Distribution	Probability Function	Mean	Variance	MGF
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y}$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	No closed form
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}$	λ	λ	$e^{\lambda(e^t-1)}$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$
Multinomial	$p(y_1,\dots,y_k) = \frac{n!}{y_1!\dots y_k!} p_1^{y_1}\dots p_k^{x_k}$	$E(Y_i) = np_i$	$V(Y_i) = np_i(1-p_i)$	
Uniform	$f(y) = \frac{1}{\theta_2-\theta_1}$	$\frac{\theta_1+\theta_2}{2}$	$\frac{(\theta_2-\theta_1)^2}{12}$	$\frac{e^{t\theta_2}-e^{t\theta_1}}{t(\theta_2-\theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2}$	μ	σ^2	$e^{\mu t + \frac{t^2\sigma^2}{2}}$
Exponential	$f(y) = \frac{1}{\beta} e^{-\frac{y}{\beta}}$	β	β^2	$(1-\beta t)^{-1}$
Gamma	$f(y) = \left(\frac{1}{\Gamma(\alpha)\beta^\alpha}\right) y^{\alpha-1} e^{-\frac{y}{\beta}}$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{y^{\frac{\nu}{2}-1} e^{-\frac{y}{2}}}{2^{\frac{\nu}{2}}\Gamma(\frac{\nu}{2})}$	ν	2ν	$(1-2t)^{-\frac{\nu}{2}}$
Beta	$f(y) = \left(\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right) y^{\alpha-1}(1-y)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	No closed form

MATH324 Formula Sheet

Julian Lore

Bias

$$Bias(\hat\theta) = E[\hat\theta] - \theta$$

$$\begin{aligned} MSE(\hat\theta) &= E[(\hat\theta-\theta)^2] \\ &= V(\hat\theta) + [Bias(\hat\theta)]^2 \end{aligned}$$

Common Point Estimators

Target Parameter θ	Sample Size(s)	Point Estimator $\hat\theta$	$E(\hat\theta)$	Standard Error $\sigma_{\hat\theta}$
μ	n	$\overline{Y}$	μ	$\frac{\sigma}{\sqrt{n}}$
p	n	$\hat p = \frac{Y}{n}$	p	$\sqrt{\frac{pq}{n}}$
$\mu_1 - \mu_2$	n_1, n_2	$\overline{Y}_1 - \overline{Y}_2$	$\mu_1 - \mu_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
$p_1 - p_2$	n_1, n_2	$\hat p_1 - \hat p_2$	$p_1 - p_2$	$\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

Confidence Intervals

$$P(\hat\theta_L \le \theta \le \hat\theta_U) = 1 - \alpha$$

$$P\left[\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}}\right] = 1 - \alpha$$

Large Sample

$$Z = \frac{\hat\theta - \theta}{\sigma_{\hat\theta}} \sim N(0,1)$$

$$z_{\alpha/2}\sigma_{\hat\theta} = B$$

$$\hat\theta_n - z_\alpha \sigma_{\hat\theta}$$

$$\hat\theta_n + z_\alpha \sigma_{\hat\theta}$$

$$\hat\theta_n \pm z_{\alpha/2} \sigma_{\hat\theta}$$

Small Sample

$$T_{n-1} = \frac{\overline{Y} - \mu}{\frac{S}{\sqrt{n}}}$$

$$P(-t_{\frac{\alpha}{2}} \le T \le t_{\frac{\alpha}{2}}) = 1 - \alpha$$

$$\overline{Y} \pm t_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right), \nu = n - 1$$

$$\left(\overline{Y_1} - \overline{Y_2} \pm t_{\alpha/2} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right), \nu = n_1 + n_2 - 2$$

