose a based off π, s. Take a, get r and s'. Comp TD error $\delta = r + \gamma V(s') - V(s)$. Update $\vec{w} = \vec{w} + \alpha\delta\nabla V(s)$. $s = s'$. For lin func approx, gradient is input feature vec

14 Misc & Ex





Local beam search with $k = 1\to$ hill climbing. LBS with 1 init state and return as many as possible → BFS. SA with $T = 0\to$ HC, SA with $T = \infty\to$ rand walk.
Knapsack prob, var for each item, domain is $\{0,1\}$ (in bag or not), constraint is sum should be less than cap.
Map of diff cities, 2 friends in diff cities, each wait for other, only 1 new node at a time. Want to meet. States: pair of cities. Succ: neighbor nodes. Cost: max dist of both friends.
Does it still make sense to use prob in deterministic world? Yes might not be fully obs. Even w/ perf inf, might have long list to reason about.