Efficiency

$$eff(\hat\theta_1, \hat\theta_2) = \frac{V(\hat\theta_2)}{V(\hat\theta_1)}$$

Hypothesis Testing

$$\begin{aligned} \alpha &= P(\text{type I error}) \\ &= P(\text{rejecting } H_0 \text{ when } H_0 \text{ is true}) \\ \beta &= P(\text{type II error}) \\ &= P(\text{accepting } H_0 \text{ when } H_a \text{ is true}) \end{aligned}$$

$$Z = \frac{\hat\theta - \theta_0}{\sigma_{\hat\theta}}$$

$$\chi^2_{n-1} = \frac{(n-1)S^2}{\sigma_0^2}$$

$$F_{n_1-1,n_2-1} = \frac{S_1^2}{S_2^2} = \frac{\left[\frac{(n_1-1)S_1^2}{\sigma^2}\right]/(n_1-1)}{\left[\frac{(n_2-1)S_2^2}{\sigma^2}\right]/(n_2-1)}$$

Small Sample

$$T_{n-1} = \frac{\overline{Y} - \mu_0}{S/\sqrt{n}}, \mu = \mu_0$$

$$T_{n_1+n_2-2} = \frac{\overline{Y}_1 - \overline{Y}_2 - D_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \mu_1 - \mu_2 = D_0$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$$

Power of Tests

$$\begin{aligned} power(\theta) &= P(W \in RR|\theta) \\ power(\theta_a) &= 1 - \beta(\theta_a) \end{aligned}$$

$$\frac{L(\theta_0)}{L(\theta_a)} < k$$

Likelihood Ratio Test

$$\lambda = \frac{L(\hat{\Omega}_0)}{L(\hat{\Omega})} = \frac{\max_{\Theta \in \Omega_0} L(\Theta)}{\max_{\Theta \in \Omega} L(\Theta)}$$

$$L(\theta; y_1, \dots, y_n) = \prod_{i=1}^{n} f(\theta; y_i)$$

Linear Regression

$$Y = \beta_0 + \beta_1 x + \varepsilon$$
$$E(Y) = \beta_0 + \beta_1 x$$
$$E(\varepsilon) = 0$$
$$Var(\varepsilon) = \sigma^2$$
$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$
$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = S_{yy} - \hat{\beta}_1 S_{xy}$$
$$S_{xy} = \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})$$
$$= \sum_{i=1}^{n} x_i y_i - \frac{1}{n} \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i$$
$$S_{xx} = \sum_{i=1}^{n} (x_i - \overline{x})^2$$
$$= \sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right)^2$$
$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$$
$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$
$$E(\hat{\beta}_1) = \beta_1$$
$$E(\hat{\beta}_0) = \beta_0$$
$$Var(\hat{\beta}_1) = \frac{\sigma^2}{S_{xx}}$$
$$Var(\hat{\beta}_0) = \frac{\sigma^2 \sum_{i=1}^{n} x_i^2}{n S_{xx}}$$
$$Cov(\hat{\beta}_0, \hat{\beta}_1) = \frac{-\overline{x}\sigma^2}{S_{xx}}$$
$$S^2 = \frac{SSE}{(n-2)}$$

Hypothesis Testing

$$T_{n-2} = \frac{\hat{\beta}_0 - \beta_{00}}{S\sqrt{\frac{\sum x_i^2}{n S_{xx}}}}$$

$$T_{n-2} = \frac{\hat{\beta}_1 - \beta_{10}}{S\sqrt{\frac{1}{s_{xx}}}}$$

Confidence Intervals

$$\beta_0 = \hat{\beta}_0 \pm z_{\frac{\alpha}{2}, n-2} \sqrt{Var(\hat{\beta}_0)}$$
$$\beta_1 = \hat{\beta}_1 \pm z_{\frac{\alpha}{2}, n-2} \sqrt{Var(\hat{\beta}_1)}$$
$$E(Y) = \beta_0 + \beta_1 x^* = \hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2, n-2} S \sqrt{\frac{1}{n} + \frac{(x^* - \overline{x})^2}{S_{xx}}}$$
$$Y = \hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2, n-2} S \sqrt{1 + \frac{1}{n} + \frac{(x^* - \overline{x})^2}{S_{xx}}}$$

Correlation

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$
$$t_{n-2} = \frac{\hat{\beta}_1 - 0}{S/\sqrt{S_{xx}}} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$
$$R^2 = \frac{SS_{reg}}{SS_{total}} = \frac{\hat{\beta}_1^2 S_{xx}}{\hat{\beta}_1^2 S_{xx} + SS_{res}}$$
$$R^2 \sim \beta\left(\frac{1}{2}, \frac{n-2}{2}\right)$$

ANOVA

Source	(SS)	df	Mean Square (MS)	F	p-val
Treatments	SST	$k-1$	$\frac{SST}{k-1}$	$F = \frac{MST}{MSE}$	
Error	SSE	$n-k$	$\frac{SSE}{n-k}$		
Total	Total SS	$n-1$			

$$Total\ SS = SST + SSE$$
$$Total\ SS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (Y_{ij} - \overline{Y})^2$$
$$SST = \sum_{i=1}^{k} n_i (\overline{Y}_{i\bullet} - \overline{Y})^2$$
$$SSE = \sum_{i=1}^{k} \sum_{j=1}^{m_i} (Y_{ij} - \overline{Y}_{i\bullet})^2 = \sum_{i=1}^{k} (n_i - 1) S_i^2$$
$$= Total\ SS - SST$$
$$S^2 = MSE = \frac{SSE}{n-k}$$
$$MST = \frac{SST}{k-1}$$
$$F_{k-1, n-k} = \frac{MST}{MSE}$$

Goodness of Fit

$$X^2 = \sum_{i=1}^{k} \frac{(n_i - E(n_i))^2}{E(n_i)} = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} \sim \chi^2_{k-1}$$

Contingency Tables

$$\hat{p}_{ij} = \frac{n_{ij}}{N}, \hat{p}_{\bullet j} = \frac{n_{\bullet j}}{N}, \hat{p}_{i\bullet} = \frac{n_{i\bullet}}{N}$$
$$\widehat{E(n_{ij})} = \frac{r_i c_j}{n}$$
$$X^2 = \sum_{j=1}^{c} \sum_{i=1}^{r} \frac{(n_{ij} - \widehat{E(n_{ij})})^2}{\widehat{E(n_{ij})}} \sim \chi^2_{(r-1)(c-1)}$$

Misc

$$F_{Y_{(n)}} = [F(Y)]^n, F_{Y_{(1)}} = [1 - F(Y)]^n$$
$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i, E(\overline{Y}) = \mu$$
$$Var(\overline{Y}) = \frac{\sigma^2}{n} = \frac{Var(Y)}{n}$$
$$S^2 = \frac{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}{n-1}$$
$$V(S^2) = \frac{2\sigma^4}{n-1}$$
$$P(g(x) \geq \lambda) \leq \frac{E[g(x)]}{\lambda}, \lambda > 0$$
$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$
$$P\left(\left|\hat{\theta}_n - \theta\right| \geq \varepsilon\right) \leq \frac{MSE(\hat{\theta}_n)}{\varepsilon^2}$$
$$Var(X) = E[(X - \mu)^2]$$
$$= E[X^2] - (E[X])^2$$
$$z_{0.025} = 1.96, z_{0.005} = 2.576$$
$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\, dx$$
$$E[E(\hat{\theta}|U)] = E[\hat{\theta}]$$
$$V(\hat{\theta}) = V[E(\hat{\theta}|U)] + E[V(\hat{\theta}|U)]$$
$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}, T = \frac{Z}{\sqrt{\chi^2_\nu/\nu}} \sim T_\nu$$
$$F_{n_1-1, n_2-1} \sim \frac{\chi^2_{n_1-1}/(n_1-1)}{\chi^2_{n_2-1}/(n_2-1)}$$
$$Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y) \pm Cov(X, Y)$$
$$Cov(X, Y) = E(XY) - E(X)E(Y)$$
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$T \sim Gamma(\nu, \theta) \implies \frac{2T}{\theta} \sim \chi^2_{2\nu}$$